



Approximate Solution of a Neutrosophic Nonlinear Van der Pol Oscillator Problem by Semi Analytical Method Using Thick Function

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Abstract

In this paper, an analytical method (Homotopy perturbation method HPM) is used for solving the initial value problem represented by a neutrosophic nonlinear Van der Pol oscillator equation (N-VDP) arising in applied dynamics using the thick function. We find the solutions of the (N-VDP) equation by HPM and then compare the numerical results with fourth order Runge-Kutta method (RK4). The results showed that the HPM lead to accurate and efficient results. Furthermore, these results of the HPM scheme and RK4 are implemented in Matlab.

Keywords: Neutrosophic Van der pol oscillator; Neutrosophic thick function; Homotopy perturbation method; Nonlinear initial value problem; Fourth Order Runge-Kutta; Approximate solution.

1. Introduction

The neutrosophic is a new idea for mathematical modeling and is considered as a generalization of the Fuzzy Logic and is an extension of the Sets Fuzzy Theory by Zadeh [1] and Atanassov [2]. The Neutrosophic was proposed first by Smarandache. F, where he proposed a neutrosophic measure, neutrosophic integral, neutrosophic sets and systems, and neutrosophic precalculus [3,4]. In addition, more researches were done in this field. Namely, Neutrosophic closed sets, Neutrosophic Crisp for Gray Scale Image and Basic structure of some classes, Neutrosophic Crips Nearly open sets by Salama. A *et al* [5-7], Cosine similarity measures of Bipolar Neutrosophic set and a novel group decision making model based on Neutrosophic sets by Abdel-Basset. M *et al* [8,9], Design of variable sampling plan for Pareto Distribution by Neutrosophic Statistical interval method by Khan *et al* [10], Agriculture production decision making by Shanmugam *et al* [11], On neutrosophic crisp generalized alpha sets by Murtadha *et al* [12], Review of generalized neutrosophic set in decision making problems by Sonali *et al* [13], Exact solution for neutrosophic system differential equations by Laplace transform [14], and finally, Single-valued Neutrosophic weak polygroups by Madeleine [15]. Recently, Malath. F studied the integration of neutrosophic thick function and some of neutrosophic differential equations by using a neutrosophic thick function [16,17].

Van der Pol equation, a kind of nonlinear oscillator, represents a good model of the first vacuum tube circuits, this equation has been studied by Kummer [18] and Darbyshire [19].

Definition 1. [14]

Consider a neutrosophic thick function as follows:

$$g : R \rightarrow P(R) ; g(t) = [g_1(t), g_2(t)]$$

where $P(R)$ is the set of all subsets of R .

Definition 2. Neutrosophic integration thick function [14]

Let $g(t) = [g_1(t), g_2(t)]$ be a neutrosophic thick function. The integration of thick function defined by:

$$\int g(t) dt = \left[\int g_1(t) dt + c_1, \int g_2(t) dt + c_2 \right]$$

2. Neutrosophic nonlinear Van der Pol oscillator problem (N-VDP)

Consider the following neutrosophic nonlinear Van der Pol oscillator problem:

$$v''(x) = [\alpha_1(x), \alpha_2(x)]v'(x) + [\alpha_3(x), \alpha_4(x)]v(x) + [\alpha_5(x), \alpha_6(x)]v^2(x)v'(x) \tag{1}$$

With the initial conditions:

$$v(0) = [\beta_1, \beta_2] \quad , \quad v'(0) = [\beta_3, \beta_4]$$

Where α_i , $i = 1, 2, \dots, 6$ are analytical functions.

$$\beta_k \in R \quad , \quad k = 1, 2, 3, 4$$

Constructing the Homotopy on (1), gives:

$$H(v, p) = v'' - p([\alpha_1, \alpha_2]v'(x) + [\alpha_3, \alpha_4]v(x) + [\alpha_5, \alpha_6]v^2(x)v'(x)) = 0 \tag{2}$$

Where $p \in [0, 1]$ is an embedding parameter.

According to the HPM technique the solution of (2) can be given as:

$$v = \sum_{i=0}^{\infty} p^i v_i \tag{3}$$

Substituting (3) into (2), leads to:

$$\sum_{i=0}^{\infty} p^i v_i'' - p \left([\alpha_1, \alpha_2] \sum_{i=0}^{\infty} p^i v_i' + [\alpha_3, \alpha_4] \sum_{i=0}^{\infty} p^i v_i + [\alpha_5, \alpha_6] \left(\sum_{i=0}^{\infty} p^i v_i \right)^2 \left(\sum_{i=0}^{\infty} p^i v_i'(x) \right) \right) = 0 \tag{4}$$

Comparing both sides of equation (4) according to the coefficients of the like power of p , yields:

$$p^0 : v_0''(x) = 0$$

$$p^1 : v_1''(x) = [\alpha_1, \alpha_2]v_0'(x) + [\alpha_3, \alpha_4]v_0(x) + [\alpha_5, \alpha_6]v_0^2(x)v_0'(x)$$

⋮

Using the initial conditions yields:

$$v_0(x) = [\beta_1 + \beta_3 x, \beta_2 + \beta_4 x]$$

$$v_1(x) = \int_0^x \int_0^t \left[\beta_1 \alpha_1(t) + (\beta_1 + \beta_3 t)\alpha_3(t) + \beta_1 (\beta_1 + \beta_3 t)^2 \alpha_5(t), \right. \\ \left. \beta_2 \alpha_2(t) + (\beta_2 + \beta_4 t)\alpha_4(t) + \beta_2 (\beta_2 + \beta_4 t)^2 \alpha_6(t) \right] dt dt$$

$$= \left[\int_0^x \int_0^t \beta_1 \alpha_1(t) + (\beta_1 + \beta_3 t)\alpha_3(t) + \beta_1 (\beta_1 + \beta_3 t)^2 \alpha_5(t) dt dt, \right. \\ \left. \int_0^x \int_0^t \beta_2 \alpha_2(t) + (\beta_2 + \beta_4 t)\alpha_4(t) + \beta_2 (\beta_2 + \beta_4 t)^2 \alpha_6(t) dt dt \right]$$

⋮

Finally, the approximate solution of (1) with the initial conditions can be written as:

$$v(x) = \sum_{i=0}^{\infty} v_i \tag{5}$$

It is well known that equation (5) is a convergent series for most of the cases [20-21].

3. Test problem

Example 1.

Substituting $\alpha_1 = -1$, $\alpha_2 = -2$, $\alpha_3 = -1$, $\alpha_4 = 3$, $\alpha_5 = -1$, $\alpha_6 = -1$ into equation (1) yields:

$$v''(x) = [-1, -2]v'(x) + [-1, 3]v(x) + [-1, -1]v^2(x)v'(x) \tag{6}$$

with the initial conditions:

$$v(0) = [0, 0], \quad v'(0) = [1, 1]$$

Now, constructing the Homotopy on (6), gives:

$$H(v, p) = v'' - p \left([-1, -2]v'(x) + [-1, 3]v(x) + [-1, -1]v^2(x)v'(x) \right) = 0$$

or:

$$v'' = p \left([-1, -2]v' + [-1, 3]v + [-1, -1]v^2v' \right) \tag{7}$$

Substituting (3) into (7), yields:

$$\sum_{i=0}^{\infty} p^i v_i'' = p \left([-1, -2] \sum_{i=0}^{\infty} p^i v_i' + [-1, 3] \sum_{i=0}^{\infty} p^i v_i + [-1, -1] \left(\sum_{i=0}^{\infty} p^i v_i \right)^2 \left(\sum_{i=0}^{\infty} p^i v_i'(x) \right) \right) \tag{8}$$

Comparing both sides of equation (8) according to the coefficients of the like power of p , yields:

$$p^0 : v_0''(x) = 0$$

$$p^1 : v_1''(x) = [-v_0 - v_0' - v_0^2 v_0', 3v_0 - 2v_0' - v_0^2 v_0']$$

$$p^2 : v_2''(x) = [-v_1 - v_1' - v_0^2 v_1' - 2v_0 v_1 v_0', 3v_1 - 2v_1' - v_0^2 v_1' - 2v_0 v_1 v_0']$$

$$p^3 : v_3''(x) = \left[\begin{array}{l} -v_2 - v_2' - v_1^2 v_0' - v_0^2 v_2' - 2v_0 v_1 v_1' - 2v_0 v_2 v_0', \\ 3v_2 - 2v_2' - v_1^2 v_0' - v_0^2 v_2' - 2v_0 v_1 v_1' - 2v_0 v_2 v_0' \end{array} \right]$$

$$p^4 : v_4''(x) = \left[\begin{array}{l} -v_3 - v_3' - v_0^2 v_3' - v_1^2 v_1' - 2v_0 v_2 v_1' - 2v_0 v_1 v_2' - 2v_1 v_2 v_0' - 2v_0 v_3 v_0', \\ 3v_3 - 2v_3' - v_0^2 v_3' - v_1^2 v_1' - 2v_0 v_2 v_1' - 2v_0 v_1 v_2' - 2v_1 v_2 v_0' - 2v_0 v_3 v_0' \end{array} \right]$$

Then:

$$v_0(x) = [x, x]$$

$$v_1(x) = \left[-\frac{x^2}{2} - \frac{x^3}{6} - \frac{x^4}{12}, -x^2 + \frac{x^3}{2} - \frac{x^4}{12} \right]$$

$$v_2(x) = \left[\frac{x^3}{6} + \frac{x^4}{12} + \frac{x^5}{8} + \frac{11x^6}{360} + \frac{x^7}{84}, \frac{2x^3}{3} - \frac{x^4}{2} + \frac{37x^5}{120} - \frac{11x^6}{120} + \frac{x^7}{84} \right]$$

$$v_3(x) = \left[\begin{array}{l} \frac{x^4}{24} + \frac{x^5}{40} + \frac{67x^6}{720} + \frac{31x^7}{720} + \frac{53x^8}{1680} + \frac{67x^9}{10080} + \frac{19x^{10}}{10080}, \\ -\frac{x^4}{3} + \frac{3x^5}{10} - \frac{31x^6}{72} + \frac{21x^7}{80} - \frac{331x^8}{3360} + \frac{67x^9}{3360} - \frac{19x^{10}}{10080} \end{array} \right]$$

$$v_4(x) = \left[\begin{array}{l} \frac{x^5}{120} + \frac{x^6}{180} + \frac{25x^7}{504} + \frac{11x^8}{336} + \frac{13921x^9}{362880} + \frac{14039x^{10}}{907200} + \frac{8347x^{11}}{1108800} \\ + \frac{1889x^{12}}{1330560} + \frac{737x^{13}}{2358720}, \frac{2x^5}{15} - \frac{2x^6}{15} + \frac{271x^7}{630} - \frac{667x^8}{1680} + \frac{30739x^9}{120960} \\ - \frac{2087x^{10}}{20160} + \frac{30533x^{11}}{1108800} - \frac{1889x^{12}}{443520} + \frac{737x^{13}}{2358720} \end{array} \right]$$

The following table.1 shows the solutions of N-VDP using RK4 method (by Matlab) and HPM for $v_0 + v_1 + v_2 + v_3$ ($n = 4$) and $v_0 + v_1 + \dots + v_4$ ($n = 5$) at different values of x .

Table1: The solutions of N-VDP based on RK4 and HPM for example 1

x	SRK4	SHPM $n = 4$	SHPM $n = 5$
0	[0 , 0]	[0 , 0]	[0 , 0]
0.001	[9.994999999584403 e-04 , 9.990011657507490 e-04]	[9.994999999584333 e-04 , 9.990011657506078 e-04]	[9.994999999584416 e-04 , 9.990011657507410 e-04]
0.003	[2.995499996652454 e-03 , 2.991031426033710 e-03]	[2.995499996649254 e-03 , 2.991031425897445 e-03]	[2.995499996651283 e-03 , 2.991031425929749 e-03]
0.005	[4.987499974292065 e-03 , 4.975145262758745 e-03]	[4.987499974269854 e-03 , 4.975145262309570 e-03]	[4.987499974295987 e-03 , 4.975145262724187 e-03]
0.007	[6.975499902123229 e-03 , 6.951397985652526 e-03]	[6.975499901631654 e-03 , 6.951397975913045 e-03]	[6.975499901772407 e-03 , 6.951397978138644 e-03]
0.009	[8.959499736023266 e-03 , 8.919844605804774 e-03]	[8.959499732496535 e-03 , 8.919844521395253 e-03]	[8.959499732991801 e-03 , 8.919844529199635 e-03]
0.01	[9.949999598566142 e-03 , 9.901157698531246 e-03]	[9.949999593270519 e-03 , 9.901157560313845 e-03]	[9.949999594109907 e-03 , 9.901157573518107 e-03]
0.03	[2.954996883890668 e-02 , 2.913077502126183 e-02]	[2.954996863373540 e-02 , 2.913077190773699 e-02]	[2.954996884139247 e-02 , 2.913077505968901 e-02]
0.05	[4.874977258284214 e-02 , 4.764033561432903 e-02]	[4.874976983118878 e-02 , 4.764028632168047 e-02]	[4.874977256226879 e-02 , 4.764032622605293 e-02]
0.07	[6.754917460956525 e-02 , 6.547933048890503 e-02]	[6.754916002527053 e-02 , 6.547912069076151 e-02]	[6.754917511367904 e-02 , 6.547933242119513 e-02]
0.09	[8.594787580537228 e-02 , 8.269440112585977 e-02]	[8.594782189656170 e-02 , 8.269368482549287 e-02]	[8.594787658536898 e-02 , 8.269442024692327 e-02]
0.1	[9.499685991377974 e-02 , 9.108179307220726 e-02]	[9.499676739594246 e-02 , 9.108058758622024 e-02]	[9.499686161251654 e-02 , 9.108182687589945 e-02]
0.2	[1.799644515361985 e-01 , 1.680671797415925 e-01]	[1.799608502450794 e-01 , 1.680311813180952 e-01]	[1.799646125839756 e-01 , 1.680699243605246 e-01]
0.3	[2.548861418854766 e-01 , 2.354916069229221 e-01]	[2.548509130224107 e-01 , 2.352264803134821 e-01]	[2.548890697917848 e-01 , 2.355257445257613 e-01]
0.4	[3.198241861015422 e-01 , 2.967879021285581 e-01]	[3.196276807517460 e-01 , 2.956803647390476 e-01]	[3.198505155723461 e-01 , 2.970009912846601 e-01]
0.5	[3.749623060327481 e-01 , 3.545848967266488 e-01]	[3.741628495473710 e-01 , 3.511884416852679 e-01]	[3.751195795002252 e-01 , 3.554892029556845 e-01]

Example 2.

Substituting $\alpha_1 = x$, $\alpha_2 = x$, $\alpha_3 = 2x$, $\alpha_4 = 3x$, $\alpha_5 = -x$, $\alpha_6 = -x$ into equation (1) yields:

$$v''(x) = [x, x]v'(x) + [2x, 3x]v(x) + [-x, -x]v^2(x)v'(x) \tag{9}$$

with the initial conditions:

$$v(0) = [0, 0] \quad , \quad v'(0) = [-1, -1]$$

Now, constructing the Homotopy on (9), gives:

$$H(v, p) = v'' - p \left([x, x]v'(x) + [2x, 3x]v(x) + [-x, -x]v^2(x)v'(x) \right) = 0$$

or:

$$v'' = p \left([x, x]v'(x) + [2x, 3x]v(x) + [-x, -x]v^2(x)v'(x) \right)$$

And the same technique as Example 1. yields:

$$v_0(x) = [-x, -x]$$

$$v_1(x) = \left[-\frac{x^3}{6} - \frac{x^4}{6} + \frac{x^5}{20}, -\frac{x^3}{6} - \frac{x^4}{4} + \frac{x^5}{20} \right]$$

$$v_2(x) = \left[-\frac{x^5}{40} - \frac{x^6}{30} + \frac{x^7}{56} + \frac{11x^8}{560} - \frac{7x^9}{1440}, -\frac{x^5}{40} - \frac{x^6}{20} + \frac{x^7}{126} + \frac{33x^8}{1120} - \frac{7x^9}{1440} \right]$$

The following table.2 shows the solutions of N-VDP using RK4 method (by Matlab) and HPM for $v_0 + v_1 + v_2$ ($n = 3$) at different values of x .

Table 2: The solutions of N-VDP based on RK4 and HPM for example 2

x	SRK4	SHPM $n = 3$
0	[0 , 0]	[0 , 0]
0.001	[-1.000000166833309 e-03 , -1.000000166916641 e-03]	[-1.000000166833308 e-03 , -1.000000166916642 e-03]
0.003	[-3.000004513493401 e-03 , -3.000004520243442 e-03]	[-3.000004513493949 e-03 , -3.000004520243961 e-03]
0.005	[-5.000020937423095 e-03 , -5.000020989506691 e-03]	[-5.000020937422394 e-03 , -5.000020989505989 e-03]
0.007	[-7.000057566336459 e-03 , -7.000057766434132 e-03]	[-7.000057566417065 e-03 , -7.000057766502367 e-03]
0.009	[-9.000122591332583 e-03 , -9.000123138214248 e-03]	[-9.000122592041403 e-03 , -9.000123138800308 e-03]
0.01	[-1.000016832981086 e-02 , -1.000016916335799 e-02]	[-1.000016833086649 e-02 , -1.000016916421658 e-02]
0.03	[-3.000463441677338 e-02 , -3.000470192914477 e-02]	[-3.000463441639667 e-02 , -3.000470192875719 e-02]
0.05	[-5.002186770564214 e-02 , -5.002238884691159 e-02]	[-5.002186769362464 e-02 , -5.002238879474148 e-09]
0.07	[-7.006113017017689 e-02 , -7.006313304083611 e-02]	[-7.006113007927786 e-02 , -7.006313294946670 e-02]
0.09	[-9.013230432186622 e-02 , -9.013778116624989 e-02]	[-9.013230415542563 e-02 , -9.013778094499849 e-02]
0.1	[-1.001831150699992 e-01 , -1.001914660632726 e-01]	[-1.001831146893849 e-01 , -1.001914655832341 e-07]
0.2	[-2.015939026937555 e-01 , -2.017284109653404 e-01]	[-2.015938569650794 e-01 , -2.017283588063492 e-06]

0.3	$[-3.058092259845875 \text{ e-01} , -3.064980562825945 \text{ e-01}]$	$[-3.058084515562500 \text{ e-01} , -3.064971268151786 \text{ e-05}]$
0.4	$[-4.147782865203336 \text{ e-01} , -4.169910811440851 \text{ e-01}]$	$[-4.147730106920635 \text{ e-01} , -4.169844280888889 \text{ e-04}]$
0.5	$[-5.308026715347419 \text{ e-01} , -5.363169501030642 \text{ e-01}]$	$[-5.307828388516865 \text{ e-01} , -5.362907288566468 \text{ e-01}]$

4. Conclusion

In this work, we used the Homotopy perturbation method scheme (HPM) for solving a new type of neutrosophic problems which is a neutrosophic nonlinear Van der Pol oscillator equation (N-VDP) by the thick function. The solutions of N-VDP obtained from Homotopy Perturbation method are good compared to RK4 numerical method. Then we show that the HPM is a good and powerful technique for obtaining approximate solution of neutrosophic nonlinear initial value problems.

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