Ambiguous Set Theory: A New Approach to Deal with Unconsciousness and Ambiguousness of Human Perception

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Abstract

Recently, precise measurement of uncertainty of data with fuzzy attributes is considered as one of the main problems. For this purpose, fuzzy sets, intuitionistic fuzzy sets, and neutrosophic set theory are extensively introduced. The problem arises in computing the complement of true or false membership values in the case of indeterminacy. Instead of indeterminacy as discussed in neutrosophic set theory, it can be completely true, partially true, or partially false. To deal with this, the theory of ambiguous sets has recently been introduced. A real-time example of dealing with unconsciousness and ambiguousness in relation to human perception is discussed, illustrating the motivation of the ambiguous set theory. Finally, this study focuses on the definition of the ambiguous set, its mathematical representation, and the associated concepts.

Keywords: Fuzzy set; intuitionistic fuzzy set; neutrosophic set; turiyam set; ambiguous set; unconsciousness; uncertainty.

1. Introduction

Recently, one of the major issues for data science researchers is the precise assessment of uncertainty and vagueness of attributes. To deal with this issue, fuzzy set (FS) theory was developed by Zadeh [1]. The FS theory employs a non-probabilistic approach to manage with the event's vagueness. This theory provides a single value membership to assess both belonging and non-belonging degrees of an event g in a particular FS in the interval $[0,1]$. This membership degree is considered the true membership degree $(\Delta t(g))$ in the case of FS theory. As a result, non-belonging degree of an event is inevitably complement to 1 of the $\Delta t(g)$. However, some of the events may contain a certain non-belonging degree as independent rather than complement only. However, it is difficult to express the non-belonging degree of an event in FS.

Atanassov [2] developed an intuitionistic fuzzy set (IFS) which can express the belonging and non-belonging degrees to an event together. In the IFS theory, these belonging and non-belonging membership degrees are called true membership degree $(\Delta t(g))$ and false membership degree $(\Delta f(g))$, respectively. In the IFS, the $(\Delta f(g))$ of an event is always complement to 1 of the $\Delta t(g)$, i.e., $\Delta t(g) = 1 - \Delta f(g)$. Here, $\Delta t(g) \in [0,1]$ and $\Delta f(g) \in [0,1]$ with the condition $0 \leq \Delta t(g) + \Delta f(g) \leq 1$. A third component in the IFS is employed to describe the uncertainty between the $\Delta t(g)$ and $\Delta f(g)$, can be called hesitant membership degree $(\Delta h(g))$. The $\Delta h(g)$ of an event is always complement to 1 of the $\Delta t(g)$ and $\Delta f(g)$, i.e., $\Delta h(g) = 1 - \Delta t(g) - \Delta f(g)$. In the case of IFS, $\Delta t(g)$, $\Delta f(g)$, and $\Delta h(g)$ are linearly dependent on one another, and express the uncertainty of an event in the interval $[0,1]$. That is, if $\Delta t(g)$ increases, then $\Delta f(g)$ must decrease and vice versa, thus creating the situation of

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indeterminacy. Nevertheless, in many situations, this might not be feasible. For illustration, consider a tennis tournament between players from Australia and Argentina, in which Australia supporters celebrate their team's victory while Argentina supporters lament their team's defeat.

Smarandache [3] introduced the concept of neutrosophic set (NS) theory. In this set theory, the true, false and hesitant part of IFS became independent in [0,1]. It means the NS can be considered as a generalization of IFS theory. In the NS, the uncertainty of an event is expressed as similarly to IFS expressing the membership degrees in terms of $\Delta t(g)$, $\Delta f(g)$, and $\Delta h(g)$. However, $\Delta h(g)$ is referred to by Smarandache as the indeterministic membership degree ($\Delta i(g)$) in the interval [0,1]. Smarandache explicitly mentioned that $\Delta t(g)$, $\Delta f(g)$, and $\Delta i(g)$ are not interdependent when the uncertainty of an event is expressed by NS. Here, $\Delta t(g)$, $\Delta f(g)$, and $\Delta i(g)$ must satisfy the condition of $^{-}0 \leq \Delta t(g) + \Delta i(g) + \Delta f(g) \leq 3^{+}$, where individually $\Delta t(g)$, $\Delta f(g)$, and $\Delta i(g)$ must belong to the interval $]^{-}0,1^{+}[$. This mathematical representation of NS creates an issue while defining the complement of $\Delta i(g)$ in the case of partially true or partially false or human turiyam consciousness where two truths exist. In this case, the complement of $\Delta i(g)$ may not be only indeterminate. It may be partially true, partially false or unknown. This leads to a real-time problem while dealing with the uncertainty of a real time event containing partially true or partially false membership values. In case of unknown uncertainty the event is liberalized which needs human turiyam consciousness for exploration. The problem becomes crucial while representation of the partially true and partially false behavior of an uncertain event creates unclear boundaries among: (a) true membership degree and partially true membership degree, and (b) false membership degree and partially false membership degree. Hence, to deal with partially true and partially false membership degrees, Singh et al. [4] have introduced an ambiguous set (AS) theory. This theory deals entirely with the unconscious behavior of human perception, in which the representation of true and false is inherently ambiguous. The AS theory provides the representation to uncertain event with respect to four membership degrees as: true membership degree $(\Pi t(g))$, false membership degree $(\Pi f(g))$, true-ambiguous membership degree $(\Pi ta(g))$, and false-ambiguous membership degree $(\Pi fa(g))$. Here, $(\Pi t(g))$, $(\Pi f(g))$, $(\Pi ta(g))$, and $(\Pi fa(g))$ are dependent on one another, and they have well-defined dimensions for unclear boundaries in the AS. Therefore, AS is applicable to the rare case of an uncertain event in which the human unconscious plays an important role. Singh and Bose [5] demonstrated the real-time application of AS by designing a clustering algorithm, called ambiguous D-means fusion clustering algorithm. Singh and Huang [6] proposed various formulas for the membership functions, set-theoretic operations, and distance measurement methods for a set of events in the ambiguous set. The AS theory is differ from the turiyam set theory [7], because this theory entirely deals with the conscious [8] behavior of human perception in quaternion [9]. Recently one of the authors tried to categorize the difference among these set theories for various applications [10]. Motivated from this study, the author tried to focus on explaining the basics of AS and its related concepts with an illustrative example.

The rest of the paper is organized as follows: Section **2** provides preliminaries about AS. Section **3** gives an example for AS in terms of human perception. Section **4** discusses various mathematical definitions of AS. Section **5** contains conclusions and future directions.

2. Ambiguous set (AS)

This section provides basic definitions of FS, IFS, NS followed by the AS.

Definition 1: (FS) [1]. Let $U = \{g\}$ be the universe for any event g, which is fixed. A FS $\tilde{A}$ for $g \in U$ is defined by:

$$\tilde{A} = \{g, \Delta t(g) \mid g \in U\} \quad (1)$$

Here, $\Delta t: U \rightarrow [0,1]$ denotes the true membership degree function for the fuzzy set $\tilde{A}$, and $\Delta t(g) \in [0,1]$ is called the membership degree of $g \in U$ in $\tilde{A}$.

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Definition 2: (IFS) [2]. Let $\mathrm{U} = \{\mathrm{g}\}$ be the universe for any event g, which is fixed. An IFS $\hat{\mathrm{I}}$ for $\mathrm{g} \in \mathrm{U}$ is defined by:

$$\hat{\mathrm{I}} = \{g, \Delta t(g), \Delta f(g) \mid g \in U\} \tag{2}$$

where, $\Delta \mathrm{t}(\mathrm{g})\colon \mathrm{U} \rightarrow [0,1]$ and $\Delta \mathrm{f}(\mathrm{g})\colon \mathrm{U} \rightarrow [0,1]$ are called the true membership degree and false membership degree, respectively. Both $\Delta \mathrm{t}(\mathrm{g})$ and $\Delta \mathrm{f}(\mathrm{g})$ must satisfy the following condition as:

$$0 \leq \Delta \mathrm{t}(\mathrm{g}) + \Delta \mathrm{f}(\mathrm{g}) \leq 1 \tag{3}$$

A hesitant membership degree $\Delta \mathrm{h}(\mathrm{g})$ is always considered in IFS $\hat{\mathrm{I}}$. Hence, an IFS $\hat{\mathrm{I}}$ with respect to $\Delta \mathrm{h}(\mathrm{g})$ can be expressed as:

$$\hat{\mathrm{I}} = \{\mathrm{g}, \Delta \mathrm{t}(\mathrm{g}), \Delta \mathrm{f}(\mathrm{g}), \Delta \mathrm{h}(\mathrm{g}) \mid \mathrm{g} \in \mathrm{U}\} \tag{4}$$

In Eq. (4), $\Delta \mathrm{t}(\mathrm{g})$, $\Delta \mathrm{f}(g)$, and $\Delta \mathrm{h}(\mathrm{g})$ must satisfy the following condition as:

$$\Delta \mathrm{t}(\mathrm{g}) + \Delta \mathrm{f}(\mathrm{g}) + \Delta \mathrm{h}(\mathrm{g}) = 1 \tag{5}$$

Definition 3: (NS) [3]. Let $\mathrm{U} = \{\mathrm{g}\}$ be the universe for any event g, which is fixed. A NS $\check{\mathrm{N}}$ for $\mathrm{g} \in \mathrm{U}$ is defined by:

$$\check{\mathrm{N}} = \{\mathrm{g}, \Delta \mathrm{t}(\mathrm{g}), \Delta \mathrm{f}(\mathrm{g}), \Delta \mathrm{i}(\mathrm{g}) \mid \mathrm{g} \in \mathrm{U}\} \tag{6}$$

where, $\Delta \mathrm{t}(\mathrm{g})\colon \mathrm{U} \rightarrow]^{-}0,1^{+}[$, $\Delta \mathrm{f}(\mathrm{g})\colon \mathrm{U} \rightarrow]^{-}0,1^{+}[$, and $\Delta \mathrm{i}(\mathrm{g})\colon \mathrm{U} \rightarrow]^{-}0,1^{+}[$ are called the true membership degree, indeterministic membership degree, and false membership degree, respectively. Here, $\Delta \mathrm{t}(\mathrm{g})$, $\Delta \mathrm{f}(\mathrm{g})$, and $\Delta \mathrm{i}(\mathrm{g})$ must satisfy the condition of ${}^{-}0 \leq \Delta \mathrm{t}(\mathrm{g}) + \Delta \mathrm{i}(\mathrm{g}) + \Delta \mathrm{f}(\mathrm{g}) \leq 3^{+}$.

In the next, definition of the AS is provided in terms of four membership degrees.

Definition 4: (AS) [4, 5]. Let $\mathrm{U} = \{\mathrm{g}\}$ be the universe for any event g, which is fixed. An AS $\acute{\mathrm{S}}$ for $\mathrm{g} \in \mathrm{U}$ is defined by:

$$\acute{\mathrm{S}} = \{\mathrm{g}, \Pi\mathrm{t}(\mathrm{g}), \Pi\mathrm{f}(\mathrm{g}), \Pi\mathrm{ta}(\mathrm{g}), \Pi\mathrm{fa}(\mathrm{g}) \mid \mathrm{g} \in \mathrm{U}\} \tag{7}$$

where, $\Pi\mathrm{t}(\mathrm{g})\colon \mathrm{U} \rightarrow [0,1]$, $\Pi\mathrm{f}(\mathrm{g})\colon \mathrm{U} \rightarrow [0,1]$, $\Pi\mathrm{ta}(\mathrm{g})\colon \mathrm{U} \rightarrow [0,1]$, and $\Pi\mathrm{fa}(\mathrm{g})\colon \mathrm{U} \rightarrow [0,1]$ are called the true membership degree (TMD), false membership degree (FMD), true-ambiguous membership degree (TAMD), and false-ambiguous membership degree (FAMD), respectively. In Eq. (7), $\Pi\mathrm{t}(\mathrm{g})$, $\Pi\mathrm{f}(\mathrm{g})$, $\Pi\mathrm{ta}(\mathrm{g})$and $\Pi\mathrm{fa}(\mathrm{g})$ must satisfy the following condition as:

$$0 \leq \Pi\mathrm{t}(\mathrm{g}) + \Pi\mathrm{f}(\mathrm{g}) + \Pi\mathrm{ta}(\mathrm{g}) + \Pi\mathrm{fa}(\mathrm{g}) \leq 2 \tag{8}$$

In Definition (4), $\Pi\mathrm{t}(\mathrm{g})$, $\Pi\mathrm{f}(\mathrm{g})$, $\Pi\mathrm{ta}(\mathrm{g})$, and $\Pi\mathrm{fa}(\mathrm{g})$ are called the true membership function (TMF), false membership function (FMF), and true-ambiguous membership function (TAMF), and false-ambiguous membership function (FAMF), respectively. All these four membership functions together are called ambiguous membership functions (AMFs). Singh and Hunag [6] proposed four different types of AMFs and called them T1AMFs, T2AMFs, T3AMFs, and T4AMFs.

3. Example for ambiguous set (AS)

Consider the following perception of human cognition while eating pizza in a restaurant to illuminate the idea of AS:

- P1: Pizza is *very tasty*.

In the case of a FS, the above perception P1 can be considered true, and it is assigned a true membership degree (i.e., $\Delta \mathrm{t}$(very tasty). In the case of a IFS, if perception P1 has a $\Delta \mathrm{t}$(tasty), there must be a false perception, which can be defined as:

- P2: Pizza is *not very tasty*.

Here, perception P2 is a contradiction to perception P1, which can be regarded as false and to which a false degree of membership, i.e., $\Delta \mathrm{f}$(not very tasty) is assigned.

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Human perception cannot fully distinguish between $very\ tasty$ and $not\ very\ tasty$ in perceptions P1 and P2 because there is an indeterminate unconsciousness between perceptions P1 and P2. Such a perception can be defined as:

- P12: Pizza is $either\ very\ tasty\ or\ not\ very\ tasty$.

In the case of NS, the perceptions P1 and P2 can be represented by the $\Delta\mathrm{t}(\text{very tasty})$ and $\Delta\mathrm{f}(\text{not very tasty})$. Perception P12, however, can be considered indeterministic, and it is assigned an indeterministic membership degree, i.e., $\Delta\mathrm{i}(\text{either very tasty or not very tasty})$. Thus, in the case of NS, indeterministic perception always leads to confusion in decision-making and final opinion. Another problem with NS is that the perceptions P1-P12 are independent, i.e., $\Delta\mathrm{t}(\text{very tasty})$, $\Delta\mathrm{f}(\text{not very tasty})$, and $\Delta\mathrm{i}(\text{either very tasty or not very tasty})$ are also independent. In this example, however, the three perceptions, namely P1-P12, are defined over the same perception. So, it is obvious that the membership degrees are interdependent. But, these membership degrees have unclear margins that make them indistinguishable from each other. For example, there is an unclear consciousness between $\Delta\mathrm{t}(\text{very tasty})$ and $\Delta\mathrm{f}(\text{not very tasty})$ in perception P12. To solve this problem of unclear margin between $\Delta\mathrm{t}(\text{very tasty})$ and $\Delta\mathrm{f}(\text{not very tasty})$, the following two additional perceptions can also be made with respect to perceptions P1 and P2:

- P3: Pizza is $a\ little\ tasty$.
- P4: Pizza is $not\ a\ little\ tasty$.

Perceptions P3-P4 may have different membership degrees in addition to the two membership degrees, i.e., $\Delta\mathrm{t}(\text{very tasty})$ and $\Delta\mathrm{f}(\text{not very tasty})$. Perception P3 is very closely related to perception P1, and it inherits unconsciousness from perception P1. Therefore, perception P3 can be considered true-ambiguous, and represented by a TAMD. Similarly, perception P4 is very closely related to perception P2, and it inherits unconsciousness from perception P2. Therefore, perception P4 can be considered as false-ambiguous, and represented by a FAMD. To solve the problem of including these four membership degrees in the analysis of perception or uncertain events, the AS theory was introduced.

To make a clear distinction in the representation of the membership degrees of FS, IFS and NS, the designations TMD, FMD, TAMD, and FAMD of AS are used (Definition (4)). According to these designations, the membership degrees for the perceptions P1, P2, P3, and P4 can be defined with AS as:

- $\Pi\mathrm{t}(\text{very tasty}) \in [0,1]$,
- $\Pi\mathrm{f}(\text{not very tasty}) \in [0,1]$,
- $\Pi\mathrm{ta}(\text{a little tasty}) \in [0,1]$, and
- $\Pi\mathrm{fa}(\text{not a little tasty}) \in [0,1]$.

The above four representations of membership degrees solve the problem of uncertain margins arises through human unconsciousness. In the case of AS, the AMFs define the four membership degrees in such a way that it must satisfy the condition (Eq. (8)) as:

$$0 \leq \Pi\mathrm{t}(\text{very tasty}) + \Pi\mathrm{f}(\text{not very tasty}) + \Pi\mathrm{ta}(\text{a little tasty}) + \Pi\mathrm{fa}(\text{not a little tasty}) \leq 2 \tag{9}$$

Suppose two customers A and B visit a restaurant and order a pizza. After eating the pizza, both customers may judge the taste of the pizza differently. The perceptions of customers A and B regarding the taste of the pizza can be denoted by ASs Ś1 and Ś2, and defined in Eq. (10) and (11), respectively, as:

$$\acute{S}1 = \{\text{tasty}, 0.46, 0.47, 0.42, 0.43 \mid g \in U\} \tag{10}$$

$$\acute{S}2 = \{\text{tasty}, 0.55, 0.38, 0.51, 0.35 \mid g \in U\} \tag{11}$$

Here, Eqs. (10) and (11) can be read as:

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{tasty, Πt(very tasty), Πf(not very tasty), Πta(a little tasty),
Πfa(not a little tasty) |tasty ∈ U}.

4. Additional mathematical definitions related to ambiguous set (AS)

This section provides various mathematical definitions for the AS.

Definition 5: (Ambiguousness) [6]. The process of assigning membership degrees to the event *g* is called an *ambiguousness* operation. An *ambifier* ∅=(Πt,Πf,Πta,Πfa) is a 4-tuple of membership functions Πt, Πf, Πta, Πfa: U → [0,1]. When applied to U, the ambifier ∅ yields an ambiguous set Ś in U as:

$$\emptyset(\mathrm{U}) = \{g, \Pi t(g), \Pi f(g), \Pi ta(g), \Pi fa(g)\} \tag{12}$$

An AS forms a region in a 2-dimensional (2D) space based on AMFs, called *ambiguous region (AR)*. Mathematically, it can be defined as:

Definition 6: (AR) [6]. For g ∈ U, an AR, denoted by $Ś_{RN}$ is the convex polygonal region with vertices $(X_1(g), 0)$, $(X_2(g), 0)$, $(0, Y_1(g))$, and $(0, Y_2(g))$.

Fig. 1(a)-(b) shows the membership degrees for Ś1 (Eq. (10)) and Ś2 (Eq. (11)) for the perceptions of customers A and B regarding the taste of the pizza. In this figure, the green shaded region is denoted as AR. This region represents the ambiguous profile of human perceptions (i.e., customers A and B) regarding the taste of the pizza.

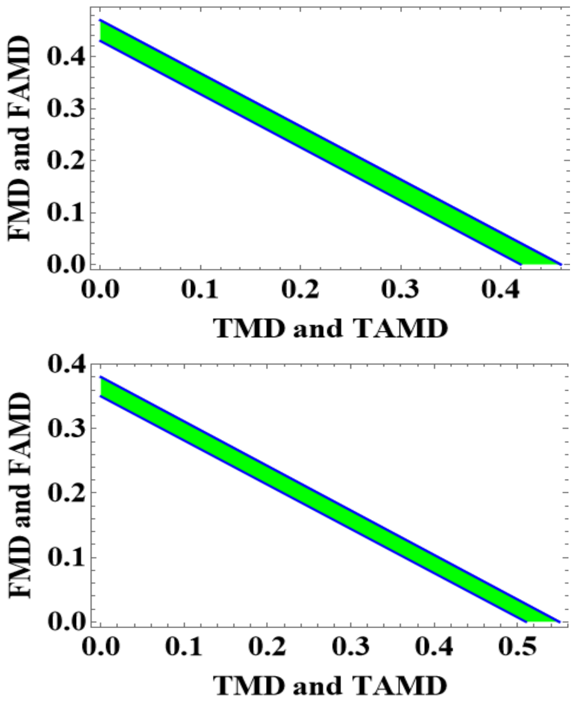

Figure 1: Membership degrees for the ASs: (a) Ś1 (top), and (b) Ś2 (bottom).

The ambiguousness arises because of unconsciousness. The measurement of this ambiguousness is possible through entropy, which is called *ambiguous entropy (AE)*. Mathematically, it can be defined as:

Definition 7: (AE). The AE *E* of an AS Ś (Eq. (7)) can be defined as:

$$\mathrm{E}(Ś) = 1 - \frac{1}{4}[\Pi t(g) + \Pi f(g)] \times [\Pi ta(g) - \Pi fa(g)]$$

(13)

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where, g is any event in the universe U. Here, $\Pi t(g): U \rightarrow [0,1]$, $\Pi f(g): U \rightarrow [0,1]$, $\Pi ta(g): U \rightarrow [0,1]$, $\Pi fa(g): U \rightarrow [0,1]$ for the AS Ś, and $E(Ś) \leq 2$.

We can join two ambiguous sets with "AND" and "OR" operators, called "connective operators". Both connective operators are defined as follows.

Definition 8: (AND operator). Two ASs Ś1 and Ś2 can be joined with AND operator, and defined as:

$$Ś1 \text{ AND } Ś2 = Ś1 \wedge Ś2 = \{g, [\Pi t1(g) \wedge \Pi t2(g)], [\Pi f1(g) \vee \Pi f2(g)], [\Pi ta1(g) \vee \Pi ta2(g)], [\Pi fa1(g) \vee \Pi fa2(g)] \mid g \in U\} \quad (14)$$

Definition 9: (OR operator). Two ASs Ś1 and Ś2 can be joined with OR operator, and defined as:

$$Ś1 \text{ OR } Ś2 = Ś1 \vee Ś2 = \{g, [\Pi t1(g) \vee \Pi t2(g)], [\Pi f1(g) \wedge \Pi f2(g)], [\Pi ta1(g) \wedge \Pi ta2(g)], [\Pi fa1(g) \wedge \Pi fa2(g)] \mid g \in U\} \quad (15)$$

In Eqs. (14) and (15), $\Pi t1(g)$, $\Pi f1(g)$, $\Pi ta1(g)$, and $\Pi fa1(g)$ belong to Ś1; whereas $\Pi t1(g)$, $\Pi f1(g)$, $\Pi ta1(g)$, and $\Pi ta1(g)$ belong to Ś2.

By following Eqs. (10) and (11), $Ś1 \wedge Ś2$ can be obtained as:

$$Ś1 \wedge Ś2 = \{tasty, [0.46 \wedge 0.55], [0.47 \vee 0.38], [0.42 \vee 0.51], [0.43 \vee 0.35] \mid tasty \in U\}$$
$$= \{tasty, 0.46, 0.47, 0.51, 0.43 \mid tasty \in U\}$$

Similarly, by following Eqs. (10) and (11), $Ś1 \vee Ś2$ can be obtained as:

$$Ś1 \vee Ś2 = \{tasty, [0.46 \vee 0.55], [0.47 \wedge 0.38], [0.42 \wedge 0.51], [0.43 \wedge 0.35] \mid tasty \in U\}$$
$$= \{tasty, 0.55, 0.38, 0.42, 0.35 \mid tasty \in U\}$$

For the two ASs Ś1 and Ś2, it satisfies the following properties:

a) **P1**: $(Ś1)^{C} = \{g, \Pi 1f(g), \Pi t1(g), 1 - \Pi t1(g), 1 - \Pi f1(g) \mid g \in U\}$,
   where, $Ś1 = \{g, \Pi t1(g), \Pi f1(g), \Pi ta1(g), \Pi fa1(g) \mid g \in U\}$. Here, "**c**" denotes the complement operator.
b) **P2**: $Ś1 \wedge Ś2 = Ś2 \wedge Ś1$, and
c) **P3**: $Ś1 \vee Ś2 = Ś2 \vee Ś1$.

5. Conclusions and future directions

In this study, the concept of AS theory was introduced to the scientific community with a clear explanation. The main goal of this theory was clearly explained with a real-time example. By applying this theory, unconsciousness and ambiguousness, which are inherent in every human perception, can be defined and represented with four membership degrees. In defining the membership degrees, their sum is limited to less than or equal to 2. In addition, the formula for measuring ambiguousness, called AE, was also provided in this study. Two set-theoretic operators, namely AND and OR, showed how to join two different ASs. Finally, in support of AS, the complement operator was presented.

In the future, various studies can be conducted to establish the AS theory in the scientific community, including time series prediction model, image segmentation algorithm, edge detection algorithm in biomedical images, multi-criteria decision making model and others.

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