A Note on the Uniqueness and Existence of the Solution to the Problem of Boundary Values for Some Partial Differential Equations of Second Order

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Abstract

Most physical mathematical problems, when solved, turn into one or more partial differential equations with imposed initial conditions or boundary conditions . This is known as the boundary value problems of differential equations. This research studies the solution of the set of Partial Differential Equations of parabolic and hyperbolic-hyperbolic type with boundary conditions imposed in different regions of the plane x o y.

This research has been proving the theorem of uniqueness and the existence of the solution.

Keywords: Boundary value; differential equation; partial differential equation

Introduction

The research deals with the theory of mixed differential equations, which is one of the theories currently being treated by partial differential equations , and although it was first treated by Tricomi in the Forties of the last century , interest in it did not begin in earnest until the late seventies of the same century , when Frenkel pointed out its importance in some dilemmas related to the movement of gases and liquids and the curves of surfaces. For more information and results, see [1-7].

This research is an extension of the research published for [1] , [3] , [7] , and Salah al-dinov , the unity theorem and the existence of the solution to the problem of boundary values of the set of Partial Differential Equations of the second order have been proved , and I have also come to find the functions that fulfill these equations in certain areas and under imposed boundary conditions , and here we also proved that when some conditions are met, the existence of the solution requires their unity . Therefore, we attributed the solution of these boundary value problems to the solution of a system of linear fredholm Integral Equations of the second type.

Main Results

Definition:

Fredholm linear equation is defined as follows:

$$.v(y)-\lambda\int_a^b k(y,ɲ)v(ɲ)dɲ=p(y) \qquad \text{(A)}$$

Transported equation is defined as follows
$v_1(y)-\lambda\int_a^b k(ɲ,y)v_1(ɲ)dɲ=p_1(y).$

Remark:

Solving kernel is defined

$$R(y,ɲ,\lambda)=H^{1(y,ɲ)}+\lambda H^{2(y,ɲ)}+\lambda^2H^{3(y,ɲ)}+\ldots\ldots+\lambda^{m-1}H_m(y,ɲ)+\ldots\ldots$$

The solution of Fredholm linear equation with respect to the solving kernel is:

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$v(y) = p(y) + \lambda \int_a^b R(y, \eta, \lambda) p(\eta) d\eta$.

The description of the problem:

Consider the following system:

$$(1) \qquad U_{x-x} - U_y + a(x,y)\, U_x + b(x,y)U = 0 \qquad ; (x,y) \in D_1$$

$$(2) \qquad U_{x-x} - U_{yy} + a_1 U_x + b_1 U_y + c_1 U = 0 \qquad ; (x,y) \in D_2$$

$$(3) \qquad U_{x-x} - U_{y-y} + a_2 U_x + b_2 U_y + c_2 U = 0 \qquad ; (x,y) \in D_3$$

Where a(x , y) is a continuous differential function one time in $\overline{D_1}$ and has the Kilder condition with coefficient $0 < \alpha \leq 1$. Also, *b(x , y)* is continuous known function with Kilder condition with coefficient α in $\overline{D_1}$.

As well as, D is open and bounded by the lines AB, BB_0, B_0A_0, A_0A, which their equations are

$y = 0, x = 1, y = 1, x = 0$

; $D = D_1 \cup AA_0 \cup BB_0 \cup D_2 \cup D_3$

If we made the following transformation to 1:

$$U(x,y) = V(x,y) \exp\ \{-\frac{1}{2} \int_0^X a(t,y)dt\ \}$$

We get:

$$V_{xx} - V_y + C(x,y)V = 0$$

Where,

$$C(x,y) = -\frac{1}{4}\ a^2(x,y) - \frac{1}{2}\frac{\partial}{\partial y} a(x,y) + \frac{1}{2}\frac{\partial}{\partial y} \int_0^X a(t,y)dt + b(x,y)$$

If we applied the following transformation to 2:

$$U(x,y) = V(x,y)\ e^{\alpha_1 x + \beta_1 y}$$

We get:

$$V_{xx} - V_{yy} + \lambda_1 V = 0$$

$$\lambda_1 = \frac{1}{4}\left(4c_1^2 - {a_1}^2 - {b_1}^2\right) \quad ; \ \alpha_1 = \frac{-a_1}{2} \ ; \ \beta_1 = +\frac{b_1}{2}$$

If we applied the following transformation to 3:

$$U(x,y) = V(x,y)\ e^{\alpha_2 x + \beta_2 y}$$

So that,

$$V_{xx} - V_{yy} + \lambda_2 V = 0$$

$$\lambda_2 = \frac{1}{4}\left(4c_2 - {a_2}^2 - {b_2}^2\right) \quad ; \ \alpha_1 = \frac{-a_2}{2} \ ; \ \beta_1 = +\frac{b_2}{2}$$

So it is sufficient to check:

$$(4) \qquad U_{xx} - U_y + C(x,y)U = 0 \qquad ; (x,y) \in D$$

$$(5) \qquad -U_{xx} - U_{yy} - \lambda_1 U = 0 \qquad ; (x,y) \in D_2$$

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$$(6) \quad -U_{xx} - U_{yy} - \lambda_2 U = 0 \qquad ; (x, y) \in D_3$$

Problem (N)

We aim to find the uniform solution U for (4), (5), (6) in D, except the points of BB_0, AA_0, and with the condition:

$$U(x,y) \in c\left(\overline{D_1}\right) \cap [c^1(D_2 \cup AA_0) \cap c^1(D_3 \cup BB_3) \cap c^1(D_1 \cup AA_0 \cup BB_0)]$$

$$(7) \qquad U|_{A_0C} = \Psi_1(y) \;\; ; \qquad \frac{1}{2} \le y \le 1$$

$$(8) \qquad U|_{BE} = \Psi_2(y) \;\; ; \qquad 0 \le y \le \frac{1}{2}$$

$$(9) \qquad U|_{y=0} = \varphi(x) \;\; ; \qquad 0 \le x \le 1$$

and

$$U(-0,y) = \alpha_1(y)U(+0,y) + \gamma_1(y)$$

$$U_X(-0,y) = \beta_1(y)U_X(+0,y) + \delta_1(y)U(+0,y) + \sigma_1(y)$$

$$U(1+0,y) = \alpha_2(y)U(1-0,y) + \gamma_2(y)$$

$$(10) \qquad U_X(1+0,y) = \beta_2(y)U_X(1-0,y) + \delta_2(y)U(1-0,y) + \sigma_2(y)$$

Under the assumption

$$\alpha_1(y), \alpha_2(y), \beta_1(y), \beta_2(y), \gamma_1(y), \gamma_2(y), \sigma_1(y), \sigma_2(y), \delta_1(y), \delta_1(y), \delta_2(y), \Psi_1(y), \Psi_2$$

φ(x) -are known continuous differentiable functions with

. $\alpha_1''(y), \varphi'(x), \Psi_2''(y), \Psi_1''(y), \gamma_1''(y), \gamma_2''(y), \beta_2''(y), \beta_1''(y), \alpha_2''$ are continuous.

Let us suppose:

$$\begin{cases} U(+0,y) = \tau_1^+(y), U_X(+0,y) = v_1^+(y), \\ U(-0,y) = \tau_1^-(y), U_x(-0,y) = v_1^-(y), \\ U(1+0,y) = \tau_2^-(y), U_X(1+0,y) = v_2^-(y), \\ U(1-0,y) = \tau_2^-(y), U_X(1-0,y) = v_2^+(y), \end{cases}$$

$$(12) \qquad \tau_1^-(y) = \rho_1(y) + \int_y^1 J_0[\lambda_1(y-t)]v_2^-(t)dt \;\;, \quad 0 < y < 1$$

$$(13) \qquad \tau_2^-(y) = \rho_2(y) + \int_0^y J_0[\lambda_2(y-t)]v_2^-(t)dt \;\;, \quad 0 < y < 1$$

$$\rho_1(y) = 2\Psi_1\left(\frac{y+1}{2}\right) - \Psi_1(1) + \int_y^1 \frac{\partial}{\partial t} J_0\left(\lambda_1\sqrt{(y-1)(y-t)}\right)\left[2\Psi_1\left(\frac{t+1}{2}\right) - \Psi_1(1)\right]dt$$

$$\rho_2(y) = 2\Psi_2\left(\frac{y}{2}\right) - \Psi_2(0) + \int_0^y \frac{\partial}{\partial t} J_0\left(\lambda_2\sqrt{t(t-y)}\right)\left[2\Psi_2\left(\frac{t}{2}\right) - \Psi_2(0)\right]dt$$

Theorem

If:

$$(14) \qquad C(x,y) \le 0 \qquad ; \qquad (X,y) \in D_1$$

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$$(15)\qquad \frac{1}{\alpha_1(0)\beta_1(0)} > 0, \frac{d}{dy}[\frac{1}{\alpha_1(y)\beta_1} \geq 0\,, \frac{\delta_1(y)}{\beta_1(y)}] \leq 0$$

$$(15)\qquad \frac{1}{\alpha_2(1)\beta_2(1)} > 0, \frac{d}{dy}[\frac{1}{\alpha_2(y)\beta_2} \leq 0\,, \frac{\delta_2(y)}{\beta_2(y)}] \geq 0$$

Then problem N has a unique solution.

Proof

Assume that $\mathrm{U(x,y)} \neq$ const in $\overline{D}$ is a solution of the following:

$$U_{XX} - U_y + C(X,y)U = 0 \qquad ; \ (X,y) \in D_1$$

$$U_{yy} - U_{XX} - \lambda_j U = 0 \quad ; \ (X,y) \in D_i \quad i = 2,3; j = 1,2$$

$$U|_{A_0C} = 0 \ ; \quad U|_{BE} = 0 \ ; \ U|_{y=0} = 0$$

$$U(-0,y) = \alpha_1(y)U(+0,y) +$$

$$U_X(-0,y) = \beta_1(y)U_X(+0,y) + \delta_1(y)U(+0,y)$$

$$U(1+0,y) = \alpha_2(y)U(1-0,y)$$

$$U_X(1+0,y) = \beta_2(y)U_X(1-0,y) + \delta_2(y)U(1-0,y)$$

$$(17)\qquad \frac{1}{2}\int_0^1 U^2(x,1)dx + \int_0^1 \tau_1^+(y)v_1^+(y)dy - \int_0^1 \tau_2^+(y)v_2^+(y)dy + \iint_{D_1} U_X^2 - c(X,Y)U^2dXdy = 0$$

We must find:

$$I_j = \int_0^1 \tau_j^+(y)v_j^+(y)dy \quad ,j = 1.2$$

$$I_1 = \int_0^1 \tau_1^+(y)v_1^+(y)dy =$$

$$= \frac{1}{\pi}\int_0^1 (1-z^2)^{-\frac{1}{2}}dz\{\frac{1}{\alpha_1(0)\beta_1(0)} \times [(\int_0^1 \cos\lambda_1 zt\overline{v}_1(t)dt)^2 +$$

$$+(\int_0^1 \sin\lambda_1 zt\overline{v}_1(t)dt)^2\,]\ \int_0^1 \left[\frac{1}{\alpha_1(y)\beta_1(y)}\right]'[(\int_y^1 \cos\lambda_1 zt\overline{v}_1(t)dt)^2 +$$

$$+(\int_0^1 \sin\lambda_1 Zt\overline{v}_1(t)dt)^2\ ]\int_0^1 \frac{\delta_1(y)}{\alpha_1^2(y)\beta_1(y)}\overline{\tau}_1(y)dy\ ,$$

$$I_2 = \int_0^1 \tau_2^+(y)v_2^+(y)dy =$$

$$\frac{1}{\pi}\int_0^1 (1-z^2)^{-\frac{1}{2}}dz \times \{\frac{1}{\alpha_2(1)\beta_2(1)} \times (\int_0^1 \cos\lambda_2 zt\overline{v}_2(t)dt)^2 +$$

$$+\int_0^1 \left[\frac{1}{\alpha_2(y)\beta_2(y)}\right]' \times (\int_0^y \cos\lambda_2 zt\overline{v}_2(t)dt)^2dy - \frac{1}{\alpha_2(1)\beta_1(1)} \times$$

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$$\times \left(\int_0^1 \sin\lambda_2 zt\overline{v}_2(t)dt\right)^2 + \int_0^1 \left[\frac{1}{\alpha_2(y)\beta_2(y)}\right]' \left(\int_0^y \sin\lambda_2 zt\overline{v}_2(t)dt\right)^2 dy\} -$$

$$-\int_0^1 \frac{\delta_2(y)}{\alpha_2^2(y)\beta_2(y)}\overline{\tau}_2(y)dy$$

Then:

$$I_1 = \int_0^1 \tau_1^+(y)v_1^+(y)dy \qquad > 0$$

$$I_2 = \int_0^1 \tau_2^+(y)v_2^+(y)dy \qquad < 0$$

thus from (17) $U_X = 0$

hence, $U(X,Y) = \mu(Y)$

since, $U(0,Y) = U(1,Y) = 0$

$\mu(y) \equiv 0$

$U(X,Y) \equiv 0 \;\; ; \;\; (X,Y) \;\in\; \overline{D}_1$

This means that:

D_3 و$U(X,Y) \equiv 0 \;\; ; \;\; (X,Y) \;\in\; \overline{D}$ which is a contradiction.

Now we prove that the solution is existed:

$$U(X,Y) = \int_0^Y G_\xi(X,Y,ր)\tau_1^+(ր)dn - \int_0^Y G_\xi(X,Y,1,ր)\tau_1^+(ր)dn + \int_0^1 G(X,Y,\xi,0)\varphi(\xi)d\xi - \int_0^1 d\xi +$$

$$(18) \qquad \times \int_0^Y C(\xi,ր)G(X,Y;\xi,ր)U(\xi,ր)dn)$$

$$G(x,y;\xi,ր) = \frac{1}{\sqrt[2]{\pi(y-ր)}} - \exp\{-\frac{(x+\xi+2n)^2}{4(y-ր)}\}] \sum_{n=-ր}^{ր} [\exp\{-\frac{(x-\xi+2n)^2}{4(y-ր)}\}$$

Where G(x,y,ξ,ր) is the mixed Green function.

$$U(x,y) = \int_0^y G_\xi(x,y;0,ր)\tau_1^+(ր)dn - \int_0^y G_\xi(x,y,1,ր)\tau_2^+(ր)dn +$$

$$+\int_0^Y \Phi_2(ր;x,y)v_2^-(ր)dn + \Psi(X,Y) + V(X,Y)$$

where

$$\emptyset_1 = (ր;x,y) = \int_ր^y \int_0^1 G_\xi(\theta,t,0,ր)R_1(x,y;\theta,t)d\theta dt \quad ;$$

$$\emptyset_2 = (ր;x,y) = -\int_ր^y \int_0^1 G_\xi(\theta,t,1,ր)R_1(x,y;\theta,t)d\theta dt \quad ;$$

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$$V(X,Y)=\int_0^1 G(x,y;\xi,0)\varphi(\xi)d\xi\;;$$

$$\Psi(x,y)=\int_0^y\int_0^1\int_0^1 R_1(x,y;\theta,t)G(\theta,t,\xi,0)\varphi(\xi)d\xi d\theta dt$$

We get:

$$U_X|_{X=0}\equiv v_1^+(y)=\int_0^Y\{-\frac{1}{\sqrt{\pi(Y-\eta)}}+\frac{2}{\sqrt{\pi(Y-\eta)}}\times$$

$$\times\sum_{n=1}^{\infty}\exp\,(-\frac{n^2}{Y-\eta})\}\tau_1^{+\prime}(\eta)dn+\int_0^y\{\frac{1}{\sqrt{\pi(Y-\eta)}}\exp\left[-\frac{1}{4\sqrt{(Y-\eta)}}\right]+$$

$$+\frac{1}{2\times\sqrt{\pi(Y-\eta)}}\times\sum_{n=-\eta}^{\eta}{}'\left[\exp\left\{-\frac{(-1+2n)^2}{4(Y-\eta)}\right\}+\exp\left\{-\frac{(1+2n)^2}{4(Y-\eta)}\right\}\right]\}\;\times$$

$$(19)\times\;\tau_2^{+\prime}(\eta)d\eta+\int_0^Y\emptyset_{1X}(\eta;0;Y)\tau_1^+(\eta)d\eta+\int_0^Y\emptyset_{2X}(\eta;0,Y)\tau_2^+(\eta;0,Y)\tau_2^+(\eta)d\eta+F_1(Y)$$

$$U_X|_{X=1}\equiv\;v_2^+(Y)=\int_0^Y\{-\frac{1}{\sqrt{\pi(Y-\eta)}}\exp\left[-\frac{1}{4(Y-\eta)}\right]-$$

$$-\frac{1}{\pi(Y-\eta)}\sum_{n=-\eta}^{\eta}{}'\exp\,[-\frac{(1+2n)^2}{4(Y-\eta)}]\}\tau_1^{+\prime}(\eta)d\eta+$$

$$+\int_0^y\{\frac{1}{2\times\sqrt{\pi(Y-\eta)}}+\frac{1}{\sqrt{\pi(Y-\eta)}}\quad\sum_{n=1}^{\infty}\exp\left(-\frac{n^2}{y-\eta}\right)+$$

$$+\frac{1}{2\times\sqrt{\pi(Y-\eta)}}\exp\left[-\frac{1}{Y-\eta}\right]+\frac{1}{2\times\sqrt{\pi(Y-\eta)}}\times$$

$$\times\sum_{n=-\eta}^{\eta}\exp\,[-\frac{(1+2n)^2}{y-\eta}]\}\tau_2^{+\prime}(\eta)d\eta+\int_0^Y\emptyset_{1X}(\eta;1,y)\tau_1^+(\eta)d\eta+$$

$$(20)\qquad+\int_0^y\emptyset_{2X}(\eta;1,y)\tau_2^+(\eta)d\eta+F_2(y)$$

where

$$F_{i+1}(Y)=[\frac{\partial\Psi(x,y)}{\partial x}+\frac{\partial V(x,y)}{\partial x}]_{x=i}\quad;\;i=0.1$$

Now, we get:

$$(22)\;\overline{v}_1(y)+\int_0^y M_1(y,\eta)\overline{v}_1(\eta)d\eta+\int_y^1 M_2(y,\eta)\overline{v}_1(\eta)d\eta+\int_0^y M_3(y,\eta)\overline{v}_2(\eta)d\eta=P_1;$$

(23) $\overline{v}_1(y)+\int_0^y M_1\,(y,\eta)\overline{v}_1(\eta)d\eta+\int_y^1 M_2(y,\eta)\overline{v}_1(\eta)d\eta+\;\int_y^1 M_2(y,\eta)\overline{v}_1(\eta)d\eta+\int_y^1 M_6(y,\eta)\overline{v}_1(\eta)d\eta=$ $P_2(y)$.

$$M_1(y,ɲ) = \frac{\beta_1(y)}{\alpha_1(ɲ)\sqrt{\pi(y-ɲ)}}[1+2\sum_{n=1}^{ɲ} \exp{(-\frac{n^2}{y-ɲ})}] + \beta_1(y) \times$$

$$\times \int_0^{ɲ} \frac{\alpha_1(t)-\alpha_1'(t)}{\alpha_1^2(t)\sqrt{\pi(y-t)}}[1+2\sum_{n=1}^{ɲ} exp(-\frac{n^2}{y-t})]\frac{\partial}{\partial t}J_0 \times$$

$$\times [\lambda_1(t-\lambda)]dt - \beta_1(y)\int_{\cap}^{ɲ} J_0[\lambda_1(t-ɲ)\emptyset_{1X}(t;0,Y)dt \ ;$$

$$M_2(y,ɲ) = \beta_1(y\int_0^{y} \frac{\alpha_1(t)-\alpha_1'(t)}{\alpha_1^2(t)\sqrt{\pi(y-t)}}[1+2\sum_{n=1}^{ɲ} exp(-\frac{n^2}{y-t})]\frac{\partial}{\partial t}J_0$$

$$\times [\lambda_1(t-ɲ)]dt - \beta_1(y)\int_0^{y} J_0[\lambda_1(t-ɲ)\emptyset_{1X}(t;0,Y)dt - \frac{\delta_1(Y)J_0[\lambda_1(Y-ɲ)]}{\alpha_1(y)} \quad ;$$

$$M_3(y,ɲ) = \frac{\beta_1(y)}{\alpha_2(ɲ)\sqrt{\pi(y-ɲ)}}\{\exp\left[-\frac{1}{4(y-ɲ)}\right] + \frac{1}{2}\sum_{n=-ɲ}^{ɲ}{}'[\exp\left\{-\frac{-(-1+2n)^2}{4(y-ɲ)}\right\} +$$

$$\beta_1(y)\int_{ɲ}^{y} \frac{\alpha_2(t)+\alpha_2'}{\alpha_2^2(t)\sqrt{\pi(y-T)}}\{exp\left[-\frac{1}{4(y-ɲ)}\right] + exp\{-\frac{(1+2n)^2}{4(y-ɲ}\}]\} +$$

$$\sum_{n=-ɲ}^{ɲ}{}'\frac{1}{2}[exp\left\{-\frac{(-1+2n)^2}{4(y-t)}\right\} + exp\{-\frac{(1+2n)^2}{4(y-t)}\}]\} \times$$

$$\times \frac{\partial}{\partial t}J_0[\lambda_2(t-)]dt + \beta_1(Y)\int_{ɲ}^{Y} J_0[\lambda_2(t-)]\emptyset_{2X}(t;0,y)dt$$

$$M_4(y,ɲ) = +\frac{\beta_2(y)}{\alpha_2(ɲ)\sqrt{2\pi(y-ɲ)}}\{1+2\sum_{n=1}^{ɲ} exp(-\frac{n^2}{y-ɲ})$$

$$\sum_{n=-ɲ}^{ɲ}{}'\exp{[-\frac{91+2n)^2}{4(y-ɲ)}]\}} + \exp{[-\frac{1}{y-ɲ}]} +$$

$$+\beta_2(y)\int_{ɲ}^{y} \frac{\alpha_2(y)+\alpha_2'(t)}{\alpha_2^2(t)\sqrt{2\pi(y-t)}}\{1+2\sum_{n=1}^{ɲ} exp\left(-\frac{n^2}{y-t}\right) + exp\left[-\frac{1}{y-t}\right] +$$

$$\lambda_2(t-ɲ)dt + + \sum_{n=-ɲ}^{ɲ}{}'\exp{[-\frac{(1+2n)^2}{4(Y-t)}]\}}\frac{\partial}{\partial t}J_0$$

$$+\frac{\delta_2(y)J_0[\lambda_2(y-ɲ)]}{\alpha_2(y)} \quad ; +\beta_2(Y)\int_{ɲ}^{Y} J_0 \ \ [\lambda_2(t-ɲ)]\emptyset_{2X}(t;1,y)dt$$

$$\sum_{n=-ɲ}^{ɲ}{}'\exp\left[-\frac{(1+2n)^2}{4(Y-t)}\right]\frac{\beta_2(y)}{\alpha_1(ɲ)\sqrt{\pi(y-ɲ)}}\{\exp\left[-\frac{1}{y-ɲ}\right] + M_5(y,ɲ) = +$$

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$$+\beta_2(y)\int_0^{\eta}\frac{\alpha_2(y)+\alpha_2'(t)}{\alpha_1^2(t)\sqrt{2\pi(y-t)}}\exp\left[-\frac{1}{4(Y-t)}\right]+$$

$$\sum_{n=-\eta}^{\eta}{}'\exp\left[-\frac{(1+2n)^2}{4(Y-\eta)}\right]\frac{\partial}{\partial t}J_0\times[\lambda_1(t-\eta)]dt-$$

$$-\beta_2(Y)\int_0^{Y}J_0\ \ [\lambda_1(t-\eta)]\emptyset_{1X}(t;1,y)dt\ \ ;$$

$$\beta_2(y)\int_0^{y}\frac{\alpha_1(y)+\alpha_1'(t)}{\alpha_1^2(t)\sqrt{\pi(y-t)}}\exp\left[-\frac{1}{4(y-t)}\right]+M_6(y,\eta)=$$

$$[\lambda_1(t-\eta)]dt-\sum_{n=-\eta}^{\eta}{}'\exp\left[-\frac{(1+2n)^2}{4(Y-\eta)}\right]\}\frac{\partial}{\partial t}J_0\times$$

$$\frac{\delta_1(Y)J_0[\lambda_1(Y-\eta)]}{\alpha_1(y)}\quad;-\beta_2(Y)\int_0^{Y}J_0\ \ [\lambda_1(t-\eta)]\emptyset_{1X}(t;1,y)dt-$$

$$P_1(y)=\beta_1(y)=F_1(y)=\sigma_1(y)+$$

$$\frac{\delta_1(y)[P_1(y)-\gamma_1(y)]}{\alpha_1(y)}+\beta_1(y)\int_0^{y}J_0\{\frac{\rho_1(t)-\gamma_1(t)}{\alpha_2(t)}+\ \emptyset_{1X}(t;0,y)+$$

$$\emptyset_{2X}=(t;0,y)\}dt-\beta_1(Y)\int_0^{Y}\frac{1}{\sqrt{\pi(Y-\eta)}}\times+\frac{\rho_2(t)-\gamma_2(t)}{\alpha_2(t)}\times$$

$$\frac{\alpha_1(\eta)\rho_1'(\eta)-\alpha_1'(\eta)\rho_1(\eta)+\alpha_1'(\eta)\gamma_1(\eta)-\alpha_1(\eta)\gamma_1'(\eta)}{\alpha_1^2(\eta)}\times$$

$$\times\left[1+2\sum_{n=1}^{\eta}exp\left(-\frac{n^2}{y-\eta}\right)\right]\}d\eta+\beta_1(Y)\int_0^{Y}\frac{1}{2\times\sqrt{\pi(Y-\eta)}}\times$$

$$\frac{\alpha_2(\eta)\rho_2'(\eta)-\alpha_2'(\eta)\rho_2(\eta)+\alpha_2'(\eta)\gamma_2(\eta)-\alpha_2(\eta)\gamma_2'(\eta)}{\alpha_2^2(\eta)}\times$$

$$\sum_{n=-\infty}^{\infty}{}'\exp\times\left\{-\frac{(1+2n)^2}{4(Y-\eta)}\right\}+\times\{\exp\left[-\frac{1}{4(Y-\eta)}\right]+$$

$$\exp\{-\frac{(1+2n)^2}{4(Y-\eta)}\}]\}\,d\eta\qquad;$$

$$P_2(y)=\beta_2(y)=F_2(y)=\sigma_2(y)+$$

$$\frac{\delta_2(y)[P_2(y)-\gamma_2(y)]}{\alpha_2(y)}+\beta_2(y)\int_0^{y}J_0\{\frac{\rho_1(t)-\gamma_1(t)}{\alpha_1(t)}+\ \emptyset_{1X}(t;1,Y)$$

$$\emptyset_{2X}(t;1,y)\}dt-\beta_2(y)\int_0^{y}\frac{1}{\sqrt{\pi(y-\eta)}}\times+\frac{\rho_2(t)-\gamma_2(t)}{\alpha_2(t)}\times$$

$$\frac{\alpha_1(\eta)\rho_1'(\eta)-\alpha_1'(\eta)\rho_1(\eta)+\alpha_1'(\eta)\gamma_1(\eta)-\alpha_1(\eta)\gamma_1'(\eta)}{\alpha_1^2(\eta)}\times$$

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$$\beta_2(Y)\int_0^Y \frac{1}{2\times\sqrt{\pi(Y-\eta)}}\times\sum_{n=-\infty}^{\infty}{}'\exp\left[-\frac{(1+2n)^2}{4(Y-\eta)}\right]\}d\eta+\times\{\exp\left[-\frac{1}{4(Y-\eta)}\right]+$$

$$\sum_{n=-\infty}^{\infty}{}'\exp\left[-\frac{(1+2n)^2}{4(Y-\eta)}\right]]\}d\eta\exp\left[-\frac{1}{(Y-\eta)}\right]+\times 1+2\sum_{n=1}^{\infty}exp\left(-\frac{n^2}{y-\eta}\right)+$$

$$\int_y^1 M_8(y,\eta)\overline{v}_1(\eta)\,d\eta \quad (24) \qquad \overline{v}_2(y)=p_3(y)-\int_0^y M_7(y,\eta)\overline{v}_1(\eta)\,d\eta$$

$$\int_y^1 M_6(t,\eta)R_2(y,t)\,dt\,; M_7(y,\eta)=M_5(y,\eta)+\int_\eta^y M_5(t,\eta)R_2(y,t)\,dt+$$

$y, M_8(y,\eta)=M_6(y,\eta)+\int_0^y M_6(t,\eta)R_2(y,t)\,dt\,;\; P_3(Y)=P_2(Y)+\int_0^y P_2(\eta)\,R_2(\eta)d\eta;$

with:

$$k(y,\eta)=\begin{cases}K_1(y,\eta) & if \quad 0\le\eta\le y\\ K_2(y,\eta) & if \quad y<\eta\le 1\end{cases}$$

$$(25) \qquad \overline{v}_1(y)+\int_0^1 K(y,\eta)\overline{v}_1(\eta)d\eta=P(y)$$

$$K_1(y,\eta)=M_1(y,\eta)-\int_\eta^y M_3(Y,t)\,M_7(t,\eta)dt-\int_0^\eta M_3(Y,t)M_8(t,\eta)dt\;;$$

$$K_2(y,\eta)=M_2(y,\eta)-\int_0^y M_3(y,t)\,M_8(t,\eta)dt;\mathrm{P}(Y)=P_1(Y)-\int_0^y M_3(y,t)P_3(t)dt$$

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