Q-Complex Neutrosophic Set

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Abstract

Most complex problems in the real-world typically involve uncertain,incomplete and indeterminate two-dimensional data i.e. information pertaining to the attributes and the periodicity of the problem parameters. To meet the demand for models that has the ability to handle these information with these characteristics, the introduction of neutrosophic sets (NSs) was followed by their extension to the complex neutrosophic sets (CNSs). In this paper, we introduce the concept of Q- complex neutrosophic set (Q-CNS) by extending the ranges of the membership functions in Q-neutrosophic set (Q-NS) from [0,1] to the unit circle in the complex plane. Q-CNS plays a key role in the decision making theory, where the extra information provided by the elements of the Q-set serve in modeling of some decision making problems. Based on this new concept we define the basic theoretical operations such as complement, equality, subset, union, intersection, Q-complex neutrosophic product and Cartesian product. Some related examples are also given to enhance the understanding of the proposed concepts. The basic properties of these operators are also verified with supporting proofs.

Kewords: Complex Neutrosophic Set; Decision Making; Neutrosophic Set; Q-Neutrosophic Set.

1 introduction

The concepts of Q-fuzzy sets (Q-FSs), intuitionistic Q-fuzzy sets (Q-IFSs) and Q-NSs are introduced by many researchers and were extensively investigated in many algebraic structures. Jun[1] introduced the notion of Q-fuzzy subalgebras of BCK/BCI-algebras. Roh et al.[2] studied intuitionistic Q-fuzzy subalgebras of BCK/BCI-algebras. Kazancı et al.[3] initiated the notion of intuitionistic Q-fuzzification of R-subgroups (subnear-rings) in a near-ring and investigate some related properties. Solairaju and Nagarajan[4] reconstructed Q-fuzzy groups and investigated some of their related properties. Muthuraj et al.[5] introduced and investigated anti Q-fuzzy group and its lower level subgroups. Nayagam et al.[6] defined the concept of an intuitionistic Q-fuzzy HX group and defined a new algebraic structure of intuitionistic Q-fuzzy HX group. Sithar Selvam et al.[7] discussed anti Q-fuzzy R-closed KU- ideal of KU-algebras and its lower level cuts in detail. Tanamoon et al.[8] established the notions of Q-fuzzy UP-ideals and Q-fuzzy UP-subalgebras of UP-algebras, and investigated their properties. Abu Qamar and Hassan[9] elaborated the concept of Q-NS to provide a way to deal with uncertain, indeterminate and inconsistent two-dimensional information. They also extended this concept to multi Q-neutrosophic set, while Thiruveni and Solairaju[10] defined neutrosopic Q-fuzzy set and derived the results on neutrosophic Q-fuzzy subgroups.

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Many researchers incorporated soft set into Q-FS, Q-IFS and Q-NS. Adam and Hassan[11] combined the concept of soft set and Q-FS to introduce the concept of Q-fuzzy soft set. Broumi[12] presented the notion of the Q-intuitionistic fuzzy soft set (Q-IFSS), and defined some basic properties and basic operations. Mahmood et al.[13] introduced the concept of Q-single value neutrosophic soft set, multi Q-single valued neutrosophic set and defined some basic results and related properties, while Abu Qamar and Hassan[14] initiated the concept of the Q-neutrosophic soft set (Q-NSS) and studied the relations between two Q-NSSs.

To represent incomplete, indeterminate, and inconsistent information, Smarandache[15,16] originally gave a concept of NS from a philosophical point of view which is a part of neutrosophy. NS is characterised independently by a truth-membership function (T-MF), indeterminacy-membership function(I-MF) and falsity-membership function(F-MF), denoted by T, I, F, respectively, where the indeterminacy is quantified explicitly. The ranges of the functions T, I and F are subsets of the real standard or nonstandard interval. To constrain them in the real standard interval [0,1] for convenient science and engineering applications, single-valued neutrosophic set (SVNS) and its operators were introduced by Wang et al.[17] as the sub-classes of the NSs.

However, in some situations where the need to the second variable in the decision process, the description of objects by NS in terms of membership functions with one variable (dimension) only is not adequate. Ali and Smarandache[18] have heightened the need for extra variable (phase term) to denote two objectives in one set simultaneously. Subsequently, they extended the NS to CNS which is also a generalisation of complex fuzzy set (CFS)[19] and complex intuitionistic fuzzy set (CIFS).[20] This model has the capability of handling the different aspects of uncertainty, such as incompleteness, indeterminacy and inconsistency, whilst simultaneously handling the periodicity aspect of the objects, all in a single set.

A CNS is defined by a complex-valued T-MF which represents uncertainty with periodicity, complex-valued I-MF which represents indeterminacy with periodicity, and a complex-valued F-MF which represents falsity with periodicity. Al-Quran and Alkhazaleh[21] derived and investigated the relations among the CNSs and used these relations to describe and handle a real decision-making problem. Ali et al.[22] then extended the CNS to the notion of the interval complex neutrosophic set and applied it to a decision-making problem for the green supplier selection. Recent years have seen the rapid utilization and usage of CNS in many real life problems.[23–28]

In this paper, we define Q-CNS, which is basically a CNS defined over a two-dimensional set. Thus, it has added advantages to CNS by treating a two-dimensional universal set, which makes it more valid in modeling real-life problems where two-dimensional sets and indeterminacy majorly appear.

2 Preliminaries

In this section, a summary of the literature on Q-FS, Q-IFS, Q-NS, NS and CNS relevant to this paper is presented.

We begin by recalling the definition of Q-FS, followed by the definitions of Q-IFS and Q-NS.

Definition 2.1. [1] A Q-fuzzy set (Q-FS) in a non empty set X is an arbitrary function $f : X \times Q \longrightarrow [0, 1]$, where Q is a non empty set and [0, 1] is the unit segment of the real line.

Definition 2.2. [2] An intuitionistic Q-fuzzy subset Λ in a non empty set X is defined as an object of the form

$$\Lambda = \{< (x, q); \mu_\Lambda(x, q), \nu_\Lambda(x, q) >: x \in X, q \in Q\},$$

where $\mu_\Lambda : X \times Q \longrightarrow [0, 1]$ and $\nu_\Lambda : X \times Q \longrightarrow [0, 1]$ are the degree of membership and non-membership of the element $(x, q) \in X \times Q$, respectively, and for every $(x, q) \in X \times Q$, $0 \leq \mu_\Lambda(x, q) + \nu_\Lambda(x, q) \leq 1$.

Definition 2.3. [14] Let X be a universal set and Q be a nonempty set. A Q-neutrosophic set (Q-NS) M in X and Q is an object of the form

$$M = \{< (x, q); T_M(x, q), I_M(x, q), F_M(x, q) >: x \in X, q \in Q\},$$

where $T_M; I_M; F_M : X \times Q \to]^-0; 1^+[$ are the T-MF, I-MF and F-MF, respectively with $^-0 \leq T_M + I_M + F_M \leq 3^+$.

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Definition 2.4. [14] Let X be a universal set and Q be a nonempty set, L be a unit interval [0, 1], n be a positive integer. A multi Q- neutrosophic set $\hat{M}$ in $X \times Q$ is a set of ordered sequences:

$$\hat{M} = \{< (x, q); T_{M_i}(x, q), I_{M_i}(x, q), F_{M_i}(x, q) >: x \in X, q \in Q\},$$

$\forall i = 1, 2, 3, ..., n,$ where $T_{M_i}, I_{M_i}, F_{M_i} : X \times Q \longrightarrow L^n, \forall i = 1, 2, 3, ..., n,$ are respectively, T-MF, I-MF and F-MF for each $x \in X$ and $q \in Q$ and satisfy the condition $0 \leq T_{M_i} + I_{M_i} + F_{M_i} \leq 3, \forall i = 1, 2, 3, ..., n,$

where n is called the dimension of $\hat{M}$.

Smarandache[16] defined the NS as follows.

Definition 2.5. Let U be a universe of discourse, and a neutrosophic set N in U is defined as:

$$A = \{< u; T_N(u); I_N(u); F_N(u) >; u \in U\},$$

where $T_N(u), I_N(u)$ and $F_N(u)$ are T-MF, I-MF and F-MF, respectively, such that $T;\ I;\ F : X \rightarrow]^{-}0; 1^{+}[$ and $^{-}0 \leq T_N(u) + I_N(u) + F_N(u) \leq 3^{+}$.

Definition 2.6. [17] Let U be a universe of discourse. A single valued neutrosophic set (SVNS) S in U is defined as:

$$S = \int_U \langle T(U), I(U), F(U)\rangle / u,\ u \in U,$$

when U is continuous and

$$S = \sum_{i=1}^{n} \langle T(U_i), I(U_i), F(U_i)\rangle / u_i,\ u_i \in U,$$

when U is discrete, where T, I, $F : U \rightarrow [0, 1]$ are T-MF, I-MF and F-MF, respectively.

Ali and Smarandache[18] conceptualized CNS and gave the basic operations in the following three definitions.

Definition 2.7. [18] Let a universe of discourse X, a CNS S in X is characterized by a T-MF $T_S(x)$, an I-MF $I_S(x)$, and a F-MF $F_S(x)$ that assign an element $x \in X$ a complex-valued grade of $T_S(x)$, $I_S(x)$, and $F_S(x)$ in S . By definition, the values $T_S(x), I_S(x), F_S(x)$ and their sum may all within the unit circle in the complex plane and are of the form, $T_S(x) = p_S(x).e^{j\mu_S(x)}$, $I_S(x) = q_S(x).e^{j\nu_S(x)}$ and $F_S(x) = r_S(x).e^{j\omega_S(x)}$, each of $p_S(x), q_S(x), r_S(x)$ and $\mu_S(x), \nu_S(x), \omega_S(x)$ are, respectively, real valued and $p_S(x), q_S(x), r_S(x) \in [0, 1]$ such that $0^{-} \leq P_S(x) + q_S(x) + r_S(x) \leq 3^{+}$.

Definition 2.8. [18] Let A and B be two CNSs, where A is characterized by a complex-valued T-MF $T_A(x) = p_A(x).e^{j\mu_A(x)}$, a complex-valued I-MF $I_A(x) = q_A(x).e^{j\nu_A(x)}$ and a complex-valued F-MF $F_A(x) = r_A(x).e^{j\omega_A(x)}$ and B is characterized by a complex-valued T-MF $T_B(x) = p_B(x).e^{j\mu_B(x)}$, a complex-valued I-MF $I_B(x) = q_B(x).e^{j\nu_B(x)}$ and a complex-valued F-MF $F_B(x) = r_B(x).e^{j\omega_B(x)}$.

We define complement of A, union and intersection of A and B as follows.

1. The complement of A , denoted as $c(A)$, is specified by functions:
   $T_{c(A)}(x) = p_{c(A)}(x).e^{j\mu_{c(A)}(x)} = r_A(x).e^{j(2\pi - \mu_A(x))}$,
   $I_{c(A)}(x) = q_{c(A)}(x).e^{j\nu_{c(A)}(x)} = (1 - q_A(x)).e^{j(2\pi - \nu_A(x))}$, and
   $F_{c(A)}(x) = r_{c(A)}(x).e^{j\omega_{c(A)}(x)} = p_A(x).e^{j(2\pi - \omega_A(x))}$.

2. A is said to be complex neutrosophic subset of B ($A \subseteq B$) if and only if the following conditions are satisfied:

   - $T_A(u) \leq T_B(u)$ such that $p_A(u) \leq p_B(u)$ and $\mu_A(u) \leq \mu_B(u)$,
   - $I_A(u) \geq I_B(u)$ such that $q_A(u) \geq q_B(u)$ and $\nu_A(u) \geq \nu_B(u)$,

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- $F_A(u) \geq F_B(u)$ such that $r_A(u) \geq r_B(u)$ and $\omega_A(u) \geq \omega_B(u)$.

3. The union(intersection) of A and B, denoted as $A \cup_N (\cap_N) B$ and T-MF $T_{A\cup(\cap)B}(x)$, I-MF $I_{A\cup(\cap)B}(x)$, and F-MF $F_{A\cup(\cap)B}(x)$ are defined as:
$T_{A\cup(\cap)B}(x) = [(p_A(x) \vee (\wedge) p_B(x))].e^{j(\mu_A(x)\vee(\wedge)\mu_B(x))}$,
$I_{A\cup(\cap)B}(x) = [(q_A(x) \wedge (\vee) q_B(x))].e^{j(\nu_A(x)\wedge(\vee)\nu_B(x))}$ and
$F_{A\cup(\cap)B}(x) = [(r_A(x) \wedge (\vee) r_B(x))].e^{j(\omega_A(x)\wedge(\vee)\omega_B(x))}$,

where $\vee$ = max and $\wedge$ = min .

Definition 2.9. [18] Let A_n be N CNSs on X, $(n = 1, 2, ..., N)$, and $T_{A_n}(x) = p_{A_n}(x).e^{j\mu_{A_n}(x)}$ be a complex-valued T-MF, $I_{A_n}(x) = q_{A_n}(x).e^{j\nu_{A_n}(x)}$ be a complex-valued I-MF and $F_{A_n}(x) = r_{A_n}(x).e^{j\omega_{A_n}(x)}$ be a complex-valued F-MF. The Cartesian products of A_n, denoted as $A_1 \times A_2 \times ... \times A_N$, are specified by the functions:

$$T_{A_1\times A_2\times...\times A_N}(x) = p_{A_1\times A_2\times...\times A_N}(x).e^{j.\mu_{A_1\times A_2\times...\times A_N}(x)}$$
$$= min(p_{A_1}(x_1), p_{A_2}(x_1), ..., p_{A_N}(x_N)).\ e^{jmin(\mu_{A_1}(x_1),\mu_{A_2}(x_2),...,\mu_{A_N}(x_N))},$$

$$I_{A_1\times A_2\times...\times A_N}(x) = q_{A_1\times A_2\times...\times A_N}(x).e^{j.\nu_{A_1\times A_2\times...\times A_N}(x)}$$
$$= max(q_{A_1}(x_1), q_{A_2}(x_1), ..., q_{A_N}(x_N)).\ e^{jmax(\nu_{A_1}(x_1),\nu_{A_2}(x_2),...,\nu_{A_N}(x_N))},$$

$$F_{A_1\times A_2\times...\times A_N}(x) = r_{A_1\times A_2\times...\times A_N}(x).e^{j.\omega_{A_1\times A_2\times...\times A_N}(x)}$$
$$= max(r_{A_1}(x_1), r_{A_2}(x_1), ..., r_{A_N}(x_N)).\ e^{jmax(\omega_{A_1}(x_1),\omega_{A_2}(x_2),...,\omega_{A_N}(x_N))}.$$

3 Q-Complex Neutrosophic Set

Definition 3.1. Let U and Q be two non empty sets. A Q-complex neutrosophic set (Q-CNS) K in U (or a Q-complex neutrosophic subset of U) is defined as follows.

$$K = \{< (u, q); T_K(u, q), I_K(u, q), F_K(u, q) >: u \in U, q \in Q\},$$

where $T_K(u, q), I_K(u, q)$ and $F_K(u, q)$ are complex-valued truth, indeterminate and false membership functions of the form $T_K(u, q) = P_K(u, q)e^{j\mu_k(u,q)}, I_K(u, q) = R_K(u, q)e^{j\nu_k(u,q)}, F_K(u, q) = S_K(u, q)$ $e^{j\omega_k(u,q)}$, each of $P_K(u, q), R_K(u, q), S_K(u, q)$ and $\mu_k(u, q), \nu_k(u, q), \omega_k(u, q)$ are, respectively, real valued and $P_K, R_K, S_K : U \times Q \longrightarrow [0, 1]$, such that $0 \leq P_K(u, q) + R_K(u, q) + S_K(u, q) \leq 3$.

In the Q-CNS above, the sum of the phase terms is naturally restricted to the interval $(0, 2\pi]$, due to the periodicity of the complex-valued membership functions, since the imaginary exponential function is periodic with period 2π.

We can write $\mu_k(u, q), \nu_k(u, q)$ and $\omega_k(u, q)$ as $\mu_k(u, q) = \delta\mu_k^{'}(u, q), \nu_k(u, q) = \delta\nu_k^{'}(u, q)$ and $\omega_k(u, q) = \delta\omega_k^{'}(u, q)$, where $\delta \in (0, 2\pi]$ is a scale factor that restricts the phase terms $\mu_k(u, q), \nu_k(u, q)$ and $\omega_k(u, q)$ to the interval $(0, 2\pi]$ and since, $\mu_k^{'}(u, q)$, $\nu_k^{'}(u, q)$ and $\omega_k^{'}(u, q)$ may represent the neutrosophic information with values belong to the interval $[0, 1]$, without loss of generality, in this research, we will consider δ equals to 2π, where $T_K(u, q), I_K(u, q)$ and $F_K(u, q)$ are complex-valued truth, indeterminate and false membership functions of the form $T_K(u, q) = P_K(u, q)e^{j2\pi\mu_k^{'}(u,q)}, I_K(u, q) = R_K(u, q)e^{j2\pi\nu_k^{'}(u,q)}, F_K(u, q) = S_K(u, q)$ $e^{j2\pi\omega_k^{'}(u,q)}$. By definition $\mu_k^{'}, \nu_k^{'}, \omega_k^{'} : U \times Q \longrightarrow [0, 1]$.

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It is to be noted that Q-CNS is essentially Q-NS characterized by an additional term called the phase term which is defined over the set of complex numbers. In order to represent any ordinary Q-NS as a Q-CNS, assume the ordinary Q-NS that is characterized by T-MF $\varphi_k(u,q)$, I-MF $\varpi_k(u,q)$ and F-MF $\chi_k(u,q)$ and set $\varphi_k(u,q) = P_K(u,q), \varpi_k(u,q) = R_K(u,q)$, $\chi_k(u,q) = S_K(u,q)$ and the phase terms $\mu_k(u,q)$, $\nu_k(u,q)$ and $\omega_k(u,q)$ equal to zero for all u and q. From this observation, it is concluded that the amplitude terms in the Q-CNS are equivalent to the membership functions in the ordinary Q-NS and play the same role of the membership functions in Q-NS. Further, without the phase terms, the Q-CNS is reduced to the ordinary Q-NS.

It should also be noted that the Q-CNS is a generalization of CNS, CIFS, CFS, SVNS, intuitionistic fuzzy set, fuzzy set, soft set and classical sets. This means that Q-CNS is an advance generalization to all the existence methods and due to this feature, the concept of Q-CNS is a distinguished and novel one.

The following example illustrates the above definition of the Q-CNS.

Example 3.2. Let $U = \{u_1, u_2, u_3\}$ and $Q = \{q_1, q_2\}$ be two non empty sets. Then, K be a Q-CNS in U as given below:

$K = \{< (u_1,q_1); 0.3e^{j2\pi(0.5)}, 0.9e^{j2\pi(0.6)}, 0.5e^{j2\pi(0.4)} >, < (u_1,q_2); 0.1e^{j2\pi(0.4)}, 0.7e^{j2\pi(0.2)}, 0.8e^{j2\pi(0.3)} >, < (u_2,q_1); 0.7e^{j2\pi(0.4)}, 0.8e^{j2\pi(0.9)}, 0.6e^{j2\pi(0.5)} >, < (u_2,q_2); 0.9e^{j2\pi(0.2)}, 0.5e^{j2\pi(0.3)}, 0.4e^{j2\pi(0.7)} >, < (u_3,q_1); 0.5e^{j2\pi(0.1)}, 0.7e^{j2\pi(0.3)}, 0.6e^{j2\pi(0.1)} >, < (u_3,q_2); 0.2e^{j2\pi(0.8)}, 0.4e^{j2\pi(0.3)}, 0.6e^{j2\pi(0.2)} >\}$.

In this part, we define the concept of multi Q-complex neutrosophic set.

Definition 3.3. Let L be a unit interval [0, 1], n be a positive integer. U and Q be two non empty sets. A multi Q-complex neutrosophic set $\hat{K}$ in U is a set of ordered sequences

$$\hat{K} = \{< (u,q); T_{K_i}(u,q), I_{K_i}(u,q), F_{K_i}(u,q) >: u \in U, q \in Q\},$$

$\forall i = 1,2,3,...,n,$ where $T_{K_i}(u,q) = P_{K_i}(u,q)e^{j2\pi\mu'_{K_i}(u,q)}, I_{K_i}(u,q) = R_{K_i}(u,q)e^{j2\pi\nu'_{K_i}(u,q)}, F_{K_i}(u,q) = S_{K_i}(u,q)e^{j2\pi\omega'_{K_i}(u,q)}$ and $P_{K_i}, R_{K_i}, S_{K_i}, \mu'_{K_i}, \nu'_{K_i}, \omega'_{K_i} : U \times Q \longrightarrow L^n$ and satisfy the condition $0 \leq P_{K_i} + R_{K_i} + S_{K_i} \leq 3, 0 \leq \mu'_{K_i} + \nu'_{K_i} + \omega'_{K_i} \leq 3, \forall i = 1,2,3,...,n,$

where n is called the dimension of $\hat{K}$.

The set of all multi Q-complex neutrosophic sets of dimension n in U is denoted by $\mathcal{M}^nQ - CNS(U)$.

Remark 3.4. In the above definition,

1. If $n = 1$, then the $\mathcal{M}^nQ - CNS$ is reduced to $Q - CNS$,
2. If $n = 1$, and $\mu'_K = \nu'_K = \omega'_K = 0$, then the $\mathcal{M}^nQ - CNS$ is reduced to Q-NS.

Now, we put forward the definition of a zero Q-CNS and the definition of a unit Q-CNS.

Definition 3.5. Let K be a Q-CNS in a non empty set U, then K is said to be zero Q-CNS, denoted by K_Φ, if $T_K(u,q) = 0, I_K(u,q) = 1, F_K(u,q) = 1$, $\forall u \in U$, $q \in Q$, and defined as:

$$K_\Phi = \{< (u,q); 0,1,1 >: u \in U, q \in Q\},$$

Example 3.6. Consider Example 3.2. Then the zero Q-CNS K is given as:

$K_\Phi = \{< (u_1,q_1); 0,1,1 >, < (u_1,q_2); 0,1,1 >, < (u_2,q_1); 0,1,1 >, < (u_2,q_2); 0,1,1 >, < (u_3,q_1); 0,1,1 >, < (u_3,q_2); 0,1,1 >\}$.

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Definition 3.7. Let K be a Q-CNS in a non empty set U, then K is said to be unit Q-CNS, denoted by K_ψ, if $T_K(u,q)=1, I_K(u,q)=0, F_K(u,q)=0, \forall u\in U, q\in Q$, and defined as:

$$K_\psi=\{<(u,q);1,0,0>: u\in U, q\in Q\},$$

Example 3.8. Consider Example 3.2. Then the unit Q-CNS K is given as:

$K_\psi=\{<(u_1,q_1);1,0,0>,<(u_1,q_2);1,0,0>,<(u_2,q_1);1,0,0>,<(u_2,q_2);1,0,0>,<(u_3,q_1);1,0,0>,<(u_3,q_2);1,0,0>\}$.

4 Basic operations on Q-CNS

Each of Q- complex neutrosophic complement, subset hood, union, intersection, product and Cartesian product is defined below along with some properties and illustrative examples.

The following is the definition of the complement of Q-CNS:

Definition 4.1. Let $K=\{<(u,q);T_K(u,q),I_K(u,q),F_K(u,q)>: u\in U, q\in Q\}$ be a Q-CNS in a non empty set U. Then, the complement of a Q-CNS K is denoted as $c(K)$ and is defined by

$c(K)=\{<(u,q);T^c_K(u,q),I^c_K(u,q),F^c_K(u,q)>: u\in U, q\in Q\}$, where,

$T^c_K(u,q)=c(P_K(u,q)e^{j2\pi\mu'_k(u,q)})=P^c_K(u,q)e^{j2\pi\mu'^c_k(u,q)}=S_K(u,q)e^{j2\pi(1-\mu'_k(u,q))}$,

$I^c_K(u,q)=c(R_K(u,q)e^{j2\pi\nu'_k(u,q)})=R^c_K(u,q)e^{j2\pi\nu'^c_k(u,q)}=(1-R_K(u,q))e^{j2\pi(1-\nu'_k(u,q))}$,

$F^c_K(u,q)=c(S_K(u,q)e^{j2\pi\omega'_k(u,q)})=S^c_K(u,q)e^{j2\pi\omega'^c_k(u,q)}=P_K(u,q)e^{j2\pi(1-\omega'_k(u,q))}$.

Example 4.2. Consider Example 3.2. By Definition 4.1, we get the complement of the Q-CNS K as:

$c(K)=\{<(u_1,q_1);0.5e^{j2\pi(0.5)},0.1e^{j2\pi(0.4)},0.3e^{j2\pi(0.6)}>,<(u_1,q_2);0.8e^{j2\pi(0.6)},0.3e^{j2\pi(0.8)},0.1e^{j2\pi(0.7)}>,<(u_2,q_1);0.6e^{j2\pi(0.6)},0.2e^{j2\pi(0.1)},0.7e^{j2\pi(0.5)}>,<(u_2,q_2);0.4e^{j2\pi(0.8)},0.5e^{j2\pi(0.7)},0.9e^{j2\pi(0.3)},<(u_3,q_1);0.6e^{j2\pi(0.9)},0.3e^{j2\pi(0.7)},0.5e^{j2\pi(0.9)}>,<(u_3,q_2);0.6e^{j2\pi(0.2)},0.6e^{j2\pi(0.7)},0.2e^{j2\pi(0.8)}>\}$.

Proposition 4.3. *Let K be a Q-CNS in a non empty set U. Then $c(c(K))=K$.*

Proof. From Definitions 4.1, we have,

$c(K)=\{<(u,q);T^c_K(u,q),I^c_K(u,q),F^c_K(u,q)>: u\in U, q\in Q\}$,

$=\{<(u,q);P^c_K(u,q)e^{j2\pi\mu'^c_k(u,q)},R^c_K(u,q)e^{j2\pi\nu'^c_k(u,q)},S^c_K(u,q)e^{j2\pi\omega'^c_k(u,q)}>: u\in U, q\in Q\}$,

$=\{<(u,q);S_K(u,q)e^{j2\pi(1-\mu'_k(u,q))},(1-R_K(u,q))e^{j2\pi(1-\nu'_k(u,q))},P_K(u,q)e^{j2\pi(1-\omega'_k(u,q))}>: u\in U, q\in Q\}$,

Thus,

$c(c(K))=\{<(u,q);S^c_K(u,q)e^{j2\pi(1-\mu'^c_k(u,q))},(1-R^c_K(u,q))e^{j2\pi(1-\nu'^c_k(u,q))},P^c_K(u,q)e^{j2\pi(1-\omega'^c_k(u,q))}>: u\in U, q\in Q\}$,

$=\{<(u,q);P_K(u,q)e^{j2\pi(1-(1-\mu'_k(u,q)))},(1-(1-R_K(u,q)))e^{j2\pi(1-(1-\nu'_k(u,q)))},S_K(u,q)e^{j2\pi(1-(1-\omega'_k(u,q)))}>: u\in U, q\in Q\}$,

$=\{<(u,q);P_K(u,q)e^{j2\pi(\mu'_k(u,q))},R_K(u,q)e^{j2\pi(\nu'_k(u,q))},S_K(u,q)e^{j2\pi(\omega'_k(u,q))}>: u\in U, q\in Q\}$,

$=\{<(u,q);T_K(u,q),I_K(u,q),F_K(u,q)>: u\in U, q\in Q\}$,

$=K$. This completes the proof. □

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Now, we put forward the definition of the subset of two Q-CNSs.

Definition 4.4. A Q-CNS A is contained in another Q-CNS B i.e. $A \subseteq B$, if and only if, $\forall u \in U$ and $q \in Q$, the following conditions are satisfied:

1. $T_A(u,q) \leq T_B(u,q)$ such that $P_A(u,q) \leq P_B(u,q)$ and $\mu'_A(u,q) \leq \mu'_B(u,q)$,
2. $I_A(u,q) \geq I_B(u,q)$ such that $R_A(u,q) \geq R_B(u,q)$ and $\nu'_A(u,q) \geq \nu'_B(u,q)$,
3. $F_A(u,q) \geq F_B(u,q)$ such that $S_A(u,q) \geq S_B(u,q)$ and $\omega'_A(u,q) \geq \omega'_B(u,q)$.

Definition 4.5. For two Q-CNSs A and B in a non empty set U, A is equal to B and it is denoted as A= B, if and only if $A \subseteq B$ and $B \subseteq A$.

We establish the definitions of the union and intersection of two Q-CNSs below.

Definition 4.6. The union of the two Q-CNSs A and B in a non empty set U is denoted as $A \cup_q B$ and the T-MF $T_{A\cup_q B}(u,q)$, the I-MF $I_{A\cup_q B}(u,q)$, and the F-MF $F_{A\cup_q B}(u,q)$ are defined as:

$$T_{A\cup_q B}(u,q) = [P_A(u,q) \vee P_B(u,q)].e^{j2\pi(\mu'_A(u,q)\vee\mu'_B(u,q))},$$

$$I_{A\cup_q B}(u,q) = [R_A(u,q) \wedge R_B(u,q)].e^{j2\pi(\nu'_A(u,q)\wedge\nu'_B(u,q))}, \text{ and}$$

$$F_{A\cup_q B}(u,q) = [S_A(u,q) \wedge S_B(u,q)].e^{j2\pi(\omega'_A(u,q)\wedge\omega'_B(u,q))}.$$

Where $\vee = max$ and $\wedge = min$.

Definition 4.7. The intersection of the two Q-CNSs A and B in a non empty set U is denoted as $A \cap_q B$ and the T-MF $T_{A\cap_q B}(u,q)$, the I-MF $I_{A\cap_q B}(u,q)$, and the F-MF $F_{A\cap_q B}(u,q)$ are defined as:

$$T_{A\cap_q B}(u,q) = [P_A(u,q) \wedge P_B(u,q)].e^{j2\pi(\mu'_A(u,q)\wedge\mu'_B(u,q))},$$

$$I_{A\cap_q B}(u,q) = [R_A(u,q) \vee R_B(u,q)].e^{j2\pi(\nu'_A(u,q)\vee\nu'_B(u,q))}, \text{ and}$$

$$F_{A\cap_q B}(u,q) = [S_A(u,q) \vee S_B(u,q)].e^{j2\pi(\omega'_A(u,q)\vee\omega'_B(u,q))}.$$

Where $\vee = max$ and $\wedge = min$.

Example 4.8. Let $U = \{u_1, u_2\}$ and $Q = \{q_1, q_2\}$ be two non empty sets. Let A and B be two Q-CNSs in U as shown below:

$$A = \{<(u_1,q_1); 0.4e^{j2\pi(0.8)}, 0.9e^{j2\pi(0.3)}, 0.7e^{j2\pi(0.9)}>, <(u_1,q_2); 0.2e^{j2\pi(0.7)}, 0.6e^{j2\pi(0.3)}, 0.5e^{j2\pi(0.3)}>, <(u_2,q_1); 0.9e^{j2\pi(0.4)}, 0.3e^{j2\pi(0.5)}, 0.6e^{j2\pi(0.4)}>, <(u_2,q_2); 0.3e^{j2\pi(0.5)}, 0.4e^{j2\pi(0.8)}, 0.1e^{j2\pi(0.9)}>\},$$

$$B = \{<(u_1,q_1); 0.9e^{j2\pi(0.7)}, 0.1e^{j2\pi(0.4)}, 0.5e^{j2\pi(0.8)}>, <(u_1,q_2); 0.6e^{j2\pi(0.8)}, 0.2e^{j2\pi(0.4)}, 0.3e^{j2\pi(0.5)}>, <(u_2,q_1); 0.7e^{j2\pi(0.9)}, 0.4e^{j2\pi(0.8)}, 0.2e^{j2\pi(0.7)}>, <(u_2,q_2); 0.5e^{j2\pi(0.4)}, 0.6e^{j2\pi(0.4)}, 0.7e^{j2\pi(0.1)}>\}.$$

Then,

$$A\cup_q B = \{<(u_1,q_1); 0.9e^{j2\pi(0.8)}, 0.1e^{j2\pi(0.3)}, 0.5e^{j2\pi(0.8)}>, <(u_1,q_2); 0.6e^{j2\pi(0.8)}, 0.2e^{j2\pi(0.3)}, 0.3e^{j2\pi(0.3)}>, <(u_2,q_1); 0.9e^{j2\pi(0.9)}, 0.3e^{j2\pi(0.5)}, 0.2e^{j2\pi(0.4)}>, <(u_2,q_2); 0.5e^{j2\pi(0.5)}, 0.4e^{j2\pi(0.4)}, 0.1e^{j2\pi(0.1)}>\},$$

$$A\cap_q B = \{<(u_1,q_1); 0.4e^{j2\pi(0.7)}, 0.9e^{j2\pi(0.4)}, 0.7e^{j2\pi(0.9)}>, <(u_1,q_2); 0.2e^{j2\pi(0.7)}, 0.6e^{j2\pi(0.4)}, 0.5e^{j2\pi(0.5)}>, <(u_2,q_1); 0.7e^{j2\pi(0.4)}, 0.4e^{j2\pi(0.8)}, 0.6e^{j2\pi(0.7)}>, <(u_2,q_2); 0.3e^{j2\pi(0.4)}, 0.6e^{j2\pi(0.8)}, 0.7e^{j2\pi(0.9)}>\}.$$

Theorem 4.9. *If A and B are two Q-CNSs in a non empty set U , then the union $A \cup_q B$ is the smallest Q-CNS which contains both these two sets.*

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Proof. The proof can be easily stated according to Definitions 4.4 and 4.6. □

Theorem 4.10. *If A and B are two Q-CNSs in a non empty set U , then the intersection $A \cap_q B$ is the largest Q-CNS, which is contained in both of these two sets.*

Proof. The proof can be easily stated according to Definitions 4.4 and 4.7. □

Proposition 4.11. *Let A, B and C be three Q-CNSs in a non empty set U. Then,*

1. $A \cap A = A, \quad A \cup A = A,$
2. $A \cap B = B \cap A, \quad A \cup B = B \cup A,$
3. $(A \cap B) \cap C = A \cap (B \cap C), \quad (A \cup B) \cup C = A \cup (B \cup C),$

Proof. The proof is straightforward by Definitions 4.6 and 4.7. □

Proposition 4.12. *Let A, B and C be three Q-CNSs in a non empty set U. Then,*

1. $(A \cup B) \cap C = (A \cap C) \cup (B \cap C),$
2. $(A \cap B) \cup C = (A \cup C) \cap (B \cup C).$

Proof. Here we only prove part 1. Let$A$, B and C be three Q-CNSs in a non empty set U and $T_A(u,q), I_A(u,q), F_A(u,q), T_B(u,q), I_B(u,q), F_B(u,q)$, and $T_C(u,q), I_C(u,q), F_C(u,q)$, respectively, be their complex-valued T-MFs, complex-valued I-MFs, and complex-valued F-MFs. Then, we have

$$T_{(A\cup B)\cap C}(u,q) = P_{(A\cup B)\cap C}(u,q).e^{j2\pi\mu'_{(A\cup B)\cap C}(u,q)},$$

$$= \min(P_{A\cup B}(u,q), P_C(u,q)).e^{j2\pi\min(\mu'_{A\cup B}(u,q),\mu'_C(u,q))},$$

$$= \min(\max(P_A(u,q), P_B(u,q)), P_C(u,q)).e^{j2\pi(\min(\max(\mu'_A(u,q),\mu'_B(u,q)),\mu'_C(u,q))},$$

$$= \max(\min(P_A(u,q), P_C(u,q)), \min(P_B(u,q), P_C(u,q))).e^{j2\pi(\max(\min(\mu'_A(u,q),\mu'_C(u,q)),\min(\mu'_B(u,q),\mu'_C(u,q)))},$$

$$= \max(P_{A\cap C}(u,q), P_{B\cap C}(u,q)).e^{j2\pi\max(\mu'_{A\cap C}(u,q),\mu'_{B\cap C}(u,q))},$$

$$= P_{(A\cap C)\cup(B\cap C)}(u,q).e^{j2\pi\mu'_{(A\cap C)\cup(B\cap C)}(u,q)},$$

$$= T_{(A\cap C)\cup(B\cap C)}(u,q).$$

Similarly, on the same lines, we can show it for $I_{(A\cup B)\cap C}(u,q)$ and $F_{(A\cup B)\cap C}(u,q)$, respectively.

□

Proposition 4.13. *Let A, B be two Q-CNSs in a non empty set U. Then,*

1. $(A \cup B) \cap A = A,$
2. $(A \cap B) \cup A = A.$

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Proof. We prove it for part 1. Let$A$ and B be two Q-CNSs in a non empty set U and $T_A(u,q), I_A(u,q), F_A(u, q)$ and $T_B(u,q), I_B(u,q), F_B(u,q)$, respectively, be their complex-valued T-MFs, complex-valued I-MFs, and complex-valued F-MFs. Then, we have

$$T_{(A\cup B)\cap A}(u,q) = P_{(A\cup B)\cap A}(u,q).e^{j2\pi\mu'_{(A\cup B)\cap A}(u,q)},$$

$$= \min(P_{A\cup B}(u,q), P_A(u,q)).e^{j2\pi\min(\mu'_{A\cup B}(u,q),\mu'_A(u,q))},$$

$$= \min(\max(P_A(u,q), P_B(u,q)), P_A(u,q)).e^{j2\pi(\min(\max(\mu'_A(u,q),\mu'_B(u,q)),\mu'_A(u,q))},$$

$$= P_A(u,q)e^{j2\pi\mu'_A(u,q)},$$

$$= T_A(u,q).$$

Similarly, on the same lines, we can show it for $I_{(A\cup B)\cap A}(u,q)$ and $F_{(A\cup B)\cap A}(u,q)$, respectively.

□

Proposition 4.14. *Let A and B be two Q-CNSs in a non empty set U. Then,*

1. $c(A\cup B) = c(A)\cap c(B)$,
2. $c(A\cap B) = c(A)\cup c(B)$.

Proof. (1) Assume that $A\cup B = D$ and $\forall u\in U$ and $q\in Q$,

$$T_D(u,q) = [P_A(u,q)\vee P_B(u,q)].e^{j2\pi(\mu'_A(u,q)\vee\mu'_B(u,q))},$$

Since $A\cup B = D$, then we have $c(A\cup B) = c(D)$. Hence $\forall u\in U$ and $q\in Q$,

$$T^c_D(u,q) = [P_A(u,q)\vee P_B(u,q)]^c.e^{j2\pi(\mu'_A(u,q)\vee\mu'_B(u,q))^c},$$

$$= [S_A(u,q)\wedge S_B(u,q)].e^{j2\pi((1-\mu'_A(u,q))\wedge(1-\mu'_B(u,q)))},$$

Suppose that $c(A)\cap c(B) = E$. Hence $\forall u\in U$ and $q\in Q$,

$$T_E(u,q) = [P^c_A(u,q)\wedge P^c_B(u,q)].e^{j2\pi(\mu'^c_A(u,q)\wedge\mu'^c_B(u,q))},$$

$$= [S_A(u,q)\wedge S_B(u,q)].e^{j2\pi((1-\mu'_A(u,q))\wedge(1-\mu'_B(u,q)))}.$$

Therefore, $c(D)$ and E are the same operators, which implies, $T_{c(A\cup B)}(u,q) = T_{c(A)\cap c(B)}(u,q)$, $\forall u\in U$ and $q\in Q$.

Similarly, on the same lines, we can show it also holds for the identity and falsity terms. Thus it follows that $c(A\cup B) = c(A)\cap c(B)$ and this completes the proof.

(2) The proof is similar to that of (1).

□

In the following, we will define the Q-complex neutrosophic product of two Q-CNSs.

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Definition 4.15. Let A and B be two Q-CNSs in a non empty set U, and $T_A(u,q) = P_A(u,q)e^{j2\pi\mu'_A(u,q)}$, $I_A(u,q) = R_A(u,q)e^{j2\pi\nu'_A(u,q)}$, $F_A(u,q) = S_A(u,q)e^{j2\pi\omega'_A(u,q)}$ and $T_B(u,q) = P_B(u,q)e^{j2\pi\mu'_B(u,q)}$, $I_B(u,q) = R_B(u,q)e^{j2\pi\nu'_B(u,q)}$, $F_B(u,q) = S_B(u,q)e^{j2\pi\omega'_B(u,q)}$, respectively, be their complex-valued T-MFs, complex-valued I-MFs, and complex-valued F-MFs. The Q-complex neutrosophic product of A and B denoted as $A \circ B$ and is specified by the functions,

$$T_{A\circ B}(u,q) = P_{A\circ B}(u,q).e^{j2\pi(\mu'_{A\circ B}(u,q))} = (P_A(u,q).P_B(u,q)).e^{j2\pi(\mu'_A(u,q).\mu'_B(u,q))},$$

$$I_{A\circ B}(u,q) = R_{A\circ B}(u,q).e^{j2\pi(\nu'_{A\circ B}(u,q))} = (R_A(u,q).R_B(u,q)).e^{j2\pi(\nu'_A(u,q).\nu'_B(u,q))},$$

$$F_{A\circ B}(u,q) = S_{A\circ B}(u,q).e^{j2\pi(\omega'_{A\circ B}(u,q))} = (S_A(u,q).S_B(u,q)).e^{j2\pi(\omega'_A(u,q).\omega'_B(u,q))}.$$

Example 4.16. Consider Example 4.8. By Definition 4.15, we get the a Q-complex neutrosophic product of A and B as:

$A\circ B = \{< (u_1, q_1); 0.36e^{j2\pi(0.56)}, 0.09e^{j2\pi(0.12)}, 0.35e^{j2\pi(0.72)} >, < (u_1, q_2); 0.12e^{j2\pi(0.56)}, 0.12e^{j2\pi(0.12)}, 0.15e^{j2\pi(0.15)} >, < (u_2, q_1); 0.63e^{j2\pi(0.36)}, 0.12e^{j2\pi(0.40)}, 0.12e^{j2\pi(0.28)} >, < (u_2, q_2); 0.15e^{j2\pi(0.20)}, 0.24 e^{j2\pi(0.32)}, 0.07e^{j2\pi(0.09)} >\}$.

The following is the definition of the Cartesian product between two Q-CNSs:

Definition 4.17. Let A and B be two Q-CNSs in U and V, respectively, and $T_A(u,q) = P_A(u,q)e^{j2\pi\mu'_A(u,q)}$, $I_A(u,q) = R_A(u,q)e^{j2\pi\nu'_A(u,q)}$, $F_A(u,q) = S_A(u,q)e^{j2\pi\omega'_A(u,q)}$ and $T_B(v,q^*) = P_B(v,q^*)e^{j2\pi\mu'_B(v,q^*)}$, $I_B(v,q^*) = R_B(v,q^*)e^{j2\pi\nu'_B(v,q^*)}$, $F_B(v,q^*) = S_B(v,q^*)e^{j2\pi\omega'_B(v,q^*)}$, respectively, be their complex-valued T-MFs, complex-valued I-MFs, and complex-valued F-MFs. The Cartesian product of A and B, denoted by $A \times B$ and defined as

$A \times B = \{< (m,n); T_{A\times B}(m,n), I_{A\times B}(m,n), F_{A\times B}(m,n) >: m \in U \times Q$ and $n \in V \times Q^*\}$, where $m = (u,q),\ \ n = (v,q^*)$ and $\forall m \in U \times Q$ and $n \in V \times Q^*$,

$$T_{A\times B}(m,n) = \min(P_A(m), P_B(n)).e^{j2\pi \min(\mu'_A(m),\mu'_B(n))},$$

$$I_{A\times B}(m,n) = \max(R_A(m), R_B(n)).e^{j2\pi \min(\nu'_A(m),\nu'_B(n))},$$

$$F_{A\times B}(m,n) = \max(S_A(m), S_B(n)).e^{j2\pi \max(\omega'_A(m),\omega'_B(n))}.$$

5 Conclusion

The notion of Q-CNS has been defined as a generalisation of Q-NS and CNS. This new model has the ability to deal with uncertain, incomplete and indeterminate data in two-dimensional space. The basic operations such as complement, equality, subset, union, intersection, Q-complex neutrosophic product and Cartesian product have been discussed. The properties of these operations are also verified with supporting proofs. Q-CNS seems to be a promising new concept, paving the way toward numerous possibilities for future research. Different algebraic structures of Q-CNS can be investigated. This concept can be also extended to Q-complex neutrosophic soft set, Q- complex plithogenic set and Q-complex neutrosophic hypersoft set and so on.

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