Algebraic Approach to Neutrosophic Confidence Intervals

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Abstract

In this paper, confidence intervals for neutrosophic statistical populations were driven in many cases. Firstly, confidence intervals for one neutrosophic normal population parameters were driven including population's mean which was driven under the assumption that variance is known, then it was driven under the assumption that variance is unknown and estimated based on the sample. Confidence interval for the neutrosophic variance was also driven based on sample's estimates. Secondly, confidence intervals for two neutrosophic normal populations were driven including confidence intervals for means differences when variance are known or unknown, also confidence intervals for variances ratio for two populations were driven. All theorems and calculations were done using the AH-Isometry. Suitable numerical examples were presented and solved successfully.

Keywords: AH-Isometry; Neutrosophic Statistics; Neutrosophic Probability; Confidence Intervals; Estimation Theory; Statistical Inference.

1. Introduction

Neutrosophic real set generated by $R \cup \{I\}$ where $I^2 = I, 0 \cdot I = 0$ is a new set of numbers presented by Florentin Smarandache as a generalization of the set of reals, also it can be considered as a generalization of the complex set of numbers when it is generated by $C \cup \{I\}$. Elements of neutrosophic set have the form $\{a + bI , I^2 = I\}$ where a, b are real or complex numbers. *[1]*

M. Abobala and A. Hatip proved that $R(I) = R \cup \{I\}$ has an isometric image in $R \times R$ and defined an isometry called AH-Isometry which transfers problems from $R(I)$ to $R \times R$, and hence it is an isometry, then it has an inverse T^{-1} transfers back the problems from $R \times R$ to $R(I)$, So, mathematical problems and mathematical structures can be built and solved with robust mathematical steps and with no mathematical lags. *[2]*

Neutrosophic geometry has been defined and built well in *[2]*, many concepts have been presented using AH-Isometry including concepts of neutrosophic circles, neutrosophic lines, etc.

In the last few years, neutrosophic theory became very important and applicable in many fields of science including artificial intelligence, numbers theory, game theory, engineering, medicine, abstract algebra, etc.[3-20]

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In neutrosophic probability field of studies, many research papers have been done and published including distributions theory, statistics, hypothesis testing, statistical inference, queueing theory, etc., all these studies were done by assuming that data or parameters of the probability distributions are neutrosophic number built on one of two forms, first one is neutrosophic interval numbers and second one is single valued neutrosophic numbers [10,21-30,37]. Rare studies have been done assuming that these parameters are of the form $N = a + bI \,; I^2 = I$ which is the classical set of neutrosophic numbers.[36]

In this paper we will provide abstract forms of neutrosophic confidence intervals using an algebraic approach in many cases of neutrosophic statistical populations including one population and two populations assuming that these populations follow neutrosophic normal distribution with classical neutrosophic parameters (not interval neutrosophic numbers or single valued neutrosophic numbers). We will provide many solved examples.

2. Preliminaries

Definition 2.1 [26,31,32]

Let U be a universe and A be a subset of U, we call $\{x, \mu_A(x), \delta_A(x), \nu_A(x)\ , x \in U\}$ a single valued neutrosophic number where $\mu_A(x) \in [0,1]$ is membership degree, $\delta_A(x) \ \in [0,1]$is indeterminacy degree and $\nu_A(x) \in [0,1]$ is non-membership degree and:

$$0 \leq \mu_A(x) + \delta_A(x) + \nu_A(x) \leq 3$$

Definition 2.2 [33]

Neutrosophic classical numbers are numbers have the form $N = a + bI$ where I represents indeterminacy and it is often an interval like $I \in [0,0.01]$ or a set like $I \in \{0.1,0.2\}$

Definition 2.3 [1]

Neutrosophic literal numbers are numbers have the form $N = a + bI$ where I is an algebraic element satisfies $I^2 = I$ and we call $R(I) = \{a + bI \,; a, b \in R, I^2 = I\}$ the neutrosophic field of reals.

Definition 2.4 [2]

Let $R(I) = \{a + bI \,; a, b \in R, I^2 = I\}$ be the neutrosophic field of reals. The one-dimensional isometry (AH-Isometry) is defined as follows:

$$T: R(I) \to R \times R$$
$$T(a + bI) = (a, a + b)$$

Remark:

T is an algebraic isomorphism between two rings and it has the following properties:

1) T is bijective.
2) T preserves addition and multiplication, i.e.:

$$T[(a + bI) + (c + dI)] = T(a + bI) + T(c + dI)$$
$$T[(a + bI) \cdot (c + dI)] = T(a + bI) \cdot T(c + dI)$$

3) T is invertible by:

$$T^{-1}: R \times R \to R(I)$$
$$T^{-1}(a, b) = a + (b - a)I$$

4) T preserves distances, i.e.:

$$T\left(\left\|\overrightarrow{AB}\right\|\right) = \left\|T(\overrightarrow{AB})\right\|$$

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Definition 2.5 [38]

Let $f: R(I) \to R(I); f = f(X)$ and $X = x + yI \in R(I)$ the f is called a neutrosophic real function with one neutrosophic variable.

3. Results and Discussion

In this section we will prove some formulas of neutrosophic confidence intervals for one neutrosophic normal distribution's parameters the for two neutrosophic normal distribution's parameters. We will depend on definition 2.3 of neutrosophic numbers which differs from previous studies which were depending on definitions 2.1 and 2.2 of neutrosophic numbers.[27,34-35]

3.1 Neutrosophic Confidence Intervals for One Population

Theorem 3.1

Let $X_{N1}, X_{N2}, \dots, X_{Nn}$ be a neutrosophic sample where $X_{Ni} = X_i + Y_i I$ are random variables driven from the neutrosophic normal distribution $N(\mu_N, \sigma_N^2)$ where σ_N^2 is known, then:

$$\mu_N \in \left[\bar{X} + \bar{Y}I - Z_{1-\frac{\alpha}{2}}\left(\frac{\sigma_x}{\sqrt{n}} + \left[\frac{\sigma_{x+y}}{\sqrt{n}} - \frac{\sigma_x}{\sqrt{n}}\right]I\right), \bar{X} + \bar{Y}I + Z_{1-\frac{\alpha}{2}}\left(\frac{\sigma_x}{\sqrt{n}} + \left[\frac{\sigma_{x+y}}{\sqrt{n}} - \frac{\sigma_x}{\sqrt{n}}\right]I\right)\right]$$

Proof:

Using the one-dimensional AH-Isometry:

$$T[\bar{X}_N] = T[\bar{X} + \bar{Y}I] = (\bar{X}, \bar{X} + \bar{Y}) \sim \left(N\left(\mu_1, \frac{\sigma_x^2}{n}\right), N\left(\mu_1 + \mu_2, \frac{\sigma_{x+y}^2}{n}\right)\right)$$

$$T\left[\frac{\bar{X}_N - \mu_N}{\frac{\sigma_N}{\sqrt{n}}}\right] = T\left[\frac{(\bar{X} + \bar{Y}I) - (\mu_1 + \mu_2 I)}{\frac{(\sigma_x + [\sigma_{x+y} - \sigma_x]I)}{\sqrt{n}}}\right] = \frac{(\bar{X}, \bar{X} + \bar{Y}) - (\mu_1, \mu_1 + \mu_2)}{\frac{(\sigma_x, \sigma_{x+y})}{(\sqrt{n}, \sqrt{n})}} = \left(\frac{\bar{X} - \mu_1}{\frac{\sigma_x}{\sqrt{n}}}, \frac{(\bar{X} + \bar{Y}) - (\mu_1 + \mu_2)}{\frac{\sigma_{x+y}}{\sqrt{n}}}\right) \sim (N(0,1), N(0,1))$$

$$\left(p\left\{-Z_{1-\frac{\alpha}{2}} < \frac{\bar{X} - \mu_1}{\frac{\sigma_x}{\sqrt{n}}} < Z_{1-\frac{\alpha}{2}}\right\} = 1 - \alpha, p\left\{-Z_{1-\frac{\alpha}{2}} < \frac{(\bar{X} + \bar{Y}) - (\mu_1 + \mu_2)}{\frac{\sigma_{x+y}}{\sqrt{n}}} < Z_{1-\frac{\alpha}{2}}\right\} = 1 - \alpha\right)$$

$$\left(p\left\{\bar{X} - Z_{1-\frac{\alpha}{2}}\frac{\sigma_x}{\sqrt{n}} < \mu_1 < \bar{X} + Z_{1-\frac{\alpha}{2}}\frac{\sigma_x}{\sqrt{n}}\right\} = 1 - \alpha, p\left\{(\bar{X} + \bar{Y}) - Z_{1-\frac{\alpha}{2}}\frac{\sigma_{x+y}}{\sqrt{n}} < \mu_1 + \mu_2 < Z_{1-\frac{\alpha}{2}} + Z_{1-\frac{\alpha}{2}}\frac{\sigma_{x+y}}{\sqrt{n}}\right\} = 1 - \alpha\right)$$

$$(\mu_1, \mu_1 + \mu_2) \in \left(\left[\bar{X} - Z_{1-\frac{\alpha}{2}}\frac{\sigma_x}{\sqrt{n}}, \bar{X} + Z_{1-\frac{\alpha}{2}}\frac{\sigma_x}{\sqrt{n}}\right], \left[(\bar{X} + \bar{Y}) - Z_{1-\frac{\alpha}{2}}\frac{\sigma_{x+y}}{\sqrt{n}}, (\bar{X} + \bar{Y}) + Z_{1-\frac{\alpha}{2}}\frac{\sigma_{x+y}}{\sqrt{n}}\right]\right)$$

Now taking the inverse isometry T^{-1}:

$$\mu_N \in \left[\bar{X} - Z_{1-\frac{\alpha}{2}}\frac{\sigma_x}{\sqrt{n}}, \bar{X} + Z_{1-\frac{\alpha}{2}}\frac{\sigma_x}{\sqrt{n}}\right] + \left[\left[(\bar{X} + \bar{Y}) - Z_{1-\frac{\alpha}{2}}\frac{\sigma_{x+y}}{\sqrt{n}}, (\bar{X} + \bar{Y}) + Z_{1-\frac{\alpha}{2}}\frac{\sigma_{x+y}}{\sqrt{n}}\right] - \left[\bar{X} - Z_{1-\frac{\alpha}{2}}\frac{\sigma_x}{\sqrt{n}}, \bar{X} + Z_{1-\frac{\alpha}{2}}\frac{\sigma_x}{\sqrt{n}}\right]\right]I$$

$$\mu_N \in \left[\bar{X} + \bar{Y}I - Z_{1-\frac{\alpha}{2}}\left(\frac{\sigma_x}{\sqrt{n}} + \left[\frac{\sigma_{x+y}}{\sqrt{n}} - \frac{\sigma_x}{\sqrt{n}}\right]I\right), \bar{X} + \bar{Y}I + Z_{1-\frac{\alpha}{2}}\left(\frac{\sigma_x}{\sqrt{n}} + \left[\frac{\sigma_{x+y}}{\sqrt{n}} - \frac{\sigma_x}{\sqrt{n}}\right]I\right)\right]$$

Example 3.1:

Let $-2 - 8I, -2 + 4I, -7 - 3I, -10, -2 - 4I, 9 - 9I, 2 + 8I, -5 + I, 4I, -9 - 10I$ be a neutrosophic random sample drawn from $N(\mu_N, \sigma_N^2)$ where $\alpha = 0.05$ and $\sigma_N = 1 + 1I$ and $n = 10$, $Z_{1-\frac{\alpha}{2}} = Z_{1-\frac{0.05}{2}} = Z_{0.975} = 1.96$.

Using theorem 3.1 we get:

$$\mu_N \in \left[\bar{X} + \bar{Y}I - Z_{1-\frac{\alpha}{2}}\left(\frac{\sigma_x}{\sqrt{n}} + \left[\frac{\sigma_{x+y}}{\sqrt{n}} - \frac{\sigma_x}{\sqrt{n}}\right]I\right), \bar{X} + \bar{Y}I + Z_{1-\frac{\alpha}{2}}\left(\frac{\sigma_x}{\sqrt{n}} + \left[\frac{\sigma_{x+y}}{\sqrt{n}} - \frac{\sigma_x}{\sqrt{n}}\right]I\right)\right]$$

$$\mu_N \in \left[-2.6 - 1.7I - 1.96\left(\frac{1}{\sqrt{10}} + \frac{1}{\sqrt{10}}I\right), -2.6 - 1.7I + 1.96\left(\frac{1}{\sqrt{10}} + \frac{1}{\sqrt{10}}I\right)\right]$$

$$\mu_N \in [-3.22 - 2.32I, -1.98 - 1.08I]$$

Theorem 3.2

Let $X_{N1}, X_{N2}, \dots, X_{Nn}$ be a neutrosophic sample where $X_{Ni} = X_i + Y_i I$ are random variables driven from the neutrosophic normal distribution $N(\mu_N, \sigma_N^2)$ where σ_N^2 is unknown, then:

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$$\mu_N \in \left[\bar{X} + \bar{Y}I - t_{1-\frac{\alpha}{2}}(n-1)\left(\frac{S_x}{\sqrt{n}} + \left[\frac{S_{x+y}}{\sqrt{n}} - \frac{S_x}{\sqrt{n}}\right]I\right), \bar{X} + \bar{Y}I + t_{1-\frac{\alpha}{2}}(n-1)\left(\frac{S_x}{\sqrt{n}} + \left[\frac{S_{x+y}}{\sqrt{n}} - \frac{S_x}{\sqrt{n}}\right]I\right)\right]$$

Proof:

Let $Z_N = \frac{\bar{X}_N - \mu_N}{\frac{\sigma_N}{\sqrt{n}}}$ and $\chi_N = \frac{(n-1)S_N^2}{\sigma_N^2} = \frac{(n-1)\left(S_x^2 + [S_{x+y}^2 - S_x^2]I\right)}{\sigma_x^2 + \left[\sigma_{x+y}^2 - \sigma_x^2\right]I}$, then taking the AH-Isometry yields to:

$$T\left[\frac{\bar{X}_N - \mu_N}{\frac{\sigma_N}{\sqrt{n}}}\right] = T\left[\frac{(\bar{X} + \bar{Y}I) - (\mu_1 + \mu_2 I)}{\frac{\sigma_x + [\sigma_{x+y} - \sigma_x]I}{\sqrt{n}}}\right] = \frac{(\bar{X}, \bar{X} + \bar{Y}) - (\mu_1, \mu_1 + \mu_2)}{\frac{(\sigma_x, \sigma_{x+y})}{(\sqrt{n}, \sqrt{n})}} = \left(\frac{\bar{X} - \mu_1}{\frac{\sigma_x}{\sqrt{n}}}, \frac{(\bar{X} + \bar{Y}) - (\mu_1 + \mu_2)}{\frac{\sigma_{x+y}}{\sqrt{n}}}\right) \sim \left(N(0,1), N(0,1)\right)$$

$$T\left[\frac{(n-1)S_N^2}{\sigma_N^2}\right] = T\left[\frac{(n-1)\left(S_x^2 + [S_{x+y}^2 - S_x^2]I\right)}{\sigma_x^2 + \left[\sigma_{x+y}^2 - \sigma_x^2\right]I}\right] = \left(\frac{(n-1, n-1)\left(S_x^2, S_{x+y}^2\right)}{\left(\sigma_x^2, \sigma_{x+y}^2\right)}\right) = \left(\frac{(n-1)S_x^2}{\sigma_x^2}, \frac{(n-1)S_{x+y}^2}{\sigma_{x+y}^2}\right) \sim \left(\chi^2(n-1), \chi^2(n-1)\right)$$

$$T\left[\frac{Z_N}{\sqrt{\frac{\chi_N}{n-1}}}\right] = T\left[\frac{\frac{\bar{X}_N - \mu_N}{\frac{\sigma_N}{\sqrt{n}}}}{\sqrt{\frac{\frac{(n-1)S_N^2}{\sigma_N^2}}{(n-1)}}}\right] = \left(\frac{\frac{(\bar{X}, \bar{X} + \bar{Y}) - (\mu_1, \mu_1 + \mu_2)}{\frac{\left(\sigma_x^2, \sigma_{x+y}^2\right)}{(\sqrt{n}, \sqrt{n})}}}{\sqrt{\frac{\frac{(n-1)\left(S_x^2, S_{x+y}^2\right)}{\left(\sigma_x^2, \sigma_{x+y}^2\right)}}{(n-1, n-1)}}}\right) = \left(\frac{\frac{\bar{X} - \mu_1}{\frac{\sigma_x}{\sqrt{n}}}}{\sqrt{\frac{\frac{(n-1)S_x^2}{\sigma_x^2}}{n-1}}}, \frac{\frac{(\bar{X} + \bar{Y}) - (\mu_1 + \mu_2)}{\frac{\sigma_{x+y}}{\sqrt{n}}}}{\sqrt{\frac{\frac{(n-1)S_{x+y}^2}{\sigma_{x+y}^2}}{n-1}}}\right)$$

$$= \left(\frac{\bar{X} - \mu_1}{\frac{S_x}{\sqrt{n}}}, \frac{(\bar{X} + \bar{Y}) - (\mu_1 + \mu_2)}{\frac{S_{x+y}}{\sqrt{n}}}\right) \sim \left(t(n-1), t(n-1)\right)$$

So:

$$\left(p\left\{-t_{1-\frac{\alpha}{2}}(n-1) < \frac{\bar{X} - \mu_1}{\frac{S_x}{\sqrt{n}}} < t_{1-\frac{\alpha}{2}}(n-1)\right\} = 1 - \alpha, p\left\{-t_{1-\frac{\alpha}{2}}(n-1) < \frac{(\bar{X} + \bar{Y}) - (\mu_1 + \mu_2)}{\frac{S_{x+y}}{\sqrt{n}}} < t_{1-\frac{\alpha}{2}}(n-1)\right\} = 1 - \alpha\right)$$

$$\left(p\left\{\bar{X} - t_{1-\frac{\alpha}{2}}(n-1)\frac{S_x}{\sqrt{n}} < \mu_1 < \bar{X} + t_{1-\frac{\alpha}{2}}(n-1)\frac{S_x}{\sqrt{n}}\right\} = 1 - \alpha, p\left\{(\bar{X} + \bar{Y}) - t_{1-\frac{\alpha}{2}}(n-1)\frac{S_{x+y}}{\sqrt{n}} < \mu_1 + \mu_2 < (\bar{X} + \bar{Y}) + t_{1-\frac{\alpha}{2}}(n-1)\frac{S_{x+y}}{\sqrt{n}}\right\} = 1 - \alpha\right)$$

$$(\mu_1, \mu_1 + \mu_2) \in \left(\left[\bar{X} - t_{1-\frac{\alpha}{2}}(n-1)\frac{S_x}{\sqrt{n}}, \bar{X} + t_{1-\frac{\alpha}{2}}(n-1)\frac{S_x}{\sqrt{n}}\right], \left[(\bar{X} + \bar{Y}) - t_{1-\frac{\alpha}{2}}(n-1)\frac{S_{x+y}}{\sqrt{n}}, (\bar{X} + \bar{Y}) + t_{1-\frac{\alpha}{2}}(n-1)\frac{S_{x+y}}{\sqrt{n}}\right]\right)$$

Now taking the inverse isometry T^{-1}:

$$\mu_N \in \left[\bar{X} - t_{1-\frac{\alpha}{2}}(n-1)\frac{S_x}{\sqrt{n}}, \bar{X} + t_{1-\frac{\alpha}{2}}(n-1)\frac{S_x}{\sqrt{n}}\right] + \left[\left[(\bar{X} + \bar{Y}) - t_{1-\frac{\alpha}{2}}(n-1)\frac{S_{x+y}}{\sqrt{n}}, (\bar{X} + \bar{Y}) + t_{1-\frac{\alpha}{2}}(n-1)\frac{S_{x+y}}{\sqrt{n}}\right] - \left[\bar{X} - t_{1-\frac{\alpha}{2}}(n-1)\frac{S_x}{\sqrt{n}}, \bar{X} + t_{1-\frac{\alpha}{2}}(n-1)\frac{S_x}{\sqrt{n}}\right]\right]I$$

$$\mu_N \in \left[\bar{X} - t_{1-\frac{\alpha}{2}}(n-1)\frac{S_x}{\sqrt{n}} + \left(\bar{Y} - t_{1-\frac{\alpha}{2}}(n-1)\left[\frac{S_{x+y}}{\sqrt{n}} - \frac{S_x}{\sqrt{n}}\right]\right)I, \bar{X} + t_{1-\frac{\alpha}{2}}(n_1-1)\frac{S_x}{\sqrt{n}} + \left(\bar{Y} + t_{1-\frac{\alpha}{2}}(n-1)\left[\frac{S_{x+y}}{\sqrt{n}} - \frac{S_x}{\sqrt{n}}\right]\right)I\right]$$

i.e.:

$$\mu_N \in \left[\bar{X} + \bar{Y}I - t_{1-\frac{\alpha}{2}}(n-1)\left(\frac{S_x}{\sqrt{n}} + \left[\frac{S_{x+y}}{\sqrt{n}} - \frac{S_x}{\sqrt{n}}\right]I\right), \bar{X} + \bar{Y}I + t_{1-\frac{\alpha}{2}}(n-1)\left(\frac{S_x}{\sqrt{n}} + \left[\frac{S_{x+y}}{\sqrt{n}} - \frac{S_x}{\sqrt{n}}\right]I\right)\right]$$

Remark:

If $n > 30$ can be used $Z_{1-\frac{\alpha}{2}}$ instead of $t_{1-\frac{\alpha}{2}}(n-1)$

Example 3.2:

Let $4 - 2I, -2, 1 + 1I, -1 + 2I, 3 + 3I$ be a neutrosophic random sample drawn from $N(\mu_N, \sigma_N^2)$ where $\alpha = 0.05$

$$t_{1-\frac{\alpha}{2}}(n-1) = t_{1-\frac{0.05}{2}}(5-1) = t_{0.975}(4) = 2.776$$

Using theorem 3.2:

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$$\mu_N \in \left[\bar{X} + \bar{Y}I - t_{1-\frac{\alpha}{2}}(n-1)\left(\frac{S_x}{\sqrt{n}} + \left[\frac{S_{x+y}}{\sqrt{n}} - \frac{S_x}{\sqrt{n}}\right]I\right), \bar{X} + \bar{Y}I + t_{1-\frac{\alpha}{2}}(n-1)\left(\frac{S_x}{\sqrt{n}} + \left[\frac{S_{x+y}}{\sqrt{n}} - \frac{S_x}{\sqrt{n}}\right]I\right)\right]$$

$$\mu_N \in \left[1 + 0.8I - 2.776\left(\frac{2.55}{\sqrt{5}} + \left[\frac{2.86}{\sqrt{5}} - \frac{2.55}{\sqrt{5}}\right]I\right), 1 + 0.8I + 2.776\left(\frac{2.55}{\sqrt{5}} + \left[\frac{2.86}{\sqrt{5}} - \frac{2.55}{\sqrt{5}}\right]I\right)\right]$$

$$\mu_N \in [-2.17 - 0.42I, 4.17 + 1.18I]$$

Theorem 3.3

Let $X_{N1}, X_{N2}, \dots, X_{Nn}$ be a neutrosophic sample where $X_{Ni} = X_i + Y_i I$ are random variables driven from the neutrosophic normal distribution $N(\mu_N, \sigma_N^2)$ where σ_N^2 is known, then:

$$\sigma_N^2 \in \left[\frac{(n-1)}{\chi^2_{1-\frac{\alpha}{2}}(n-1)}\left(S_x^2 + [S_{x+y}^2 - S_x^2]I\right), \frac{(n-1)}{\chi^2_{\frac{\alpha}{2}}(n-1)}\left(S_x^2 + [S_{x+y}^2 - S_x^2]I\right)\right]$$

Proof:

$$T\left[\frac{(n-1)S_N^2}{\sigma_N^2}\right] = T\left[\frac{(n-1)\left(S_x^2+[S_{x+y}^2-S_x^2]I\right)}{\sigma_x^2+\left[\sigma_{x+y}^2-\sigma_x^2\right]I}\right] = \frac{(n-1,n-1)\left(S_x^2,S_{x+y}^2\right)}{\left(\sigma_x^2,\sigma_{x+y}^2\right)} = \left(\frac{(n-1)S_x^2}{\sigma_x^2}, \frac{(n-1)S_{x+y}^2}{\sigma_{x+y}^2}\right) \sim \left(\chi^2(n-1), \chi^2(n-1)\right)$$

$$\left(p\left\{\chi^2_{\frac{\alpha}{2}}(n-1) < \frac{(n-1)S_x^2}{\sigma_x^2} < \chi^2_{1-\frac{\alpha}{2}}(n-1)\right\} = 1-\alpha, p\left\{\chi^2_{\frac{\alpha}{2}}(n-1) < \frac{(n-1)S_{x+y}^2}{\sigma_{x+y}^2} < \chi^2_{1-\frac{\alpha}{2}}(n-1)\right\} = 1-\alpha\right)$$

$$\left(p\left\{\frac{(n-1)S_x^2}{\chi^2_{1-\frac{\alpha}{2}}(n-1)} < \sigma_x^2 < \frac{(n-1)S_x^2}{\chi^2_{\frac{\alpha}{2}}(n-1)}\right\} = 1-\alpha, p\left\{\frac{(n-1)S_{x+y}^2}{\chi^2_{1-\frac{\alpha}{2}}(n-1)} < \sigma_{x+y}^2 < \frac{(n-1)S_{x+y}^2}{\chi^2_{\frac{\alpha}{2}}(n-1)}\right\} = 1-\alpha\right)$$

$$\left(\sigma_x^2, \sigma_{x+y}^2\right) \in \left(\left[\frac{(n-1)S_x^2}{\chi^2_{1-\frac{\alpha}{2}}(n-1)}, \frac{(n-1)S_x^2}{\chi^2_{\frac{\alpha}{2}}(n-1)}\right], \left[\frac{(n-1)S_{x+y}^2}{\chi^2_{1-\frac{\alpha}{2}}(n-1)}, \frac{(n-1)S_{x+y}^2}{\chi^2_{\frac{\alpha}{2}}(n-1)}\right]\right)$$

Now taking the inverse isometry T^{-1}:

$$\sigma_N^2 \in \left[\frac{(n-1)S_x^2}{\chi^2_{1-\frac{\alpha}{2}}(n-1)}, \frac{(n-1)S_x^2}{\chi^2_{\frac{\alpha}{2}}(n-1)}\right] + \left[\left[\frac{(n-1)S_{x+y}^2}{\chi^2_{1-\frac{\alpha}{2}}(n-1)}, \frac{(n-1)S_{x+y}^2}{\chi^2_{\frac{\alpha}{2}}(n-1)}\right] - \left[\frac{(n-1)S_x^2}{\chi^2_{1-\frac{\alpha}{2}}(n-1)}, \frac{(n-1)S_x^2}{\chi^2_{\frac{\alpha}{2}}(n-1)}\right]\right]I$$

$$\sigma_N^2 \in \left[\frac{(n-1)S_x^2}{\chi^2_{1-\frac{\alpha}{2}}(n-1)} + \left(\frac{(n-1)S_{x+y}^2}{\chi^2_{1-\frac{\alpha}{2}}(n-1)} - \frac{(n-1)S_x^2}{\chi^2_{1-\frac{\alpha}{2}}(n-1)}\right)I, \frac{(n-1)S_x^2}{\chi^2_{\frac{\alpha}{2}}(n-1)} + \left(\frac{(n-1)S_{x+y}^2}{\chi^2_{\frac{\alpha}{2}}(n-1)} - \frac{(n-1)S_x^2}{\chi^2_{\frac{\alpha}{2}}(n-1)}\right)I\right]$$

$$\sigma_N^2 \in \left[\frac{(n-1)S_x^2}{\chi^2_{1-\frac{\alpha}{2}}(n-1)} + \frac{(n-1)\left(S_{x+y}^2 - S_x^2\right)}{\chi^2_{1-\frac{\alpha}{2}}(n-1)}I, \frac{(n-1)S_x^2}{\chi^2_{\frac{\alpha}{2}}(n-1)} + \frac{(n-1)\left(S_{x+y}^2 - S_x^2\right)}{\chi^2_{\frac{\alpha}{2}}(n-1)}I\right]$$

$$\sigma_N^2 \in \left[\frac{(n-1)}{\chi^2_{1-\frac{\alpha}{2}}(n-1)}\left(S_x^2 + [S_{x+y}^2 - S_x^2]I\right), \frac{(n-1)}{\chi^2_{\frac{\alpha}{2}}(n-1)}\left(S_x^2 + [S_{x+y}^2 - S_x^2]I\right)I\right]$$

Example 3.3:

Let $4-2I, -2, 1+1I, -1+2I, 3+3I$ be a neutrosophic random sample drawn from $N(\mu_N, \sigma_N^2)$ where $\alpha = 0.05, \chi^2_{1-\frac{\alpha}{2}}(n-1) = \chi^2_{1-\frac{0.05}{2}}(5-1) = \chi^2_{0.975}(4) = 11.1433$ and $\chi^2_{\frac{\alpha}{2}}(n-1) = \chi^2_{\frac{0.05}{2}}(5-1) = \chi^2_{0.025}(4) = 0.4844$

Using theorem 3.3:

$$\sigma_N^2 \in \left[\frac{(n-1)}{\chi^2_{1-\frac{\alpha}{2}}(n-1)}\left(S_x^2 + [S_{x+y}^2 - S_x^2]I\right), \frac{(n-1)}{\chi^2_{\frac{\alpha}{2}}(n-1)}\left(S_x^2 + [S_{x+y}^2 - S_x^2]I\right)I\right]$$

$$\sigma_N^2 \in \left[\frac{(5-1)}{11.1433}(6.5 + [8.18 - 6.5]I), \frac{(5-1)}{0.4844}(6.5 + [8.18 - 6.5]I)I\right]$$

$$\sigma_N^2 \in [2.33 + 0.60I, 53.67 + 13.87I]$$

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4. Neutrosophic Confidence Intervals for Two Populations

Theorem 4.1:

Let $X_{11N}, X_{12N}, \dots, X_{1nN}$ be a neutrosophic sample driven from $X_{1N} \sim N(\mu_{1N}, \sigma_{1N}^2)$ and $X_{21N}, X_{22N}, \dots, X_{2nN}$ be another sample driven from $X_{2N} \sim N(\mu_{2N}, \sigma_{2N}^2)$ where $X_{1N} = X_1 + Y_1 I, X_{2N} = X_2 + Y_2 I,\ \mu_{1N} = \mu_{11} + \mu_{12} I,\ \mu_{2N} = \mu_{21} + \mu_{22} I, \sigma_{1N}^2 = \sigma_{x_1}^2 + [\sigma_{x_1+y_1}^2 - \sigma_{x_1}^2] I, \sigma_{1N} = \sigma_{x_1} + [\sigma_{x_1+y_1} - \sigma_{x_1}] I, \sigma_{2N}^2 = \sigma_{x_2}^2 + [\sigma_{x_2+y_2}^2 - \sigma_{x_2}^2] I, \sigma_{2N} = \sigma_{x_2} + [\sigma_{x_2+y_2} - \sigma_{x_2}] I$ then the confidence interval of $\mu_{1N} - \mu_{2N}$ will be:

$$\mu_{1N} - \mu_{2N} \in \left[(\bar{X}_1 - \bar{X}_2) + (\bar{Y}_1 - \bar{Y}_2)I - Z_{1-\frac{\alpha}{2}}\left(\sqrt{\frac{\sigma_{x_1}^2}{n_1} + \frac{\sigma_{x_2}^2}{n_2}} + \left[\sqrt{\frac{\sigma_{x_1+y_1}^2}{n_1} + \frac{\sigma_{x_2+y_2}^2}{n_2}} - \sqrt{\frac{\sigma_{x_1}^2}{n_1} + \frac{\sigma_{x_2}^2}{n_2}}\right] I\right), (\bar{X}_1 - \bar{X}_2) + (\bar{Y}_1 - \bar{Y}_2)I + Z_{1-\frac{\alpha}{2}}\left(\sqrt{\frac{\sigma_{x_1}^2}{n_1} + \frac{\sigma_{x_2}^2}{n_2}} + \left[\sqrt{\frac{\sigma_{x_1+y_1}^2}{n_1} + \frac{\sigma_{x_2+y_2}^2}{n_2}} - \sqrt{\frac{\sigma_{x_1}^2}{n_1} + \frac{\sigma_{x_2}^2}{n_2}}\right] I\right)\right]$$

Proof:

$$T[\bar{X}_{1N}] = T[\bar{X}_1 + \bar{Y}_1 I] = (\bar{X}_1, \bar{X}_1 + \bar{Y}_1) \sim \left(N\left(\mu_{11}, \frac{\sigma_{x_1}^2}{\sqrt{n_1}}\right), N\left(\mu_{11} + \mu_{12}, \frac{\sigma_{x_1+y_1}^2}{\sqrt{n_1}}\right)\right)$$

$$T[\bar{X}_{2N}] = T[\bar{X}_2 + \bar{Y}_2 I] = (\bar{X}_2, \bar{X}_2 + \bar{Y}_2) \sim \left(N\left(\mu_{21}, \frac{\sigma_{x_2}^2}{\sqrt{n_2}}\right), N\left(\mu_{21} + \mu_{22}, \frac{\sigma_{x_2+y_2}^2}{\sqrt{n_2}}\right)\right)$$

$$T\left[\frac{\bar{X}_{1N} - \mu_{1N}}{\frac{\sigma_{1N}}{\sqrt{n_1}}}\right] = \left(\frac{\bar{X}_1 - \mu_{11}}{\frac{\sigma_{x_1}}{\sqrt{n_1}}}, \frac{(\bar{X}_1 + \bar{Y}_1) - (\mu_{11} + \mu_{12})}{\frac{\sigma_{x_1+y_1}}{\sqrt{n_1}}}\right) \sim (N(0,1), N(0,1))$$

$$T\left[\frac{\bar{X}_{2N} - \mu_{2N}}{\frac{\sigma_{2N}}{\sqrt{n_2}}}\right] = \left(\frac{\bar{X}_2 - \mu_{21}}{\frac{\sigma_{x_2}}{\sqrt{n_2}}}, \frac{(\bar{X}_2 + \bar{Y}_2) - (\mu_{21} + \mu_{22})}{\frac{\sigma_{x_2+y_2}}{\sqrt{n_2}}}\right) \sim (N(0,1), N(0,1))$$

$$T[\bar{X}_{1N} - \bar{X}_{2N}] = T[(\bar{X}_1 + \bar{Y}_1 I) - (\bar{X}_2 + \bar{Y}_2 I)] = T[\bar{X}_1 + \bar{Y}_1 I] - T[\bar{X}_2 + \bar{Y}_2 I] = (\bar{X}_1, \bar{X}_1 + \bar{Y}_1) - (\bar{X}_2, \bar{X}_2 + \bar{Y}_2)$$
$$= (\bar{X}_1 - \bar{X}_2, (\bar{X}_1 + \bar{Y}_1) - (\bar{X}_2 + \bar{Y}_2)) \sim \left(N\left(\mu_{11} - \mu_{21}, \frac{\sigma_{x_1}^2}{n_1} + \frac{\sigma_{x_2}^2}{n_2}\right), N\left((\mu_{11} + \mu_{12}) - (\mu_{21} + \mu_{22}), \frac{\sigma_{x_1+y_1}^2}{n_1} + \frac{\sigma_{x_2+y_2}^2}{n_2}\right)\right)$$

$$\left(\frac{(\bar{X}_1 - \bar{X}_2) - (\mu_{11} - \mu_{21})}{\sqrt{\frac{\sigma_{x_1}^2}{n_1} + \frac{\sigma_{x_2}^2}{n_2}}}, \frac{((\bar{X}_1 + \bar{Y}_1) - (\bar{X}_2 + \bar{Y}_2)) - ((\mu_{11} + \mu_{12}) - (\mu_{21} + \mu_{22}))}{\sqrt{\frac{\sigma_{x_1+y_1}^2}{n_1} + \frac{\sigma_{x_2+y_2}^2}{n_2}}}\right) \sim (N(0,1), N(0,1))$$

$$L: \left(p\left\{-Z_{1-\frac{\alpha}{2}} < \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_{11} - \mu_{21})}{\sqrt{\frac{\sigma_{x_1}^2}{n_1} + \frac{\sigma_{x_2}^2}{n_2}}} < Z_{1-\frac{\alpha}{2}}\right\} = 1 - \alpha\right)$$

$$p\left\{(\bar{X}_1 - \bar{X}_2) - Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\sigma_{x_1}^2}{n_1} + \frac{\sigma_{x_2}^2}{n_2}} < (\mu_{11} - \mu_{21}) < (\bar{X}_1 - \bar{X}_2) + Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\sigma_{x_1}^2}{n_1} + \frac{\sigma_{x_2}^2}{n_2}}\right\} = 1 - \alpha$$

$$(\mu_{11} - \mu_{21}) \in \left[(\bar{X}_1 - \bar{X}_2) - Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\sigma_{x_1}^2}{n_1} + \frac{\sigma_{x_2}^2}{n_2}}, (\bar{X}_1 - \bar{X}_2) + Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\sigma_{x_1}^2}{n_1} + \frac{\sigma_{x_2}^2}{n_2}}\right]$$

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$$R: \left(p\left\{ -Z_{1-\frac{\alpha}{2}} < \frac{\big((\bar{X}_1+\bar{Y}_1)-(\bar{X}_2+\bar{Y}_2)\big)-\big((\mu_{11}+\mu_{12})-(\mu_{21}+\mu_{22})\big)}{\sqrt{\frac{\sigma^2_{x_1+y_1}}{n_1}+\frac{\sigma^2_{x_2+y_2}}{n_2}}} < Z_{1-\frac{\alpha}{2}} \right\} = 1-\alpha \right)$$

$$p\Bigg\{(\bar{X}_1+\bar{Y}_1)-(\bar{X}_2+\bar{Y}_2)-Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\sigma^2_{x_1+y_1}}{n_1}+\frac{\sigma^2_{x_2+y_2}}{n_2}} < (\mu_{11}+\mu_{12})-(\mu_{21}+\mu_{22})$$

$$< (\bar{X}_1+\bar{Y}_1)-(\bar{X}_2+\bar{Y}_2)+Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\sigma^2_{x_1+y_1}}{n_1}+\frac{\sigma^2_{x_2+y_2}}{n_2}}\Bigg\} = 1-\alpha$$

$$(\mu_{11}+\mu_{12})-(\mu_{21}+\mu_{22}) \in \Bigg[(\bar{X}_1+\bar{Y}_1)-(\bar{X}_2+\bar{Y}_2)-Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\sigma^2_{x_1+y_1}}{n_1}+\frac{\sigma^2_{x_2+y_2}}{n_2}}, (\bar{X}_1+\bar{Y}_1)-(\bar{X}_2+\bar{Y}_2)+$$

$$Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\sigma^2_{x_1+y_1}}{n_1}+\frac{\sigma^2_{x_2+y_2}}{n_2}}\Bigg]$$

$$\big((\mu_{11}-\mu_{21}),(\mu_{11}+\mu_{12})-(\mu_{21}+\mu_{22})\big) \in (L,R)$$

$$\mu_{1N}-\mu_{2N} \in L+[R-L]I$$

$$\mu_{1N}-\mu_{2N} \in \Bigg[(\bar{X}_1-\bar{X}_2)-Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\sigma^2_{x_1}}{n_1}+\frac{\sigma^2_{x_2}}{n_2}}+\Bigg((\bar{Y}_1-\bar{Y}_2)-Z_{1-\frac{\alpha}{2}}\Bigg(\sqrt{\frac{\sigma^2_{x_1+y_1}}{n_1}+\frac{\sigma^2_{x_2+y_2}}{n_2}}-\sqrt{\frac{\sigma^2_{x_1}}{n_1}+\frac{\sigma^2_{x_2}}{n_2}}\Bigg)\Bigg)I, (\bar{X}_1-$$

$$\bar{X}_2)+Z_{1-\frac{\alpha}{2}}\sqrt{\frac{\sigma^2_{x_1}}{n_1}+\frac{\sigma^2_{x_2}}{n_2}}+\Bigg((\bar{Y}_1-\bar{Y}_2)+Z_{1-\frac{\alpha}{2}}\Bigg(\sqrt{\frac{\sigma^2_{x_1+y_1}}{n_1}+\frac{\sigma^2_{x_2+y_2}}{n_2}}-\sqrt{\frac{\sigma^2_{x_1}}{n_1}+\frac{\sigma^2_{x_2}}{n_2}}\Bigg)\Bigg)I\Bigg]$$

$$\mu_{1N}-\mu_{2N} \in \Bigg[(\bar{X}_1-\bar{X}_2)+(\bar{Y}_1-\bar{Y}_2)I-Z_{1-\frac{\alpha}{2}}\Bigg(\sqrt{\frac{\sigma^2_{x_1}}{n_1}+\frac{\sigma^2_{x_2}}{n_2}}+\Bigg[\sqrt{\frac{\sigma^2_{x_1+y_1}}{n_1}+\frac{\sigma^2_{x_2+y_2}}{n_2}}-\sqrt{\frac{\sigma^2_{x_1}}{n_1}+\frac{\sigma^2_{x_2}}{n_2}}\Bigg]I\Bigg), (\bar{X}_1-\bar{X}_2)$$

$$+(\bar{Y}_1-\bar{Y}_2)I+Z_{1-\frac{\alpha}{2}}\Bigg(\sqrt{\frac{\sigma^2_{x_1}}{n_1}+\frac{\sigma^2_{x_2}}{n_2}}+\Bigg[\sqrt{\frac{\sigma^2_{x_1+y_1}}{n_1}+\frac{\sigma^2_{x_2+y_2}}{n_2}}-\sqrt{\frac{\sigma^2_{x_1}}{n_1}+\frac{\sigma^2_{x_2}}{n_2}}\Bigg]I\Bigg)\Bigg]$$

Example 4.1:

$$X_{1N}=x_1+y_1I, X_{2N}=x_2+y_2I$$

$$X_{1N}: 3.75+1.25I, 2.75+0.25I, 2.5+1.5I, 2.25+1.75I, 2.15+0.85I, 3.15-1.65I, 3-0.5I, 3.7-1.7I, 3.75$$
$$-0.75I, 3+1I$$

$$X_{2N}: 3.5-0.5I, 3.5+0.5I, 2.75+1.25I, 3.75-0.75I, 2.5-0.5I, 2.75+1.25I, 3.75+0.25I, 3.5-0.5I$$

$$\sigma^2_{1N}=1+2I \Rightarrow \sigma^2_{x_1}=1, \sigma^2_{x_1+y_1}=3$$

$$, \sigma^2_{2N}=1.5+2.5I \Rightarrow \sigma^2_{x_2}=1.5, \sigma^2_{x_2+y_2}=4$$

Solve:

Using theorem 4.1:

$$\mu_{1N}-\mu_{2N} \in \Bigg[(\bar{X}_1-\bar{X}_2)+(\bar{Y}_1-\bar{Y}_2)I-Z_{1-\frac{\alpha}{2}}\Bigg(\sqrt{\frac{\sigma^2_{x_1}}{n_1}+\frac{\sigma^2_{x_2}}{n_2}}+\Bigg[\sqrt{\frac{\sigma^2_{x_1+y_1}}{n_1}+\frac{\sigma^2_{x_2+y_2}}{n_2}}-\sqrt{\frac{\sigma^2_{x_1}}{n_1}+\frac{\sigma^2_{x_2}}{n_2}}\Bigg]I\Bigg), (\bar{X}_1-$$

$$\bar{X}_2)+(\bar{Y}_1-\bar{Y}_2)I+Z_{1-\frac{\alpha}{2}}\Bigg(\sqrt{\frac{\sigma^2_{x_1}}{n_1}+\frac{\sigma^2_{x_2}}{n_2}}+\Bigg[\sqrt{\frac{\sigma^2_{x_1+y_1}}{n_1}+\frac{\sigma^2_{x_2+y_2}}{n_2}}-\sqrt{\frac{\sigma^2_{x_1}}{n_1}+\frac{\sigma^2_{x_2}}{n_2}}\Bigg]I\Bigg)\Bigg]$$

$$\mu_{1N}-\mu_{2N} \in \Bigg[(3-3.25)+(0.2-0.125)I-1.96\Bigg(\sqrt{\frac{1}{10}+\frac{1.5}{8}}+\Bigg[\sqrt{\frac{3}{10}+\frac{4}{8}}-\sqrt{\frac{1}{10}+\frac{1.5}{8}}\Bigg]I\Bigg), (3-3.25)+$$

$$(0.2-0.125)I+1.96\Bigg(\sqrt{\frac{1}{10}+\frac{1.5}{8}}+\Bigg[\sqrt{\frac{3}{10}+\frac{4}{8}}-\sqrt{\frac{1}{10}+\frac{1.5}{8}}\Bigg]I\Bigg)\Bigg]$$

$$\mu_{1N}-\mu_{2N} \in [-1.30-0.63I, 0.80+0.78I]$$

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Theorem 4.2:

Let $X_{11N}, X_{12N}, \dots, X_{1nN}$ be a neutrosophic sample driven from $X_{1N} \sim N(\mu_{1N}, \sigma_{1N}^2)$ and $X_{21N}, X_{22N}, \dots, X_{2nN}$ be another sample driven from $X_{2N} \sim N(\mu_{2N}, \sigma_{2N}^2)$ where $X_{1N} = X_1 + Y_1 I, X_{2N} = X_2 + Y_2 I,\ \mu_{1N} = \mu_{11} + \mu_{12} I,\ \mu_{2N} = \mu_{21} + \mu_{22} I$ and with unknown variances, then the confidence interval of $\mu_{1N} - \mu_{2N}$ will be:

If $n_1, n_2 < 30$:

$$\mu_{1N} - \mu_{2N} \in \Bigg[(\bar{X}_1 - \bar{X}_2) + (\bar{Y}_1 - \bar{Y}_2)I$$

$$- t(n_1 + n_1 - 2)\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}\left(S_{p_{x_1,x_2}} + \left[S_{p_{x_1+y_1,x_2+y_2}} - S_{p_{x_1,x_2}}\right] I\right), (\bar{X}_1 - \bar{X}_2) + (\bar{Y}_1 - \bar{Y}_2)I$$

$$+ t(n_1 + n_1 - 2)\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}\left(S_{p_{x_1,x_2}} + \left[S_{p_{x_1+y_1,x_2+y_2}} - S_{p_{x_1,x_2}}\right] I\right)\Bigg]$$

If $n_1, n_2 \geq 30$:

$$\mu_{1N} - \mu_{2N} \in \left[(\bar{X}_1 - \bar{X}_2) + (\bar{Y}_1 - \bar{Y}_2)I - Z_{1-\frac{\alpha}{2}}\left(\sqrt{\frac{S_{x_1}^2}{n_1} + \frac{S_{x_2}^2}{n_2}} + \left[\sqrt{\frac{S_{x_1+y_1}^2}{n_1} + \frac{S_{x_2+y_2}^2}{n_2}} - \sqrt{\frac{S_{x_1}^2}{n_1} + \frac{S_{x_2}^2}{n_2}}\right] I\right), (\bar{X}_1 - \bar{X}_2)\right.$$

$$\left. + (\bar{Y}_1 - \bar{Y}_2)I - Z_{1-\frac{\alpha}{2}}\left(\sqrt{\frac{S_{x_1}^2}{n_1} + \frac{S_{x_2}^2}{n_2}} + \left[\sqrt{\frac{S_{x_1+y_1}^2}{n_1} + \frac{S_{x_2+y_2}^2}{n_2}} - \sqrt{\frac{S_{x_1}^2}{n_1} + \frac{S_{x_2}^2}{n_2}}\right] I\right)\right]$$

Proof:

If $n_1, n_2 < 30$:

$$T\left[S_{pN}^2\right] = T\left[\frac{(n_1 - 1)S_{x_{1N}}^2 + (n_2 - 1)S_{x_{2N}}^2}{n_1 + n_2 - 2}\right]$$

$$= T\left[\frac{\left((n_1-1)\left(S_{x_1}^2 + [S_{x_1+y_1}^2 - S_{x_1}^2]I\right) + (n_2-1)\left(S_{x_2}^2 + [S_{x_2+y_2}^2 - S_{x_2}^2]I\right)\right)}{n_1+n_2-2}\right] = \left(\frac{(n_1-1)S_{x_1}^2 + (n_2-1)S_{x_2}^2}{n_1+n_2-2}, \frac{(n_1-1)S_{x_1+y_1}^2 + (n_2-1)S_{x_2+y_2}^2}{n_1+n_2-2}\right)$$

$$S_{pN}^2 = \frac{(n_1-1)S_{x_1}^2 + (n_2-1)S_{x_2}^2}{n_1+n_2-2} + \left[\frac{(n_1-1)S_{x_1+y_1}^2 + (n_2-1)S_{x_2+y_2}^2}{n_1+n_2-2} - \frac{(n_1-1)S_{x_1}^2 + (n_2-1)S_{x_2}^2}{n_1+n_2-2}\right] I$$

$$= S_{p_{x_1,x_2}}^2 + \left[S_{p_{x_1+y_1,x_2+y_2}}^2 - S_{p_{x_1,x_2}}^2\right] I$$

$$S_{pN} = \sqrt{\frac{(n_1-1)S_{x_1}^2 + (n_2-1)S_{x_2}^2}{n_1+n_2-2}} + \left[\sqrt{\frac{(n_1-1)S_{x_1+y_1}^2 + (n_2-1)S_{x_2+y_2}^2}{n_1+n_2-2}} - \sqrt{\frac{(n_1-1)S_{x_1}^2 + (n_2-1)S_{x_2}^2}{n_1+n_2-2}}\right] I$$

$$= S_{p_{x_1,x_2}} + \left[S_{p_{x_1+y_1,x_2+y_2}} - S_{p_{x_1,x_2}}\right] I$$

$$T\left[\frac{(\bar{X}_{1N} - \bar{X}_{2N}) - (\mu_{1N} - \mu_{2N})}{S_{pN}\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}\right] = T\left[\frac{\left((\bar{X}_1 + \bar{Y}_1 I) - (\bar{X}_2 + \bar{Y}_2 I)\right) - \left((\mu_{11} + \mu_{12} I) - (\mu_{21} + \mu_{22} I)\right)}{\left(S_{p_{x_1,x_2}} + \left[S_{p_{x_1+y_1,x_2+y_2}} - S_{p_{x_1,x_2}}\right] I\right)\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}\right]$$

$$= \left(\frac{(\bar{X}_1 - \bar{X}_2) - (\mu_{11} - \mu_{21})}{S_{p_{x_1,x_2}}\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}, \frac{\left((\bar{X}_1 + \bar{Y}_1) - (\bar{X}_2 + \bar{Y}_2)\right) - \left((\mu_{11} + \mu_{12}) - (\mu_{21} + \mu_{22})\right)}{S_{p_{x_1+y_1,x_2+y_2}}\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}\right) \sim \left(t(n_1 + n_1 - 2), t(n_1 + n_1 - 2)\right)$$

$$L: P\left\{-t(n_1 + n_2 - 2) < \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_{11} - \mu_{21})}{S_{p_{x_1,x_2}}\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} < t(n_1 + n_2 - 2)\right\} = 1 - \alpha$$

$$L: P\left\{(\bar{X}_1 - \bar{X}_2) - t(n_1 + n_2 - 2)S_{p_{x_1,x_2}}\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} < \mu_{11} - \mu_{21} < (\bar{X}_1 - \bar{X}_2) + t(n_1 + n_2 - 2)S_{p_{x_1,x_2}}\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}\right\} = 1 - \alpha$$

$$\mu_{11} - \mu_{21} \in \left[(\bar{X}_1 - \bar{X}_2) - t(n_1 + n_1 - 2)S_{p_{x_1,x_2}}\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}, (\bar{X}_1 - \bar{X}_2) + t(n_1 + n_1 - 2)S_{p_{x_1,x_2}}\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}\right]$$

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$$R:P\left\{-t(n_1+n_1-2)<\frac{\left((\bar{X}_1+\bar{Y}_1)-(\bar{X}_2+\bar{Y}_2)\right)-\left((\mu_{11}+\mu_{12})-(\mu_{21}+\mu_{22})\right)}{S_{p_{x_1+y_1,x_2+y_2}}\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}<t(n_1+n_1-2)\right\}=1-\alpha$$

$$R:P\left\{\left((\bar{X}_1+\bar{Y}_1)-(\bar{X}_2+\bar{Y}_2)\right)-t(n_1+n_1-2)S_{p_{x_1+y_1,x_2+y_2}}\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}<(\mu_{11}+\mu_{12})-(\mu_{21}+\mu_{22})< \left((\bar{X}_1+\bar{Y}_1)-(\bar{X}_2+\bar{Y}_2)\right)+t(n_1+n_1-2)S_{p_{x_1+y_1,x_2+y_2}}\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}\right\}=1-\alpha$$

$$(\mu_{11}+\mu_{12})-(\mu_{21}+\mu_{22})\in\left[\left((\bar{X}_1+\bar{Y}_1)-(\bar{X}_2+\bar{Y}_2)\right)-t(n_1+n_1-2)S_{p_{x_1+y_1,x_2+y_2}}\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)},\left((\bar{X}_1+\bar{Y}_1)-(\bar{X}_2+\bar{Y}_2)\right)+t(n_1+n_1-2)S_{p_{x_1+y_1,x_2+y_2}}\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}\right]$$

$$\left((\mu_{11}-\mu_{21}),(\mu_{11}+\mu_{12})-(\mu_{21}+\mu_{22})\right)\in(L,R)$$

$$\mu_{1N}-\mu_{2N}\in L+[R-L]I$$

$$\mu_{1N}-\mu_{2N}\in\left[(\bar{X}_1-\bar{X}_2)-t(n_1+n_1-2)S_{p_{x_1,x_2}}\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)},(\bar{X}_1-\bar{X}_2)+t(n_1+n_1-2)S_{p_{x_1,x_2}}\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}\right]+\left[\left[\left((\bar{X}_1+\bar{Y}_1)-(\bar{X}_2+\bar{Y}_2)\right)-t(n_1+n_1-2)S_{p_{x_1+y_1,x_2+y_2}}\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)},\left((\bar{X}_1+\bar{Y}_1)-(\bar{X}_2+\bar{Y}_2)\right)+t(n_1+n_1-2)S_{p_{x_1+y_1,x_2+y_2}}\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}\right]-\left[(\bar{X}_1-\bar{X}_2)-t(n_1+n_1-2)S_{p_{x_1,x_2}}\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)},(\bar{X}_1-\bar{X}_2)+t(n_1+n_1-2)S_{p_{x_1,x_2}}\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}\right]\right]I$$

$$\mu_{1N}-\mu_{2N}\in\left[(\bar{X}_1-\bar{X}_2)-t(n_1+n_1-2)S_{p_{x_1,x_2}}\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}+\left((\bar{Y}_1-\bar{Y}_2)-t(n_1+n_1-2)\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}\left(S_{p_{x_1+y_1,x_2+y_2}}-S_{p_{x_1,x_2}}\right)\right)I,(\bar{X}_1-\bar{X}_2)+t(n_1+n_1-2)S_{p_{x_1,x_2}}\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}+\left((\bar{Y}_1-\bar{Y}_2)+t(n_1+n_1-2)\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}\left(S_{p_{x_1+y_1,x_2+y_2}}-S_{p_{x_1,x_2}}\right)\right)I\right]$$

$$\mu_{1N}-\mu_{2N}\in\left[(\bar{X}_1-\bar{X}_2)+(\bar{Y}_1-\bar{Y}_2)I-t(n_1+n_1-2)\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}\left(S_{p_{x_1,x_2}}+\left[S_{p_{x_1+y_1,x_2+y_2}}-S_{p_{x_1,x_2}}\right]I\right),(\bar{X}_1-\bar{X}_2)+(\bar{Y}_1-\bar{Y}_2)I+t(n_1+n_1-2)\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}\left(S_{p_{x_1,x_2}}+\left[S_{p_{x_1+y_1,x_2+y_2}}-S_{p_{x_1,x_2}}\right]I\right)\right]$$

Now let’s prove the second case where $n_1,n_2\geq 30$ letting by definition and using AH-Isometry properties:

$$S_{1N}^2=S_{x_1}^2+\left[S_{x_1+y_1}^2-S_{x_1}^2\right]I$$

$$S_{2N}^2=S_{x_2}^2+\left[S_{x_2+y_2}^2-S_{x_2}^2\right]I$$

$$\sqrt{\frac{S_{1N}^2}{n_1}+\frac{S_{2N}^2}{n_2}}=\sqrt{\frac{S_{x_1}^2}{n_1}+\frac{S_{x_2}^2}{n_2}}+\left[\sqrt{\frac{S_{x_1+y_1}^2}{n_1}+\frac{S_{x_2+y_2}^2}{n_2}}-\sqrt{\frac{S_{x_1}^2}{n_1}+\frac{S_{x_2}^2}{n_2}}\right]I$$

$$\frac{S_{1N}^2}{n_1}+\frac{S_{2N}^2}{n_2}=\frac{S_{x_1}^2}{n_1}+\frac{S_{x_2}^2}{n_2}+\left[\frac{S_{x_1+y_1}^2}{n_1}+\frac{S_{x_2+y_2}^2}{n_2}-\frac{S_{x_1}^2}{n_1}+\frac{S_{x_2}^2}{n_2}\right]I$$

Then:

$$T\left[\frac{(\bar{X}_{1N}-\bar{X}_{2N})-(\mu_{1N}-\mu_{2N})}{\sqrt{\frac{S_{1N}^2}{n_1}+\frac{S_{2N}^2}{n_2}}}\right]=T\left[\frac{\left((\bar{X}_1+\bar{Y}_1I)-(\bar{X}_2+\bar{Y}_2I)\right)-\left((\mu_{11}+\mu_{12}I)-(\mu_{21}+\mu_{22}I)\right)}{\sqrt{\frac{S_{x_1}^2}{n_1}+\frac{S_{x_2}^2}{n_2}}+\left[\sqrt{\frac{S_{x_1+y_1}^2}{n_1}+\frac{S_{x_2+y_2}^2}{n_2}}-\sqrt{\frac{S_{x_1}^2}{n_1}+\frac{S_{x_2}^2}{n_2}}\right]I}\right]$$

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$$=\left(\frac{(\bar{X}_1-\bar{X}_2)-(\mu_{11}-\mu_{21})}{\sqrt{\frac{S_{x_1}^2}{n_1}+\frac{S_{x_2}^2}{n_2}}},\frac{\left((\bar{X}_1+\bar{Y}_1)-(\bar{X}_2+\bar{Y}_2)\right)-\left((\mu_{11}+\mu_{12})-(\mu_{21}+\mu_{22})\right)}{\sqrt{\frac{S_{x_1+y_1}^2}{n_1}+\frac{S_{x_2+y_2}^2}{n_2}}}\right)\sim\left(N(0,1),N(0,1)\right)$$

$$L.H.S=\ p\left\{-Z_{1-\frac{\alpha}{2}}<\frac{(\bar{X}_1-\bar{X}_2)-(\mu_{11}-\mu_{21})}{\sqrt{\frac{S_{x_1}^2}{n_1}+\frac{S_{x_2}^2}{n_2}}}<Z_{1-\frac{\alpha}{2}}\right\}=1-\alpha$$

$$p\left\{(\bar{X}_1-\bar{X}_2)-Z_{1-\frac{\alpha}{2}}\sqrt{\frac{S_{x_1}^2}{n_1}+\frac{S_{x_2}^2}{n_2}}<(\mu_{11}-\mu_{21})<(\bar{X}_1-\bar{X}_2)+Z_{1-\frac{\alpha}{2}}\sqrt{\frac{S_{x_1}^2}{n_1}+\frac{S_{x_2}^2}{n_2}}\right\}=1-\alpha$$

$$(\mu_{11}-\mu_{21})\in\left[(\bar{X}_1-\bar{X}_2)-Z_{1-\frac{\alpha}{2}}\sqrt{\frac{S_{x_1}^2}{n_1}+\frac{S_{x_2}^2}{n_2}},(\bar{X}_1-\bar{X}_2)+Z_{1-\frac{\alpha}{2}}\sqrt{\frac{S_{x_1}^2}{n_1}+\frac{S_{x_2}^2}{n_2}}\right]$$

$$R.H.S=p\left\{-Z_{1-\frac{\alpha}{2}}<\frac{\left((\bar{X}_1+\bar{Y}_1)-(\bar{X}_2+\bar{Y}_2)\right)-\left((\mu_{11}+\mu_{12})-(\mu_{21}+\mu_{22})\right)}{\sqrt{\frac{S_{x_1+y_1}^2}{n_1}+\frac{S_{x_2+y_2}^2}{n_2}}}<Z_{1-\frac{\alpha}{2}}\right\}=1-\alpha$$

$$p\left\{(\bar{X}_1+\bar{Y}_1)-(\bar{X}_2+\bar{Y}_2)-Z_{1-\frac{\alpha}{2}}\sqrt{\frac{S_{x_1+y_1}^2}{n_1}+\frac{S_{x_2+y_2}^2}{n_2}}<(\mu_{11}+\mu_{12})-(\mu_{21}+\mu_{22})<(\bar{X}_1+\bar{Y}_1)-(\bar{X}_2+\bar{Y}_2)+Z_{1-\frac{\alpha}{2}}\sqrt{\frac{S_{x_1+y_1}^2}{n_1}+\frac{S_{x_2+y_2}^2}{n_2}}\right\}=1-\alpha$$

$$(\mu_{11}+\mu_{12})-(\mu_{21}+\mu_{22})\in\left[(\bar{X}_1+\bar{Y}_1)-(\bar{X}_2+\bar{Y}_2)-Z_{1-\frac{\alpha}{2}}\sqrt{\frac{S_{x_1+y_1}^2}{n_1}+\frac{S_{x_2+y_2}^2}{n_2}},(\bar{X}_1+\bar{Y}_1)-(\bar{X}_2+\bar{Y}_2)+Z_{1-\frac{\alpha}{2}}\sqrt{\frac{S_{x_1+y_1}^2}{n_1}+\frac{S_{x_2+y_2}^2}{n_2}}\right]$$

So:

$$\left((\mu_{11}-\mu_{21}),(\mu_{11}+\mu_{12})-(\mu_{21}+\mu_{22})\right)\in(L.H.S,R.H.S)$$

$$\mu_{1N}-\mu_{2N}\in L.H.S+[R.H.S-L.H.S]I$$

$$\mu_{1N}-\mu_{2N}\in\left[(\bar{X}_1-\bar{X}_2)+(\bar{Y}_1-\bar{Y}_2)I-Z_{1-\frac{\alpha}{2}}\left(\sqrt{\frac{S_{x_1}^2}{n_1}+\frac{S_{x_2}^2}{n_2}}+\left[\sqrt{\frac{S_{x_1+y_1}^2}{n_1}+\frac{S_{x_2+y_2}^2}{n_2}}-\sqrt{\frac{S_{x_1}^2}{n_1}+\frac{S_{x_2}^2}{n_2}}\right]I\right),(\bar{X}_1-\bar{X}_2)+(\bar{Y}_1-\bar{Y}_2)I-Z_{1-\frac{\alpha}{2}}\left(\sqrt{\frac{S_{x_1}^2}{n_1}+\frac{S_{x_2}^2}{n_2}}+\left[\sqrt{\frac{S_{x_1+y_1}^2}{n_1}+\frac{S_{x_2+y_2}^2}{n_2}}-\sqrt{\frac{S_{x_1}^2}{n_1}+\frac{S_{x_2}^2}{n_2}}\right]I\right)\right]$$

Theorem 4.3:

Let $X_{11N},X_{12N},\dots,X_{1nN}$ be a neutrosophic sample driven from $X_{1N}\sim N(\mu_{1N},\sigma_{1N}^2)$ and $X_{21N},X_{22N},\dots,X_{2nN}$ be another sample driven from $X_{2N}\sim N(\mu_{2N},\sigma_{2N}^2)$ where $X_{1N}=X_1+Y_1I,X_{2N}=X_2+Y_2I,\ \mu_{1N}=\mu_{11}+\mu_{12}I,\ \mu_{2N}=\mu_{21}+\mu_{22}I$ and with unknown variances, then:

$$\frac{\sigma_{2N}^2}{\sigma_{1N}^2}\in\left[F_{\frac{\alpha}{2}}(n_1-1,n_2-1)\left[\frac{S_{x_2}^2}{S_{x_1}^2}+\left(\frac{S_{x_2+y_2}^2}{S_{x_1+y_1}^2}-\frac{S_{x_2}^2}{S_{x_1}^2}\right)I\right],F_{1-\frac{\alpha}{2}}(n_1-1,n_2-1)\left[\frac{S_{x_2}^2}{S_{x_1}^2}+\left(\frac{S_{x_2+y_2}^2}{S_{x_1+y_1}^2}-\frac{S_{x_2}^2}{S_{x_1}^2}\right)I\right]\right]$$

Proof:

$$T[X_{1N}^2]=T\left[\frac{(n_1-1)S_{1N}^2}{\sigma_{1N}^2}\right]=\left(\frac{(n_1-1)S_{x_1}^2}{\sigma_{x_1}^2},\frac{(n_1-1)S_{x_1+y_1}^2}{\sigma_{x_1+y_1}^2}\right)\sim\left(\chi^2(n_1-1),\chi^2(n_1-1)\right)$$

$$T[X_{2N}^2] = T\left[\frac{(n_2-1)S_{1N}^2}{\sigma_{2N}^2}\right] = \left(\frac{(n_2-1)S_{x_2}^2}{\sigma_{x_2}^2}, \frac{(n_2-1)S_{x_2+y_2}^2}{\sigma_{x_2+y_2}^2}\right) \sim \left(\chi^2(n_2-1), \chi^2(n_2-1)\right)$$

$$T\left[\frac{\frac{(n_1-1)S_{1N}^2}{\sigma_{1N}^2}/(n_1-1)}{\frac{(n_2-1)S_{2N}^2}{\sigma_{2N}^2}/(n_2-1)}\right] = T\left[\frac{S_{1N}^2\sigma_{2N}^2}{S_{2N}^2\sigma_{1N}^2}\right] = \left(\frac{S_{x_1}^2\sigma_{x_2}^2}{S_{x_2}^2\sigma_{x_1}^2}, \frac{S_{x_1+y_1}^2\sigma_{x_2+y_2}^2}{S_{x_2+y_2}^2\sigma_{x_1+y_1}^2}\right) \sim \left(F(n_1-1,n_2-1), F(n_1-1,n_2-1)\right)$$

$$L.H.S = p\left(F_{\frac{\alpha}{2}}(n_1-1,n_2-1) \le \frac{S_{x_1}^2\sigma_{x_2}^2}{S_{x_2}^2\sigma_{x_1}^2} \le F_{1-\frac{\alpha}{2}}(n_1-1,n_2-1)\right) = 1-\alpha$$

$$L.H.S = p\left(\frac{S_{x_2}^2}{S_{x_1}^2}F_{\frac{\alpha}{2}}(n_1-1,n_2-1) \le \frac{\sigma_{x_2}^2}{\sigma_{x_1}^2} \le \frac{S_{x_2}^2}{S_{x_1}^2}F_{1-\frac{\alpha}{2}}(n_1-1,n_2-1)\right) = 1-\alpha$$

$$\Rightarrow \frac{\sigma_{x_2}^2}{\sigma_{x_1}^2} \in \left[\frac{S_{x_2}^2}{S_{x_1}^2}F_{\frac{\alpha}{2}}(n_1-1,n_2-1), \frac{S_{x_2}^2}{S_{x_1}^2}F_{1-\frac{\alpha}{2}}(n_1-1,n_2-1)\right]$$

$$R.H.S = p\left(F_{\frac{\alpha}{2}}(n_1-1,n_2-1) \le \frac{S_{x_1+y_1}^2\sigma_{x_2+y_2}^2}{S_{x_2+y_2}^2\sigma_{x_1+y_1}^2} \le F_{1-\frac{\alpha}{2}}(n_1-1,n_2-1)\right) = 1-\alpha$$

$$R.H.S = p\left(\frac{S_{x_2+y_2}^2}{S_{x_1+y_1}^2}F_{\frac{\alpha}{2}}(n_1-1,n_2-1) \le \frac{\sigma_{x_2+y_2}^2}{\sigma_{x_1+y_1}^2} \le \frac{S_{x_2+y_2}^2}{S_{x_1+y_1}^2}F_{1-\frac{\alpha}{2}}(n_1-1,n_2-1)\right) = 1-\alpha$$

$$\Rightarrow \frac{\sigma_{x_2+y_2}^2}{\sigma_{x_1+y_1}^2} \in \left[\frac{S_{x_2+y_2}^2}{S_{x_1+y_1}^2}F_{\frac{\alpha}{2}}(n_1-1,n_2-1), \frac{S_{x_2+y_2}^2}{S_{x_1+y_1}^2}F_{1-\frac{\alpha}{2}}(n_1-1,n_2-1)\right]$$

$$\left(\frac{\sigma_{x_2}^2}{\sigma_{x_1}^2}, \frac{\sigma_{x_2+y_2}^2}{\sigma_{x_1+y_1}^2}\right) \in (L.H.S, R.H.S)$$

$$\frac{\sigma_{2N}^2}{\sigma_{1N}^2} \in L.H.S + [R.H.S - L.H.S]I$$

$$\frac{\sigma_{2N}^2}{\sigma_{1N}^2} \in \left[\frac{S_{x_2}^2}{S_{x_1}^2}F_{\frac{\alpha}{2}}(n_1-1,n_2-1), \frac{S_{x_2}^2}{S_{x_1}^2}F_{1-\frac{\alpha}{2}}(n_1-1,n_2-1)\right] + \left[\left[\frac{S_{x_2+y_2}^2}{S_{x_1+y_1}^2}F_{\frac{\alpha}{2}}(n_1-1,n_2-1), \frac{S_{x_2+y_2}^2}{S_{x_1+y_1}^2}F_{1-\frac{\alpha}{2}}(n_1 - \right.\right.$$
$$\left.\left. 1,n_2-1)\right] - \left[\frac{S_{x_2}^2}{S_{x_1}^2}F_{\frac{\alpha}{2}}(n_1-1,n_2-1), \frac{S_{x_2}^2}{S_{x_1}^2}F_{1-\frac{\alpha}{2}}(n_1-1,n_2-1)\right]\right]I$$

$$\frac{\sigma_{2N}^2}{\sigma_{1N}^2} \in \left[\frac{S_{x_2}^2}{S_{x_1}^2}F_{\frac{\alpha}{2}}(n_1-1,n_2-1) + \left[\frac{S_{x_2+y_2}^2}{S_{x_1+y_1}^2}F_{\frac{\alpha}{2}}(n_1-1,n_2-1) - \frac{S_{x_2}^2}{S_{x_1}^2}F_{\frac{\alpha}{2}}(n_1-1,n_2-1)\right]I, \frac{S_{x_2}^2}{S_{x_1}^2}F_{1-\frac{\alpha}{2}}(n_1-1,n_2 - \right.$$
$$\left. 1) + \left[\frac{S_{x_2+y_2}^2}{S_{x_1+y_1}^2}F_{1-\frac{\alpha}{2}}(n_1-1,n_2-1) - \frac{S_{x_2}^2}{S_{x_1}^2}F_{1-\frac{\alpha}{2}}(n_1-1,n_2-1)\right]\right]$$

$$\frac{\sigma_{2N}^2}{\sigma_{1N}^2} \in \left[F_{\frac{\alpha}{2}}(n_1-1,n_2-1)\left[\frac{S_{x_2}^2}{S_{x_1}^2} + \left(\frac{S_{x_2+y_2}^2}{S_{x_1+y_1}^2} - \frac{S_{x_2}^2}{S_{x_1}^2}\right)I\right], F_{1-\frac{\alpha}{2}}(n_1-1,n_2-1)\left[\frac{S_{x_2}^2}{S_{x_1}^2} + \left(\frac{S_{x_2+y_2}^2}{S_{x_1+y_1}^2} - \frac{S_{x_2}^2}{S_{x_1}^2}\right)I\right]\right]$$

Example 4.2:

Let:

X_{1N}: $3.75 + 1.25I, 2.75 + 0.25I, 2.5 + 1.5I, 2.25 + 1.75I, 2.15 + 0.85I, 3.15 - 1.65I, 3 - 0.5I, 3.7 - 1.7I, 3.75 - 0.75I, 3 + 1I$

X_{2N}: $3.5 - 0.5I, 3.5 + 0.5I, 2.75 + 1.25I, 3.75 - 0.75I, 2.5 - 0.5I, 2.75 + 1.25I, 3.75 + 0.25I, 3.5 - 0.5I$

And let $\alpha = 0.05$

Using theorem 4.2:

$$t_{1-\frac{\alpha}{2}}(n_1+n_1-2) = t_{0.975}(16) = 2.12$$

$$\mu_{1N} - \mu_{2N} \in \left[(\bar{X}_1 - \bar{X}_2) + (\bar{Y}_1 - \bar{Y}_2)I - t(n_1+n_1-2)\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}\left(S_{p_{x_1,x_2}} + \left[S_{p_{x_1+y_1,x_2+y_2}} - \right.\right.\right.$$
$$\left.\left.\left. S_{p_{x_1,x_2}}\right]I\right), (\bar{X}_1 - \bar{X}_2) + (\bar{Y}_1 - \bar{Y}_2)I + t(n_1+n_1-2)\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}\left(S_{p_{x_1,x_2}} + \left[S_{p_{x_1+y_1,x_2+y_2}} - S_{p_{x_1,x_2}}\right]I\right)\right]$$

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$$\mu_{1N}-\mu_{2N}\in\left[(3-3.25)+(0.2-0.125)I-2.12\sqrt{\left(\frac{1}{10}+\frac{1}{8}\right)}(0.56+[0.93-0.56]I),(3-3.25)+\right.$$

$$\left.(0.2-0.125)I+2.12\sqrt{\left(\frac{1}{10}+\frac{1}{8}\right)}(0.56+[0.93-0.56]I)\right]$$

$$\mu_{1N}-\mu_{2N}\in[-0.81-0.3I,0.31+0.45I]$$

$$F_{\frac{\alpha}{2}}(n_1-1,n_2-1)=F_{0.025}(9,7)=\frac{1}{F_{0.975}(7,9)}=\frac{1}{4.20}=0.24$$

$$F_{1-\frac{\alpha}{2}}(n_1-1,n_2-1)=F_{0.975}(9,7)=4.82$$

Using theorem 4.3:

$$\frac{\sigma_{2N}^2}{\sigma_{1N}^2}\in\left[F_{\frac{\alpha}{2}}(n_1-1,n_2-1)\left[\frac{S_{x_2}^2}{S_{x_1}^2}+\left(\frac{S_{x_2+y_2}^2}{S_{x_1+y_1}^2}-\frac{S_{x_2}^2}{S_{x_1}^2}\right)I\right],F_{1-\frac{\alpha}{2}}(n_1-1,n_2-1)\left[\frac{S_{x_2}^2}{S_{x_1}^2}+\left(\frac{S_{x_2+y_2}^2}{S_{x_1+y_1}^2}-\frac{S_{x_2}^2}{S_{x_1}^2}\right)I\right]\right]$$

$$\frac{\sigma_{2N}^2}{\sigma_{1N}^2}\in\left[0.24\left[\frac{0.25}{0.36}+\left(\frac{0.55}{1.12}-\frac{0.25}{0.36}\right)I\right],4.82\left[\frac{0.25}{0.36}+\left(\frac{0.55}{1.12}-\frac{0.25}{0.36}\right)I\right]\right]$$

$$\frac{\sigma_{2N}^2}{\sigma_{1N}^2}\in[0.17-0.05I,3.35-0.98I]$$

6. Conclusions and future research directions

In this paper, a new approach to neutrosophic confidence intervals for neutrosophic normal populations was introduced and many related theorems and formulas were proved successfully using algebraic properties of the one-dimensional AH-Isometry. Also, many solved examples were presented and done successfully. We conclude from the driven formulas that neutrosophic confidence intervals are very complicated and differ from interval valued neutrosophic confidence intervals. Content of this work is very helpful in defining and solving many open problems in statistical inference theory including hypothesis testing, goodness of fit tests, etc. We aim to do lots of works in these branches in the future.

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