A Study of a Neutrosophic Bernoulli's and Recati equations by Using the One-Dimensional Geometric AH-Isometry

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Abstract

In this paper, the definition of a Neutrosophic Differential Equation by Using the One-Dimensional Geometric AH-Isometry. The main objective is define a Neutrosophic Bernoulle's and Recati identical linear differential equation and find solutions for this equation.

Keywords: One-Dimensional Geometric AH-Isometry; Neutrosophic linear Differential Equation; Neutrosophic real number.

1.Introduction:

Neutrosophic logic. Neutrosophy, Neutrosophic set, Neutrosophic probability and alike, are recently creations of Smarandache, being characterized by having the indeterminacy as component of their framework,and a notable feature of neutrosophic logic is that can be considered a generaliazation of fuzzy logics, encompassing the classical logic as well [1]. Also. F. Smarandache, has defined the concept of continuation of a neutrosophic function in year 2015 in [1], and neutrosophic mereo-limit [1], mereo-continuity. Moreover, in 2014, he has defined the concept of a neutro-oscillator differential in [3], and mereo-derivative. Finally in 2013 he introduced neutrosophic integration in [2], and mereo-integral, besides the classical defintions of limit, continuity, deverative, and integral respectively.
Among the recent applications there are: neutrosophic crisp set theory in image processing [4,5,9], neutrosophic sets medical field [6,7,8], in information geographic systems [10,11] and possible applications to database [12]. Also, neutrosophic triplet group application to physics [13,16,19]. Morever Several researches have made multiple contributions to neutrosophic topological [14,15,18,20], Also More researches have made multiple contributions to neutrosophic analysis [21]. Finally the neutrosophic integration may have application in calculus the areas between two neutrosophic functions.

2. Preliminaries

Definition 2.1. Neutrosophic Real Number: [1]

Suppose that w is a neutrosophic number, then it takes the following standard form: $w = a + bI$ where a, b are real coefficients, and I represents the indeterminacy, where $0.I = 0$ and $I^n = I$ for all positive integers n.

For example:

$$w = 1 + 2I, w = 3 = 3 + 0I.$$

Definition 2.2. **Division of neutrosophic real numbers**: [2]

Suppose that w_1, w_2 are two neutrosophic number,where

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$$w_1 = a_1 + b_1 I, w_2 = a_2 + b_2 I$$

Then:

$$\frac{w_1}{w_2} = \frac{a_1 + b_1 I}{a_2 + b_2 I} = \frac{a_1}{a_2} + \frac{a_2 b_1 - a_1 b_2}{a_2(a_2 + b_2)} I$$

Definition 2.3

Let $R(I) = \{a + bI\ ; a, b \in R\}$ where $I^2 = I$ be the neutrosophic field of reals. The one-dimensional isometry (AH-Isometry) is defined as follows: [21]

$$T: R(I) \to R \times R$$
$$T(a + bI) = (a, a + b)$$

Remark 2.4. [21]

T is an algebraic isomorphism between two rings, it has the following properties:

1) T is bijective.
2) T preserves addition and multiplication, i.e.:

$$T[(a + bI) + (c + dI)] = T(a + bI) + T(c + dI)$$

And

$$T[(a + bI) \cdot (c + dI)] = T(a + bI) \cdot T(c + dI)$$

3) Since T is bijective, then it is invertible by:

$$T^{-1}: R \times R \to R(I)$$
$$T^{-1}(a, b) = a + (b - a)I$$

4) T preserves distances, i.e.:

The distance on $R(I)$ can be defined as follows:

Let $A = a + bI, B = c + dI$ be two neutrosophic real numbers, then $L = \|\overrightarrow{AB}\| = d[(a + bI, c + dI)] = |a + bI - (c + dI)| = |(a - c) + I(b - d)| = |a - c| + I[|a + b - c - d| - |a - c|]$.

On the other hand, we have:

$T\left(\|\overrightarrow{AB}\|\right) = (|a - c|, |(a + b) - (c + d)|) = \left(d(a, c), d(a + b, c + d)\right) = d[(a, a + b), (c, c + d)]=d\left(T(a + bI), T(c + dI)\right)$

$=\|T(\overrightarrow{AB})\|$.

This implies that the distance is preserved up to isometry. i.e.$\|T(AB)\| = T(\|AB\|)$

Definition 2.5.

Let $f: R(I) \to R(I); f = f(X)$ and $X = x + yI \in R(I)$ the f is called a neutrosophic real function with one neutrosophic variable.

a neutrosophic real function $f(X)$ written as follows:

$$f(X) = f(x + yI) = f(x) + I[f(x + y) - f(x)]$$

Theorem 2.6. any neutrosophic real function into two classical real functions, i.e., to the classical Euclidean plane $R \times R$.

Proof.

Let $f(X) = f(x + yI) = f(x) + I[f(x + y) - f(x)]$ a neutrosophic real function.

Now, Using the one-dimensional AH-isometry, we have.

$T\left(f(X)\right) = T(f(x) + I[f(x + y) - f(x)])$, then.

$(f_1, f_2) = \left(f(x), f(x + y)\right)$, then, we have.

$$\begin{cases} f_1 = f(x) \\ f_2 = f(x + y) \end{cases}$$

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the functions $f(x), f(x+y)$ are a real functions.

Definition 2.7. Computing Powers in $R(I)$

To compute such equation: $(a+bI)^{c+dI}$; $a, b, c, d \in R$ we need the one-dimensional isometry again:
$T[(a+bI)^{c+dI}] = (a, a+b)^{(c,c+d)} = (a^c, (a+b)^{c+d})$,
Which results:
$(a+bI)^{c+dI} = T^{-1}(a^c, (a+b)^{c+d}) = a^c + I[(a+b)^{c+d} - a^c]$.

Definition 2.8.

Let $f(X) = f(x+yI) = f(x) + I[f(x+y) - f(x)]$ a neutrosophic function on $R(I)$, the we define a derivative of a neutrosophic function $f(X)$ as follows:

$$f'(X) = f'(x+yI) = f'(x) + I[f'(x+y) - f'(x)]$$

Definition 2.9.

Let $f(X) = f(x+yI) = f(x) + I[f(x+y) - f(x)]$ a neutrosophic function on $R(I)$, the we define a integration of a neutrosophic function $f(X)$ as follows:

$$\int f(X)\, dX = \int f(x)\, dx + I\left[\int f(x+y)\, d(x+y) - \int f(x)\, dx\right] + (a+bI)$$

Where $(a+bI)$ is a neutrosophic constant number, and $\int f(X)\, dX = F(X) = F(x) + I[F(x+y) - F(x)]$.

3. Neutrosophic Bernoulli's equation.

In this section is defined a Neutrosophic Bernoulli's equation by Using the One-Dimensional Geometric AH-Isometry and solutions are found for this equation.

Definition 3.1 .

Let $Y = y_1 + y_2 I$, $X = x_1 + x_2 I$ We define the Neutrosophic Bernoulli's equation by Using the One-Dimensional Geometric AH-Isometry as form:

$$\acute{Y} + f(X)Y = g(X)Y^n$$

This equation can be written as follow:

$$(y_1{}' + I[(y_1+y_2)' - y_1{}'])(f(x_1) + I[f(x_1+x_2) - f(x_1)])(y_1 + y_2 I) = (g(x_1) + I[g(x_1+x_2) - g(x_1)])(y_1 + y_2 I)^n$$

$$\Rightarrow y_1' + f(x_1)y_1 + I[(y_1+y_2)' - f(x_1+x_2)(y_1+y_2) - (y_1' + f(x_1)y_1)] = (g(x_1) + I[g(x_1+x_2) - g(x_1)])((y_1)^n + I[(y_1+y_2)^n - (y_1)^n])$$

Method of solution.

1. Take AH-Isometry for the differential equation, we have.

$$T(y_1' + f(x_1)y_1 + I[(y_1+y_2)' - f(x_1+x_2)(y_1+y_2) - (y_1' + f(x_1)y_1)]) = (g(x_1) + I[g(x_1+x_2) - g(x_1)])((y_1)^n + I[(y_1+y_2)^n - (y_1)^n])$$

$$T(y_1' + f(x_1)y_1 + I[(y_1+y_2)' - f(x_1+x_2)(y_1+y_2) - (y_1' + f(x_1)y_1)]) = T(g(x_1) + I[g(x_1+x_2) - g(x_1)]).T((y_1)^n + I[(y_1+y_2)^n - (y_1)^n])$$

$$[y_1' + f(x_1)y_1, (y_1+y_2)' - f(x_1+x_2)(y_1+y_2)] = [g(x_1), g(x_1+x_2)].[(y_1)^n, (y_1+y_2)^n]$$

Then.

$$\begin{cases} y_1' + f(x_1)y_1 = g(x_1)(y_1)^n \dots\dots (1) \\ (y_1+y_2)' - f(x_1+x_2)(y_1+y_2) = g(x_1+x_2)(y_1+y_2)^n \dots\dots (2) \end{cases}$$

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The equations (1) and (2) are two Bernoulli's differential equation classical.

2. We find the solution to the equations classical (1) and (2), we have.

y_1 the solution to the equation (1).

$(y_1 + y_2)$ the solution to the equation (2).

3. We Take invertible AH-Isometry, then, we have the solution of a Neutrosophic identical linear differential equation .

$$Y = y_1 + y_2 I = T^{-1}(y_1, y_1 + y_2) = y_1 + \left((y_1 + y_2) - y_1\right)I$$

Example 3.2. Find a solution to the equation:

$$\acute{Y} + \frac{1}{X}Y = XY^3$$

Solution.

Let $Y = y_1 + y_2 I, X = x_1 + x_2 I$. Then.

$$y_1' + \frac{1}{x_1}y_1 + I\left[(y_1 + y_2)' - \frac{1}{(x_1 + x_2)}(y_1 + y_2) - \left(y_1' + \frac{1}{x_1}y_1\right)\right]$$
$$= (x_1 + I[(x_1 + x_2) - (x_1)])((y_1)^3 + I[(y_1 + y_2)^3 - (y_1)^3])$$

Now, Take AH-Isometry for the differential equation, we have.

$$\begin{cases} y_1' + \frac{1}{x_1}y_1 = x_1(y_1)^3 \dots\dots (3) \\ (y_1 + y_2)' - \frac{1}{x_1 + x_2}(y_1 + y_2) = (x_1 + x_2)(y_1 + y_2)^3 \dots\dots (4) \end{cases}$$

The solution of equation (3) written as follow:

$$y_1 = \left\{\frac{1}{\mu(x_1)}\left(a + \int \mu(x_1)g(x_1)dx_1\right)\right\}^{\frac{1}{-n+1}}$$

$$y_1 = \left\{\frac{1}{(x_1)^2}\left(a + \int -2(x_1)^3 d(x_1)\right)\right\}^{\frac{-1}{2}}$$

$$y_1 = \left\{\frac{1}{(x_1)^2}\left(a - \frac{(x_1)^4}{2}\right)\right\}^{\frac{-1}{2}} = \left\{\frac{a}{(x_1)^2} - \frac{(x_1)^2}{2}\right\}^{\frac{-1}{2}}, \text{ where } a \in R.$$

The solution of equation (4) written as follow:

$$(y_1 + y_2) = \left\{\frac{1}{(x_1+x_2)^2}\left(b - \frac{(x_1+x_2)^4}{2}\right)\right\}^{\frac{-1}{2}} = \left\{\frac{b}{(x_1+x_2)^2} - \frac{(x_1+x_2)^2}{2}\right\}^{\frac{-1}{2}}, \text{ where } b \in R$$

Now, Take invertible AH-Isometry, then, we have the solution of a Neutrosophic identical linear differential equation .

$$Y = y_1 + y_2 I = T^{-1}\left(\left\{\frac{a}{(x_1)^2} - \frac{(x_1)^2}{2}\right\}^{\frac{-1}{2}}, \left\{\frac{b}{(x_1 + x_2)^2} - \frac{(x_1 + x_2)^2}{2}\right\}^{\frac{-1}{2}}\right)$$

$$Y = y_1 + y_2 I = \left\{\frac{a}{(x_1)^2} - \frac{(x_1)^2}{2}\right\}^{\frac{-1}{2}} + \left(\left\{\frac{b}{(x_1 + x_2)^2} - \frac{(x_1 + x_2)^2}{2}\right\}^{\frac{-1}{2}} - \left\{\frac{a}{(x_1)^2} - \frac{(x_1)^2}{2}\right\}^{\frac{-1}{2}}\right)I$$

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By Definition 2.7, we have.

$$Y = y_1 + y_2 I = \left\{\frac{a}{(x_1)^2} - \frac{(x_1)^2}{2} + \left(\frac{b}{(x_1+x_2)^2} - \frac{(x_1+x_2)^2}{2}\right) I\right\}^{\frac{-1}{2}}$$

$$Y = y_1 + y_2 I = \left\{\frac{a}{(x_1)^2} + \left(\frac{b}{(x_1+x_2)^2}\right) I - \frac{(x_1)^2}{2} - \left(\frac{(x_1+x_2)^2}{2}\right) I\right\}^{\frac{-1}{2}}$$

$$Y = y_1 + y_2 I = \left\{\frac{a}{(x_1)^2} + \left(\frac{b}{(x_1+x_2)^2}\right) I - \left[\frac{(x_1)^2}{2} + \left(\frac{(x_1+x_2)^2}{2}\right)\right] I\right\}^{\frac{-1}{2}}$$

By Definition 2.5, we have.

$$\frac{a}{(x_1)^2} + \left(\frac{b}{(x_1+x_2)^2}\right) I = \frac{a}{(x_1)^2} + \frac{b}{(x_1+x_2)^2} I = \frac{a+bI}{(x_1+x_2 I)^2}$$

Then.

$$Y = y_1 + y_2 I = \left\{\frac{a+bI}{(x_1+x_2 I)^2} + \frac{-(x_1+x_2 I)^2}{2}\right\}^{\frac{-1}{2}} = \left\{\frac{a+bI}{X^2} + \frac{-X^2}{2}\right\}^{\frac{-1}{2}}$$

So that,

$$Y = y_1 + y_2 I = \left\{\frac{a+bI}{X^2} + \frac{-X^2}{2}\right\}^{\frac{-1}{2}}$$

where $a + bI \in R(I)$.

Example 3.3. Find a solution to the equation:

$$\acute{Y} + tan(X)Y = sin(X)Y^2$$

Solution.

Let $Y = y_1 + y_2 I, X = x_1 + x_2 I$. Then.

$$y_1' + tan(x_1)y_1 + I[(y_1+y_2)' + tan(x_1+x_2)(y_1+y_2) - (y_1' + tan(x_1)y_1)] = (sin(x_1) + I[sin(x_1+x_2) - sin(x_1)])((y_1)^2 + I[(y_1+y_2)^2 - (y_1)^2])$$

Now, Take AH-Isometry for the differential equation, we have.

$$\begin{cases} y_1' + tan(x_1)y_1 = sin(x_1)(y_1)^2 \ldots\ldots (5) \\ (y_1+y_2)' + tan(x_1+x_2)(y_1+y_2) = sin(x_1+x_2)(y_1+y_2)^2 \ldots\ldots (6) \end{cases}$$

The solution of equation (5s) written as follow:

$$y_1 = \left\{\frac{1}{\mu(x_1)}\left(a + \int \mu(x_1) g(x_1) dx_1\right)\right\}^{\frac{1}{-n+1}}$$

$$y_1 = \left\{\frac{1}{cos(x_1)}\left(a + \int cos(x_1).sin(x_1) d(x_1)\right)\right\}^{-1}$$

$$y_1 = \left\{\frac{1}{cos(x_1)}\left(a + \frac{1}{4}cos2(x_1)\right)\right\}^{-1} = \left\{\frac{a}{cos(x_1)} + \frac{1}{4}\frac{cos2(x_1)}{cos(x_1)}\right\}^{-1}, \text{ where } a \in R.$$

The solution of equation (6) written as follow:

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$(y_1+y_2)=\left\{\frac{1}{cos(x_1+x_2)}\left(b+\frac{1}{4}cos2(x_1+x_2)\right)\right\}^{-1}=\left\{\frac{b}{cos(x_1+x_2)}+\frac{1}{4}\frac{cos2(x_1+x_2)}{cos(x_1)}\right\}^{-1}$, where $b\in R$

Now, Take invertible AH-Isometry, then, we have the solution of a Neutrosophic identical linear differential equation .

$$Y=y_1+y_2I=T^{-1}\left(\left\{\frac{a}{cos(x_1)}+\frac{1}{4}c\frac{cos2(x_1)}{cos(x_1)}\right\}^{-1},\left\{\frac{b}{cos(x_1+x_2)}+\frac{1}{4}\frac{cos2(x_1+x_2)}{cos(x_1)}\right\}^{-1}\right)$$

$$Y=y_1+y_2I=\left\{\frac{a}{cos(x_1)}+\frac{1}{4}\frac{cos2(x_1)}{cos(x_1)}\right\}^{-1}+\left(\left\{\frac{b}{cos(x_1+x_2)}+\frac{1}{4}\frac{cos2(x_1+x_2)}{cos(x_1)}\right\}^{-1}-\left\{\frac{a}{cos(x_1)}+\frac{1}{4}\frac{cos2(x_1)}{cos(x_1)}\right\}^{-1}\right)I$$

By Definition 2.7, we have.

$$Y=y_1+y_2I=\left\{\frac{a}{cos(x_1)}+\frac{1}{4}\frac{cos2(x_1)}{cos(x_1)}+\left(\frac{b}{cos(x_1+x_2)}+\frac{1}{4}\frac{cos2(x_1+x_2)}{cos(x_1)}\right)I\right\}^{-1}$$

$$Y=y_1+y_2I=\left\{\frac{a}{cos(x_1)}+\left(\frac{b}{cos(x_1+x_2)}\right)I+\frac{1}{4}\frac{cos2(x_1)}{cos(x_1)}+\left(\frac{1}{4}\frac{cos2(x_1+x_2)}{cos(x_1)}\right)I\right\}^{-1}$$

By Definition 2.5, we have.

$$\frac{a}{cos(x_1)}+\left(\frac{b}{cos(x_1+x_2)}\right)I=\frac{a}{cos(x_1)}+\frac{b}{cos(x_1+x_2)}I=\frac{a+bI}{cos(x_1+x_2I)}$$

Then.

$$Y=y_1+y_2I=\left\{\frac{a+bI}{cos(x_1+x_2I)}+\frac{1}{4}\frac{cos2(x_1+x_2I)}{cos(x_1+x_2I)}\right\}^{-1}=\left\{\frac{a+bI}{cos(X)}+\frac{1}{4}\frac{cos2X}{cosX}\right\}^{-1}$$

So that,

$$Y=y_1+y_2I=\left\{\frac{a+bI}{cos(X)}+\frac{1}{4}\frac{cos2X}{cosX}\right\}^{-1}$$

where $a+bI\in R(I)$.

4 Neutrosophic Recati equation.

In this section is defined a Neutrosophic Recati equation by Using the One-Dimensional Geometric AH-Isometry and solutions are found for this equation.

Definition 4.1 .

Let $Y=y_1+y_2I, X=x_1+x_2I$ We define the Neutrosophic Recati equation by Using the One-Dimensional Geometric AH-Isometry as form:

$$\acute{Y}+f(X)Y^2+g(X)Y+h(X)=0$$

And takes a particular solution:

$$Z=z_1+z_2I=r(X)=r(x_1+x_2I)$$

This equation can be written as follow:

$$(y_1'+I[(y_1+y_2)'-y_1'])(f(x_1)+I[f(x_1+x_2)-f(x_1)])(y_1+y_2I)^2+(g(x_1)+I[g(x_1+x_2)-g(x_1)])(y_1+y_2I)+h(x_1)+I[h(x_1+x_2)-h(x_1)]=0$$

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$$y_1' + f(x_1)(y_1)^2 + I[(y_1+y_2)' - f(x_1+x_2)(y_1+y_2) - (y_1' + f(x_1)(y_1)^2)] + (g(x_1) + I[g(x_1+x_2) - g(x_1)])((y_1) + I[(y_1+y_2) - (y_1)]) + h(x_1) + I[h(x_1+x_2) - h(x_1)] = 0$$

And:

$$Z = z_1 + z_2 I = r(x_1 + x_2 I) = r(x_1) + I[r(x_1+x_2) - r(x_1)]$$

Method of solution.

1. Take AH-Isometry for the differential equation, and Take AH-Isometry for a particular solution, we have.

$$T\big(y_1' + f(x_1)(y_1)^2 + I[(y_1+y_2)' - f(x_1+x_2)(y_1+y_2) - (y_1' + f(x_1)(y_1)^2)] + (g(x_1) + I[g(x_1+x_2) - g(x_1)])((y_1) + I[(y_1+y_2) - (y_1)]) + h(x_1) + I[h(x_1+x_2) - h(x_1)]\big) = T(0)$$

$$[y_1' + f(x_1)(y_1)^2 + g(x_1)y_1 + h(x_1), (y_1+y_2)' + f(x_1+x_2)(y_1+y_2)^2 + g(x_1+x_2)(y_1+y_2) + h(x_1+x_2)] = [0,0]$$

Then.

$$\begin{cases} y_1' + f(x_1)(y_1)^2 + g(x_1)y_1 + h(x_1) = 0 \ldots\ldots (7) \\ (y_1+y_2)' + f(x_1+x_2)(y_1+y_2)^2 + g(x_1+x_2)(y_1+y_2) + h(x_1+x_2) = 0 \ldots\ldots (8) \end{cases}$$

And:

$$T(z_1 + z_2 I) = T(r(x_1) + I[r(x_1+x_2) - r(x_1)])$$

$$[z_1, (z_1+z_2)] = [r(x_1), r(x_1+x_2)]$$

$$\begin{cases} z_1 = r(x_1) \ldots\ldots (9) \\ (z_1+z_2) = r(x_1+x_2) \ldots\ldots (10) \end{cases}$$

The equations (7) and (8) are two Recati differential equation classical with two a particular solution (9) and (10).

2. We find the solution to the equations classical (7) and (8), we have.

y_1 the solution to the equation (7).

(y_1+y_2) the solution to the equation (8).

3. We Take invertible AH-Isometry, then, we have the solution of a Neutrosophic identical linear differential equation .

$$Y = y_1 + y_2 I = T^{-1}(y_1, y_1+y_2) = y_1 + ((y_1+y_2) - y_1)I$$

Example 4.2. Find the general solution for the following neutrosophic ricati equation:

$$\acute{Y} + \left(\frac{cosX}{1-sinX\ cosX}\right)Y^2 + \left(\frac{-1}{1-sinX\ cosX}\right)Y + \frac{sinX}{1-sinX\ cosX} = 0$$

If a particular solution is:

$$Z = z_1 + z_2 I = cosX$$

Solution.

Let $Y = y_1 + y_2 I, X = x_1 + x_2 I$. Then.

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$$y_1' + \frac{cosx_1}{1 - sinx_1 . cosx_1}(y_1)^2 + I\left[(y_1+y_2)' + \frac{cos(x_1+x_2)}{1 - sin(x_1+x_2).cos(x_1+x_2)}(y_1+y_2) - \left(y_1' + \frac{cosx_1}{1 - sinx_1 . cosx_1}(y_1)^2\right)\right] + \left(\frac{-1}{1 - sinx_1 . cosx_1} + I\left[\frac{-1}{1 - sin(x_1+x_2).cos(x_1+x_2)} - \left(\frac{-1}{1 - sinx_1 . cosx_1}\right)\right]\right)((y_1) + I[(y_1+y_2) - (y_1)]) + \frac{sinx_1}{1 - sinx_1 . cosx_1} + I\left[\frac{sin(x_1+x_2)}{1 - sin(x_1+x_2).cos(x_1+x_2)} - \frac{sinx_1}{1 - sinx_1 . cosx_1}\right] = 0$$

And:

$$Z = z_1 + z_2 I = cosX = cosx_1 + I[cos(x_1+x_2) - cosx_1]$$

Now, Take AH-Isometry for the differential equation, and Take AH-Isometry for a particular solution, we have..

$$\begin{cases} y_1' + \frac{cosx_1}{1 - sinx_1 . cosx_1}(y_1)^2 - \frac{1}{1 - sinx_1 . cosx_1} y_1 + \frac{sinx_1}{1 - sinx_1 . cosx_1} = 0 \ldots\ldots (11) \\ (y_1+y_2)' + \frac{cos(x_1+x_2)}{1 - sin(x_1+x_2).cos(x_1+x_2)}(y_1+y_2)^2 - \frac{1}{1 - sin(x_1+x_2).cos(x_1+x_2)}(y_1+y_2) + \frac{sin(x_1+x_2)}{1 - sin(x_1+x_2).cos(x_1+x_2)} = 0 \ldots\ldots (12) \end{cases}$$

And:

$$\begin{cases} z_1 = cosx_1 \\ (z_1+z_2) = cos(x_1+x_2) \end{cases}$$

The solution of equation (11) written as follow:

$y_1 = cosx_1 + \left\{\frac{1}{1 - sinx_1 . cosx_1}(a + sinx_1)\right\}^{-1}$, where $a \in R$.

By the method same, The solution of equation (12) written as follow:

$(y_1+y_2) = cos(x_1+x_2) + \left\{\frac{1}{1 - sin(x_1+x_2).cos(x_1+x_2)}(b + sin(x_1+x_2))\right\}^{-1}$, where $b \in R$

Now, Take invertible AH-Isometry, then, we have the solution of a Neutrosophic identical linear differential equation .

$$Y = y_1 + y_2 I = T^{-1}\left(cosx_1 + \left\{\frac{1}{1 - sinx_1 . cosx_1}(a + sinx_1)\right\}^{-1}, cos(x_1+x_2) + \left\{\frac{1}{1 - sin(x_1+x_2).cos(x_1+x_2)}(b + sin(x_1+x_2))\right\}^{-1}\right)$$

$$Y = y_1 + y_2 I = cosx_1 + \left\{\frac{1}{1 - sinx_1 . cosx_1}(a + sinx_1)\right\}^{-1} + I\left[cos(x_1+x_2) + \left\{\frac{1}{1 - sin(x_1+x_2).cos(x_1+x_2)}(b + sin(x_1+x_2))\right\}^{-1} - \left(cosx_1 + \left\{\frac{1}{1 - sinx_1 . cosx_1}(a + sinx_1)\right\}^{-1}\right)\right]$$

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$$Y = y_1 + y_2 I = cosx_1 + I(cos(x_1 + x_2) - cosx_1) + \left\{\frac{1}{1 - sinx_1 \,.\, cosx_1}(a + sinx_1)\right\}^{-1} + I\left[\left\{\frac{1}{1 - sin(x_1 + x_2).\, cos(x_1 + x_2)}(b + sin(x_1 + x_2))\right\}^{-1} - \left(\left\{\frac{1}{1 - sinx_1 \,.\, cosx_1}(a + sinx_1)\right\}^{-1}\right)\right]$$

$$Y = y_1 + y_2 I = cos(x_1 + x_2 I) + \left\{\frac{a}{1 - sinx_1 \,.\, cosx_1} + \frac{sinx_1}{1 - sinx_1 \,.\, cosx_1}\right\}^{-1} + I\left[\left\{\frac{b}{1 - sin(x_1 + x_2).\, cos(x_1 + x_2)} + \frac{sin(x_1 + x_2)}{1 - sin(x_1 + x_2).\, cos(x_1 + x_2)}\right\}^{-1} - \left\{\frac{a}{1 - sinx_1 \,.\, cosx_1} + \frac{sinx_1}{1 - sinx_1 \,.\, cosx_1}\right\}^{-1}\right]$$

$$Y = y_1 + y_2 I = cos(X) + \left\{\frac{a}{1 - sinx_1 \,.\, cosx_1} + \frac{sinx_1}{1 - sinx_1 \,.\, cosx_1}\right\}^{-1} + I\left[\left\{\frac{b}{1 - sin(x_1 + x_2).\, cos(x_1 + x_2)} + \frac{sin(x_1 + x_2)}{1 - sin(x_1 + x_2).\, cos(x_1 + x_2)}\right\}^{-1} - \left\{\frac{a}{1 - sinx_1 \,.\, cosx_1} + \frac{sinx_1}{1 - sinx_1 \,.\, cosx_1}\right\}^{-1}\right]$$

By Definition 2.7, we have.

$$\left\{\frac{a}{1 - sinx_1 \,.\, cosx_1} + \frac{sinx_1}{1 - sinx_1 \,.\, cosx_1}\right\}^{-1} + I\left[\left\{\frac{b}{1 - sin(x_1 + x_2).\, cos(x_1 + x_2)} + \frac{sin(x_1 + x_2)}{1 - sin(x_1 + x_2).\, cos(x_1 + x_2)}\right\}^{-1} - \left\{\frac{a}{1 - sinx_1 \,.\, cosx_1} + \frac{sinx_1}{1 - sinx_1 \,.\, cosx_1}\right\}^{-1}\right]$$
$$= \left\{\frac{a}{1 - sinx_1 \,.\, cosx_1} + \frac{sinx_1}{1 - sinx_1 \,.\, cosx_1} + \left(\frac{b}{1 - sin(x_1 + x_2).\, cos(x_1 + x_2)} + \frac{sin(x_1 + x_2)}{1 - sin(x_1 + x_2).\, cos(x_1 + x_2)}\right) I\right\}^{-1}$$

$$\left\{\frac{a}{1 - sinx_1 \,.\, cosx_1} + \frac{sinx_1}{1 - sinx_1 \,.\, cosx_1} + \frac{b}{1 - sin(x_1 + x_2).\, cos(x_1 + x_2)} I + \frac{sin(x_1 + x_2)}{1 - sin(x_1 + x_2).\, cos(x_1 + x_2)} I\right\}^{-1}$$

$$= \left\{\frac{a}{1 - sinx_1 \,.\, cosx_1} + \frac{b}{1 - sin(x_1 + x_2).\, cos(x_1 + x_2)} I + \frac{sinx_1}{1 - sinx_1 \,.\, cosx_1} + \frac{sin(x_1 + x_2)}{1 - sin(x_1 + x_2).\, cos(x_1 + x_2)} I\right\}^{-1}$$

By Definition 2.5, we have.

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$$\frac{a}{1 - sinx_1 \, . cosx_1} + \frac{b}{1 - sin(x_1 + x_2). cos(x_1 + x_2)} I = \frac{a + bI}{1 - sin(x_1 + x_2 I) \, . cos(x_1 + x_2 I)}$$
$$= \frac{a + bI}{1 - sin(X) \, . cos(X)}$$

And.

$$\frac{sinx_1}{1 - sinx_1 \, . cosx_1} + \frac{sin(x_1 + x_2)}{1 - sin(x_1 + x_2). cos(x_1 + x_2)} I = \frac{sin(x_1 + x_2 I)}{1 - sin(x_1 + x_2 I) \, . cos(x_1 + x_2 I)} =$$
$$= \frac{sin(X)}{1 - sin(X) \, . cos(X)}$$

So that,

$$Y = y_1 + y_2 I = cos(X) + \left\{ \frac{a + bI}{1 - sin(X) \, . cos(X)} + \frac{sin(X)}{1 - sin(X) \, . cos(X)} \right\}^{-1}$$

where $a + bI \in R(I)$.

5. Conclusion

In this paper, a new type of neutrosophic differential has been defined by using AH-Isometry, Moreover, we studied a linear differential equation based on AH-Isometry and found solutions to this equation.

References

[1] Smarandache F. Neutrosophic Precalculus and Neutrosophic calclus, University of New Mexico, 705 Gurley Ave. Gallup, NM 87301, USA, 2015.

[2] Smarandache F. Neutrosophic Measure and Neutrosophic Integral , In Neutrosophic Sets and Systems, 3 – 7, Vol. 1, 2013.

[3] A. A Salama, Smarandache F. Kroumov, Neutrosophic Closed Set and Continuous Functions, in Neutrosophic Sets and Systems, Vol.4, 4- 8, 2014.

[4] A. A Salama; I. M Hanafy; Hewayda Elghawalby Dabash M.S, Neutrosophic Crisp Closed RRegion and Neutrosophic Crisp Continuous Functions, New Trends in Neutrosophic Theory and Applications.

[5] A. A Salama; Hewayda Elghawalby; M.S, Dabash; A.M. NASR, Retrac Neutrosophic Crisp System For Gray Scale Image, Asian Journal Of Mathematics and Computer Research, Vol 24, 104-117, (2018).

[6] F. smarandache. "Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, neutrosophic Logic, Set, Probability, and Statistics" University of New Mexico, Gallup, NM87301, USA 2002.

[7] M. Abdel-Basset; E. Mai. Mohamed; C. Francisco; H. Z. Abd EL-Nasser. "Cosine similarity measures of bipolar neutrosophic set for diagnosis of bipolar disorder diseases" Artificial Intelligence in Medicine Vol. 101, 101735, (2019).

[8] M. Abdel-Basset; E. Mohamed; G. Abdullah; and S. Florentin. "A novel model for evaluation Hospital medical care systems based on plithogenic sets" Artificial Intelligence in Medicine 100 (2019), 101710.

[9] M. Abdel-Basset; G. Gunasekaran Mohamed; G. Abdullah. C. Victor, "A Novel Intelligent Medical Decision Support Model Based on soft Computing and Iot" IEEE Internet of Things Journal, Vol. 7, (2019).

[10] M. Abdel-Basset; E. Mohamed; G. Abdullah; G. Gunasekaran; L. Hooang Viet." A novel group decision making model based on neutrosophic sets for heart disease diagnosis" Multimedia Tools and Applications, 1-26, (2019).

[11] A. A Salama. Basic Structure of Some Classes of Neutrosophic Crisp Nearly Open Sets and Possible Application to GIS Topology. Neutrosophic Sets and Systems, Vol. 7, 18-22, (2015).

[12] A. A Salama; F. Smarandache. Neutrosophic Set Theory, Neutrosophic Sets and Systems, Vol. 5, 1-9, (2014).

[13] F. Smarandache, The Neutrosophic Triplet Group and its Application to physics, Seminar Universidad National de Quilmes , Department of science and Technology, Beunos Aires, Argentina, 20 June 2014.

DOI: https://doi.org/10.54216/JNFS.050104
Received: August 08, 2022 Accepted: December 11, 2022

[14] A. B.AL-Nafee; R.K. Al-Hamido; F.Smarandache. "Separation Axioms In Neutrosophic Crisp Topological Spaces", Neutrosophic Sets and Systems, vol. 25, 25-32, (2019).

[15] R.K. Al-Hamido, Q. H. Imran, K. A. Alghurabi, T. Gharibah, "On Neutrosophic Crisp Semi Alpha Closed Sets", Neutrosophic Sets and Systems", vol. 21, 28-35, (2018).

[16] Q. H. Imran, F. Smarandache, R.K. Al-Hamido, R. Dhavasselan, "On Neutrosophic Semi Alpha open Sets", Neutrosophic Sets and Systems, vol. 18, 37-42, (2017).

[17] Al-Hamido, R. K.; "A study of multi-Topological Spaces", PhD Theses, AlBaath university , Syria, (2019).

[18] Al-Hamido, R. K.; "Neutrosophic Crisp Supra Bi-Topological Spaces", International Journal of Neutrosophic Science, Vol. 1, 66-73, (2018).

[19] R.K. Al-Hamido, "Neutrosophic Crisp Bi-Topological Spaces", Neutrosophic Sets and Systems, vol. 21, 66-73, (2018).

[20] R.K. Al-Hamido, T. Gharibah, S. Jafari F.Smarandache, "On Neutrosophic Crisp Topology via N-Topology", Neutrosophic Sets and Systems, vol. 21, 96-109, (2018).

[21] A. Hatip, "The Special Neutrosophic Functions," International Journal of Neutrosophic Science (IJNS), p. 13, 12 May 202

DOI: https://doi.org/10.54216/JNFS.050104
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