



Integrated Multi-Criteria Decision Making Via Trapezoidal Neutrosophic Sets to Evaluate the Risks of Failure Mode

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Abstract

The purpose of a failure mode and effect analysis (FMEA) is to improve the safety and dependability of a system, product, procedure, or facility by identifying potential points of failure and determining the consequences of such failures. The assessment of failure modes, the weighting of risk factors, and the ranking of failure modes are all areas where the conventional FMEA falls short when put to use in the real world. To assess the hazard of failure modes in a trapezoidal neutrosophic sets environment, this research proposes a model that combines the neutrosophic sets and MCDM technique such as WASPAS. The WASPAS MCDM method is used to calculate the weights of standards and order the alternatives. Advantages of trapezoidal neutrosophic numbers in dealing with uncertainty, ambiguity, and incompleteness are combined with the benefits of WASPAS to create the suggested risk prioritization strategy.

Keywords: Failure mode; MCDM; Trapezoidal Neutrosophic Sets; Risk assessment;

1. Introduction

To boost the dependability and care of goods, procedures, and services, failure mode and effect analysis (FMEA) is a viewpoint risk analysis method for pinpointing and eradicating probable sources of failure, issue, and mistake. Instead of trying to fix problems after they've already affected customers, FMEA focuses on preventing the most common causes of failure in the first place. Since its inception as a formalized design philosophy in the 1960s, FMEA has been used extensively in the aircraft industry to enhance product quality. Since then, FMEA has evolved into a robust tool utilised by a broad variety of sectors, counting the nuclear, motorised, mechanical, and healthcare sectors, among many others[1]–[3].

Traditional FMEA analyses potential points of failure in terms of three risk factors: how often they are to occur, how severe their consequences, and how likely they are to go unnoticed (D). Failure modes' risk levels are prioritised using hazard numbers (RPNs) derived by multiplying the occurrence, severity, and distribution (OSD) of an event[4], [5]. Despite its usefulness as a screening instrument, the classic FMEA has been heavily criticised in the literature for a number of shortcomings. As a result of the FMEA team members' subjective and nebulous assessments, it is challenging to provide an accurate and comprehensive assessment of the risk variables[6], [7]. The old-style FMEA uses just the three risk variables O, S, and D to evaluate failure modes, which may overlook other crucial aspects of the process or system. This is not a realistic approach because (3) the relative relevance of risk variables is ignored. (4) There is some debate about whether or not RPNs should be used to rank the risks associated with various failure scenarios[8], [9].

To further illustrate the genesis and nature of neutralities across disciplines, a neutrosophic set has been developed as a generalisation of classical sets, fuzzy sets, intuitionistic fuzzy sets, and so on. F. Smarandache first proposed this multidimensional logic, which incorporates the concept of indeterminacy and is more informative than fuzzy logic. The results are superior than those obtained using fuzzy logic. Each statement in neutrosophic logic is assigned a truth value (T), an indeterminacy value (I), and a falsity value (F), where T, I, and F are either normal or non-normal subsets of $[0, 1+]$. It's challenging to work with data that doesn't fit neatly within a predetermined interval, which is why the $[0, 1]$ based single-valued neutrosophic set was developed.

Smarandache proposed the concepts of neutrosophic scale, neutrosophic total, and neutrosophic probability. There are many different forms of indeterminacies that might arise in practise, hence the neutrosophic total and neutrosophic likelihood are defined in many different ways. The logic of neutrosophy has been utilised by many scientists in many different areas.

The WASPAS approach was developed as a means of improving upon the efficacy of standard FMEA techniques by addressing their shortcomings[10]–[13].

One of the many MCDM approaches is the WASPAS technique. Zavadskas et al. created it[14]–[16]. The WASPAS technique is a novel hybrid of two common MCDM methods: the Weighted Sum Model (WSM) and the Weighted Product Model (WPM)[17]. Whereas WSM uses a simple weight matrix of the criterion values to get at a total rating, WPM uses the combination of the help analyze of every criterion multiplied by the weight of a specific criterion to arrive at an overall score. WASPAS also utilises similar techniques but aims for more precision by optimising a weighted aggregation function.

2. Trapezoidal Neutrosophic Sets

The trapezoidal neutrosophic sets is a type of neutrosophic sets[18]–[20], [20].

Definition 1:

$$T_A(x) = \begin{cases} \left(\frac{x-a}{b-a}\right) \gamma_A & a \leq x \leq b \\ \gamma_A & b \leq x \leq c \\ \left(\frac{d-x}{d-c}\right) \gamma_A & c \leq x \leq d \\ 0 & \text{otherwise} \end{cases}$$

$$I_A(x) = \begin{cases} \left(\frac{b-x}{b-a}\right) \beta_A & a \leq x \leq b \\ \beta_A & b \leq x \leq c \\ \left(\frac{d-x}{d-c}\right) \beta_A & c \leq x \leq d \\ 0 & \text{otherwise} \end{cases}$$

$$F_A(x) = \begin{cases} \left(\frac{b-x}{b-a}\right) \kappa_A & a \leq x \leq b \\ \kappa_A & b \leq x \leq c \\ \left(\frac{d-x}{d-c}\right) \kappa_A & c \leq x \leq d \\ 0 & \text{otherwise} \end{cases}$$

Definition 2[21]–[24]:

The derivative of a fuzzy valued function

$f: (a, b) \rightarrow R$ at x_0 is defined as follows:

$$f'(x_0) = \lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

$\langle \begin{matrix} f'(x_0) \text{ is } D_1 - \text{differentiable at } x_0 \text{ if } [f'(x_0)]\alpha = [f'_1(x_0, \alpha), f'_2(x_0, \alpha)] \\ f \text{ and } f'(x_0) \text{ is } D_2 - \text{differentiable at } x_0 \text{ if } [f'(x_0)]\alpha = [f'_2(x_0, \alpha), f'_1(x_0, \alpha)] \\ \text{all } \alpha \in [0,1] \end{matrix} \rangle$

Definition 3[25], [26]:

(y_1, y_2, y_3) a trapezoidal neutrosophic number ATRN
 $= \langle (a, b, c, d); \vartheta A, \nu A, \kappa A \rangle$ is defined as follows:

$$Ay_1, y_2, y_3 = [A_1(y_1), A_2(y_1)];$$

$$\langle \begin{matrix} [A_1(y_2), A_2(y_1)]; \\ [A_1(y_3), A_2(y_3)], \end{matrix} \rangle,$$

$$0 \leq y_1 + y_2 + y_3 \leq 3$$

$$A_1(y_1), A_2(y_1)] = [A_1(y_2), A_2(y_1)]$$

$$= [A_1(y_3), A_2(y_3)] = [(y_1 + y_1(b - y_1))\vartheta A,$$

where $\langle \begin{matrix} (d - y_1(d - c))\vartheta A, [(b - y_2(b - a))\nu A, \\ (c + y_2(d - c))\nu A], \\ [(b - y_3(b - a))\kappa A, (c + y_3(d - c))\kappa A \end{matrix} \rangle$

Definition 4:

$$y1(x, \alpha) = [(u1 + \alpha(u2 - u1))\vartheta1]cosq\sqrt{l} + 1sinq\sqrt{l}\{[(v1 + \alpha(v2 - v1))\vartheta2]$$

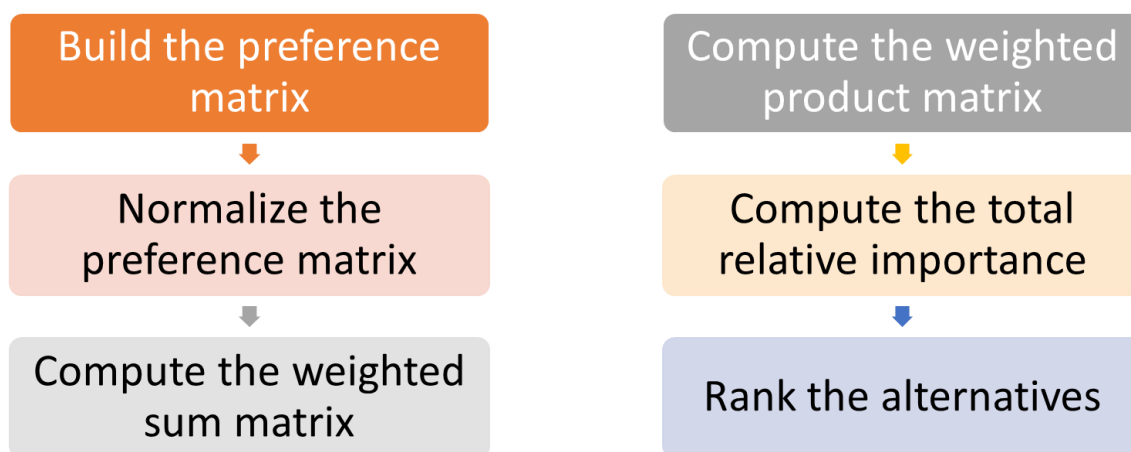


Figure 1: The proposed methodology.

3. The WASPAS method

A set of m viable options $A_i = 1, 2, \dots, m$ and n conditions $C_j (j = 1, 2, \dots, n)$ are also assumed in the WASPAS technique. The application phases of WASPAS technique are provided in the following

Step 1: Build the preference matrix

Let experts to assess the factors and options by the trapezoidal neutrosophic numbers. Then calculate the weights of criteria by the average technique.

Step 2: Normalize the preference matrix

The normalization is made on the positive and negative criteria as:

$$x_{ij}^* = \frac{x_{ij}}{\max_i x_{ij}} \quad i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n$$

$$x_{ij}^* = \frac{\min_i x_{ij}}{x_{ij}}$$

Step 3: Compute the weighted sum matrix

$$Q_i^1 = \sum_{j=1}^n x_{ij}^* * W_j$$

Step 4: Calculate the weighted product matrix

$$Q_i^2 = \prod_{j=1}^n (x_{ij}^*)^{W_j}$$

Step 5: Compute the total relative importance as:

$$Q_i = \lambda Q_i^1 + (1 - \lambda) Q_i^2$$

Step 6: Rank the alternatives

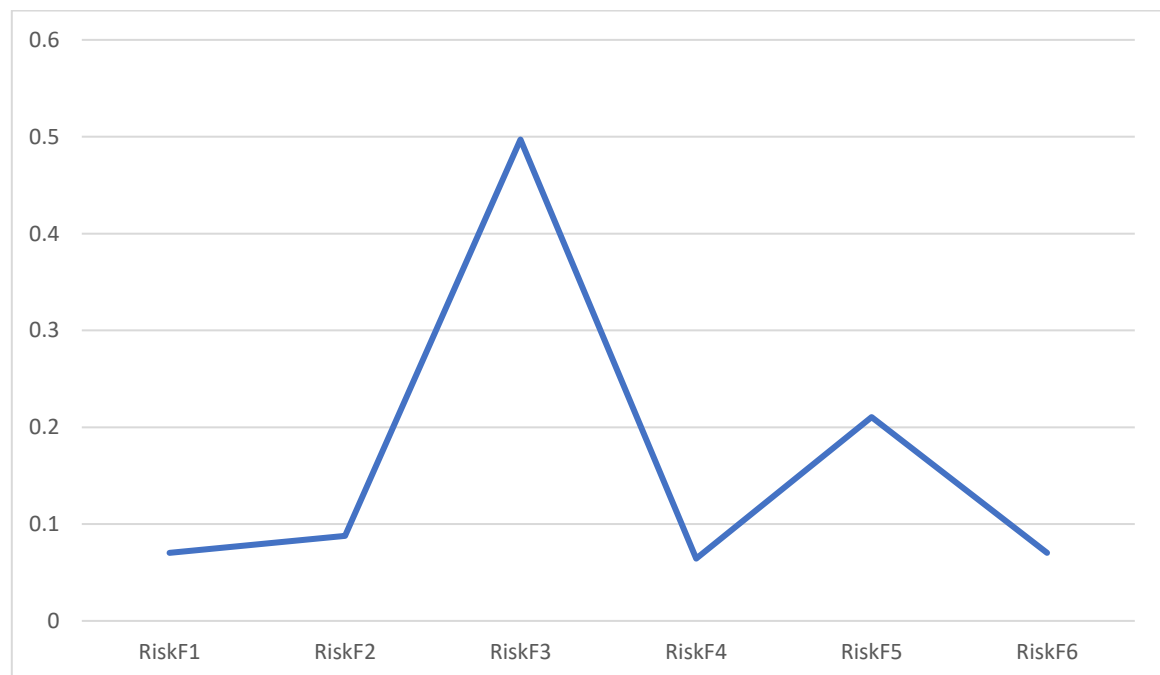


Figure 2: The weights of criteria.

4. The Application

Step 1: Build the preference matrix

The criteria (risks factors) are collected from the previous works such as: frequency, impact, repeatability, customer satisfaction, interdependency, and systematic detection. The alternatives (failures modes) failures mode 1,2,3,4. The weights of criteria are computed from the average method as figure 2. The criterion 3 is the highest weight and criterion 4 is the lowest weight.

Step 2: Normalize the preference matrix

Table 1 show the normalization matrix.

Table 1: The normalization matrix.

	RiskF1	RiskF2	RiskF3	RiskF4	RiskF5	RiskF6
RiskA1	1	0.92	0.789474	0.965909	1	0.75
RiskA2	0.5	0.84	1	0.75	0.619565	0.8125
RiskA3	0.444444	1	0.978947	0.875	0.630435	0.9375
RiskA4	0.416667	0.28	0.968421	1	0.75	1

Step 3: Compute the weighted sum matrix

Table 2.shows the values of WSM

Table 2. The WSM values.

	RiskF1	RiskF2	RiskF3	RiskF4	RiskF5	RiskF6
RiskA1	0.028879	0.025034	0.021913	0.026966	0.032083	0.025412
RiskA2	0.014439	0.022857	0.027756	0.020938	0.019878	0.02753
RiskA3	0.012835	0.027211	0.027172	0.024428	0.020226	0.031765
RiskA4	0.012033	0.007619	0.02688	0.027917	0.024062	0.033883

Step 4: Compute the weighted product matrix

Table 3. shows the WPM values.

Table 3: The WPM values.

	RiskF1	RiskF2	RiskF3	RiskF4	RiskF5	RiskF6
RiskA1	1	0.997734	0.99346	0.999032	1	0.9903
RiskA2	0.980182	0.995267	1	0.992001	0.984758	0.992989
RiskA3	0.976854	1	0.99941	0.996279	0.985308	0.997816
RiskA4	0.975035	0.965954	0.99911	1	0.990813	1

Step 5: Compute the total relative importance as:

The total relative importance is computed

Step 6: Rank the alternatives

Figure 3. shows the rank of alternatives. The risk 1 is the highest risk and risk 4 is the lowest risk.

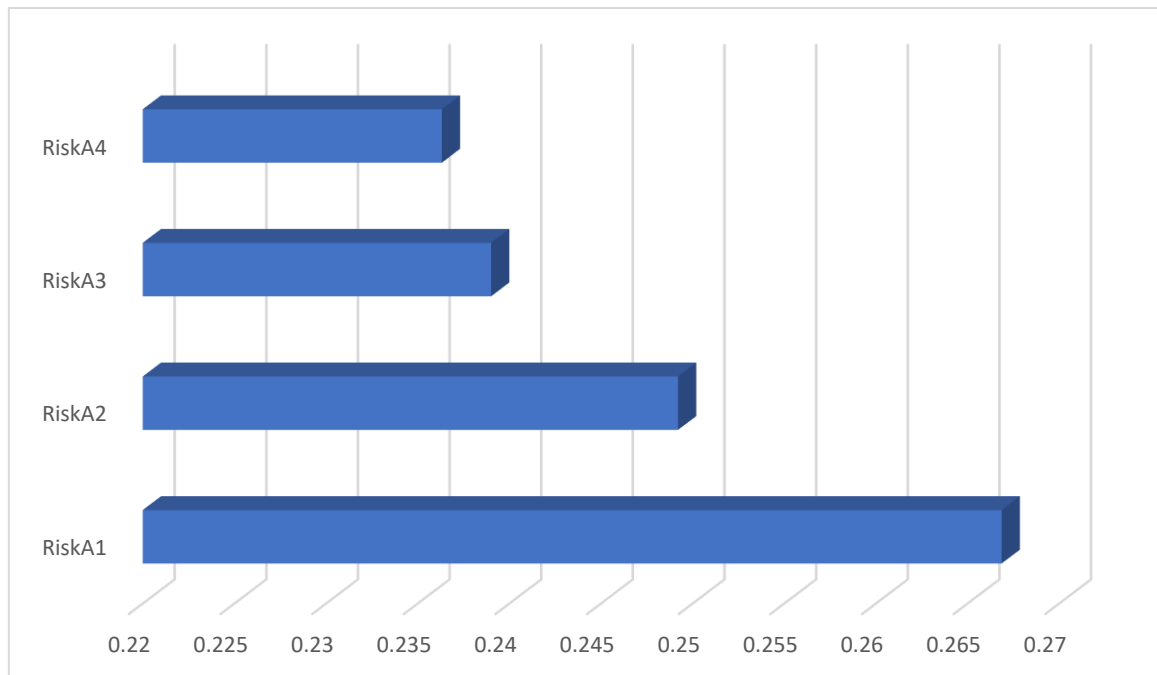


Figure 3: The rank of alternatives.

5. Conclusion

In order to assess and priorities the risk of failure modes in a trapezoidal neutrosophic sets environment, we present a novel FMEA model that combines the neutrosophic sets with the WASPAS method. The suggested risk assessment approach gets beyond the drawbacks of the standard FMEA by giving you a methodical and organized way to rate the severity of potential failures. The FMEA team provides all assessment information in natural language phrases, which are subsequently translated into trapezoidal neutrosophic numbers. After that, we apply the the neutrosophic WASPAS method to compute the rank of alternatives, which takes into account the relative relevance of risk variables, is then used to rank the risks associated with various failure scenarios. The benefits and applications of the proposed FMEA model were shown via the presentation of a application Also, the results demonstrate that the suggested hybrid MCDM model is more accurate, effective, and practical for risk assessment in FMEA in complex and uncertain settings.

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