



## A Study of a Neutrosophic Differential Equation by Using the One-Dimensional Geometric AH-Isometry

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### Abstract

In this paper, the definition of a Neutrosophic Differential Equation by Using the One-Dimensional Geometric AH-Isometry. The main objective is define a Neutrosophic identical linear differential equation and Neutrosophic non-homogeneous linear differential equation and find solutions for this equation.

**Keywords:** One-Dimensional Geometric AH-Isometry; Neutrosophic linear Differential Equation; Neutrosophic real number.

### 1. Introduction :

Neutrosophic logic. Neutrosophy, Neutrosophic set, Neutrosophic probability and alike, are recently creations of Smarandache, being characterized by having the indeterminacy as component of their framework, and a notable feature of neutrosophic logic is that can be considered a generaliazation of fuzzy logics, encompassing the classical logic as well [1]. Also. F. Smarandache, has defined the concept of continuation of a neutrosophic function in year 2015 in [1], and neutrosophic mereo-limit [1], mereo-continuity. Moreover, in 2014, he has defined the concept of a neutro-oscillator differential in [3], and mereo-derivative. Finally in 2013 he introduced neutrosophic integration in [2], and mereo-integral, besides the classical defintions of limit, continuity, deverative, and integral respectively.

Among the recent applications there are: neutrosophic crisp set theory in image processing [4][5], neutrosophic sets medical field [6,7,8,9,10], in information geographic systems [11] and possible applications to database [12]. Also, neutrosophic triplet group application to physics [13]. Moreover Several researches have made multiple contributions to neutrosophic topological [14,14,16,17,18,19,20]. Also More researches have made multiple contributions to neutrosophic analysis [21]. Finally the neutrosophic integration may have application in calculus the areas between two neutrosophic functions.

### 2. Preliminaries

#### Definition 2.1. Neutrosophic Real Number: [1]

Suppose that  $w$  is a neutrosophic number, then it takes the following standard form:  $w = a + bI$  where  $a, b$  are real coefficients, and  $I$  represents the indeterminacy, where  $0.I = 0$  and  $I^n = I$  for all positive integers  $n$ .

For example:  $w = 1 + 2I$ ,  $w = 3 = 3 + 0I$ .

#### Definition 2.2. Division of neutrosophic real numbers: [2]

Suppose that  $w_1, w_2$  are two neutrosophic number, where  $w_1 = a_1 + b_1I, w_2 = a_2 + b_2I$  Then:

$$\frac{w_1}{w_2} = \frac{a_1 + b_1I}{a_2 + b_2I} = \frac{a_1}{a_2} + \frac{a_2b_1 - a_1b_2}{a_2(a_2 + b_2)}I$$

**Definition 2.3**

Let  $R(I) = \{a + bI; a, b \in R\}$  where  $I^2 = I$  be the neutrosophic field of reals. The one-dimensional isometry (AH-Isometry) is defined as follows: [21]

$$T: R(I) \rightarrow R \times R$$

$$T(a + bI) = (a, a + b)$$

**Remark 2.4.** [21]

$T$  is an algebraic isomorphism between two rings, it has the following properties:

- 1)  $T$  is bijective.
- 2)  $T$  preserves addition and multiplication, i.e.:  
 $T[(a + bI) + (c + dI)] = T(a + bI) + T(c + dI)$  And  $T[(a + bI) \cdot (c + dI)] = T(a + bI) \cdot T(c + dI)$
- 3) Since  $T$  is bijective, then it is invertible by:  
 $T^{-1}: R \times R \rightarrow R(I)$ ,  $T^{-1}(a, b) = a + (b - a)I$
- 4)  $T$  preserves distances, i.e.:

The distance on  $R(I)$  can be defined as follows: Let  $A = a + bI, B = c + dI$  be two neutrosophic real numbers, then:

$$L = \|\overline{AB}\| = d[(a + bI, c + dI)] = |a + bI - (c + dI)| = |(a - c) + I(b - d)| = |a - c| + I[|a + b - c - d| - |a - c|].$$

On the other hand, we have:

$$T(\|\overline{AB}\|) = (|a - c|, |(a + b) - (c + d)|) = (d(a, c), d(a + b, c + d)) = d[(a, a + b), (c, c + d)] = d(T(a + bI), T(c + dI)) = \|T(\overline{AB})\|.$$

This implies that the distance is preserved up to isometry. i.e.  $\|T(AB)\| = T(\|AB\|)$

**Definition 2.5.**

Let  $f: R(I) \rightarrow R(I); f = f(X)$  and  $X = x + yI \in R(I)$  the  $f$  is called a neutrosophic real function with one neutrosophic variable.

a neutrosophic real function  $f(X)$  written as follows:

$$f(X) = f(x + yI) = f(x) + I[f(x + y) - f(x)]$$

**Theorem 2.6.** any neutrosophic real function into two classical real functions, i.e., to the classical Euclidean plane  $R \times R$ .

**Proof:**

Let  $f(X) = f(x + yI) = f(x) + I[f(x + y) - f(x)]$  a neutrosophic real function.

Now, Using the one-dimensional AH-isometry, we have.

$T(f(X)) = T(f(x) + I[f(x + y) - f(x)])$ , then.  $(f_1, f_2) = (f(x), f(x + y))$ , then, we have.

$$\begin{cases} f_1 = f(x) \\ f_2 = f(x + y) \end{cases}$$

the functions  $f(x), f(x + y)$  are a real functions.

**Definition 2.7.**

Let  $f(X) = f(x + yI) = f(x) + I[f(x + y) - f(x)]$  a neutrosophic function on  $R(I)$ , the we define a derivative of a neutrosophic function  $f(X)$  as follows:

$$f'(X) = f'(x + yI) = f'(x) + I[f'(x + y) - f'(x)]$$

**Definition 2.8.**

Let  $f(X) = f(x + yI) = f(x) + I[f(x + y) - f(x)]$  a neutrosophic function on  $R(I)$ , the we define a integration of a neutrosophic function  $f(X)$  as follows:

$$\int f(X) dX = \int f(x) dx + I \left[ \int f(x + y) d(x + y) - \int f(x) dx \right] + (a + bI)$$

Where  $(a + bI)$  is a neutrosophic constant number, and  $\int f(X) dX = F(X) = F(x) + I[F(x + y) - F(x)]$ .

**3. Neutrosophic linear differential equation.**

In this section is defined a linear differential equation by Using the One-Dimensional Geometric AH-Isometry and solutions are found for this equation.

**3.1 Neutrosophic identical linear differential equation.**

**Definition 3.2.**

Let  $Y = y_1 + y_2I$ ,  $X = x_1 + x_2I$  We define the Neutrosophic identical linear differential equation by Using the One-Dimensional Geometric AH-Isometry as form:

$$\dot{Y} + f(x)Y = 0$$

This equation can be written as follow:

$$\begin{aligned} (y_1' + I[(y_1 + y_2)' - y_1'])f(x_1) + I[f(x_1 + x_2) - f(x_1)](y_1 + y_2I) &= 0 \\ \Rightarrow y_1' + f(x_1)y_1 + I[(y_1 + y_2)' - f(x_1 + x_2)(y_1 + y_2) - (y_1' + f(x_1)y_1)] &= 0 \end{aligned}$$

**Method of solution.**

1. Take AH-Isometry for the differential equation, we have.

$$\begin{aligned} T(y_1' + f(x_1)y_1 + I[(y_1 + y_2)' - f(x_1 + x_2)(y_1 + y_2) - (y_1' + f(x_1)y_1)]) &= T(0) \\ \Rightarrow (y_1' + f(x_1)y_1, (y_1 + y_2)' - f(x_1 + x_2)(y_1 + y_2)) &= (0,0) \end{aligned}$$

Then.

$$\begin{cases} y_1' + f(x_1)y_1 = 0 \dots \dots (1) \\ (y_1 + y_2)' - f(x_1 + x_2)(y_1 + y_2) = 0 \dots \dots (2) \end{cases}$$

The equations (1) and (2) are two differential equation classical.

2. We find the solution to the equations classical (1) and (2), we have.

$y_1$  the solution to the equation (1).

$(y_1 + y_2)$  the solution to the equation (2).

3. We Take invertible AH-Isometry, then, we have the solution of a Neutrosophic identical linear differential equation .

$$Y = y_1 + y_2I = T^{-1}(y_1, y_1 + y_2) = y_1 + ((y_1 + y_2) - y_1)I$$

**Example 3.3.** Find a solution to the equation:

$$\dot{Y} + \frac{1}{X}Y = 0$$

**Solution.**

Let  $Y = y_1 + y_2I$ ,  $X = x_1 + x_2I$ . Then.

$$y_1' + \frac{1}{x_1}y_1 + I\left[(y_1 + y_2)' - \frac{1}{(x_1 + x_2)}(y_1 + y_2) - \left(y_1' + \frac{1}{x_1}y_1\right)\right] = (0)$$

Now, Take AH-Isometry for the differential equation, we have.

$$\begin{cases} y_1' + \frac{1}{x_1}y_1 = 0 \dots \dots (3) \\ (y_1 + y_2)' - \frac{1}{x_1 + x_2}(y_1 + y_2) = 0 \dots \dots (4) \end{cases}$$

The solution of equation (3) written as follow:

$$y_1 = e^{-\int \frac{1}{x_1} dx_1} + a = e^{-\ln x_1} + a = \frac{1}{x_1} + a, \text{ where } a \in R.$$

The solution of equation (4) written as follow:

$$(y_1 + y_2) = e^{-\int \frac{1}{x_1 + x_2} d(x_1 + x_2)} + b = e^{-\ln(x_1 + x_2)} + b = \frac{1}{x_1 + x_2} + b, \text{ where } b \in R$$

Now, Take invertible AH-Isometry, then, we have the solution of a Neutrosophic identical linear differential equation

$$\begin{aligned} Y = y_1 + y_2I &= T^{-1}\left(\frac{1}{x_1} + a, \frac{1}{x_1 + x_2} + b\right) = \frac{1}{x_1} + a + \left(\frac{1}{x_1 + x_2} + b - \frac{1}{x_1} - a\right)I \\ Y = y_1 + y_2I &= \frac{1}{x_1} + \left(\frac{1}{x_1 + x_2} - \frac{1}{x_1}\right)I + a + (b - a)I \end{aligned}$$

By Definition 2.2, we have.

$$\frac{1}{x_1} + \left(\frac{1}{x_1 + x_2} - \frac{1}{x_1}\right)I = \frac{1}{x_1} + \left(\frac{-x_2}{x_1(x_1 + x_2)}\right)I = \frac{1}{x_1 + x_2I} = \frac{1}{X}$$

So that,

$$Y = y_1 + y_2I = \frac{1}{X} + (a + bI)$$

where  $a + bI \in R(I)$ .

**Example 3.4.** Find a solution to the equation:

$$\dot{Y} + 2XY = 0$$

**Solution.**

Let  $Y = y_1 + y_2I$ ,  $X = x_1 + x_2I$ . Then.  $(y_1' + 2x_1y_1, (y_1 + y_2)' - 2(x_1 + x_2)(y_1 + y_2)) = (0)$

Now, Take AH-Isometry for the differential equation, we have.

$$\begin{cases} y_1' + 2x_1y_1 = 0 \dots \dots (5) \\ (y_1 + y_2)' - 2(x_1 + x_2)(y_1 + y_2) = 0 \dots \dots (6) \end{cases}$$

The solution of equation (5) written as follow:

$$y_1 = e^{-\int 2x_1 dx_1} + a = e^{-(x_1)^2} + a, \text{ where } a \in R.$$

The solution of equation (6) written as follow:

$$(y_1 + y_2) = e^{-\int 2(x_1+x_2) d(x_1+x_2)} + b = e^{-(x_1+x_2)^2} + b, \text{ where } b \in R$$

Now, Take invertible AH-Isometry, then, we have the solution of a Neutrosophic identical linear differential equation

$$Y = y_1 + y_2I = T^{-1}(e^{-(x_1)^2} + a, e^{-(x_1+x_2)^2} + b) = \frac{1}{x_1} + a + (e^{-(x_1+x_2)^2} + b - e^{-(x_1)^2} - a)I$$

$$Y = y_1 + y_2I = e^{-(x_1)^2} + (e^{-(x_1+x_2)^2} - e^{-(x_1)^2})I + a + (b - a)I$$

By Definition 2.5, we have.

$$e^{-(x_1)^2} + (e^{-(x_1+x_2)^2} - e^{-(x_1)^2})I = e^{-X^2}$$

So that,

$$Y = y_1 + y_2I = e^{-X^2} + (a + bI)$$

where  $a + bI \in R(I)$ .

**3.5 Neutrosophic non-homogeneous linear differential equation.****Definition 3.6.**

We define the Neutrosophic non- homogeneous linear differential equation is taken the following forms:

$$\dot{Y} + f(X)Y = g(X)$$

This equation can be written as follow:

$$\begin{aligned} (y_1' + I[(y_1 + y_2)' - y_1'])(f(x_1) + I[f(x_1 + x_2) - f(x_1)])(y_1 + y_2I) &= g(x_1) + I[g(x_1 + x_2) - g(x_1)] \\ \Rightarrow y_1' + f(x_1)y_1 + I[(y_1 + y_2)' - f(x_1 + x_2)(y_1 + y_2) - (y_1' + f(x_1)y_1)] &= g(x_1) + I[g(x_1 + x_2) - g(x_1)] \end{aligned}$$

**Method of solution.**

1. Take AH-Isometry for the differential equation, we have.

$$\begin{aligned} T(y_1' + f(x_1)y_1 + I[(y_1 + y_2)' - f(x_1 + x_2)(y_1 + y_2) - (y_1' + f(x_1)y_1)]) &= T(g(x_1) + I[g(x_1 + x_2) - g(x_1)]) \\ \Rightarrow (y_1' + f(x_1)y_1, (y_1 + y_2)' - f(x_1 + x_2)(y_1 + y_2)) &= (g(x_1), g(x_1 + x_2)) \end{aligned}$$

Then.

$$\begin{cases} y_1' + f(x_1)y_1 = g(x_1) \dots \dots (7) \\ (y_1 + y_2)' - f(x_1 + x_2)(y_1 + y_2) = g(x_1 + x_2) \dots \dots (8) \end{cases}$$

The equations (7) and (8) are two differential equation classical.

2. We find the solution to the equations classical (7) and (8), we have.  $y_1$  the solution to the equation (7).  $(y_1 + y_2)$  the solution to the equation (8).
3. We Take invertible AH-Isometry, then, we have the solution of a Neutrosophic identical linear differential equation .

$$Y = y_1 + y_2I = T^{-1}(y_1, y_1 + y_2) = y_1 + ((y_1 + y_2) - y_1)I$$

**Example 3.7.** Find the general solution for the following neutrosophic non- homogeneous linear differential equation:

$$\dot{Y} + 2XY = X$$

**Solution.**

Let  $Y = y_1 + y_2I$ ,  $X = x_1 + x_2I$ . Then.  $(y_1' + 2x_1y_1, (y_1 + y_2)' - 2(x_1 + x_2)(y_1 + y_2)) = (x_1, x_1 + x_2)$

Now, Take AH-Isometry for the differential equation, we have.

$$\begin{cases} y_1' + 2x_1y_1 = x_1 \dots \dots (9) \\ (y_1 + y_2)' - 2(x_1 + x_2)(y_1 + y_2) = x_1 + x_2 \dots \dots (10) \end{cases}$$

The solution of equation (10) written as follow:

$$y_1 = \frac{1}{\mu(x_1)}(a + \int x_1\mu(x_1)dx_1) = \frac{1}{e^{(x_1)^2}}(a + \int x_1e^{(x_1)^2}dx) = \frac{1}{e^{(x_1)^2}}(a + \frac{1}{2}e^{(x_1)^2}), \text{ where } a \in R.$$

By the method same, The solution of equation (10) written as follow:

$$(y_1 + y_2) = \frac{1}{e^{(x_1+x_2)^2}}(b + \frac{1}{2}e^{(x_1+x_2)^2}), \text{ where } b \in R$$

Now, Take invertible AH-Isometry, then, we have the solution of a Neutrosophic identical linear differential equation

$$\begin{aligned}
 Y = y_1 + y_2I &= T^{-1} \left( \frac{1}{e^{(x_1)^2}} \left( a + \frac{1}{2} e^{(x_1)^2} \right), \frac{1}{e^{(x_1+x_2)^2}} \left( b + \frac{1}{2} e^{(x_1+x_2)^2} \right) \right) \\
 &= T^{-1} \left( \frac{1}{e^{(x_1)^2}}, \frac{1}{e^{(x_1+x_2)^2}} \right) \cdot \left[ T^{-1}(a, b) + T^{-1} \left( \frac{1}{2} e^{(x_1)^2}, \frac{1}{2} e^{(x_1+x_2)^2} \right) \right] \\
 Y = y_1 + y_2I &= \left[ \frac{1}{e^{(x_1)^2}} + \left( \frac{1}{e^{(x_1+x_2)^2}} - \frac{1}{e^{(x_1)^2}} \right) I \right] \cdot \left[ a + (b - a)I + \frac{1}{2} e^{(x_1)^2} + \left( \frac{1}{2} e^{(x_1+x_2)^2} - \frac{1}{2} e^{(x_1)^2} \right) I \right]
 \end{aligned}$$

By Definition 2.5, we have.

$$\begin{aligned}
 \frac{1}{e^{(x_1)^2}} + \left( \frac{1}{e^{(x_1+x_2)^2}} - \frac{1}{e^{(x_1)^2}} \right) I &= \frac{1}{e^{(x_1+x_2)^2}} = \frac{1}{e^{X^2}} \\
 \text{And } \frac{1}{2} e^{(x_1)^2} + \left( \frac{1}{2} e^{(x_1+x_2)^2} - \frac{1}{2} e^{(x_1)^2} \right) I &= \frac{1}{2} \left[ e^{(x_1)^2} + (e^{(x_1+x_2)^2} - e^{(x_1)^2}) I \right] = \frac{1}{2} e^{(x_1+x_2)^2} = \frac{1}{2} e^{X^2}
 \end{aligned}$$

So that,

$$Y = y_1 + y_2I = \frac{1}{e^{X^2}} \left[ a + bI + \frac{1}{2} e^{X^2} \right]$$

where  $a + bI \in R(I)$ .

**Example 3.8.** Find the general solution for the following neutrosophic non- homogeneous linear differential equation:

$$\dot{Y} + \cot g(X)Y = \sin(X)$$

**Solution.**

Let  $Y = y_1 + y_2I, X = x_1 + x_2I$ . Then.

$$(y_1' + \cot g(x_1)y_1, (y_1 + y_2)' + \cot g(x_1 + x_2)(y_1 + y_2)) = (\sin(x_1), \sin(x_1 + x_2))$$

Now, Take AH-Isometry for the differential equation, we have.

$$\begin{cases}
 y_1' + \cot g(x_1)y_1 = \sin(x_1) \dots \dots (11) \\
 (y_1 + y_2)' + \cot g(x_1 + x_2)(y_1 + y_2) = \sin(x_1 + x_2) \dots \dots (12)
 \end{cases}$$

The solution of equation (11) written as follow:

$$y_1 = \frac{1}{\mu(x_1)} \left( a + \int x_1 \mu(x_1) dx_1 \right) = \frac{1}{\sin(x_1)} \left( a + \int \sin(x_1) \cdot \sin(x_1) dx \right) = \frac{1}{\sin(x_1)} \left( a + \int \sin^2(x_1) dx \right)$$

$$y_1 = \frac{1}{\sin(x_1)} \left( a + \frac{1}{2} x_1 + \frac{1}{4} \sin(2x_1) \right), \text{ where } a \in R.$$

By the method same, The solution of equation (12) written as follow:

$$(y_1 + y_2) = \frac{1}{\sin(x_1+x_2)} \left( b + \frac{1}{2} (x_1 + x_2) + \frac{1}{4} \sin(2(x_1 + x_2)) \right), \text{ where } b \in R$$

Now, Take invertible AH-Isometry, then, we have the solution of a Neutrosophic identical linear differential equation

$$Y = y_1 + y_2I = T^{-1} \left( \frac{1}{\sin(x_1)} \left( a + \frac{1}{2} x_1 + \frac{1}{4} \sin(2x_1) \right), \frac{1}{\sin(x_1 + x_2)} \left( b + \frac{1}{2} (x_1 + x_2) + \frac{1}{4} \sin(2(x_1 + x_2)) \right) \right)$$

$$\begin{aligned}
 Y = y_1 + y_2I &= T^{-1} \left( \frac{1}{\sin(x_1)}, \frac{1}{\sin(x_1 + x_2)} \right) \cdot \left[ T^{-1}(a, b) + T^{-1} \left( \frac{1}{2} x_1, \frac{1}{2} (x_1 + x_2) \right) \right. \\
 &\quad \left. + T^{-1} \left( \frac{1}{4} \sin(2x_1), \frac{1}{4} \sin(2(x_1 + x_2)) \right) \right]
 \end{aligned}$$

$$\begin{aligned}
 Y = y_1 + y_2I &= \left[ \frac{1}{\sin(x_1)} + \left( \frac{1}{\sin(x_1 + x_2)} - \frac{1}{\sin(x_1)} \right) I \right] \cdot \left[ a + (b - a)I + \frac{1}{2} x_1 + \left( \frac{1}{2} (x_1 + x_2) - \frac{1}{2} x_1 \right) I \right. \\
 &\quad \left. + \frac{1}{4} \sin(2x_1) + \left( \frac{1}{4} \sin(2(x_1 + x_2)) - \frac{1}{4} \sin(2x_1) \right) I \right]
 \end{aligned}$$

By Definition 2.5, we have.

$$\frac{1}{\sin(x_1)} + \left( \frac{1}{\sin(x_1 + x_2)} - \frac{1}{\sin(x_1)} \right) I = \frac{1}{\sin(x_1 + x_2)} = \frac{1}{\sin(X)}$$

$$\text{And } \frac{1}{2} x_1 + \left( \frac{1}{2} (x_1 + x_2) - \frac{1}{2} x_1 \right) I = \frac{1}{2} \left[ x_1 + ((x_1 + x_2) - x_1) I \right] = \frac{1}{2} (x_1 + x_2) I = \frac{1}{2} X$$

$$\begin{aligned}
 \text{And } \frac{1}{4} \sin(2x_1) + \left( \frac{1}{4} \sin(2(x_1 + x_2)) - \frac{1}{4} \sin(2x_1) \right) I &= \frac{1}{4} \left[ \sin(2x_1) + (\sin(2(x_1 + x_2)) - \sin(2x_1)) I \right] = \\
 \frac{1}{4} \sin 2(x_1 + x_2) I &= \frac{1}{4} \sin 2X
 \end{aligned}$$

So that,

$$Y = y_1 + y_2 I = \frac{1}{\sin(X)} \left[ a + bI + \frac{1}{2} X + \frac{1}{4} \sin 2X \right]$$

where  $a + bI \in R(I)$ .

#### 4. Conclusion

In this paper, a new type of neutrosophic integration has been defined by using the thick function, Moreover, we studied a linear differential equation based on the thick function and found solutions to this equation. Also solutions of other types of neutrosophic differential equations can be found depending on the thick function such as Bernoulli's equation. We will work on this in the future.

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