



## A Study of Neutrosophic Real Analysis by Using the One-Dimensional Geometric AH-Isometry

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### Abstract

The objective of this paper is to study and define the neutrosophic real functions with one neutrosophic variable depending on the geometric isometry (AH-Isometry), with a lot of concepts from real analysis including continuity, differentiability, integrability. We have presented the formal forms of different popular functions in neutrosophic environment like logarithmic function, exponential function, trigonometric functions. Raising neutrosophic numbers to any power is well defined including rising to neutrosophic powers.

**Keywords:** Neutrosophic real analysis; AH-isometry; Integration; Differentiation; Continuity.

### 1. Introduction

Neutrosophy is a new branch of philosophy concerns with the indeterminacy in all areas of life and science. It has become a useful tool in generalizing many classical systems such as equations [30], number theory, topology, linear spaces, modules, and ring of matrices [1-7,12-21].

In the literature, we find many studies about neutrosophic calculus, where some definitions and properties were presented about neutrosophic real functions and numbers [9-11,23-33].

The neutrosophic real functions with one variable were defined only in a special case, as follows:

$f(x) = g(x) + h(x)I$  where  $I$  takes an interval value defining what is called by neutrosophic thick functions. For example  $f(x) = 2x + 5xI, I \in [0, 0.01]$  is a neutrosophic real thick function.

The problem with this definition, that it does not consider the general case  $f: R(I) \rightarrow R(I); f = f(X)$  and  $X = x + yI \in R(I)$ .

Recently, Abobala et.al, have presented the concept of two-dimensional AH-isometry to study the correspondence between neutrosophic plane  $R(I) \times R(I)$  and the classical module  $R^2 \times R^2$ . Also, the one-dimensional AH-isometry between  $R(I)$  and  $R \times R$ . This isometry was useful in defining inner products and norms, ordering, and neutrosophic geometrical shapes.

In this work, we use the one-dimensional AH-isometry to turn the general case of neutrosophic real functions with one variable into two classical real functions so we will go from  $R(I)$  space into  $R \times R$  space, we study the properties of our functions then we go back to  $R(I)$  space using AH-isometry.

This work will provide for the first time an algorithm to compute the neutrosophic powers of neutrosophic numbers including neutrosophic powers which wasn't studied before, we will present and define neutrosophic continuity, differentiation, integration and lots of popular neutrosophic functions like neutrosophic exponential function, neutrosophic logarithmic function and neutrosophic trigonometric functions.

Definitions and theorems presented in this paper are very useful to define mathematically lots of concepts including differential equations, integral equations, probability distribution functions,... etc.

## 2. Neutrosophic Functions on $R(I)$

### Definition 2.1

Let  $R(I) = \{a + bI ; a, b \in R\}$  where  $I^2 = I$  be the neutrosophic field of reals. The one-dimensional isometry (AH-Isometry) is defined as follows: [49]

$$T: R(I) \rightarrow R \times R$$

$$T(a + bI) = (a, a + b)$$

#### Remark:

$T$  is an algebraic isomorphism between two rings, it has the following properties:

1)  $T$  is bijective.

2)  $T$  preserves addition and multiplication, i.e.:

$$T[(a + bI) + (c + dI)] = T(a + bI) + T(c + dI)$$

And

$$T[(a + bI) \cdot (c + dI)] = T(a + bI) \cdot T(c + dI)$$

3) Since  $T$  is bijective, then it is invertible by:

$$T^{-1}: R \times R \rightarrow R(I)$$

$$T^{-1}(a, b) = a + (b - a)I$$

4)  $T$  preserves distances, i.e.:

The distance on  $R(I)$  can be defined as follows:

$$\text{Let } A = a + bI, B = c + dI \text{ be two neutrosophic real numbers, then } L = \|\overline{AB}\| = d[(a + bI, c + dI)] = |a + bI - (c + dI)| = |(a - c) + I(b - d)| = |a - c| + I[|a + b - c - d| - |a - c|].$$

On the other hand, we have:

$$T(\|\overline{AB}\|) = (|a - c|, |(a + b) - (c + d)|) = (d(a, c), d(a + b, c + d)) = d[(a, a + b), (c, c + d)] = d(T(a + bI), T(c + dI)) = \|T(\overline{AB})\|.$$

This implies that the distance is preserved up to isometry. i.e.  $\|T(AB)\| = T(\|AB\|)$

### Definition 2.2

Let  $f: R(I) \rightarrow R(I); f = f(X)$  and  $X = x + yI \in R(I)$  the  $f$  is called a neutrosophic real function with one neutrosophic variable.

#### Example:

Take  $f: R(I) \rightarrow R(I); f(X) = X^2 + IX + 2I = (x + yI)^2 + I(x + yI) + 2I = x^2 + I(y^2 + 2xy + x + y + 2)$

In the following we are going to show how to study analytical properties of the function defined in definition 2.2

#### Remark:

Using the one-dimensional AH-isometry we can turn any neutrosophic real function into two classical real functions, i.e., to the classical Euclidean plane  $R \times R$ .

#### Example:

Consider the following neutrosophic real function

$$f: R(I) \rightarrow R(I) : f(X) = IX - 1 + 3I,$$

We can write:  $T(f(X)) = T(I)T(X) + T(-1 + 3I)$

$$= (0,1)(x, x + y) + (-1,2),$$

$$= (-1, x + y + 2),$$

Assuming that  $X = 1 + I$ , i.e.,  $x = y = 1$  we get:

$$T(f(1 + I)) = (-1,4) \Rightarrow f(1 + I) = T^{-1}(-1,4) = -1 + 5I,$$

By direct computing we can find:

$$f(1 + I) = I(1 + I) - 1 + 3I = -1 + 5I,$$

### Theorem 2.1

Let  $f: R(I) \rightarrow R(I)$  be a neutrosophic real function with one variable,  $X = x + yI \in R(I)$  then  $f$  can be turned into two classical real functions.

**Proof**

Since the direct image of the variable  $X$  by the one-dimensional AH-isometry is  $T(X) = (x, x + y)$  we get that  $T(f(X)) = (h(x), g(x + y))$  where:  $h, g: R \rightarrow R$  are classical real functions.

**Definition 2.3**

Let  $f: R(I) \rightarrow R(I)$  be a neutrosophic real function with one variable. We say that  $f$  is integrable, continuous, or differentiable on  $]a + bI, c + dI[$  iff  $T(f(X))$  is integrable, continuous, or differentiable on  $T( ]a + bI, c + dI[ )$

**Example:**

Let  $f: R(I) \rightarrow R(I); f(X) = \frac{1}{2}X^2 + I$  we have:

- 1)  $1 + I \leq 1 + 2I$  because  $1 \leq 1, 1 + 1 \leq 1 + 2$
- 2)  $f$  is integrable on  $[1 + I, 1 + 2I]$  that is because

$$\begin{aligned} T(f(X)) &= T\left(\frac{1}{2}\right)T(X^2) + T(I), \\ &= \left(\frac{1}{2}, \frac{1}{2}\right)(x^2, (x + y)^2) + (0, 1), \\ &= \left(\frac{1}{2}x^2, \frac{1}{2}(x + y)^2 + 1\right), \\ T([1 + I, 1 + 2I]) &= [(1, 2), (1, 3)] = ([1, 1], [2, 3]), \end{aligned}$$

- 3) Now, we integrate  $f$  as following:

- a) by using  $T$ :

We have:

$$\int_1^{1+2I} \frac{1}{2}x^2 dx = 0,$$

Also:

$$\begin{aligned} &\int_2^3 \left[\frac{1}{2}(x + y)^2 + 1\right] d(x + y), \\ &= \left[\frac{1}{6}(x + y)^3 + (x + y)\right]_2^3, \\ &= \frac{1}{6}27 + 3 - \frac{1}{6}8 - 2 = \frac{25}{6}, \end{aligned}$$

So:

$$T^{-1}\left(0, \frac{25}{6}\right) = \frac{25}{6}I = \int_{1+I}^{1+2I} f(X) dX,$$

- b) by direct computing:

$$\begin{aligned} \int_{1+I}^{1+2I} \left(\frac{1}{2}X^2 + I\right) dX &= \left[\frac{1}{6}X^3 + IX\right]_{1+I}^{1+2I}, \\ &= \frac{1}{6}(1 + 2I)^3 + I(1 + 2I) - \frac{1}{6}(1 + I)^3 - I(1 + I), \\ &= \frac{1}{6}(1 + 6I + 12I + 8I) + 3I - \frac{1}{6}(1 + 3I + 3I + I), \\ &= \frac{25}{6}I, \end{aligned}$$

Notice that we get the same result!

**3. Computing Powers in  $R(I)$**

To compute such equation:  $(a + bI)^{c+dI}; a, b, c, d \in R$  we need the one-dimensional isometry again:

$$T[(a + bI)^{c+dI}] = (a, a + b)^{(c, c+d)} = (a^c, (a + b)^{c+d}),$$

Which results:

$$\begin{aligned} (a + bI)^{c+dI} &= T^{-1}(a^c, (a + b)^{c+d}), \\ &= a^c + I[(a + b)^{c+d} - a^c]. \end{aligned}$$

**Example:**

let  $A = (2 + I)^I$ , we have:

$$T(A) = (2, 3)^{(0, 1)} = (2^0, 3^1) = (1, 3).$$

$$\Rightarrow A = T^{-1}(1, 3) = 1 + 2I.$$

**4. Neutrosophic Trigonometric Functions:**

In the following theorem, we are going to represent and provide formulas of trigonometric functions with neutrosophic angle of the form  $\theta_N = a + bI$  and some properties according to it.

**Theorem 4.1:**

Let  $R(I)$  be the neutrosophic field of reals, we have:

1.  $\sin(a + bI) = \sin a + I[\sin(a + b) - \sin a]$
2.  $\cos(a + bI) = \cos a + I[\cos(a + b) - \cos a]$
3.  $\tan(a + bI) = \tan a + I[\tan(a + b) - \tan a]$
4.  $\sin^2(a + bI) + \cos^2(a + bI) = 1$

**Proof:**

$$\begin{aligned}
 1. \quad \sin(a + bI) &= \frac{e^{a+bI} - e^{-(a+bI)}}{2i} = A, i^2 = -1. \\
 T(A) &= T\left(\frac{1}{2i}\right)T(e^{a+bI} - e^{-(a+bI)}) \\
 &= \left(\frac{1}{2i}\right)[(e, e)^{(a, a+b)} - (e, e)^{(-a, -a-b)}] \\
 &= \frac{1}{2i}[(e^a, e^{a+b}) - (e^{-a}, e^{-a-b})] \\
 \Rightarrow A &= \frac{1}{2i}(e^a + (e^{a+b} - e^a)I - e^{-a} - I(e^{-a-b} - e^{-a})) \\
 &= \frac{e^a - e^{-a}}{2i} + I\left[\frac{e^{a+b} - e^{-a-b}}{2i} - \frac{e^a - e^{-a}}{2i}\right] \\
 &= \sin a + I[\sin(a + b) - \sin a]
 \end{aligned}$$

Notice that we can compute  $\sin(a + bI)$  directly as follows:

$$\begin{aligned}
 T(\sin(a + bI)) &= \sin[T(a + bI)] = \sin(a, a + b) = (\sin a, \sin(a + b)), \\
 \Rightarrow \sin(a + bI) &= \sin a + I[\sin(a + b) - \sin a],
 \end{aligned}$$

2. similar to proof 1
3. similar to proof 1
4.  $\sin^2(a + bI) = (\sin a + I[\sin(a + b) - \sin a])^2$   
 $= \sin^2 a + 2I \sin a \sin(a + b) - 2I \sin^2 a + I(\sin^2(a + b) - 2 \sin(a + b) \sin a + \sin^2 a),$   
 $= \sin^2 a + I[2 \sin a \sin(a + b) - 2 \sin^2 a + \sin^2(a + b) - 2 \sin(a + b) \sin a + \sin^2 a],$   
 $= \sin^2 a + I[\sin^2(a + b) - \sin^2 a],$

Similarly, we find that:

$$\cos^2(a + bI) = \cos^2 a + I[\cos^2(a + b) - \cos^2 a],$$

So:

$$\begin{aligned}
 \sin^2(a + bI) + \cos^2(a + bI) &= \sin^2 a + \cos^2 a + I[\sin^2(a + b) - \sin^2 a + \cos^2(a + b) - \cos^2 a], \\
 &= 1 + I[1 - 1], \\
 &= 1.
 \end{aligned}$$

**Theorem 4.2:**

1.  $-1 \leq \sin(a + bI) \leq 1$
2.  $-1 \leq \cos(a + bI) \leq 1$

**Proof:**

1. We have  $(-1, -1) \leq T[\sin(a + bI)] = (\sin a, \sin(a + b)) \leq (1, 1)$ , thus  $T^{-1}(-1, -1) \leq \sin(a + bI) \leq T^{-1}(1, 1)$ , i. e.  $-1 \leq \sin(a + bI) \leq 1$ .
2. The proof is similar to 1.

**Remarks:**

1.  $\sin I\pi = \sin 0 + I[\sin \pi - \sin 0] = 0$
2.  $\cos I\pi = \cos 0 + I[\cos \pi - \cos 0] = 1 + I[-1 - 1] = 1 - 2I$
3.  $\sin \frac{\pi}{2}I = \sin 0 + I\left[\sin \frac{\pi}{2} - \sin 0\right] = I$
4.  $\cos \frac{\pi}{2}I = \cos 0 + I\left[\cos \frac{\pi}{2} - \cos 0\right] = 1 - I$
5.  $\sin \frac{\pi}{4}I = \frac{1}{\sqrt{2}}I$
6.  $\cos \frac{\pi}{4}I = 1 + \left(\frac{1}{\sqrt{2}} - 1\right)I$
7.  $\tan \frac{\pi}{4}I = I = \frac{\frac{1}{\sqrt{2}}I}{1 + \left(\frac{1}{\sqrt{2}} - 1\right)I}$

**5. Neutrosophic exponential and logarithmic functions:**

In this section we are going to define and find the mathematical form of the function  $f(X) = e^X$  where  $X = x + Iy$ :

**Theorem 5.1**

Let  $R(I)$  be the neutrosophic field of reals, we have:

1.  $e^{x+Iy} = e^x + I(e^{x+y} - e^x)$
2.  $\ln(x + Iy) = \ln x + I(\ln(x + y) - \ln(x))$ , where  $x + yI > 0$ .

**Proof:**

1. Let  $A = e^X$ :

$$T(A) = T(e^X) = T(e^{x+Iy}) = e^{T(x+Iy)} = e^{(x, x+y)} = (e^x, e^{x+y}) \Rightarrow A = T^{-1}(e^x, e^{x+y}) = e^x + I(e^{x+y} - e^x).$$

2. Let:

$$a_1 + a_2I = \ln X$$

$$\begin{aligned} \Rightarrow X = x + Iy &= e^{a_1+a_2I} = e^{a_1} + I(e^{a_1+a_2} - e^{a_1}) \\ \Rightarrow x = e^{a_1}, y &= e^{a_1+a_2} - e^{a_1} \\ \Rightarrow a_1 = \ln(x), y &= x \cdot e^{a_2} - x \\ &\Rightarrow \frac{y}{x} + 1 = e^{a_2} \\ \Rightarrow a_2 &= \ln\left(\frac{x+y}{x}\right) = \ln(x+y) - \ln(x) \end{aligned}$$

Which yields:

$$\ln(x + Iy) = \ln x + I(\ln(x + y) - \ln(x))$$

**Example 5.1:**

Let's calculate  $e^I, \ln(1 + I)$ :

$$\begin{aligned} e^I &= e^{0+I} = e^0 + (e - e^0)I = 1 + (e - 1)I \\ \ln(1 + I) &= \ln(1) + I(\ln(2) - \ln(1)) = \ln(2)I \end{aligned}$$

It is easy to check that the properties of exponential/logarithmic function are still true in the neutrosophic case.

**6. Conclusions**

In this paper we have studied some concepts of neutrosophic real analysis depending on the one-dimensional AH-isometry. We have provided a strict definition of continuity, integration and differentiation of a neutrosophic functions with neutrosophic variables. We have presented an algorithm to compute powers of neutrosophic numbers to neutrosophic powers. Definitions of trigonometric functions, exponential functions and logarithmic functions were presented and lots of their properties had been proved.

**Future research directions**

Content of this work is very helpful in defining and solving many open problems in neutrosophic theory including differential equations, integral equations and probability distribution functions. We aim to do a lots of works in these branches in the future.

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**References**

- [1] Abobala, M., "AH-Subspaces in Neutrosophic Vector Spaces", International Journal of Neutrosophic Science, Vol. 6 , pp. 80-86. 2020.
- [2] Abobala, M.,. "A Study of AH-Substructures in *n*-Refined Neutrosophic Vector Spaces", International Journal of Neutrosophic Science", Vol. 9, pp.74-85. 2020.
- [3] Alhamido, R., and Abobala, M., "AH-Substructures in Neutrosophic Modules", International Journal of Neutrosophic Science, Vol. 7, pp. 79-86 . 2020.
- [4] Smarandache, F., " A Unifying Field in Logics: Neutrosophic Logic, Neutrosophy, Neutrosophic Set, Neutrosophic Probability", American Research Press. Rehoboth, 2003.
- [5] Suresh, R., and S. Palaniammal,. "Neutrosophic Weakly Generalized open and Closed Sets", Neutrosophic Sets and Systems, Vol. 33, pp. 67-77,. 2020.
- [6] Olgun, N., and Hatip, A., "The Effect Of The Neutrosophic Logic On The Decision Making, in Quadruple Neutrosophic Theory And Applications", Belgium, EU, Pons Editions Brussels,pp. 238-253. 2020.
- [7] Abobala, M., "Classical Homomorphisms Between *n*-refined Neutrosophic Rings", International Journal of Neutrosophic Science", Vol. 7, pp. 74-78. 2020.

- [8] Smarandache, F., and Abobala, M., n-Refined neutrosophic Rings, International Journal of Neutrosophic Science, Vol. 5 , pp. 83-90, 2020.
- [9] Abobala, M., On Some Special Substructures of Neutrosophic Rings and Their Properties, International Journal of Neutrosophic Science", Vol. 4 , pp. 72-81, 2020.
- [10] Abobala, M., "On Some Special Substructures of Refined Neutrosophic Rings", International Journal of Neutrosophic Science, Vol. 5, pp. 59-66. 2020.
- [11] Sankari, H., and Abobala, M., " AH-Homomorphisms In neutrosophic Rings and Refined Neutrosophic Rings", Neutrosophic Sets and Systems, Vol. 38, pp. 101-112, 2020.
- [12] Smarandache, F., and Kandasamy, V.W.B., " Finite Neutrosophic Complex Numbers",.Source: arXiv. 2011.
- [13] Agboola, A.A.A., Akwu, A.D., and Oyebo, Y.T., " Neutrosophic Groups and Subgroups", International .J .Math. Combin, Vol. 3, pp. 1-9. 2012.
- [14] Smarandache, F., " *n*-Valued Refined Neutrosophic Logic and Its Applications in Physics", Progress in Physics, 143-146, Vol. 4, 2013.
- [15] Adeleke, E.O., Agboola, A.A.A.,and Smarandache, F., "Refined Neutrosophic Rings I", International Journal of Neutrosophic Science, Vol. 2(2), pp. 77-81. 2020.
- [16] Adeleke, E.O., Agboola, A.A.A., and Smarandache, F., "Refined Neutrosophic Rings II", International Journal of Neutrosophic Science, Vol. 2(2), pp. 89-94. 2020.
- [17] Abobala, M., On Refined Neutrosophic Matrices and Their Applications In Refined Neutrosophic Algebraic Equations, Journal Of Mathematics, Hindawi, 2021
- [18] Giorgio, N, Mehmood, A., and Broumi, S., " Single Valued neutrosophic Filter", International Journal of Neutrosophic Science, Vol. 6, 2020.
- [19] Abobala, M., "On The Characterization of Maximal and Minimal Ideals In Several Neutrosophic Rings", Neutrosophic Sets and Systems, Vol. 45, 2021.
- [20] Milles, S, Barakat, M, and Latrech, A., " Completeness and Compactness In Standard Single Valued neutrosophic Metric Spaces", International Journal of Neutrosophic Science, Vol.12 , 2021.
- [21] Abobala, M., "On Some Algebraic Properties of n-Refined Neutrosophic Elements and n-Refined Neutrosophic Linear Equations", Mathematical Problems in Engineering, Hindawi, 2021
- [22] Kandasamy V, Smarandache F., and Kandasamy I., Special Fuzzy Matrices for Social Scientists . Printed in the United States of America,2007, book, 99 pages.
- [23] Khaled, H., and Younus, A., and Mohammad, A., " The Rectangle Neutrosophic Fuzzy Matrices", Faculty of Education Journal Vol. 15, 2019. (Arabic version).
- [24] F. Smarandache, *Neutrosophic Theory and Applications*, Le Quy Don Technical University, Faculty of Information technology, Hanoi, Vietnam, 17<sup>th</sup> May 2016.
- [25] Ibrahim, M., and Abobala, M., "An Introduction To Refined Neutrosophic Number Theory", Neutrosophic Sets and Systems, Vol. 45, 2021.
- [26] Ibrahim, M.A., Agboola, A.A.A, Badmus, B.S., and Akinleye, S.A., "On refined Neutrosophic Vector Spaces II", International Journal of Neutrosophic Science, Vol. 9, pp. 22-36. 2020.
- [27] Abobala, M, "*n*-Cyclic Refined Neutrosophic Algebraic Systems Of Sub-Indeterminacies, An Application To Rings and Modules", International Journal of Neutrosophic Science, Vol. 12, pp. 81-95 . 2020.
- [28] Smarandache, F., "Neutrosophic Set a Generalization of the Intuitionistic Fuzzy Sets", Inter. J. Pure Appl. Math., pp. 287-297. 2005.

- [29] Merkepci, H., and Abobala, M., " The Application of AH-isometry In The Study Of Neutrosophic Conic Sections", *Galoitica Journal Of Mathematical Structure And Applications*, Vol.2, 2022.
- [30] Aswad, F, M., " A Study of Neutrosophic Complex Number and Applications", *Neutrosophic Knowledge*, Vol. 1, 2020.
- [31] Aswad, F, M., " A Study of neutrosophic Bi Matrix", *Neutrosophic Knowledge*, Vol. 2, 2021.
- [32] Aswad, M., " A Study of The Integration Of Neutrosophic Thick Function", *International journal of neutrosophic Science*, 2020.
- [33] Ali, R, "Neutrosophic Matrices and their Properties", *Hal- Archives*, 2021.
- [34] Abobala, M., and Hatip, A., "An Algebraic Approach to Neutrosophic Euclidean Geometry", *Neutrosophic Sets and Systems*, Vol. 43, 2021.
- [35] Aswad, M., " A Study Of neutrosophic Differential Equation By using A Neutrosophic Thick Function", *neutrosophic knowledge*, Vol. 1, 2020.
- [36] Olgun, N., Hatip, A., Bal, M., and Abobala, M., " A Novel Approach To Necessary and Sufficient Conditions For The Diagonalization of Refined Neutrosophic Matrices", *International Journal of Neutrosophic Science*, Vol. 16, pp. 72-79, 2021.
- [37] Khaled, H., Smarandache, F., and Essa, A., "A Neutrosophic Binomial Factorial Theorem With Their Refrains", *Neutrosophic Sets and Systems*, Vol. 14, 2016.
- [38] Zeina, M. B., and Hatip, A., "Neutrosophic Random Variables ", *Neutrosophic Sets and Systems*, Vol. 39 , 2021.
- [39] Zeina, M. B., "Erlang Service Queueing Model with Neutrosophic Parameters", *International Journal of Neutrosophic Science*, Vol. 6 ,2020.
- [40] Zeina, M. B., "Neutrosophic Event-Based Queueing Model", *International Journal of Neutrosophic Science*, Vol. 6 , 2020.
- [41] Ali, R., "A Short Note On The Solution of n-Refined Neutrosophic Linear Diophantine Equations", *International Journal Of Neutrosophic Science*, Vol. 15, 2021.
- [42] Hajjari, A., and Ali, R., " A Contribution To Kothe's Conjecture In Refined Neutrosophic Rings", *International Journal of Neutrosophic Science*", Vol. 16, 2021.
- [43] Merkepci, M.; Sarkis, M. An Application of Pythagorean Circles in Cryptography and Some Ideas for Future Non Classical Systems. *Galoitica Journal of Mathematical Structures and Applications* **2022**, 2, 28-30, doi: 10.54216/GJMSA.020205.
- [44] Merkepci, M., and Ali, R., " A Study of Some Neutrosophic Algebraic Games and Their Winning Stratigies", *Galoitica Journal Of Mathematical Structure And Applications*, Vol.2, 2022.
- [45] Merkepci, H., and Ahmad, K., " On The Conditions Of Imperfect Neutrosophic Duplets and Imperfect Neutrosophic Triplets", *Galoitica Journal Of Mathematical Structure And Applications*, Vol.2, 2022.