Direct Product of Neutrosophic h-ideal in INK-Algebra

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Abstract

In this paper, "we first define the belief of direct product from neutrosophic sets in INK algebras, neutrosophic set, neutrosophic h-ideals, neutrosophic INK-subalgebra and direct product of neutrosophic h-ideals in INK algebras. Let's prove some theorems that show that there is some connection between these principles. Finally, we define the INK subalgebra of the INK algebra and then offer the ideal theorem approximately the connection between its pix and the direct product from the neutrosophic h-ideals.

Keywords: INK-algebra; the direct product of neutrosophic INK-sub algebra; direct product of neutrosophic h-ideal.

1. Introduction

In 1986, "Atanassov Introduced the Intuitionistic fuzzy set, and the later intuitionistic fuzzy set was applied in BCI/BCK-algebra, Introduced by Imai and Iseki in the 1980s. Following this, various researchers published articles using the intuitionistic fuzzy set concept. In 2005, Smarandache invented the new notion of the neutrosophic set in 1998 and it is a common code from the intuitionistic fuzzy set [1-8] and [15-20]. This has been followed by a lot of researchers publishing various articles over the last few years. In [9], [10], [11], [13], [14], and [12] Kaviyarasu et. al published an article using the fuzzy concept set in INK-algebra and later in solve the neutrosophic set in INK algebra. In this paper, we have introduced a new code using two different neutrosophic sets called a direct product of neutrosophic h-ideal in INK algebra. We are also examining the relationship between neutrosophic INK- sub-algebra and neutrosophic h-ideal and its conditions.

2. Preliminary

Definition 2.1[11] An algebra $(X,*, 0)$ is called an INK-algebra if you meet the ensuing conditions for every
$x, y, z \in X$.

1. $((x*y)*(x*z))*(z*y) = 0$
2. $((x*z)*(y*z))*(x*y) = 0$
3. $x * 0 = x$
4. $x * y = 0$ and $y*x = 0$ imply $x = y$.

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Definition 2.2[11] Let (X,∗, 0) be an INK-algebra. A nonempty subset I of X is called an ideal of X if it satisfies

1. $0 \in I$
2. $x * y \in I$ and $y \in I$ imply $x \in I$ for all $x, y \in X$.

Any ideal I has the property that $y \in I$ and $x \leq y$ imply $x \in I$.

Definition 2.3[19] Let (X,∗, 0) be a BCI-algebra. A nonempty subset I of X is called an h-ideal of X if it satisfies

1. $0 \in I$
2. $x * (y*z) \in I$ and $y \in I$ imply $x \in I$ for all $x, y, z \in X$.

Definition 2.4 [7] Let X be a non-empty set. A fuzzy set can be defined as an object of the form Ϙ = {(X, Ϙ (x)): x∈ X }, where the function ϑ: X ⟶ [0, 1] is the degree of membership.

Definition 2.5 [11] A fuzzy set Ϙ in an INK-algebra X is called a fuzzy subalgebra of X if
Ϙ (x∗y) ≥ min{Ϙ(x), Ϙ(y)}, for all x, y ∈ X.

Definition 2.7[11] Let X be an INK-algebra. A fuzzy subset Ϙ in X is called a fuzzy ideal of X if it satisfies the following conditions:

1. Ϙ(0) ≥ Ϙ(x)
2. Ϙ(x) ≥ min{Ϙ(x∗y)}, for all x, y ∈ X.

Definition 2.8 [19] Let μ be a fuzzy set in BCI-algebra X. μ is called a fuzzy h-ideal if it satisfies the:

1. $0 \in I$
2. $\mu(x) \geq \min\{\mu((x * (y * z)), \mu(y)\}, \forall x, y, z \in X$.

Definition 2.9 [2] An intuitionistic fuzzy set Ã in a non-empty set X is an object haϐing a form Ã = {X, $ϙ_{Ã}$ (x), $ϐ_{Ã}$ (x): x ∈ X}, where the function $ϙ_{Ã}$: X → [0, 1] and $ϐ_{Ã}$: X → [0, 1] denote the degree of membership and the degree of non-membership of each element x ∈ X to set A respectiϐely, and 0 ≤ $ϙ_{Ã}$ (x)+ $ϐ_{Ã}$ (x) ≤ 1, for all x ∈ X. For the sake of simplicity, the symbol Ã = (X, $ϙ_{Ã}$, $ϐ_{Ã}$) is used for the IF_S Ã = {X, $ϙ_{Ã}$ (x), $ϐ_{Ã}$ (x): x ∈ X}.

Definition 2.10 [2] An intuitionistic fuzzy set Ã in X is called an intuitionistic fuzzy sub-algebra X if

1. $ϙ_{Ã}$ (x∗y) ≥ min {$ϙ_{Ã}$(x), $ϙ_{Ã}$(y)}
2. $ϐ_{Ã}$ (x∗y) ≤ max {$ϐ_{Ã}$(x), $ϐ_{Ã}$(y)} for all x, y ∈ X.

Definition 2.11[1] An intuitionistic fuzzy set Ã in X is called an intuitionistic fuzzy ideal of X if it satisfies

for all $x, y \in X$,

1. $ϙ_{Ã}$(0) ≥ $ϙ_{Ã}$(x) and $ϐ_{Ã}$ (0) ≤ $ϐ_{Ã}$(x)
2. $ϙ_{Ã}$ (x) ≥ min {$ϙ_{Ã}$(x∗y), $ϙ_{Ã}$(y)}
3. $ϐ_{Ã}$ (x) ≤ max {$ϐ_{Ã}$(x∗y), $ϐ_{Ã}$(y)}

Definition 2.12[1] An intuitionistic fuzzy set Ã in X is called an intuitionistic fuzzy h- the ideal of X if it satisfies for all x, y, z ∈ X,

1. $ϙ_{Ã}$(0) ≥ $ϙ_{Ã}$(x) and $ϐ_{Ã}$ (0) ≤ $ϐ_{Ã}$(x)
2. $ϙ_{Ã}$ (x) ≥ min {$ϙ_{Ã}$(x∗(y∗z)), $ϙ_{Ã}$(z)}
3. $ϐ_{Ã}$ (x) ≤ max{$ϐ_{Ã}$(x∗(y∗z)), $ϐ_{Ã}$(z)}.

Definition 2.13[11] Let Ǻ = ⟨$ϙ_{Ǻ}$, $ϐ_{Ǻ}$⟩ and È = ⟨$ϙ_{È}$, $ϐ_{È}$⟩ be two intuitionistic fuzzy sets in INK-algebras X_1 and X_2 respectively .Then direct product of intuitionistic fuzzy sets Ǻ and È is denoted by
Ǻ x È = ⟨$ϙ_{Ǻ x È}$, $ϐ_{Ǻ x È}$⟩ and defined as

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1. $\varphi_{\acute{A} \times \grave{E}}(x, y) = \min\{\varphi_{\acute{A}}(x), \mu_{\grave{E}}(y)\}$ and
2. $\beta_{\acute{A} \times \grave{E}}(x, y) = \max\{\beta_{\acute{A}}(x), \beta_{\grave{E}}(y)\}$, for all $(x, y) \in X_1 \times X_2$.

Definition 2.14[11] An IFS $\acute{A} \times \grave{E} = \langle \varphi_{\acute{A} \times \grave{E}}, \beta_{\acute{A} \times \grave{E}} \rangle$ of $X_1 \times X_2$ is called an intuitionistic fuzzy sub algebra of $X_1 \times X_2$ if
1. $\varphi_{\acute{A} \times \grave{E}}((x_1,y_1)*(x_2,y_2)) \geq \min\{\varphi_{\acute{A} \times \grave{E}}(x_1,y_1), \varphi_{\acute{A} \times \grave{E}}(x_2, y_2)\}$
2. $\beta_{\acute{A} \times \grave{E}}((x_1,y_1)*(x_2,y_2)) \leq \max\{\beta_{\acute{A} \times \grave{E}}(x_1, y_1), \beta_{\acute{A} \times \grave{E}}(x_2, y_2)\}$, for all $(x_1, y_1), (x_2, y_2) \in X_1 \times X_2$.

Definition 2.15[19] An IFS $\acute{A} \times \grave{E} = \langle \varphi_{\acute{A} \times \grave{E}}, \beta_{\acute{A} \times \grave{E}} \rangle$ of $X1 \times X_2$ is called an intuitionistic fuzzy h-ideal of $X_1 \times X_2$ if,
1 $\varphi_{\acute{A} \times \grave{E}}(0, 0) \geq \varphi_{\acute{A} \times \grave{E}}(x, y)$ and $\beta_{\acute{A} \times \grave{E}}(0, 0) \leq \beta_{\acute{A} \times \grave{E}}(x, y)$.
2 $\varphi_{\acute{A} \times \grave{E}}(x_1,y_1) \geq \min\{\varphi_{\acute{A} \times \grave{E}}((x_1,y_1)*(x_2,y_2)*(x_3,y_3)), \varphi_{\acute{A} \times \grave{E}}(x_2, y_2)\}$
3 $\beta_{\acute{A} \times \grave{E}}(x_1, y_1) \leq \max\{\beta_{\acute{A} \times \grave{E}}((x_1,y_1)*(x_2,y_2)*(x_3,y_3)), \beta_{\acute{A} \times \grave{E}}(x_2, y_2)\}$,
for all $(x_1,y_1), (x_2, y_2), (x_3, y_3) \in X_1 \times X_2$.

Definition 2.16[1] A neutrosophic set φ in a nonempty set X is a structure of the form $\varphi = \{X, \varphi_T(x), \varphi_I(x), \varphi_F(x) | x \in X\}$, Where φ_T: X→ [0,1] is a truth membership function, φ_I: X→ [0,1] Is an indeterminacy membership function and φ_F: X→ [0,1] is a false membership function.

Definition 2.17[12] A neutrosophic set φ in X is called a neutrosophic INK-sub algebra of X if it satisfies the following condition, for all x, y, z ∈ X.
1. $\varphi_T(x*y) \geq min\{\varphi_T(x), \varphi_T(y)\}$
2. $\varphi_I(x*y) \geq min\{\varphi_I(x), \varphi_I(y)\}$
3. $\varphi_F(x*y) \leq max\{\varphi_F(x), \varphi_F(y)\}$.

Definition 2.18[12] A neutrosophic set φ in X is called a neutrosophic ideal of X if it satisfies the following condition, for all $x, y \in X$

1. $\varphi_T(0) \geq \varphi_T(x)$, $\varphi_I(0)\} \geq \varphi_I(x)$, and $\varphi_F(0) \leq \varphi_F(x)$
2. $\varphi_T(x) \geq min\{\varphi_T(x*y), \varphi_T(y)\}$
3. $\varphi_I(x) \geq min\{\varphi_I(x*y), \varphi_I(y)\}$
4. $\varphi_F(x) \leq max\{\varphi_F(x*y), \varphi_F(y)\}$

3. The direct product of neutrosophic INK-sub algebra

Definition 3.1 Let $\acute{A} = (T_{\acute{A}}, I_{\acute{A}}, F_{\acute{A}})$ and $\grave{E} = (T_{\grave{E}}, I_{\grave{E}}, F_{\grave{E}})$ be two neutrosophic sets in INK-algebra X and Y respectively .Then direct product of neutrosophic sets $\acute{A}$ and $\grave{E}$ is denoted by $\acute{A} \times \grave{E} = (T_{\acute{A}\times\grave{E}}, I_{\acute{A}\times\grave{E}}, F_{\acute{A}\times\grave{E}})$ and defined as $(T_{\acute{A}\times\grave{E}}, I_{\acute{A}\times\grave{E}}, F_{\acute{A}\times\grave{E}})$ and defined as

$$T_{\acute{A}\times\grave{E}}(x, y) = \min\{T_{\acute{A}}(x), T_{\grave{E}}(y)\}$$
$$I_{\acute{A}\times\grave{E}}(x, y) = \min\{I_{\acute{A}}(x), I_{\grave{E}}(y)\}$$
$$F_{\acute{A}\times\grave{E}}(x, y) = \max\{F_{\acute{A}}(x), F_{\grave{E}}(y)\}, \text{ For all } x, y \in X \times Y.$$

Definition 3.2 A neutrosophic set $\acute{A} \times \grave{E} = (T_{\acute{A}\times\grave{E}}, I_{\acute{A}\times\grave{E}}, F_{\acute{A}\times\grave{E}})$ of INK- algebra $X \times Y$ is called a direct product of neutrosophic INK- sub algebra of $X \times Y$.

1. $T_{\acute{A}\times\grave{E}}\left((x_1,y_1)*(x_2,y_2)\right) \geq \min\{T_{\acute{A}\times\grave{E}}(x_1,y_1), T_{\acute{A}\times\grave{E}}(x_2,y_2)\}$
2. $I_{\acute{A}\times\grave{E}}\left((x_1,y_1)*(x_2,y_2)\right) \geq \min\{I_{\acute{A}\times\grave{E}}(x_1,y_1), I_{\acute{A}\times\grave{E}}(x_2,y_2)\}$
3. $F_{\acute{A}\times\grave{E}}\left((x_1,y_1)*(x_2,y_2)\right) \leq \max\{F_{\acute{A}\times\grave{E}}(x_1,y_1), F_{\acute{A}\times\grave{E}}(x_2,y_2)\}$

Theorem.3.3 Let $\acute{A} = (T_{\acute{A}}, I_{\acute{A}}, F_{\acute{A}})$ and $\grave{E} = (T_{\grave{E}}, I_{\grave{E}}, F_{\grave{E}})$ be two neutrosophic INK sub algebras of INK-algebras X and Y respectively. Then $\acute{A} \times \grave{E} = (T_{\acute{A}\times\grave{E}}, I_{\acute{A}\times\grave{E}}, F_{\acute{A}\times\grave{E}})$ is also neutrosophic sub algebra of INK-algebra of $X \times Y$.

Proof. For any $(x_1, y_1), (x_2, y_2) \in X \times Y$.

Then $T_{\acute{A}\times\grave{E}}\left((x_1,y_1)*(x_2,y_2)\right) = T_{\acute{A}\times\grave{E}}(x_1*x_2, y_1*y_2)$
$$= \min\{T_{\acute{A}}(x_1*x_2), T_{\grave{E}}(y_1*y_2)\}$$

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$$\geq \min\{\min\{Ţ_{\tilde{A}}(x_1), Ţ_{\tilde{A}}(x_2)\}, \min\{Ţ_{\tilde{E}}(y_1), Ţ_{\tilde{E}}(y_2)\}\}$$
$$= \min\{\min(Ţ_{\tilde{A}}(x_1), Ţ_{\tilde{E}}(y_1))\}, \min\{Ţ_{\tilde{A}}(x_2), Ţ_{\tilde{E}}(y_2)\}$$
$$= \min\{Ţ_{\tilde{A}\times\tilde{E}}(x_1, y_1), Ţ_{\tilde{A}\times\tilde{E}}(x_2, y_2)\}$$

Similarly

$$ɭ_{\tilde{A}\times\tilde{E}} = \min\{ɭ_{\tilde{A}\times\tilde{E}}(x_1, y_1), ɭ_{\tilde{A}\times\tilde{E}}(x_2, y_2)\}$$

And

$$Ƒ_{\tilde{A}\times\tilde{E}}\{(x_1, y_1)*(x_2, y_2)\} = \max\{Ƒ_{\tilde{A}\times\tilde{E}}(x_1*x_2), Ƒ_{\tilde{A}\times\tilde{E}}(y_1*y_2)\}$$
$$= \max\{\max\{Ƒ_{\tilde{A}}(x_1), Ƒ_{\tilde{E}}(x_2)\}, m\tilde{A}x\{Ƒ_{\tilde{A}}(y_1), Ƒ_{\tilde{E}}(y_2)\}\}$$
$$\leq \max\{\max\{Ƒ_{\tilde{A}}(x_1), Ƒ_{\tilde{E}}(y_1)\}, \max\{Ƒ_{\tilde{A}}(x_2), Ƒ_{\tilde{E}}(y_2)\}\}$$
$$Ƒ_{\tilde{A}\times\tilde{E}}\{(x_1, y_1)*(x_2, y_2)\} = \max\{Ƒ_{\tilde{A}\times\tilde{E}}(x_1, y_1), Ƒ_{\tilde{A}\times\tilde{E}}(x_2, y_2)\}.$$

Theorm.3.4 Let Á=($Ţ_{\tilde{A}}, ɭ_{\tilde{A}}, Ƒ_{\tilde{A}}$) and Ȅ=($Ţ_{\tilde{E}}, ɭ_{\tilde{E}}, Ƒ_{\tilde{E}}$) be Two neutrosophic INK-sub algebras of INK-algebra X and Y repectively . Then

1. $Ţ_{\tilde{A}\times\tilde{E}}(0,0) \geq Ţ_{\tilde{A}\times\tilde{E}}(x, y)$
2. $ɭ_{\tilde{A}\times\tilde{E}}(0,0) \geq ɭ_{\tilde{A}\times\tilde{E}}(x, y)$
3. $Ƒ_{\tilde{A}\times\tilde{E}}(0,0) \leq Ƒ_{\tilde{A}\times\tilde{E}}(x, y)$ For any $(x, y) \in X \times Y$.

Proof. By Definition $Ţ_{\tilde{A}\times\tilde{E}}(0,0) = Ţ_{\tilde{A}\times\tilde{E}}\{(x, y)*(x, y)\}$

$$\geq \min\{Ţ_{\tilde{A}\times\tilde{E}}(x, y), Ţ_{\tilde{A}\times\tilde{E}}(x, y)\}$$
$$\geq Ţ_{\tilde{A}\times\tilde{E}}(x, y)$$

Therefore, $Ţ_{\tilde{A}\times\tilde{E}}(0,0) \geq Ţ_{\tilde{A}\times\tilde{E}}(x, y)$

$$ɭ_{\tilde{A}\times\tilde{E}}(0,0) = ɭ_{\tilde{A}\times\tilde{E}}\{(x, y)*(x, y)\}$$
$$\geq \min\{ɭ_{\tilde{A}\times\tilde{E}}(x, y), ɭ_{\tilde{A}\times\tilde{E}}(x, y)\}$$
$$\geq ɭ_{\tilde{A}\times\tilde{E}}(x, y)$$

Therefore $ɭ_{\tilde{A}\times\tilde{E}}(0,0) \geq ɭ_{\tilde{A}\times\tilde{E}}(x, y)$

And $Ƒ_{\tilde{A}\times\tilde{E}}(0,0) = Ƒ_{\tilde{A}\times\tilde{E}}\{(x, y)*(x, y)\}$

$$\leq \max\{Ƒ_{\tilde{A}\times\tilde{E}}(x, y), Ƒ_{\tilde{A}\times\tilde{E}}(x, y)\}$$
$$\leq Ƒ_{\tilde{A}\times\tilde{E}}(x, y)$$

Therefore $Ƒ_{\tilde{A}\times\tilde{E}}(0,0) \leq Ƒ_{\tilde{A}\times\tilde{E}}(x, y)$ For all $(x, y) \in X \times Y$.

Lemma. 3.5 Let Á=($Ţ_{\tilde{A}}, ɭ_{\tilde{A}}, Ƒ_{\tilde{A}}$) and Ȅ=($Ţ_{\tilde{E}}, ɭ_{\tilde{E}}, Ƒ_{\tilde{E}}$) be two neutrosophic INK-sub algebra of INK – algebra X and Y respectively . Then the following are true.

1. $Ţ_{\tilde{A}}(0) \geq Ţ_{\tilde{E}}(y)$ and $Ţ_{\tilde{E}}(0) \geq Ţ_{\tilde{A}}(x)$
2. $ɭ_{\tilde{A}}(0) \geq ɭ_{\tilde{E}}(y)$ and $ɭ_{\tilde{E}}(0) \geq ɭ_{\tilde{A}}(x)$
3. $Ƒ_{\tilde{A}}(0) \leq Ƒ_{\tilde{A}}(y)$ and $Ƒ_{\tilde{E}}(0) \leq Ƒ_{\tilde{A}}(x), \forall x \in X, y \in Y$.

Proof. Assume That $Ţ_{\tilde{E}}(y) > Ţ_{\tilde{A}}(0)$and $Ţ_{\tilde{A}}(x) > Ţ_{\tilde{E}}(0)$

Then, $Ţ_{\tilde{A}\times\tilde{E}}(x, y) = \min\{Ţ_{\tilde{A}}(x), Ţ_{\tilde{E}}(y)\}$

$$\geq \min\{Ţ_{\tilde{E}}(0), Ţ_{\tilde{E}}(y)\}$$
$$Ţ_{\tilde{A}\times\tilde{E}}(x, y) = Ţ_{\tilde{A}\times\tilde{E}}(0,0)$$

That is contradiction.

Similarly, Let $ɭ_{\tilde{E}}(y) > ɭ_{\tilde{E}}(0)$ and $ɭ_{\tilde{E}}(x) > ɭ_{\tilde{E}}(0)$

Then, $ɭ_{\tilde{A}\times\tilde{E}}(x, y) = \min\{ɭ_{\tilde{A}}(x), ɭ_{\tilde{E}}(y)\}$

$$\geq \min\{ɭ_{\tilde{E}}(0), ɭ_{\tilde{A}}(0)\}$$
$$ɭ_{\tilde{A}\times\tilde{E}}(x, y) = ɭ_{\tilde{A}\times\tilde{E}}(0,0)$$

That is contradiction

$$Ƒ_{\tilde{A}\times\tilde{E}}(x, y) = \max\{Ƒ_{\tilde{A}}(x), Ƒ_{\tilde{E}}(y)\}$$
$$\leq \max\{Ƒ_{\tilde{E}}(0), Ƒ_{\tilde{A}}(0)\}$$
$$Ƒ_{\tilde{A}\times\tilde{E}}(x, y) = Ƒ_{\tilde{A}\times\tilde{E}}(0,0)$$

That is contradiction. Thus proving the result.

Theorm.3.6 If $\tilde{A} \times \tilde{E}$ Is a neutrosophic sub algebra of $\tilde{A} \times \tilde{E}$, Then either Á or Ȅ Is a neutrosophic INK-sub algebra of $X \times Y$.

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Proof. Since $\tilde{A} \times \grave{E}$ Is a neutrosophic INK-sub algebra of $X \times Y$.
We have

$$\mathrm{T}_{\tilde{A}\times\grave{E}}\big((x_1,y_1)*(x_2,y_2)\big) \geq \min\{\mathrm{T}_{\tilde{A}\times\grave{E}}(x_1,y_1), \mathrm{T}_{\tilde{A}\times\grave{E}}(x_2,y_2)\}$$

By putting $x_1 = x_2 = 0$, we get

$$\mathrm{T}_{\tilde{A}\times\grave{E}}\Big((0,y_1)*(0,y_2) \geq \min\{\mathrm{T}_{\tilde{A}\times\grave{E}}(0,y_1), \mathrm{T}_{\tilde{A}\times\grave{E}}(0,y_2)\}\Big) \ldots\ldots\ldots\ldots (1)$$

Also

$$\begin{aligned}\mathrm{T}_{\tilde{A}\times\grave{E}}\big((0,y_1)*(0,y_2)\big) &= \mathrm{T}_{\tilde{A}\times\grave{E}}\big((0*0),(y_1*y_2)\big)\\ &= \min\{\mathrm{T}_{\tilde{A}}(0*0), \mathrm{T}_{\grave{E}}(y_1*y_2)\}\\ &= \mathrm{T}_{\grave{E}}(y_1*y_2) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)\end{aligned}$$

By using lemma, we have,

$$\min\{\mathrm{T}_{\tilde{A}\times\grave{E}}(0,y_1), \mathrm{T}_{\tilde{A}\times\grave{E}}(0,y_2)\} = \min\{\mathrm{T}_{\tilde{A}}(y_1), \mathrm{T}_{\grave{E}}(y_2)\} \ldots\ldots\ldots\ldots\ldots (3)$$

So From (1) , (2) and (3) we get

$$\mathrm{T}_{\grave{E}}(y_1*y_2) \geq \min\{\mathrm{T}_{\tilde{A}}(y_1), \mathrm{T}_{\grave{E}}(y_2)\},$$

$$\mathrm{I}_{\tilde{A}\times\grave{E}}\big((x_1,y_1)*(x_2,y_2)\big) \geq \min\{\mathrm{T}_{\tilde{A}\times\grave{E}}(x_1,y_1), \mathrm{T}_{\tilde{A}\times\grave{E}}(x_2,y_2)\}$$

By putting $x_1 = x_2 = 0$, We get

$$\mathrm{I}_{\tilde{A}\times\grave{E}}\big((0,y_1)*(0,y_2)\big) \geq \min\{\mathrm{I}_{\tilde{A}\times\grave{E}}(0,y_1), \mathrm{I}_{\tilde{A}\times\grave{E}}(0,y_2)\} \quad \ldots\ldots\ldots (4)$$

Also,

$$\begin{aligned}\mathrm{I}_{\tilde{A}\times\grave{E}}\big((0,y_1)*(0,y_2)\big) &= \mathrm{I}_{\tilde{A}\times\grave{E}}\big((0*0),(y_1*y_2)\big)\\ &= \min\{\mathrm{I}_{\tilde{A}}(0*0), \mathrm{I}_{\grave{E}}(y_1*y_2)\}\\ &= \mathrm{I}_{\grave{E}}(y_1*y_2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)\end{aligned}$$

By using lemma, we have

$$\min\{\mathrm{I}_{\tilde{A}\times\grave{E}}(0,y_1), \mathrm{I}_{\tilde{A}\times\grave{E}}(0,y_2)\} = \min\{\mathrm{I}_{\tilde{A}}(y_1), \mathrm{I}_{\grave{E}}(y_2)\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

So From (4), (5) and (6) we get

$$\mathrm{I}_{\grave{E}}(y_1*y_2) \geq \min\{\mathrm{I}_{\tilde{A}}(y_1), \mathrm{I}_{\grave{E}}(y_2)\}$$

And, $\mathrm{F}_{\tilde{A}\times\grave{E}}\big((x_1,y_1)*(x_2,y_2)\big) \leq \max\{\mathrm{F}_{\tilde{A}\times\grave{E}}(x_1,y_1), \mathrm{F}_{\tilde{A}\times\grave{E}}(x_2,y_2)\}$

By Putting $x_1 = x_2 = 0$, we get

$$\mathrm{F}_{\tilde{A}\times\grave{E}}\big((0,y_1)*(0,y_2)\big) \leq \max\{\mathrm{F}_{\tilde{A}\times\grave{E}}(0,y_1), \mathrm{F}_{\tilde{A}\times\grave{E}}(0,y_2)\} \quad \ldots\ldots (7)$$

Also,

$$\begin{aligned}\mathrm{F}_{\tilde{A}\times\grave{E}}\big((0,y_1)*(0,y_2)\big) &= \mathrm{F}_{\tilde{A}\times\grave{E}}\big((0*0),(y_1*y_2)\big)\\ &= \max\{\mathrm{F}_{\tilde{A}}(0*0), \mathrm{F}_{\grave{E}}(y_1*y_2)\}\\ &= \mathrm{F}_{\grave{E}}(y_1*y_2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)\end{aligned}$$

By using lemma, we have,

$$\max\{\mathrm{F}_{\tilde{A}\times\grave{E}}(0,y_1), \mathrm{F}_{\tilde{A}\times\grave{E}}(0,y_2)\} = \max\{\mathrm{F}_{\tilde{A}}(y_1), \mathrm{F}_{\grave{E}}(y_2)\} \ldots\ldots\ldots\ldots (9)$$

So From (7) , (8) and (9) , we get

$$\mathrm{F}_{\grave{E}}(y_1*y_2) \leq \max\{\mathrm{F}_{\tilde{A}}(y_1), \mathrm{F}_{\grave{E}}(y_2)\}$$

Hence $\grave{E}$ is a neutrosophic INK- sub-algebra of $X \times Y$.

4. Direct product Neutrosophic h-Ideal

Definition 4.1 A Neutrosophic set $\tilde{A} \times \grave{E} = (\mathrm{T}_{\tilde{A}\times\grave{E}}, \mathrm{I}_{\tilde{A}\times\grave{E}}, \mathrm{F}_{\tilde{A}\times\grave{E}})$ of INK-algebra $X \times Y$ Is called a neutrosophic h- Ideal of $X \times Y$, For all $(x_1,y_1),(x_2,y_2),(x_3,y_3) \in X \times Y$.

1. $\mathrm{T}_{\tilde{A}\times\grave{E}}(0,0) \geq \mathrm{T}_{\tilde{A}\times\grave{E}}(x,y)$, $\mathrm{I}_{\tilde{A}\times\grave{E}}(0,0) \geq \mathrm{I}_{\tilde{A}\times\grave{E}}(x,y)$ and $\mathrm{F}_{\tilde{A}\times\grave{E}}(0,0) \leq \mathrm{F}_{\tilde{A}\times\grave{E}}(x,y)$.
2. $\mathrm{T}_{\tilde{A}\times\grave{E}}\big((x_1,y_1)*(x_2,y_2)\big) \geq \min\Big\{\mathrm{T}_{\tilde{A}\times\grave{E}}\Big((x_1,y_1)*\big((x_2,y_2)*(x_3,y_3)\big)\Big), \mathrm{T}_{\tilde{A}\times\grave{E}}(x_2,y_2)\Big\}$
3. $\mathrm{I}_{\tilde{A}\times\grave{E}}\big((x_1,y_1)*(x_3,y_3)\big) \geq \min\{\mathrm{I}_{\tilde{A}\times\grave{E}}\big((x_1,y_1)*((x_2,y_2)*(x_3,y_3))\big), \mathrm{I}_{\tilde{A}\times\grave{E}}(x_2,y_2)\}$
4. $\mathrm{F}_{\tilde{A}\times\grave{E}}\big((x_1,y_1)*(x_3,y_3)\big) \leq \max\{\mathrm{F}_{\tilde{A}\times\grave{E}}\big((x_1,y_1)*((x_2,y_2)*(x_3,y_3))\big), \mathrm{F}_{\tilde{A}\times\grave{E}}(x_2,y_2)\}$.

Theorm 4.2 Let $\tilde{A} = (\mathrm{T}_{\tilde{A}}, \mathrm{I}_{\tilde{A}}, \mathrm{F}_{\tilde{A}})$ and $\grave{E} = (\mathrm{T}_{\grave{E}}, \mathrm{I}_{\grave{E}}, \mathrm{F}_{\grave{E}})$ be two neutrosophic h- Ideals of INK -algebras X and Y respectively. Then $\tilde{A} \times \grave{E} = (\mathrm{T}_{\tilde{A}\times\grave{E}}, \mathrm{I}_{\tilde{A}\times\grave{E}}, \mathrm{F}_{\tilde{A}\times\grave{E}})$ is a nutrosophic h-Ideal of INK – algebra $X \times Y$.

Proof. For any $(x,y) \in X \times Y$.

$$\begin{aligned}\mathrm{T}_{\tilde{A}\times\grave{E}}(0,0) &= \min\{\mathrm{T}_{\tilde{A}}(0), \mathrm{T}_{\grave{E}}(0)\}\\ &\geq \min\{\mathrm{T}_{\tilde{A}}(x), \mathrm{T}_{\grave{E}}(y)\}\\ &= \mathrm{T}_{\tilde{A}\times\grave{E}}(x,y)\end{aligned}$$

$$\mathrm{I}_{\tilde{A}\times\grave{E}}(0,0) = \min\{\mathrm{I}_{\tilde{A}}(0), \mathrm{I}_{\grave{E}}(0)\}$$

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$\geq \min\{\mathrm{I}_{\tilde{A}}(x), \mathrm{I}_{\tilde{E}}(y)\}$
$= \mathrm{I}_{\tilde{A}\times\tilde{E}}(x,y)$

And $\mathrm{F}_{\tilde{A}\times\tilde{E}}(0,0) = \max\{\mathrm{F}_{\tilde{A}}(0), \mathrm{F}_{\tilde{E}}(0)\}$
$\leq \min\{\mathrm{F}_{\tilde{A}}(x), \mathrm{F}_{\tilde{E}}(y)\}$
$= \mathrm{F}_{\tilde{A}\times\tilde{E}}(x,y)$

Now For any $(x_1, y_1), (x_2, y_2), (x_3, y_3) \in X \times Y$

$\mathrm{T}_{\tilde{A}\times\tilde{E}}((x_1,y_1)*(x_3,y_3)) = \mathrm{T}_{\tilde{A}\times\tilde{E}}(x_1*x_3; y_1*y_3)$
$= \min\{\mathrm{T}_{\tilde{A}}(x_1*x_3), \mathrm{T}_{\tilde{E}}(y_1*y_3)\}$
$= \min\{\min\{\mathrm{T}_{\tilde{A}}(x_1*(x_2*x_3)), \mathrm{T}_{\tilde{A}}(x_2)\}, \min\{\mathrm{T}_{\tilde{E}}(y_1*(y_2*y_3), \mathrm{T}_{\tilde{E}}(y_2)\}\}$
$= \min\{\min\{\mathrm{T}_{\tilde{A}}(x_1*(x_2*x_3)), \mathrm{T}_{\tilde{E}}(y_1*(y_2*y_3))\}, \min\{\mathrm{T}_{\tilde{A}}(x_2), \mathrm{T}_{\tilde{E}}(y_2)\}\}$
$= \min\{\mathrm{T}_{\tilde{A}\times\tilde{E}}\{(x_1*(x_2*x_3)), (y_1*(y_2*y3))\}, \mathrm{T}_{\tilde{A}\times\tilde{E}}(x_2,y_2)\}$
$= \min\{\mathrm{T}_{\tilde{A}\times\tilde{E}}(x_1,y_1)*(x_2,y_2)*(x_3,y_3))), \mathrm{T}_{\tilde{A}\times\tilde{E}}(x_2,y_2)\}$

$\mathrm{I}_{\tilde{A}\times\tilde{E}}((x_1,y_1)*(x_3,y_3)) = \mathrm{I}_{\tilde{A}\times\tilde{E}}(x_1*x_3; y_1*y_3)$
$= \min\{\mathrm{I}_{\tilde{A}}(x_1*x_3), \mathrm{I}_{\tilde{E}}(y_1*y_3)\}$
$= \min\{\min\{\mathrm{I}_{\tilde{A}}(x_1*(x_2*x_3)), \mathrm{I}_{\tilde{A}}(x_2)\}, \min\{\mathrm{I}_{\tilde{E}}(y_1*(y_2*y_3), \mathrm{I}_{\tilde{E}}(y_2)\}\}$
$= \min\{\min\{\mathrm{I}_{\tilde{A}}(x_1*(x_2*x_3)), \mathrm{I}_{\tilde{E}}(y_1*(y_2*y_3))\}, \min\{\mathrm{I}_{\tilde{A}}(x_2), \mathrm{I}_{\tilde{E}}(y_2)\}\}$
$= \min\{\mathrm{I}_{\tilde{A}\times\tilde{E}}\{(x_1*(x_2*x_3)), (y_1*(y_2*y_3))\}, \mathrm{I}_{\tilde{A}\times\tilde{E}}(x_2,y_2)\}$
$= \min\{\mathrm{I}_{\tilde{A}\times\tilde{E}}(x_1,y_1)*(x_2,y_2)*(x_3,y_3))), \mathrm{I}_{\tilde{A}\times\tilde{E}}(x_2,y_2)\}$

And

$\mathrm{F}_{\tilde{A}\times\tilde{E}}((x_1,y_1)*(x_3,y_3)) = \mathrm{F}_{\tilde{A}\times\tilde{E}}(x_1*x_3; y_1*y_3)$
$= \max\{\mathrm{F}_{\tilde{A}}(x_1*x_3), \mathrm{F}_{\tilde{E}}(y_1*y_3)\}$
$= \max\{\max\{\mathrm{F}_{\tilde{A}}(x_1*(x_2*x_3)), \mathrm{F}_{\tilde{A}}(x_2)\}, \max\{\mathrm{F}_{\tilde{E}}(y_1*(y_2*y_3), \mathrm{F}_{\tilde{E}}(y_2)\}\}$
$= \max\{\max\{\mathrm{F}_{\tilde{A}}(x_1*(x_2*x_3)), \mathrm{F}_{\tilde{E}}(y_1*(y_2*y_3))\}, \min\{\mathrm{F}_{\tilde{A}}(x_2), \mathrm{F}_{\tilde{E}}(y_2)\}\}$
$= \max\{\mathrm{F}_{\tilde{A}\times\tilde{E}}\{(x_1*(x_2*x_3)), (y_1*(y_2*y_3))\}, \mathrm{F}_{\tilde{A}\times\tilde{E}}(x_2,y_2)\}$
$= \max\{\mathrm{F}_{\tilde{A}\times\tilde{E}}(x_1,y_1)*(x_2,y_2)*(x_3,y_3))), \mathrm{F}_{\tilde{A}\times\tilde{E}}(x_2,y_2)\}$

Hence, $\tilde{A}\times\tilde{E} = (\mathrm{T}_{\tilde{A}\times\tilde{E}}, \mathrm{I}_{\tilde{A}\times\tilde{E}}, \mathrm{F}_{\tilde{A}\times\tilde{E}})$ is a neutrosophic h-Ideal of $X \times Y$.

Theorem.4.3 Let $\tilde{A} = (\mathrm{T}_{\tilde{A}}, \mathrm{I}_{\tilde{A}}, \mathrm{F}_{\tilde{A}})$ and $\tilde{E} = (\mathrm{T}_{\tilde{E}}, \mathrm{I}_{\tilde{E}}, \mathrm{F}_{\tilde{E}})$ be two neutrosophic h-Ideals of INK-algebra X and Y respectively. If $\tilde{A}\times\tilde{E}$ Is a neutrosophic h-Ideal of $X \times Y$, Then $\tilde{A}\times\tilde{E}$ must be a neutrosophic subalgebra of $X \times Y$.

Proof. Since $\tilde{A}\times\tilde{E}$ is a neutrosophic h-Ideal of $X \times Y$, Then For all $(x_1,y_1), (x_2,y_2), (x_3,y_3) \in X \times Y$.

We have $\mathrm{T}_{\tilde{A}\times\tilde{E}}((x_1,y_1)*(x_3,y_3)) \geq \min\{\mathrm{T}_{\tilde{A}\times\tilde{E}}((x_1,y_1)*(x_2,y_2)*(x_3,y_3)), \mathrm{T}_{\tilde{A}\times\tilde{E}}(x_2,y_2)\}$

By putting $x_3 = y_3 = 0$, we get

$\mathrm{T}_{\tilde{A}\times\tilde{E}}(x_1,y_1) \geq \min\{\mathrm{T}_{\tilde{A}\times\tilde{E}}((x_1,y_1)*(x_2,y_2)), \mathrm{T}_{\tilde{A}\times\tilde{E}}(x_2,y_2)\}$...... (1)

Again Since, $((x_1,y_1)*(x_2,y_2)) \geq (x_1,y_1)$, For all $(x_1,y_1), (x_2,y_2) \in X \times Y$.

$\mathrm{T}_{\tilde{A}\times\tilde{E}}((x_1,y_1)*(x_2,y_2)) \geq \mathrm{T}_{\tilde{A}\times\tilde{E}}(x_1,y_1)$ ....(2)

Hence From (1) and (2) we get,

$\mathrm{T}_{\tilde{A}\times\tilde{E}}((x_1,y_1)*(x_2,y_2)) \geq \mathrm{T}_{\tilde{A}\times\tilde{E}}(x_1,y_1)$
$\geq \min\{\mathrm{T}_{\tilde{A}\times\tilde{E}}((x_1*y_1)*(x_2,y_2)), \mathrm{T}_{\tilde{A}\times\tilde{E}}(x_2,y_2)\}$
$\geq \min\{\mathrm{T}_{\tilde{A}\times\tilde{E}}(x_1,y_1), \mathrm{T}_{\tilde{A}\times\tilde{E}}(x_2,y_2)\}$,

For all $(x_1,y_1), (x_2,y_2) \in X \times Y$.

Similarly we can prove that,

$\mathrm{I}_{\tilde{A}\times\tilde{E}}((x_1,y_1)*(x_2,y_2)) \geq \min\{\mathrm{I}_{\tilde{A}\times\tilde{E}}(x_1,y_1), \mathrm{I}_{\tilde{A}\times\tilde{E}}(x_2,y_2)\}$

$\mathrm{F}_{\tilde{A}\times\tilde{E}}((x_1,y_1)*(x_2,y_2)) \leq \max\{\mathrm{F}_{\tilde{A}\times\tilde{E}}(x_1,y_1), \mathrm{F}_{\tilde{A}\times\tilde{E}}(x_2,y_2)\}$

For all $(x_1,y_1), (x_2,y_2) \in X \times Y$. Thus $\tilde{A}\times\tilde{E}$ is a neutrosophic INK-sub algebra of $X \times Y$.

Lemma.4.4 IF $\tilde{A}\times\tilde{E} = (\mathrm{T}_{\tilde{A}\times\tilde{E}}, \mathrm{I}_{\tilde{A}\times\tilde{E}}, \mathrm{F}_{\tilde{A}\times\tilde{E}})$ is a neutrosophic h-Ideal of INK-algebra $X \times Y$. IF $(a,b) \leq (x,y)$, Then $\mathrm{T}_{\tilde{A}\times\tilde{E}}(x,y) \geq \mathrm{T}_{\tilde{A}\times\tilde{E}}(a,b)$, $\mathrm{I}_{\tilde{A}\times\tilde{E}}(x,y) \geq \mathrm{I}_{\tilde{A}\times\tilde{E}}(a,b)$ and $\mathrm{F}_{\tilde{A}\times\tilde{E}}(x,y) \leq \mathrm{F}_{\tilde{A}\times\tilde{E}}(a,b)$, For all $(a,b), (x,y) \in X \times Y$.

Proof. Let $(a,b), (x,y) \in X \times Y$, such that $(a,b) \leq (x,y)$ implies $(a,b)*(x,y) = (0,0)$

Now, $\mathrm{T}_{\tilde{A}\times\tilde{E}}(x,y) = \mathrm{T}_{\tilde{A}\times\tilde{E}}((x,y)*(0,0))$
$\geq \min\{\mathrm{T}_{\tilde{A}\times\tilde{E}}((x,y)*(a,b)*(0,0)), \mathrm{T}_{\tilde{A}\times\tilde{E}}(a,b)\}$

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$= \min\{\mathrm{T}_{\tilde{A}\times\tilde{E}}((x,y)*(a,b)), \mathrm{T}_{\tilde{A}\times\tilde{E}}(a,b)\}$
$= \mathrm{T}_{\tilde{A}\times\tilde{E}}(a,b)$
$\mathrm{I}_{\tilde{A}\times\tilde{E}}(x,y) = \mathrm{I}_{\tilde{A}\times\tilde{E}}\{(x_1,y_1)*(0,0)\}$
$\geq \min\left\{\mathrm{I}_{\tilde{A}\times\tilde{E}}\left((x,y)*((a,b)*(0,0))\right), \mathrm{I}_{\tilde{A}\times\tilde{E}}(a,b)\right\}$
$= \min\{\mathrm{I}_{\tilde{A}\times\tilde{E}}((x,y)*(a,b)), \mathrm{I}_{\tilde{A}\times\tilde{E}}(a,b)\}$
$= \mathrm{I}_{\tilde{A}\times\tilde{E}}(a,b).$

And, $\mathrm{F}_{\tilde{A}\times\tilde{E}}(x,y) = \mathrm{F}_{\tilde{A}\times\tilde{E}}((x,y)*(0,0))$
$\leq \max\left\{\mathrm{F}_{\tilde{A}\times\tilde{E}}\left((x,y)*((a,b)*(0,0))\right), \mathrm{F}_{\tilde{A}\times\tilde{E}}(a,b)\right\}$
$= \max\{\mathrm{F}_{\tilde{A}\times\tilde{E}}((x,y)*(a,b)), \mathrm{F}_{\tilde{A}\times\tilde{E}}(a,b)\}$
$= \mathrm{F}_{\tilde{A}\times\tilde{E}}(a,b).$

This completes the proof.

Theorm 4.5 Let $\tilde{A} = (\mathrm{T}_{\tilde{A}}, \mathrm{I}_{\tilde{A}}, \mathrm{F}_{\tilde{A}})$ and $\tilde{E} = (\mathrm{T}_{\tilde{E}}, \mathrm{I}_{\tilde{E}}, \mathrm{F}_{\tilde{E}})$ be two neutrosophic h-Ideals of INK- algebra X and Y repectively. Then $\pi(\tilde{A}\times\tilde{E}) = (\mathrm{T}_{\tilde{A}\times\tilde{E}}, \overline{\mathrm{T}}_{\tilde{A}\times\tilde{E}})$ Is a neutrosophic h-Ideal of $X \times Y$, where $\overline{\mathrm{T}}_{\tilde{A}\times\tilde{E}} = 1 - \mathrm{T}_{\tilde{A}\times\tilde{E}}$.

Proof. We show That $\tilde{A}\times\tilde{E}$ Is a neutrosophic h-Ideal of $X\times Y$. Hence For any $(x,y) \in X \times Y$

$\mathrm{T}_{\tilde{A}\times\tilde{E}}(0,0) \geq \mathrm{T}_{\tilde{A}\times\tilde{E}}(x,y)$
1-$\mathrm{T}_{\tilde{A}\times\tilde{E}}(0,0) \leq 1 - \mathrm{T}_{\tilde{A}\times\tilde{E}}(x,y).$
$\overline{\mathrm{T}}_{\tilde{A}\times\tilde{E}}(0,0) \leq \overline{\mathrm{T}}_{\tilde{A}\times\tilde{E}}(x,y)$

Now For any $(x_1,y_1),(x_2,y_2),(x_3,y_3) \in X\times Y$.
We have,

$\mathrm{T}_{\tilde{A}\times\tilde{E}}((x_1,y_1)*(x_3,y_3)) \geq \min\{\mathrm{T}_{\tilde{A}\times\tilde{E}}((x_1,y_1)*(x_3,y_3))\}$
$\geq \min\{\mathrm{T}_{\tilde{A}\times\tilde{E}}((x_1,y_1)*(x_2,y_2)*(x_3,y_3)), \mathrm{T}_{\tilde{A}\times\tilde{E}}(x_2,y_2)\}$
$1 - \mathrm{T}_{\tilde{A}\times\tilde{E}}((x_1,y_1)*(x_3,y_3)) \leq 1 - \min\left\{\mathrm{T}_{\tilde{A}\times\tilde{E}}\left((x_1,y_1)*((x_2,y_2)*(x_3,y_3))\right), \mathrm{T}_{\tilde{A}\times\tilde{E}}(x_2,y_2)\right\}$
$\overline{\mathrm{T}}_{\tilde{A}\times\tilde{E}}((x_1,y_1)*(x_3,y_3)) \leq \max\left\{1 - \mathrm{T}_{\tilde{A}\times\tilde{E}}\left((x_1,y_1)*((x_2,y_2)*(x_3,y_3))\right), 1 - \mathrm{T}_{\tilde{A}\times\tilde{E}}(x_2,y_2)\right\}$

Finally,

$\overline{\mathrm{T}}_{\tilde{A}\times\tilde{E}}((x_1,y_1)*(x_3,y_3)) \leq \max\left\{\overline{\mathrm{T}}_{\tilde{A}\times\tilde{E}}\left((x_1,y_1)*((x_2,y_2)*(x_3,y_3))\right), \overline{\mathrm{T}}_{\tilde{A}\times\tilde{E}}(x_2,y_2)\right\}.$

Theorm.4.6 Let $\tilde{A} = (\mathrm{T}_{\tilde{A}}, \mathrm{I}_{\tilde{A}}, \mathrm{F}_{\tilde{A}})$ and $\tilde{E} = (\mathrm{T}_{\tilde{E}}, \mathrm{I}_{\tilde{E}}, \mathrm{F}_{\tilde{E}})$ be two neutrosophic h-Ideals of INK-algebra X and Y respectively. Then $\pi(\tilde{A}\times\tilde{E}) = (\overline{\mathrm{I}}_{\tilde{A}\times\tilde{E}}, \mathrm{I}_{\tilde{A}\times\tilde{E}})$ is a neutrosophic h-Ideal of $X\times Y$, Where $\overline{\mathrm{I}}_{\tilde{A}\times\tilde{E}} = 1 - \mathrm{I}_{\tilde{A}\times\tilde{E}}$.

Proof

The result follows the previous Theorem.

Theorm.4.7 Let $\tilde{A} = (\mathrm{T}_{\tilde{A}}, \mathrm{I}_{\tilde{A}}, \mathrm{F}_{\tilde{A}})$ and $\tilde{E} = (\mathrm{T}_{\tilde{E}}, \mathrm{I}_{\tilde{E}}, \mathrm{F}_{\tilde{E}})$ be two neutrosophic h-Ideal of INK-algebra X and Y respectively. Then $(\tilde{A}\times\tilde{E}) = (\overline{\mathrm{F}}_{\tilde{A}\times\tilde{E}}, \mathrm{F}_{\tilde{A}\times\tilde{E}})$ is a neutrosophic h-Ideal of $X\times Y$, where $\overline{\mathrm{F}}_{\tilde{A}\times\tilde{E}} = 1 - \mathrm{F}_{\tilde{A}\times\tilde{E}}$.

Proof. We know That $\tilde{A}\times\tilde{E}$ is a neutrosophic h-Ideal of $X\times Y$, For any $(x,y) \in X\times Y$.

$\mathrm{F}_{\tilde{A}\times\tilde{E}}(0,0) \leq \mathrm{F}_{\tilde{A}\times\tilde{E}}(x,y)$. Hence,
1-$\mathrm{F}_{\tilde{A}\times\tilde{E}}(0,0) \leq 1 - \mathrm{F}_{\tilde{A}\times\tilde{E}}(x,y).$
$\overline{\mathrm{F}}_{\tilde{A}\times\tilde{E}}(0,0) \leq \overline{\mathrm{F}}_{\tilde{A}\times\tilde{E}}(x,y).$

Now For any $(x_1,y_1),(x_2,y_2),(x_3,y_3) \in X\times Y$.We have,

$\mathrm{F}_{\tilde{A}\times\tilde{E}}((x_1,y_1)*(x_3,y_3)) \leq \max\{\mathrm{F}_{\tilde{A}\times\tilde{E}}((x_1,y_1)*(x_2,y_2)*(x_3,y_3)), \mathrm{F}_{\tilde{A}\times\tilde{E}}(x_2,y_2)\}.$
1-$\mathrm{F}_{\tilde{A}\times\tilde{E}}((x_1,y_1)*(x_3,y_3)) \geq 1 - \max\{\mathrm{F}_{\tilde{A}\times\tilde{E}}(x_1,y_1)*((x_2,y_2)*(x_3,y_3)), \mathrm{F}_{\tilde{A}\times\tilde{E}}(x_2,y_2)\}$
$\overline{\mathrm{F}}_{\tilde{A}\times\tilde{E}}((x_1,y_1)*(x_3,y_3)) \geq \min\left\{1 - \mathrm{F}_{\tilde{A}\times\tilde{E}}\left((x_1,y_1)*((x_2,y_2)*(x_3,y_3))\right), 1 - \mathrm{F}_{\tilde{A}\times\tilde{E}}(x_2,y_2)\right\}$

Finally,

$\overline{\mathrm{F}}_{\tilde{A}\times\tilde{E}}((x_1,y_1)*(x_3,y_3)) \geq \min\left\{1 - \mathrm{F}_{\tilde{A}\times\tilde{E}}\left((x_1,y_1)*((x_2,y_2)*(x_3,y_3))\right), 1 - \overline{\mathrm{F}}_{\tilde{A}\times\tilde{E}}(x_2,y_2)\right\}.$

Hence,$(\tilde{A}\times\tilde{E})$ is a neutrosophic h-Ideal of $X\times Y$.

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Proposition.4.8 Let a neutrosophic set $\tilde{A} \times \tilde{E} = (T_{\tilde{A}\times\tilde{E}}, I_{\tilde{A}\times\tilde{E}}, F_{\tilde{A}\times\tilde{E}})$ be a neutrosophic h-ideal of a INK-algebra $X \times Y$. Then $T_{\tilde{A}\times\tilde{E}}\left((0,0)*((0,0)*(x,y))\right) \geq T_{\tilde{A}\times\tilde{E}}(x,y)$,

$I_{\tilde{A}\times\tilde{E}}\left((0,0)*((0,0)*(x,y))\right) \geq I_{\tilde{A}\times\tilde{E}}(x,y)$

and $F_{\tilde{A}\times\tilde{E}}\left((0,0)*((0,0)*(x,y))\right) \leq F_{\tilde{A}\times\tilde{E}}(x,y)$, For all $(x,y) \in X \times Y$.

Proof. $T_{\tilde{A}\times\tilde{E}}\left((0,0)*((0,0)*(x,y))\right) \geq \min\left\{T_{\tilde{A}\times\tilde{E}}\left((0,0)*\left((x,y)*((0,0)*(x,y))\right)\right), T_{\tilde{A}\times\tilde{E}}(x,y)\right\}$

$$= \min\left\{T_{\tilde{A}\times\tilde{E}}\left((0,0)*((x,y)*(0,0))\right), T_{\tilde{A}\times\tilde{E}}(x,y)\right\}$$
$$= \min\{T_{\tilde{A}\times\tilde{E}}((0,0)*(x,y)), T_{\tilde{A}\times\tilde{E}}(x,y)\}$$
$$= \min\{T_{\tilde{A}\times\tilde{E}}(0,0), T_{\tilde{A}\times\tilde{E}}(x,y)\}$$
$$= T_{\tilde{A}\times\tilde{E}}(x,y), \text{ For all } (x,y) \in X \times Y.$$

Therefore $T_{\tilde{A}\times\tilde{E}}\left((0,0)*((0,0)*(x,y))\right) \geq T_{\tilde{A}\times\tilde{E}}(x,y)$

Similarly $I_{\tilde{A}\times\tilde{E}}\left((0,0)*((0,0)*(x,y))\right) \geq I_{\tilde{A}\times\tilde{E}}(x,y)$

And

$$F_{\tilde{A}\times\tilde{E}}\left((0,0)*((0,0)*(x,y))\right) \leq \min\left\{F_{\tilde{A}\times\tilde{E}}\left((0,0)*((x,y)*(0,0)*(x,y))\right), F_{\tilde{A}\times\tilde{E}}(x,y)\right\}$$
$$= \max\left\{F_{\tilde{A}\times\tilde{E}}\left((0,0)*((x,y)*(0,0))\right), F_{\tilde{A}\times\tilde{E}}(x,y)\right\}$$
$$= \max\{F_{\tilde{A}\times\tilde{E}}((0,0)*(x,y)), F_{\tilde{A}\times\tilde{E}}(x,y)\}$$
$$= \max\{F_{\tilde{A}\times\tilde{E}}(0,0), F_{\tilde{A}\times\tilde{E}}(x,y)\}$$
$$= F_{\tilde{A}\times\tilde{E}}(x,y), \text{ For all } (x,y) \in X \times Y$$

Therefore, $F_{\tilde{A}\times\tilde{E}}\left((0,0)*((0,0)*(x,y))\right) \leq F_{\tilde{A}\times\tilde{E}}(x,y)\ \forall\, x,y \in X \times Y$.

5. Conclusion

In this paper, we applied the notion of the direct product of the neutrosophic set to the h-ideal of INK algebra. We have introduced the direct product of the concept of neutrosophic INK-algebra and a direct product of closed neutrosophic h-ideal, and have investigated several properties. We have provided conditions for a direct product of neutrosophic set to be a direct product of neutrosophic h-ideal in INK-algebra."

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References

[1] Smarandache, F., Neutrosophic set a generalizations of the intuitionistic fuzzy sets. inter. J. Pure Appl. Math., 24, 287 – 297, 2005.
[2] Atanassov, K. Intuitionistic Fuzzy sets. Fuzzy Sets and Systems, 1986, 20(1), 87-96.
[3] Jun, Y.B. Neutrosophic sub algebras of several Types in BCK/BCl-algebras. Annals of Fuzzy Mathematics. information, 2017, 14(1), 75–86.
[4] Songsaeng, M.; lampan, A. Neutrosophic Set Theory Applied to UP-Algebra. European Journal of Pure and Applied Mathematics.. 2019, 12(4), 1382-1409.
[5] Jun, Y. B.; Smarandache, F.; Bordbar. H. Neutrosophic N-structures applied to BCK/BCl-algebras, intormation. 2017, 8(4),128.
[6] Jun, Y. B.; Smarandache, F.; Song, S-Z.; Khan, M. Neutrosophic positive implicative N-ideals in BCK-algebras. Axioms, 2018, 7(1), 3.
[7] Zadeh, L. A. Fuzzy sets, information control. 1965, 8, 338-353.
[8] Ozturk, M. A.; Jun, Y. B. Neutrosophic ideals ln BCK/BCl-algebras based on neutrosophic polnTs, Journal of the international Mathematical Virtual institute, 2018, 8, 1–17.
[9] Kaviyarasu, M.; Indhira, K.; Chandrasekaran, V.M. Fuzzy p-ideal in INK-algebra, *Journal of XI'an University of Architecture & Technology*, **2018**, 12 (3), 4746-4752.

[10] Kaviyarasu, M.; Indhira, K.; Chandrasekaran, V.M. Intuitionistic fuzzy translation on INK-Algebra, *Advances in Mathematics: Scientific Journal*, **2020**, 9 (1), 295–303.

[11] Kaviyarasu, M.; Indhira, K.; Chandrasekaran, V.M. Direct product of intuitionistic fuzzy INK-ideals of INK-algebras, *Material Today: Proceedings,* **2019**, 16 (2), 449-455. doI.org/10.1016/j.matpr.2019.05.114.

[12] Kaviyarasu, M.; Indhira, K.; Chandrasekaran, V.M. Neutrosophic set in INK-Algebra, *Advances in Mathematics: Scientific Journal*, **2020**, 9 (7), 4345-4352.

[13] Kaviyarasu, M.; Indhira, K.; Chandrasekaran, V.M. Fuzzy sub algebras and fuzzy INK-ideals in INK-algebras, *International Journal of Pure and Applied Mathematics*, **2017**, 113(6), 47-55.

[14] Kaviyarasu, M; Indhira, K. Review on BCI/BCK-algebras and development, *International Journal of Pure and Applied Mathematics*, **2017,** 117(11), 377-383.

[15] Florentin Smarandache, NeutroAlgebra is a Generalization of Partial Algebra, International Journal of Neutrosophic Science, Vol. 2, No. 1 , (2020) : 08-17

[16] Smarandache, F. Introduction to Neutro Algebraic Structures and Anti Algebraic Structures, *Neutrosophic Sets and Systems*, **2020**, 31, 1-16.

[17] Salama, A. A., Smarandache, F., & Kroumov, V., Neutrosophic crisp Sets & Neutrosophic crisp Topological Spaces. Sets and Systems, 2(1), 25-30, 2014.

[18] iorgio Nordo , Arif Mehmood , Said Broumi, Single Valued Neutrosophic Filters, International Journal of Neutrosophic Science, Vol. 6 , No. 1 , (2020) : 08-21

[19] R.Jayasudha. (2018). On intuitionistic fuzzy P-ideals and H-ideals in BCI-Algebras, International Journal of Engineering Science Invention, 07(03), 56-62.

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