Medical diagnosis decision making using type-II generalized Pythagorean neutrosophic interval valued soft sets

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Abstract

The theory of type-II generalized Pythagorean neutrosophic interval valued soft set (Type-II PyNSIVS) and its application to real problems are introduced in this study. Additionally, we define a few operations using the type-II PyNSIVS set. The Pythagorean neutrosophic interval valued soft (PyNSIVS) set and Pythagorean fuzzy soft set are both generalized to form the type-II PyNSIVS set. Complement, union, intersection, AND, and OR are some examples of operations that we define. In particular, we demonstrate the applicability of De Morgan's laws, associative laws, and distributive laws in type-II PyNSIVS set. The proposed similarity measure of type-II GPyNSIVS sets serves as the foundation for a strategy we provide for a medical diagnosis challenge. This method of comparing two type-II GPyNSIVS sets can be used to determine if a sick person has a particular disease or not. We support a method using the type-II generalized soft set model to tackle the decision making (DM) problem. We describe the use of a similarity measure between two type-II GPyNSIVS sets in a medical diagnosis situation. To demonstrate how they can be utilized to successfully address issues with uncertainties, illustrative examples are given.

Keywords: Type-II GPyNSIVS set; PyNSIVS set; decision making problem.

1 Introduction

Following Zadeh's demonstration of fuzzy set (FS), they have gained enormous popularity across practically all scientific disciplines. This shows that decision makers should take membership degree (MD) into account when resolving ambiguous issues.[26] Atanassov[4] established the idea of an intuitionistic fuzzy set (IFS), which is indicated by a MD and non-membership degree (NMD) that meets the requirement that the summation of their MD and NMD does not exceed unity. However, if the summation of MD and NMD for a given attribute exceeds unity, we may encounter a problem while DM. Yager introduction of the idea of Pythagorean fuzzy set (PFS).[24] The square summation of its MD and NMD has been extended from IFSs, and it is indicated by the restriction that it does not exceed unity. The idea of picture fuzzy sets,[5] which are extensions of FSs and IFSs. Cuong created picture fuzzy sets in 2015. Models based on picture fuzzy sets may be appropriate when

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dealing with human opinions that involve multiple types of responses, such as yes, abstain, no, and refuse. Human voters can be divided into four categories: those who vote for, those who abstain, those who vote against, and those who refuse to cast a ballot. Voting can serve as a good example. These sets enable decision-makers to assign MD, NMD, and reluctance degrees over a wider area. Cuong et al.,[6] the inventor of image FS logic, employed three pointers: positive MD, neutral MD, and negative MD, with the summation of these three grades not exceeding 1. Finally, for a few applications, it offers more advantages than IFS and PFS.

Neutrosophy were established by Smarandache[23] to address the issues of ambiguous and inconsistent information because this collection presents several difficulties in terms of applicability. A brand-new theory known as NS logic and sets has recently been put forth. Neutrosophy is the study of neutral cognition, and this neutral is the main distinction between a FS and an IFS. Smarandache[23] is recognized for developing NS reasoning. A further generalization of the FS and IFS is the neutrosophic (NS) set which ranges from $[0, 1]$ in the NS set. From a philosophical standpoint, it has been established that a NS set generalizes a classical set, FS, IVFS, etc. Recently, Jansi et al. engaged with the idea of a Pythagorean NS set.[8]

Molodtsov[14] made the theory of soft sets his main contribution. When compared to other uncertain theories, soft sets more accurately capture the objectivity and complexity of decision-making in real-world contexts. Additionally, a crucial topic for research is the integration of soft sets with other mathematical models. The fuzzy soft set (FSS)[11] and intuitionistic fuzzy soft set (IFSS)[12] concepts were proposed by Maji. These two theories are used to address a range of DM issues. Picture fuzzy soft set[25] talked Yong Yang. FSS has been expanded to include Pythagorean fuzzy soft set by Peng[22] in recent years. This methodology resolved a group of MADM problems where the summation of the MD and NMD is greater than one but the summation of the squares is equal to or not greater than one. Majumdar talked about generalized FSS.[13] Shawkat et al.[1] covered the idea of a generalized interval valued fuzzy soft set. According to Alkhazaleh et al.[2] possibility fuzzy soft set is a novel concept with practical applications. After, Using a decision making technique, Karaaslan proposed the reasoning for possibility neutrosophic soft sets.[9] In order to parameterize the type-II GPyNSIV set using the generalized soft set model, this article extends the idea of generalized IVFSSs. On the basis of this generalized soft set model, we will go on to build a similarity measure. A decision making problem is resolved as an application after relations on type-II GPyNSIVS sets are specified, their properties are explored, and their relations are defined. Many researchers discussed with real applications based on neutrosophic set.[3,7,10] Palanikumar et al. discussed various ideals structures and its applications.[15–21]

This article aims to investigate the type-II GPyNSIVS set. The following five elements make up the article. The introduction is in Section 1. The Generalized IVS set and PyNSIVS set are in Section 2. Type-II GPyNSIVFS set is conceptualized in Section 3. Method for type-II GPyNSIVFS sets similarity measure in Section 4. The idea of a medical diagnosis problem solved by a type-II GPyNSIVS set model is described in Section 5. The section 6 provides the conclusion. If you are evaluating the type-II GPyNSIVS set model, give some numerical examples.

2 Preliminaries

In this part, we review and introduce some ideas interrelated to the well-known literary concepts of the Pythagorean neutrosophic and generalized fuzzy soft set.

Definition 2.1. [8] Let $\mathscr{X}$ be the universe, Pythagorean neutrosophic interval valued (PyNSIV) set P in $\mathscr{X}$ is

$$\overbrace{\widetilde{P}} = \{c\widetilde{\vartheta}_P(\chi), \widetilde{\varpi}_P(\chi), \widetilde{\tau}_P(\chi) | \chi \in \mathscr{X}\},$$

where $\widetilde{\vartheta}_P(\chi) = [\vartheta_P^{\mathscr{L}}(\chi), \vartheta_P^{\mathscr{U}}(\chi)]$, $\widetilde{\varpi}_P(\chi) = [\varpi_P^{\mathscr{L}}(\chi), \varpi_P^{\mathscr{U}}(\chi)]$, and $\widetilde{\tau}_P(\chi) = [\tau_P^{\mathscr{L}}(\chi), \tau_P^{\mathscr{U}}(\chi)]$ represent the degree of truth membership, indeterminacy membership and falsity membership of P, respectively. The function $\widetilde{\vartheta}_P : \mathscr{X} \to D[0, 1]$, $\widetilde{\varpi}_P : \mathscr{X} \to D[0, 1]$, $\widetilde{\tau}_P : \mathscr{X} \to D[0, 1]$ and $0 \preceq (\widetilde{\vartheta}_P(\chi))^2 + (\widetilde{\varpi}_P(\chi))^2 + (\widetilde{\tau}_P(\chi))^2 \preceq 2$ means $0 \preceq (\vartheta_P^{\mathscr{U}}(\chi))^2 + (\varpi_P^{\mathscr{U}}(\chi))^2 + (\tau_P^{\mathscr{U}}(\chi))^2 \preceq 2$. Here the PyNSIV $\overbrace{\widetilde{P}} = \left\langle [\vartheta_P^{\mathscr{L}}, \vartheta_P^{\mathscr{U}}], [\varpi_P^{\mathscr{L}}, \varpi_P^{\mathscr{U}}], [\tau_P^{\mathscr{L}}, \tau_P^{\mathscr{U}}] \right\rangle$ is called a Pythagorean neutrosophic interval valued number (PyNSIVN).

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Definition 2.2. [8] Given that $\overbrace{\widetilde{\gamma_1}} = \langle \vartheta_{\widetilde{\gamma_1}}, \varpi_{\widetilde{\gamma_1}}, \tau_{\widetilde{\gamma_1}} \rangle$, $\overbrace{\widetilde{\gamma_2}} = \langle \vartheta_{\widetilde{\gamma_2}}, \varpi_{\widetilde{\gamma_2}}, \tau_{\widetilde{\gamma_2}} \rangle$ and $\overbrace{\widetilde{\gamma_3}} = \langle \vartheta_{\widetilde{\gamma_3}}, \varpi_{\widetilde{\gamma_3}}, \tau_{\widetilde{\gamma_3}} \rangle$ are any three PyNSIVNs over $(\mathscr{X}, E)$. Then

(i) $\overbrace{\widetilde{\gamma_1^c}} = \langle \tau_{\widetilde{\gamma_1}}, \varpi_{\widetilde{\gamma_1}}, \vartheta_{\widetilde{\gamma_1}} \rangle$.

(ii) $\overbrace{\widetilde{\gamma_1}} \sqcup \overbrace{\widetilde{\gamma_2}} = \Big\langle \max(\vartheta_{\widetilde{\gamma_1}}, \vartheta_{\widetilde{\gamma_2}}), \min(\varpi_{\widetilde{\gamma_1}}, \varpi_{\widetilde{\gamma_2}}), \min(\tau_{\widetilde{\gamma_1}}, \tau_{\widetilde{\gamma_2}}) \Big\rangle$.

(iii) $\overbrace{\widetilde{\gamma_1}} \sqcap \overbrace{\widetilde{\gamma_2}} = \Big\langle \min(\vartheta_{\widetilde{\gamma_1}}, \vartheta_{\widetilde{\gamma_2}}), \min(\varpi_{\widetilde{\gamma_1}}, \varpi_{\widetilde{\gamma_2}}), \max(\tau_{\widetilde{\gamma_1}}, \tau_{\widetilde{\gamma_2}}) \Big\rangle$.

(iv) $\overbrace{\widetilde{\gamma_1}} \preceq \overbrace{\widetilde{\gamma_2}}$ if and only if $\vartheta_{\widetilde{\gamma_1}} \preceq \vartheta_{\widetilde{\gamma_2}}$ and $\varpi_{\widetilde{\gamma_1}} \preceq \varpi_{\widetilde{\gamma_2}}$ and $\tau_{\widetilde{\gamma_1}} \succeq \tau_{\widetilde{\gamma_2}}$.

(v) $\overbrace{\widetilde{\gamma_1}} = \overbrace{\widetilde{\gamma_2}}$ if and only if $\vartheta_{\widetilde{\gamma_1}} = \vartheta_{\widetilde{\gamma_2}}$ and $\varpi_{\widetilde{\gamma_1}} = \varpi_{\widetilde{\gamma_2}}$ and $\tau_{\widetilde{\gamma_1}} = \tau_{\widetilde{\gamma_2}}$.

Definition 2.3. [1] Let $\mathscr{X} = \{\chi_1, \chi_2, ..., \chi_n\}$ and $\mathscr{E} = \{\varepsilon_1, \varepsilon_2, ..., \varepsilon_m\}$ be the universe and set of parameters, respectively. The notation $(\mathscr{X}, \mathscr{E})$ is a soft universe. The function $\widetilde{\mathscr{U}} : \mathscr{E} \to D(I)^{\mathscr{X}}$ and $\widetilde{\xi}$ be an IVF subset of $\mathscr{E}$, i.e., $\widetilde{\xi} : \mathscr{E} \to I = D[0,1]$. Let $\widetilde{\mathscr{U}_\xi} : \mathscr{E} \to D(I)^{\mathscr{X}} \times D(I)$ be the function defined as $\widetilde{\mathscr{U}_\xi}(\varepsilon) = \Big(\widetilde{\mathscr{U}}(\varepsilon)(\chi), \widetilde{\xi}(\varepsilon)\Big), \forall \chi \in \mathscr{X}$. Then $\widetilde{\mathscr{U}_\xi}$ is called a generalized interval valued fuzzy soft (GIVFS) set on $(\mathscr{X}, \mathscr{E})$. For each parameter ε_i, $\widetilde{\mathscr{U}_\xi}(\varepsilon_i) = \Big(\widetilde{\mathscr{U}}(\varepsilon_i)(\chi), \widetilde{\xi}(\varepsilon_i)\Big), \forall \chi \in \mathscr{X}$ not simply the degree to which the constituent parts of $\mathscr{X}$ in $\widetilde{\mathscr{U}}(\varepsilon_i)$ but moreover the interval valued likelihood of such belongingness, which is given by $\widetilde{\xi}(\varepsilon_i)$. So we can write $\widetilde{\mathscr{U}_\xi}(\varepsilon_i)$ as $\widetilde{\mathscr{U}_\xi}(\varepsilon_i) = \Big(\Big\{\frac{\chi_1}{\widetilde{\mathscr{U}}(\varepsilon_i)(\chi_1)}, \frac{\chi_2}{\widetilde{\mathscr{U}}(\varepsilon_i)(\chi_2)}, ..., \frac{\chi_n}{\widetilde{\mathscr{U}}(\varepsilon_i)(\chi_n)}\Big\}, \widetilde{\xi}(\varepsilon_i)\Big)$, where $\widetilde{\mathscr{U}}(\varepsilon_i)(\chi_1)$, $\widetilde{\mathscr{U}}(\varepsilon_i)(\chi_2)$,...,$\widetilde{\mathscr{U}}(\varepsilon_i)(\chi_n)$ are the degrees of belongingness and The degree of such belongingness interval valued possibility is $\widetilde{\xi}(\varepsilon_i)$.

Definition 2.4. [2] Let $\mathscr{X} = \{\chi_1, \chi_2, ..., \chi_n\}$ and $\mathscr{E} = \{\varepsilon_1, \varepsilon_2, ..., \varepsilon_m\}$ be the universe and set of parameters, respectively. The notation $(\mathscr{X}, \mathscr{E})$ is a soft universe. The function $\widetilde{\mathscr{U}} : \mathscr{E} \to \widetilde{\mathscr{U}}(\mathscr{X})$ and $\widetilde{\xi}$ be an IVF subset of $\mathscr{E}$, i.e., $\widetilde{\xi} : \mathscr{E} \to \widetilde{\mathscr{U}}(\mathscr{X})$. Let $\widetilde{\mathscr{U}_\xi} : \mathscr{E} \to \widetilde{\mathscr{U}}(\mathscr{X}) \times \widetilde{\mathscr{U}}(\mathscr{X})$ be a function defined as $\widetilde{\mathscr{U}_\xi}(\varepsilon) = (\widetilde{\mathscr{U}}(\varepsilon)(\chi), \widetilde{\xi}(\varepsilon)(\chi)), \forall \chi \in \mathscr{X}$. Then $\widetilde{\mathscr{U}_\xi}$ is called a possibility interval valued fuzzy soft (PIVFS) set on $(\mathscr{X}, \mathscr{E})$. For each parameter ε_i, $\widetilde{\mathscr{U}_\xi}(\varepsilon_i) = (\widetilde{\mathscr{U}}(\varepsilon_i)(\chi), \widetilde{\xi}(\varepsilon_i)(\chi))$ not merely how much the components of $\mathscr{X}$ belong together in $\widetilde{\mathscr{U}}(\varepsilon_i)$, also the degree of IV possibility of such belongingness which is $\widetilde{\xi}(\varepsilon_i)$. Hence, $\widetilde{\mathscr{U}_\xi}(\varepsilon_i) = \Big\{ \Big(\frac{\chi_1}{\widetilde{\mathscr{U}}(\varepsilon_i)(\chi_1)}, \widetilde{\xi}(\varepsilon_i)(\chi_1)\Big), \Big(\frac{\chi_2}{\widetilde{\mathscr{U}}(\varepsilon_i)(\chi_2)}, \widetilde{\xi}(\varepsilon_i)(\chi_2)\Big), ..., \Big(\frac{\chi_n}{\widetilde{\mathscr{U}}(\varepsilon_i)(\chi_n)}, \widetilde{\xi}(\varepsilon_i)(\chi_n)\Big) \Big\}$.

3 Type-II GPyNSIVFS sets

The idea of type-II generalized Pythagorean neutrosophic interval valued soft sets is only being started.

Definition 3.1. Let $\mathscr{X} = \{\chi_1, \chi_2, ..., \chi_n\}$ and $\mathscr{E} = \{\varepsilon_1, \varepsilon_2, ..., \varepsilon_m\}$ be the universal and set of parameters, respectively. The notation $(\mathscr{X}, \mathscr{E})$ represents a soft universe. Consider $\overbrace{\widetilde{\mathscr{U}}} : \mathscr{E} \to \overbrace{S\widetilde{\mathscr{U}}(\mathscr{X})}$ and $\widetilde{u}$ is a PyNSIV subset of $\mathscr{E}$. That is $\overbrace{\widetilde{u}} : \mathscr{E} \to \overbrace{D[0,1]}$, $\overbrace{S\widetilde{\mathscr{U}}(\mathscr{X})}$ denotes the collection of all PyNSIV subsets of $\mathscr{X}$. If $\overbrace{\widetilde{\mathscr{U}_u}} : \mathscr{E} \to \overbrace{S\widetilde{\mathscr{U}}(\mathscr{X})} \times \overbrace{D[0,1]}$ is a function defined as $\overbrace{\widetilde{\mathscr{U}_u}(\varepsilon)} = \left(\overbrace{\widetilde{\mathscr{U}}(\varepsilon)(\chi)}, \overbrace{\widetilde{u}(\varepsilon)}\right), \chi \in \mathscr{X}$, then $\overbrace{\widetilde{\mathscr{U}_u}}$ is a type-II generalized Pythagorean neutrosophic interval valued soft (Type-II GPyNSIVS) set on $(\mathscr{X}, \mathscr{E})$. For each parameter e,

$$\overbrace{\widetilde{\mathscr{U}_u}(\varepsilon_i)} = \Big(\Big\{\frac{\chi_1}{(\vartheta_{\widetilde{\mathscr{U}}(\varepsilon)}(\chi_1), \varpi_{\widetilde{\mathscr{U}}(\varepsilon)}(\chi_1), \tau_{\widetilde{\mathscr{U}}(\varepsilon)}(\chi_1))}, ..., \frac{\chi_n}{(\vartheta_{\widetilde{\mathscr{U}}(\varepsilon)}(\chi_n), \varpi_{\widetilde{\mathscr{U}}(\varepsilon)}(\chi_n), \tau_{\widetilde{\mathscr{U}}(\varepsilon)}(\chi_n))}\Big\}, (\widetilde{u}_1(\varepsilon_i), \widetilde{u}_2(\varepsilon_i), \widetilde{u}_3(\varepsilon_i))\Big).$$

Example 3.2. Let $\mathscr{X} = \{\chi_1, \chi_2, \chi_3\}$ be a set of three heart patients with symptoms, $\mathscr{E} = \{\varepsilon_1$: hyper tension, ε_2 : highly blood pressure, and ε_3 : weight loss$\}$. Consider $\overbrace{\widetilde{\mathscr{U}_u}} : \mathscr{E} \to \overbrace{S\widetilde{\mathscr{U}}(\mathscr{X})} \times \overbrace{D[0,1]}$ is given by

$$\overbrace{\widetilde{\mathscr{U}_u}(\varepsilon_1)} = \left(\left\{\begin{array}{c} \frac{\chi_1}{([0.40,0.50],[0.10,0.15],[0.50,0.65])} \\ \frac{\chi_2}{([0.55,0.60],[0.10,0.20],[0.45,0.50])} \\ \frac{\chi_3}{([0.35,0.40],[0.25,0.30],[0.45,0.55])} \end{array}\right\}, ([0.45,0.55],[0.45,0.50],[0.15,0.20])\right);$$

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$$\overbrace{\widetilde{\mathscr{U}}_u(\varepsilon_2)} = \left(\left\{ \begin{array}{c} \frac{\chi_1}{([0.35,0.40],[0.15,0.20],[0.55,0.65])} \\ \frac{\chi_2}{([0.40,0.50],[0.25,0.35],[0.45,0.55])} \\ \frac{\chi_3}{([0.35,0.45],[0.15,0.25],[0.50,0.65])} \end{array} \right\}, ([0.25,0.35],[0.15,0.25],[0.35,0.45]) \right);$$

$$\overbrace{\widetilde{\mathscr{U}}_u(\varepsilon_3)} = \left(\left\{ \begin{array}{c} \frac{\chi_1}{([0.10,0.15],[0.35,0.40],[0.55,0.65])} \\ \frac{\chi_2}{([0.20,0.25],[0.45,0.50],[0.40,0.45])} \\ \frac{\chi_3}{([0.10,0.15],[0.20,0.30],[0.50,0.55])} \end{array} \right\}, ([0.30,0.45],[0.25,0.35],[0.35,0.45]) \right).$$

Definition 3.3. Let $\mathscr{X}$ and $\mathscr{E}$ be the universal and set of parameters, respectively. Let $\widehat{\widetilde{\mathscr{U}}_u}$ and $\widehat{\widetilde{\mathscr{V}}_v}$ be the type-II GPyNSIVS sets on $(\mathscr{X}, \mathscr{E})$. Now $\widehat{\widetilde{\mathscr{U}}_v}$ is a type-II GPyNSIVS subset of $\widehat{\widetilde{\mathscr{V}}_v}$ if and only if
(i) $\overbrace{\widetilde{\mathscr{U}}(\varepsilon)(\chi)} \sqsubseteq \overbrace{\widetilde{\mathscr{V}}(\varepsilon)(\chi)}$ if $\vartheta_{\widetilde{\mathscr{U}}(\varepsilon)}(\chi) \preceq \vartheta_{\widetilde{\mathscr{V}}(\varepsilon)}(\chi)$, $\varpi_{\widetilde{\mathscr{U}}(\varepsilon)}(\chi) \preceq \varpi_{\widetilde{\mathscr{V}}(\varepsilon)}(\chi)$, $\tau_{\widetilde{\mathscr{U}}(\varepsilon)}(\chi) \succeq \tau_{\widetilde{\mathscr{V}}(\varepsilon)}(\chi)$,
(ii) $\widehat{\widetilde{u}(\varepsilon)} \preceq \widehat{\widetilde{v}(\varepsilon)}$, $\forall \varepsilon \in \mathscr{E}$ and $\forall \chi \in \mathscr{X}$.

Example 3.4. Consider the type-II GPyNSIVS set $\widehat{\widetilde{\mathscr{U}}_u}$ in Example 3.2. Let $\widehat{\widetilde{\mathscr{V}}_v}$ be another type-II GPyNSIVS set is established as:

$$\overbrace{\widetilde{\mathscr{V}}_v(\varepsilon_1)} = \left(\left\{ \begin{array}{c} \frac{\chi_1}{([0.50,0.60],[0.15,0.35],[0.30,0.35])} \\ \frac{\chi_2}{([0.60,0.75],[0.15,0.35],[0.10,0.15])} \\ \frac{\chi_3}{([0.45,0.65],[0.35,0.55],[0.20,0.25])} \end{array} \right\}, ([0.50,0.60],[0.55,0.60],[0.10,0.15]) \right);$$

$$\overbrace{\widetilde{\mathscr{V}}_v(\varepsilon_2)} = \left(\left\{ \begin{array}{c} \frac{\chi_1}{([0.45,0.55],[0.25,0.45],[0.15,0.20])} \\ \frac{\chi_2}{([0.50,0.65],[0.40,0.65],[0.05,0.10])} \\ \frac{\chi_3}{([0.45,0.75],[0.35,0.45],[0.20,0.25])} \end{array} \right\}, ([0.40,0.45],[0.25,0.30],[0.30,0.35]) \right);$$

$$\overbrace{\widetilde{\mathscr{V}}_v(\varepsilon_3)} = \left(\left\{ \begin{array}{c} \frac{\chi_1}{([0.30,0.45],[0.45,0.60],[0.10,0.15])} \\ \frac{\chi_2}{([0.40,0.55],[0.65,0.65],[0.10,0.15])} \\ \frac{\chi_3}{([0.30,0.55],[0.40,0.50],[0.15,0.20])} \end{array} \right\}, ([0.35,0.50],[0.30,0.40],[0.15,0.25]) \right).$$

Definition 3.5. Let $\mathscr{X}$ and $\mathscr{E}$ be the universe and set of parameters, respectively. Let $\widehat{\widetilde{\mathscr{U}}_u}$ and $\widehat{\widetilde{\mathscr{V}}_v}$ be the type-II GPyNSIVS sets on $(\mathscr{X}, \mathscr{E})$. These two type-II GPyNSIVS sets are equal if and only if $\widehat{\widetilde{\mathscr{U}}_u} \sqsubseteq \widehat{\widetilde{\mathscr{V}}_v}$ and $\widehat{\widetilde{\mathscr{U}}_u} \sqsupseteq \widehat{\widetilde{\mathscr{V}}_v}$.

Definition 3.6. Let $\mathscr{X}$ and $\mathscr{E}$ be the universe and set of parameters, respectively. Let $\widehat{\widetilde{\mathscr{V}}_v}$ be the type-II GPyNSIVS set on $(\mathscr{X}, \mathscr{E})$. The complement of $\widehat{\widetilde{\mathscr{V}}_v}$ is established as $\widehat{\widetilde{\mathscr{V}}_v^c} = \left(\overbrace{\widetilde{\mathscr{V}}^c(\varepsilon)(\chi)}, \overbrace{\widetilde{v}^c(\varepsilon)} \right)$, where $\overbrace{\widetilde{\mathscr{V}}^c(\varepsilon)(\chi)} = \left\{ \frac{x}{\left(\tau_{\widetilde{\mathscr{V}}(\varepsilon)}(\chi), \varpi_{\widetilde{\mathscr{V}}(\varepsilon)}(\chi), \vartheta_{\widetilde{\mathscr{V}}(\varepsilon)}(\chi)\right)} \right\}$ and $\overbrace{\widetilde{v}^c(\varepsilon)} = (\widetilde{v}_3(\varepsilon), \widetilde{v}_2(\varepsilon), \widetilde{v}_1(\varepsilon))$. Clearly, $\left(\widehat{\widetilde{\mathscr{V}}_v^c}\right)^c = \widehat{\widetilde{\mathscr{V}}_v}$.

Definition 3.7. Let $\mathscr{X}$ and $\mathscr{E}$ be the universe and set of parameters, respectively. Let $\widehat{\widetilde{\mathscr{U}}_u}$ and $\widehat{\widetilde{\mathscr{V}}_v}$ be the type-II GPyNSIVS sets on $(\mathscr{X}, \mathscr{E})$. The union and intersection of $\widehat{\widetilde{\mathscr{U}}_u}$ and $\widehat{\widetilde{\mathscr{V}}_v}$ over $(\mathscr{X}, \mathscr{E})$ are denoted by $\widehat{\widetilde{\mathscr{U}}_u} \sqcup \widehat{\widetilde{\mathscr{V}}_v}$ and $\widehat{\widetilde{\mathscr{U}}_u} \sqcap \widehat{\widetilde{\mathscr{V}}_v}$, and calculated by $\widehat{\widetilde{Z}_z} : \mathscr{E} \to \overbrace{S\widetilde{\mathscr{U}}(\mathscr{X})} \times \overbrace{D[0,1]}$, $\widehat{\widetilde{Y}_y} : \mathscr{E} \to$

$\overbrace{S\widetilde{\mathscr{U}}(\mathscr{X})} \times \overbrace{D[0,1]}$ such that $\overbrace{\widetilde{Z}_z(\varepsilon)(\chi)} = \left(\overbrace{\widetilde{Z}(\varepsilon)(\chi)}, \overbrace{\widetilde{z}(\varepsilon)}\right)$, $\overbrace{\widetilde{Y}_y(\varepsilon)(\chi)} = \left(\overbrace{\widetilde{Y}(\varepsilon)(\chi)}, \overbrace{\widetilde{y}(\varepsilon)}\right)$, $\overbrace{\widetilde{Z}(\varepsilon)(\chi)} = \overbrace{\widetilde{\mathscr{U}}(\varepsilon)(\chi)} \sqcup \overbrace{\widetilde{\mathscr{V}}(\varepsilon)(\chi)}$, $\overbrace{\widetilde{z}(\varepsilon)} = \overbrace{\widetilde{u}(\varepsilon)} \sqcup \overbrace{\widetilde{v}(\varepsilon)}$, $\overbrace{\widetilde{Y}(\varepsilon)(\chi)} = \overbrace{\widetilde{\mathscr{U}}(\varepsilon)(\chi)} \sqcap \overbrace{\widetilde{\mathscr{V}}(\varepsilon)(\chi)}$ and $\overbrace{\widetilde{y}(\varepsilon)} = \overbrace{\widetilde{u}(\varepsilon)} \sqcap \overbrace{\widetilde{v}(\varepsilon)}, \forall \chi \in \mathscr{X}$.

Example 3.8. Let $\overbrace{\mathscr{U}_u}$ and $\overbrace{\mathscr{V}_v}$ be the type-II GPyNSIVS sets on $(\mathscr{X}, \mathscr{E})$ is established as

$$\overbrace{\widetilde{\mathscr{U}}_u(\varepsilon_1)} = \left(\left\{\begin{array}{c}\frac{\chi_1}{([0.40,0.60],[0.30,0.40],[0.40,0.60])}\\ \frac{\chi_2}{([0.30,0.50],[0.60,0.70],[0.40,0.50])}\\ \frac{\chi_3}{([0.40,0.70],[0.30,0.50],[0.30,0.40])}\end{array}\right\}, ([0.35,0.40],[0.40,0.50],[0.50,0.65])\right);$$

$$\overbrace{\widetilde{\mathscr{U}}_u(\varepsilon_2)} = \left(\left\{\begin{array}{c}\frac{\chi_1}{([0.30,0.60],[0.10,0.20],[0.60,0.70])}\\ \frac{\chi_2}{([0.40,0.50],[0.20,0.40],[0.60,0.70])}\\ \frac{\chi_3}{([0.50,0.70],[0.20,0.30],[0.40,0.60])}\end{array}\right\}, ([0.40,0.60],[0.30,0.35],[0.35,0.40])\right);$$

and

$$\overbrace{\widetilde{\mathscr{V}}_v(\varepsilon_1)} = \left(\left\{\begin{array}{c}\frac{\chi_1}{([0.40,0.50],[0.30,0.50],[0.50,0.70])}\\ \frac{\chi_2}{([0.30,0.40],[0.10,0.30],[0.60,0.80])}\\ \frac{\chi_3}{([0.50,0.60],[0.40,0.50],[0.30,0.40])}\end{array}\right\}, ([0.40,0.50],[0.45,0.65],[0.25,0.35])\right);$$

$$\overbrace{\widetilde{\mathscr{V}}_v(\varepsilon_2)} = \left(\left\{\begin{array}{c}\frac{\chi_1}{([0.20,0.50],[0.30,0.40],[0.60,0.70])}\\ \frac{\chi_2}{([0.30,0.40],[0.40,0.50],[0.50,0.60])}\\ \frac{\chi_3}{([0.30,0.50],[0.40,0.60],[0.50,0.60])}\end{array}\right\}, ([0.30,0.40],[0.25,0.45],[0.60,0.70])\right).$$

Thus, $\overbrace{\mathscr{U}_u} \sqcup \overbrace{\mathscr{V}_v}$ is

$$\overbrace{\widetilde{\mathscr{U}}_u(\varepsilon_1)} \sqcup \overbrace{\widetilde{\mathscr{V}}_v(\varepsilon_1)} = \left(\left\{\begin{array}{c}\frac{\chi_1}{([0.40,0.60],[0.30,0.40],[0.40,0.60])}\\ \frac{\chi_2}{([0.30,0.50],[0.10,0.30],[0.40,0.50])}\\ \frac{\chi_3}{([0.50,0.70],[0.30,0.50],[0.30,0.40])}\end{array}\right\}, ([0.40,0.50],[0.40,0.50],[0.25,0.35])\right);$$

$$\overbrace{\widetilde{\mathscr{U}}_u(\varepsilon_2)} \sqcup \overbrace{\widetilde{\mathscr{V}}_v(\varepsilon_2)} = \left(\left\{\begin{array}{c}\frac{\chi_1}{([0.30,0.60],[0.10,0.20],[0.60,0.70])}\\ \frac{\chi_2}{([0.40,0.50],[0.20,0.40],[0.50,0.60])}\\ \frac{\chi_3}{([0.50,0.70],[0.20,0.30],[0.40,0.60])}\end{array}\right\}, ([0.40,0.60],[0.25,0.35],[0.35,0.40])\right).$$

Thus, $\overbrace{\mathscr{U}_u} \sqcap \overbrace{\mathscr{V}_v}$ is

$$\overbrace{\widetilde{\mathscr{U}}_u(\varepsilon_1)} \sqcap \overbrace{\widetilde{\mathscr{V}}_v(\varepsilon_1)} = \left(\left\{\begin{array}{c}\frac{\chi_1}{([0.40,0.50],[0.30,0.40],[0.50,0.70])}\\ \frac{\chi_2}{([0.30,0.40],[0.10,0.30],[0.60,0.80])}\\ \frac{\chi_3}{([0.40,0.60],[0.30,0.50],[0.30,0.40])}\end{array}\right\}, ([0.35,0.40],[0.40,0.50],[0.50,0.65])\right);$$

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$$\overbrace{\widetilde{\mathscr{U}_u}(\varepsilon_2)} \sqcap \overbrace{\widetilde{\mathscr{V}_v}(\varepsilon_2)} = \left(\left\{ \begin{array}{c} \frac{\chi_1}{([0.20, 0.50], [0.10, 0.20], [0.60, 0.70])} \\ \frac{\chi_2}{([0.30, 0.40], [0.20, 0.40], [0.60, 0.70])} \\ \frac{\chi_3}{([0.30, 0.50], [0.20, 0.30], [0.50, 0.60])} \end{array} \right\}, ([0.30, 0.40], [0.25, 0.35], [0.60, 0.70]) \right).$$

Definition 3.9. A type-II GPyNSIVS set $\overbrace{\widetilde{\emptyset}_\theta(\varepsilon)(\chi)} = \left(\overbrace{\widetilde{\emptyset}(\varepsilon)(\chi)}, \overbrace{\widetilde{\theta}(\varepsilon)}\right)$ is called as type-II generalized null Pythagorean neutrosophic interval valued soft set $\overbrace{\widetilde{\emptyset}_\theta} : \mathscr{E} \to \overbrace{S\widetilde{\mathscr{U}}(\mathscr{X})} \times \overbrace{D[0,1]}$, where $\overbrace{\widetilde{\emptyset}(\varepsilon)(\chi)} = (\widetilde{0}, \widetilde{0}, \widetilde{1})$ and $\overbrace{\widetilde{\theta}(\varepsilon)} = (\widetilde{0}, \widetilde{0}, \widetilde{0})$, where $\widetilde{0} = [0,0] = 0$ and $\widetilde{1} = [1,1] = 1, \forall \chi \in \mathscr{X}$.

Definition 3.10. A type-II GPyNSIVS set $\overbrace{\widetilde{\Omega}_\Lambda(\varepsilon)(\chi)} = \left(\overbrace{\widetilde{\Omega}(\varepsilon)(\chi)}, \overbrace{\widetilde{\Lambda}(\varepsilon)}\right)$ is called as type-II generalized absolute Pythagorean neutrosophic interval valued soft set $\overbrace{\widetilde{\Omega}_\Lambda} : \mathscr{E} \to \overbrace{S\widetilde{\mathscr{U}}(\mathscr{X})} \times \overbrace{D[0,1]}$, where $\overbrace{\widetilde{\Omega}(\varepsilon)(\chi)} = (\widetilde{1}, \widetilde{0}, \widetilde{0})$, $\overbrace{\widetilde{\Lambda}(\varepsilon)} = (\widetilde{1}, \widetilde{1}, \widetilde{1}), \forall \chi \in \mathscr{X}$.

Theorem 3.11. *Let $\overbrace{\widetilde{\mathscr{U}_u}}$ be the type-II GPyNSIVS set on $(\mathscr{X}, \mathscr{E})$. Then*
(i) $\overbrace{\widetilde{\mathscr{U}_u}} = \overbrace{\widetilde{\mathscr{U}_u}} \sqcup \overbrace{\widetilde{\mathscr{U}_u}}$, $\overbrace{\widetilde{\mathscr{U}_u}} = \overbrace{\widetilde{\mathscr{U}_u}} \sqcap \overbrace{\widetilde{\mathscr{U}_u}}$,
(ii) $\overbrace{\widetilde{\mathscr{U}_u}} \sqsubseteq \overbrace{\widetilde{\mathscr{U}_u}} \sqcup \overbrace{\widetilde{\mathscr{U}_u}}$, $\overbrace{\widetilde{\mathscr{U}_u}} \sqsubseteq \overbrace{\widetilde{\mathscr{U}_u}} \sqcap \overbrace{\widetilde{\mathscr{U}_u}}$,
(iii) $\overbrace{\widetilde{\mathscr{U}_u}} \sqcup \overbrace{\widetilde{\emptyset}_\theta} = \overbrace{\widetilde{\mathscr{U}_u}}$, $\overbrace{\widetilde{\mathscr{U}_u}} \sqcap \overbrace{\widetilde{\emptyset}_\theta} = \overbrace{\widetilde{\emptyset}_\theta}$,
(iv) $\overbrace{\widetilde{\mathscr{U}_u}} \sqcup \overbrace{\widetilde{\Omega}_\Lambda} = \overbrace{\widetilde{\Omega}_\Lambda}$, $\overbrace{\widetilde{\mathscr{U}_u}} \sqcap \overbrace{\widetilde{\Omega}_\Lambda} = \overbrace{\widetilde{\mathscr{U}_u}}$.

Theorem 3.12. *Let $\overbrace{\widetilde{\mathscr{U}_u}}$ and $\overbrace{\widetilde{\mathscr{V}_v}}$ are any two type-II GPyNSIVS sets over $(\mathscr{X}, \mathscr{E})$. Then*
(i) $\overbrace{\widetilde{\mathscr{U}_u}} \sqcup \overbrace{\widetilde{\mathscr{V}_v}} = \overbrace{\widetilde{\mathscr{V}_v}} \sqcup \overbrace{\widetilde{\mathscr{U}_u}}$,
(ii) $\overbrace{\widetilde{\mathscr{U}_u}} \sqcap \overbrace{\widetilde{\mathscr{V}_v}} = \overbrace{\widetilde{\mathscr{V}_v}} \sqcap \overbrace{\widetilde{\mathscr{U}_u}}$,
(iii) $\left(\overbrace{\widetilde{\mathscr{U}_u}} \sqcup \overbrace{\widetilde{\mathscr{V}_v}}\right)^c = \overbrace{\widetilde{\mathscr{U}_u^c}} \sqcap \overbrace{\widetilde{\mathscr{V}_v^c}}$,
(iv) $\left(\overbrace{\widetilde{\mathscr{U}_u}} \sqcap \overbrace{\widetilde{\mathscr{V}_v}}\right)^c = \overbrace{\widetilde{\mathscr{U}_u^c}} \sqcup \overbrace{\widetilde{\mathscr{V}_v^c}}$.

Theorem 3.13. *Let $\overbrace{\widetilde{\mathscr{U}_u}}$, $\overbrace{\widetilde{\mathscr{V}_v}}$ and $\overbrace{\widetilde{\mathscr{W}_w}}$ are three type-II GPyNSIVS sets over $(\mathscr{X}, \mathscr{E})$. Then*
(i) $\overbrace{\widetilde{\mathscr{U}_u}} \sqcup (\overbrace{\widetilde{\mathscr{V}_v}} \sqcup \overbrace{\widetilde{\mathscr{W}_w}}) = (\overbrace{\widetilde{\mathscr{U}_u}} \sqcup \overbrace{\widetilde{\mathscr{V}_v}}) \sqcup \overbrace{\widetilde{\mathscr{W}_w}}$,
(ii) $\overbrace{\widetilde{\mathscr{U}_u}} \sqcap (\overbrace{\widetilde{\mathscr{V}_v}} \sqcap \overbrace{\widetilde{\mathscr{W}_w}}) = (\overbrace{\widetilde{\mathscr{U}_u}} \sqcap \overbrace{\widetilde{\mathscr{V}_v}}) \sqcap \overbrace{\widetilde{\mathscr{W}_w}}$,
(iii) $\overbrace{\widetilde{\mathscr{U}_u}} \sqcup (\overbrace{\widetilde{\mathscr{V}_v}} \sqcap \overbrace{\widetilde{\mathscr{W}_w}}) = (\overbrace{\widetilde{\mathscr{U}_u}} \sqcup \overbrace{\widetilde{\mathscr{V}_v}}) \sqcap (\overbrace{\widetilde{\mathscr{U}_u}} \sqcup \overbrace{\widetilde{\mathscr{W}_w}})$,
(iv) $\overbrace{\widetilde{\mathscr{U}_u}} \sqcap (\overbrace{\widetilde{\mathscr{V}_v}} \sqcup \overbrace{\widetilde{\mathscr{W}_w}}) = (\overbrace{\widetilde{\mathscr{U}_u}} \sqcap \overbrace{\widetilde{\mathscr{V}_v}}) \sqcup (\overbrace{\widetilde{\mathscr{U}_u}} \sqcap \overbrace{\widetilde{\mathscr{W}_w}})$,
(v) $(\overbrace{\widetilde{\mathscr{U}_u}} \sqcup \overbrace{\widetilde{\mathscr{V}_v}}) \sqcap \overbrace{\widetilde{\mathscr{U}_u}} = \overbrace{\widetilde{\mathscr{U}_u}}$,
(vi) $(\overbrace{\widetilde{\mathscr{U}_u}} \sqcap \overbrace{\widetilde{\mathscr{V}_v}}) \sqcup \overbrace{\widetilde{\mathscr{U}_u}} = \overbrace{\widetilde{\mathscr{U}_u}}$.

Definition 3.14. Let $(\overbrace{\widetilde{\mathscr{U}_u}}, \mathscr{P})$ and $(\overbrace{\widetilde{\mathscr{V}_v}}, \mathscr{Q})$ be the type-II GPyNSIVS sets on $(\mathscr{X}, \mathscr{E})$. Then the operation "$(\overbrace{\widetilde{\mathscr{U}_u}}, \mathscr{P})$ AND $(\overbrace{\widetilde{\mathscr{V}_v}}, \mathscr{Q})$" is defined as $(\overbrace{\widetilde{\mathscr{U}_u}}, \mathscr{P}) \wedge (\overbrace{\widetilde{\mathscr{V}_v}}, \mathscr{Q}) = (\overbrace{\widetilde{\mathscr{W}_w}}, \mathscr{P} \times \mathscr{Q})$, where $\overbrace{\widetilde{\mathscr{W}_r}(\kappa, \gamma)} = \left(\overbrace{\widetilde{\mathscr{W}_w}(\kappa, \gamma)(\chi)}, \overbrace{\widetilde{w}(\kappa, \gamma)}\right)$ such that $\overbrace{\widetilde{\mathscr{W}}(\kappa, \gamma)} = \overbrace{\widetilde{\mathscr{U}}(\kappa)} \sqcap \overbrace{\widetilde{\mathscr{V}}(\gamma)}$ and $\overbrace{\widetilde{w}(\kappa, \gamma)} = \overbrace{\widetilde{u}(\kappa)} \sqcap \overbrace{\widetilde{v}(\gamma)}, \forall (\kappa, \gamma) \in \mathscr{P} \times \mathscr{Q}$.

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Definition 3.15. Let $(\overbrace{\widetilde{\mathscr{U}_u}}, \mathscr{P})$ and $(\overbrace{\widetilde{\mathscr{V}_v}}, \mathscr{Q})$ be the type-II GPyNSIVS sets on $(\mathscr{X}, \mathscr{E})$. Then the operation "$(\overbrace{\widetilde{\mathscr{U}_u}}, \mathscr{P})$ OR $(\overbrace{\widetilde{\mathscr{V}_v}}, \mathscr{Q})$" is defined as $(\overbrace{\widetilde{\mathscr{U}_u}}, \mathscr{P}) \vee (\overbrace{\widetilde{\mathscr{V}_v}}, \mathscr{Q}) = (\overbrace{\widetilde{\mathscr{W}_w}}, \mathscr{P} \times \mathscr{Q})$, where $\overbrace{\widetilde{\mathscr{W}_r}(\kappa,\gamma)} = \left(\overbrace{\widetilde{\mathscr{W}_w}(\kappa,\gamma)(\chi)}, \overbrace{\widetilde{w}(\kappa,\gamma)}\right)$ such that $\overbrace{\widetilde{\mathscr{W}}(\kappa,\gamma)} = \overbrace{\widetilde{\mathscr{U}}(\kappa)} \sqcup \overbrace{\widetilde{\mathscr{V}}(\gamma)}$ and $\overbrace{\widetilde{w}(\kappa,\gamma)} = \overbrace{\widetilde{u}(\kappa)} \sqcup \overbrace{\widetilde{v}(\gamma)}, \forall(\kappa,\gamma) \in \mathscr{P} \times \mathscr{Q}$.

Theorem 3.16. *Let $(\overbrace{\widetilde{\mathscr{U}_u}}, \mathscr{P})$ and $(\overbrace{\widetilde{\mathscr{V}_v}}, \mathscr{Q})$ be the type-II GPyNSIVS sets on $(\mathscr{X}, \mathscr{E})$. Then*

(i) $\left((\overbrace{\widetilde{\mathscr{U}_u}}, \mathscr{P}) \wedge (\overbrace{\widetilde{\mathscr{V}_v}}, \mathscr{Q})\right)^c = \left(\overbrace{\widetilde{\mathscr{U}_u}}, \mathscr{P}\right)^c \vee \left(\overbrace{\widetilde{\mathscr{V}_v}}, \mathscr{Q}\right)^c$,

(ii) $\left((\overbrace{\widetilde{\mathscr{U}_u}}, \mathscr{P}) \vee (\overbrace{\widetilde{\mathscr{V}_v}}, \mathscr{Q})\right)^c = \left(\overbrace{\widetilde{\mathscr{U}_u}}, \mathscr{P}\right)^c \wedge \left(\overbrace{\widetilde{\mathscr{V}_v}}, \mathscr{Q}\right)^c$.

Proof. (i) Suppose that $(\overbrace{\widetilde{\mathscr{U}_u}}, \mathscr{P}) \wedge (\overbrace{\widetilde{\mathscr{V}_v}}, \mathscr{Q}) = (\overbrace{\widetilde{\mathscr{W}_w}}, \mathscr{P} \times \mathscr{Q})$ and $\left((\overbrace{\widetilde{\mathscr{U}_u}}, \mathscr{P}) \wedge (\overbrace{\widetilde{\mathscr{V}_v}}, \mathscr{Q})\right)^c = (\overbrace{\widetilde{\mathscr{W}^c_w}}, \mathscr{P} \times \mathscr{Q})$. Now, $\overbrace{\widetilde{\mathscr{W}^c_w}(\kappa,\gamma)} = \left(\overbrace{\widetilde{\mathscr{W}^c}(\kappa,\gamma)(\chi)}, \overbrace{\widetilde{w}^c(\kappa,\gamma)}\right), \forall(\kappa,\gamma) \in \mathscr{P} \times \mathscr{Q}$. By Theorem 3.12 and Definition 3.14, $\overbrace{\widetilde{\mathscr{W}^c}(\kappa,\gamma)} = \left(\overbrace{\widetilde{\mathscr{U}}(\kappa)} \sqcap \overbrace{\widetilde{\mathscr{V}}(\gamma)}\right)^c = \overbrace{\widetilde{\mathscr{U}^c}(\kappa)} \sqcup \overbrace{\widetilde{\mathscr{V}^c}(\gamma)}$ and $\overbrace{\widetilde{w}^c(\kappa,\gamma)} = \left(\overbrace{\widetilde{u}(\kappa)} \sqcap \overbrace{\widetilde{v}(\gamma)}\right)^c = \overbrace{\widetilde{u}^c(\kappa)} \sqcup \overbrace{\widetilde{v}^c(\gamma)}$. Also, $\left(\overbrace{\widetilde{\mathscr{U}_u}}, \mathscr{P}\right)^c \vee \left(\overbrace{\widetilde{\mathscr{V}_v}}, \mathscr{Q}\right)^c = (\overbrace{\widetilde{\Lambda}_o}, \mathscr{P} \times \mathscr{Q})$, where $\overbrace{\widetilde{\Lambda}_o(\kappa,\gamma)} = \left(\overbrace{\widetilde{\Lambda}(\kappa,\gamma)(\chi)}, \overbrace{\widetilde{o}(\kappa,\gamma)}\right)$ such that $\overbrace{\widetilde{\Lambda}(\kappa,\gamma)} = \overbrace{\widetilde{\mathscr{U}^c}(\kappa)} \sqcup \overbrace{\widetilde{\mathscr{V}^c}(\gamma)}$ and $\overbrace{\widetilde{o}(\kappa,\gamma)} = \overbrace{\widetilde{u}^c(\kappa)} \sqcup \overbrace{\widetilde{v}^c(\gamma)}, \forall(\kappa,\gamma) \in \mathscr{P} \times \mathscr{Q}$. Thus, $\overbrace{\widetilde{\mathscr{W}^c_w}} = \overbrace{\widetilde{\Lambda}_o}$. Hence $\left((\overbrace{\widetilde{\mathscr{U}_u}}, \mathscr{P}) \wedge (\overbrace{\widetilde{\mathscr{V}_v}}, \mathscr{Q})\right)^c = \left(\overbrace{\widetilde{\mathscr{U}_u}}, \mathscr{P}\right)^c \vee \left(\overbrace{\widetilde{\mathscr{V}_v}}, \mathscr{Q}\right)^c$.

(ii) Suppose that $(\overbrace{\widetilde{\mathscr{U}_u}}, \mathscr{P}) \vee (\overbrace{\widetilde{\mathscr{V}_v}}, \mathscr{Q}) = (\overbrace{\widetilde{\mathscr{W}_w}}, \mathscr{P} \times \mathscr{Q})$ and $\left((\overbrace{\widetilde{\mathscr{U}_u}}, \mathscr{P}) \vee (\overbrace{\widetilde{\mathscr{V}_v}}, \mathscr{Q})\right)^c = (\overbrace{\widetilde{\mathscr{W}^c_w}}, \mathscr{P} \times \mathscr{Q})$. Now, $\overbrace{\widetilde{\mathscr{W}^c_w}(\kappa,\gamma)} = \left(\overbrace{\widetilde{\mathscr{W}^c}(\kappa,\gamma)(\chi)}, \overbrace{\widetilde{w}^c(\kappa,\gamma)}\right), \forall(\kappa,\gamma) \in \mathscr{P} \times \mathscr{Q}$. By Theorem 3.12 and Definition 3.14, $\overbrace{\widetilde{\mathscr{W}^c}(\kappa,\gamma)} = \left(\overbrace{\widetilde{\mathscr{U}}(\kappa)} \sqcup \overbrace{\widetilde{\mathscr{V}}(\gamma)}\right)^c = \overbrace{\widetilde{\mathscr{U}^c}(\kappa)} \sqcap \overbrace{\widetilde{\mathscr{V}^c}(\gamma)}$ and $\overbrace{\widetilde{w}^c(\kappa,\gamma)} = \left(\overbrace{\widetilde{u}(\kappa)} \sqcup \overbrace{\widetilde{v}(\gamma)}\right)^c = \overbrace{\widetilde{u}^c(\kappa)} \sqcap \overbrace{\widetilde{v}^c(\gamma)}$. Also, $\left(\overbrace{\widetilde{\mathscr{U}_u}}, \mathscr{P}\right)^c \wedge \left(\overbrace{\widetilde{\mathscr{V}_v}}, \mathscr{Q}\right)^c = (\overbrace{\widetilde{\Lambda}_o}, \mathscr{P} \times \mathscr{Q})$, where $\overbrace{\widetilde{\Lambda}_o(\kappa,\gamma)} = \left(\overbrace{\widetilde{\Lambda}(\kappa,\gamma)(\chi)}, \overbrace{\widetilde{o}(\kappa,\gamma)}\right)$ such that $\overbrace{\widetilde{\Lambda}(\kappa,\gamma)} = \overbrace{\widetilde{\mathscr{U}^c}(\kappa)} \sqcap \overbrace{\widetilde{\mathscr{V}^c}(\gamma)}$ and $\overbrace{\widetilde{o}(\kappa,\gamma)} = \overbrace{\widetilde{u}^c(\kappa)} \sqcap \overbrace{\widetilde{v}^c(\gamma)}, \forall(\kappa,\gamma) \in \mathscr{P} \times \mathscr{Q}$. Thus, $\overbrace{\widetilde{\mathscr{W}^c_w}} = \overbrace{\widetilde{\Lambda}_o}$. Hence $\left((\overbrace{\widetilde{\mathscr{U}_u}}, \mathscr{P}) \vee (\overbrace{\widetilde{\mathscr{V}_v}}, \mathscr{Q})\right)^c = \left(\overbrace{\widetilde{\mathscr{U}_u}}, \mathscr{P}\right)^c \wedge \left(\overbrace{\widetilde{\mathscr{V}_v}}, \mathscr{Q}\right)^c$. □

4 Method for similarity measure

In this section, method for similarity measure between type-II GPyNSIVS sets is given below.

Definition 4.1. Let $\mathscr{X} = \{\chi_1, \chi_2, ..., \chi_m\}$ and $\mathscr{E} = \{\varepsilon_1, \varepsilon_2, ..., \varepsilon_n\}$ be the universe and set of parameters, respectively. Let $\overbrace{\widetilde{\mathscr{P}_u}}$ and $\overbrace{\widetilde{\mathscr{Q}_v}}$ be the type-II GPyNSIVS sets on $(\mathscr{X}, \mathscr{E})$. The similarity measure between two type-II GPyNSIVS sets $\overbrace{\widetilde{\mathscr{P}_u}}$ and $\overbrace{\widetilde{\mathscr{Q}_v}}$ is defined as $\mathrm{Sim}(\overbrace{\widetilde{\mathscr{P}_u}}, \overbrace{\widetilde{\mathscr{Q}_v}}) = \varphi(\overbrace{\widetilde{\mathscr{P}}}, \overbrace{\widetilde{\mathscr{Q}}}) \cdot \psi(\widetilde{u}, \widetilde{v})$. where $\varphi(\overbrace{\widetilde{\mathscr{P}}}, \overbrace{\widetilde{\mathscr{Q}}}) =$

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$$\frac{1}{m}\sum_{j=1}^{m}\begin{bmatrix}\min\left\{\mathscr{T}_1^{\mathscr{L}}\left(\overbrace{\widetilde{\mathscr{P}}(\varepsilon)(\chi_j)},\overbrace{\widetilde{\mathscr{Q}}(\varepsilon)(\chi_j)}\right),\mathscr{T}_2^{\mathscr{L}}\left(\overbrace{\widetilde{\mathscr{P}}(\varepsilon)(\chi_j)},\overbrace{\widetilde{\mathscr{Q}}(\varepsilon)(\chi_j)}\right),S^{\mathscr{L}}\left(\overbrace{\widetilde{\mathscr{P}}(\varepsilon)(\chi_j)},\overbrace{\widetilde{\mathscr{Q}}(\varepsilon)(\chi_j)}\right)\right\},\\ \max\left\{\mathscr{T}_1^{\mathscr{U}}\left(\overbrace{\widetilde{\mathscr{P}}(\varepsilon)(\chi_j)},\overbrace{\widetilde{\mathscr{Q}}(\varepsilon)(\chi_j)}\right),\mathscr{T}_2^{\mathscr{U}}\left(\overbrace{\widetilde{\mathscr{P}}(\varepsilon)(\chi_j)},\overbrace{\widetilde{\mathscr{Q}}(\varepsilon)(\chi_j)}\right),S^{\mathscr{U}}\left(\overbrace{\widetilde{\mathscr{P}}(\varepsilon)(\chi_j)},\overbrace{\widetilde{\mathscr{Q}}(\varepsilon)(\chi_j)}\right)\right\}\end{bmatrix}$$

and

$$\widetilde{\mathscr{T}_1}\left(\overbrace{\widetilde{\mathscr{P}}(\varepsilon)(\chi_j)},\overbrace{\widetilde{\mathscr{Q}}(\varepsilon)(\chi_j)}\right)=$$

$$\left[\frac{\sum_{i=1}^{n}\left(\vartheta^{\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\vartheta^{\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)}{\sum_{i=1}^{n}\left(1-\sqrt{\left(1-\vartheta^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\vartheta^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)}\right)},\ \frac{\sum_{i=1}^{n}\left(\vartheta^{\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\vartheta^{\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)}{\sum_{i=1}^{n}\left(1-\sqrt{\left(1-\vartheta^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\vartheta^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)}\right)}\right],$$

$$\widetilde{\mathscr{T}_2}\left(\overbrace{\widetilde{\mathscr{P}}(\varepsilon)(\chi_j)},\overbrace{\widetilde{\mathscr{Q}}(\varepsilon)(\chi_j)}\right)=$$

$$\left[\frac{\sum_{i=1}^{n}\left(\varpi^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\varpi^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)}{\sum_{i=1}^{n}\left(1-\sqrt{\left(1-\varpi^{4\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\varpi^{4\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)}\right)},\ \frac{\sum_{i=1}^{n}\left(\varpi^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\varpi^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)}{\sum_{i=1}^{n}\left(1-\sqrt{\left(1-\varpi^{4\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\varpi^{4\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)}\right)}\right],$$

$$\widetilde{\mathscr{S}}\left(\overbrace{\widetilde{\mathscr{P}}(\varepsilon)(\chi_j)},\overbrace{\widetilde{\mathscr{Q}}(\varepsilon)(\chi_j)}\right)=$$

$$\left[1-\sqrt{\left|\left[\frac{\sum_{i=1}^{n}\left(\tau^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)-\tau^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)}{\sum_{i=1}^{n}1+\left(\tau^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\tau^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)},\ \frac{\sum_{i=1}^{n}\left(\tau^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)-\tau^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)}{\sum_{i=1}^{n}1+\left(\tau^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\tau^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)}\right]\right|}\right],$$

for $j=1,2,...,m$.

and $\psi(\widetilde{u},\widetilde{v})=1-\left[\frac{\sum_{i=1}^{n}\min\left\{\left[u^{\mathscr{L}}(\varepsilon_i),v^{\mathscr{L}}(\varepsilon_i)\right]\right\}}{\sum_{i=1}^{n}\left(u^{\mathscr{U}}(\varepsilon_i)+v^{\mathscr{U}}(\varepsilon_i)\right)},\ \frac{\sum_{i=1}^{n}\max\left\{\left[u^{\mathscr{U}}(\varepsilon_i),v^{\mathscr{U}}(\varepsilon_i)\right]\right\}}{\sum_{i=1}^{n}\left(u^{\mathscr{L}}(\varepsilon_i)+v^{\mathscr{L}}(\varepsilon_i)\right)}\right]$.

Theorem 4.2. *Let* $\overbrace{\widetilde{\mathscr{P}}_u}$, $\overbrace{\widetilde{\mathscr{Q}}_v}$ *and* $\overbrace{\widetilde{\mathscr{R}}_w}$ *be the any three type-II GPyNSIVS sets over* $(\mathscr{X},\mathscr{E})$*. Then* $\overbrace{\widetilde{\mathscr{P}}_u}\sqsubseteq\overbrace{\widetilde{\mathscr{Q}}_v}\sqsubseteq\overbrace{\widetilde{\mathscr{R}}_w}\implies\mathrm{Sim}(\overbrace{\widetilde{\mathscr{P}}_u},\overbrace{\widetilde{\mathscr{R}}_w})\preceq\mathrm{Sim}(\overbrace{\widetilde{\mathscr{Q}}_v},\overbrace{\widetilde{\mathscr{R}}_w})$.

Proof. For $j=1,2,...,m$,

$$\overbrace{\widetilde{\mathscr{P}}_u}\sqsubseteq\overbrace{\widetilde{\mathscr{Q}}_v}\implies\begin{Bmatrix}\left[\vartheta^{\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j),\vartheta^{\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right]\preceq\left[\vartheta^{\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j),\vartheta^{\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right]\\ \left[\varpi^{\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j),\varpi^{\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right]\preceq\left[\varpi^{\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j),\varpi^{\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right]\\ \left[\tau^{\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j),\tau^{\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right]\succeq\left[\tau^{\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j),\tau^{\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right]\\ \left[u^{\mathscr{L}}(\varepsilon_i),u^{\mathscr{U}}(\varepsilon_i)\right]\preceq\left[v^{\mathscr{L}}(\varepsilon_i),v^{\mathscr{U}}(\varepsilon_i)\right]\end{Bmatrix}$$

$$\overbrace{\widetilde{\mathscr{P}}_u}\sqsubseteq\overbrace{\widetilde{\mathscr{R}}_w}\implies\begin{Bmatrix}\left[\vartheta^{\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j),\vartheta^{\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right]\preceq\left[\vartheta^{\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j),\vartheta^{\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right]\\ \left[\varpi^{\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j),\varpi^{\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right]\preceq\left[\varpi^{\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j),\varpi^{\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right]\\ \left[\tau^{\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j),\tau^{\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right]\succeq\left[\tau^{\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j),\tau^{\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right]\\ \left[u^{\mathscr{L}}(\varepsilon_i),u^{\mathscr{U}}(\varepsilon_i)\right]\preceq\left[w^{\mathscr{L}}(\varepsilon_i),w^{\mathscr{U}}(\varepsilon_i)\right]\end{Bmatrix}$$

$$\overbrace{\widetilde{\mathscr{Q}}_v}\sqsubseteq\overbrace{\widetilde{\mathscr{R}}_w}\implies\begin{Bmatrix}\left[\vartheta^{\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j),\vartheta^{\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right]\preceq\left[\vartheta^{\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j),\vartheta^{\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right]\\ \left[\varpi^{\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j),\varpi^{\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right]\preceq\left[\varpi^{\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j),\varpi^{\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right]\\ \left[\tau^{\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j),\tau^{\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right]\succeq\left[\tau^{\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j),\tau^{\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right]\\ \left[v^{\mathscr{L}}(\varepsilon_i),v^{\mathscr{U}}(\varepsilon_i)\right]\preceq\left[w^{\mathscr{L}}(\varepsilon_i),w^{\mathscr{U}}(\varepsilon_i)\right]\end{Bmatrix}.$$

Clearly,

$$\begin{aligned}&\left[\left(\vartheta^{\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\vartheta^{\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right),\left(\vartheta^{\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\vartheta^{\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]\\ \preceq\ &\left[\left(\vartheta^{\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\cdot\vartheta^{\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right),\left(\vartheta^{\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\cdot\vartheta^{\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]\end{aligned}$$

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implies

$$\begin{aligned} &\left[\sum_{i=1}^{n}\left(\vartheta^{\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\vartheta^{\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right),\sum_{i=1}^{n}\left(\vartheta^{\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\vartheta^{\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]\\ \preceq\ &\left[\sum_{i=1}^{n}\left(\vartheta^{\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\cdot\vartheta^{\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right),\sum_{i=1}^{n}\left(\vartheta^{\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\cdot\vartheta^{\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right] \end{aligned} \tag{1}$$

for $j = 1, 2, ..., m$.
Clearly,

$$\left[\vartheta^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j),\ \vartheta^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right]\preceq\left[\vartheta^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j),\ \vartheta^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right]\preceq\left[\vartheta^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j),\ \vartheta^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right]$$

implies

$$\left[-\vartheta^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j),\ -\vartheta^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right]\succeq\left[-\vartheta^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j),\ -\vartheta^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right]\succeq\left[-\vartheta^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j),\ -\vartheta^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right]$$

and

$$\left[1-\vartheta^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j),\ 1-\vartheta^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right]\succeq\left[1-\vartheta^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j),\ 1-\vartheta^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right]\succeq\left[1-\vartheta^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j),\ 1-\vartheta^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right]$$

and

$$\begin{aligned} &\left[\left(\left(1-\vartheta^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\vartheta^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right),\ \left(\left(1-\vartheta^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\vartheta^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)\right]\\ \succeq\ &\left[\left(\left(1-\vartheta^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\vartheta^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right),\ \left(\left(1-\vartheta^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\vartheta^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)\right] \end{aligned}$$

and

$$\begin{aligned} &\left[\sqrt{\left(\left(1-\vartheta^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\vartheta^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)},\ \sqrt{\left(\left(1-\vartheta^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\vartheta^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right]\\ \succeq\ &\left[\sqrt{\left(\left(1-\vartheta^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\vartheta^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)},\ \sqrt{\left(\left(1-\vartheta^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\vartheta^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right] \end{aligned}$$

and

$$\begin{aligned} &1-\left[\sqrt{\left(\left(1-\vartheta^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\vartheta^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)},\ \sqrt{\left(\left(1-\vartheta^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\vartheta^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right]\\ \preceq\ &1-\left[\sqrt{\left(\left(1-\vartheta^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\vartheta^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)},\ \sqrt{\left(\left(1-\vartheta^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\vartheta^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right] \end{aligned}$$

and

$$\begin{aligned} &\left[1-\sqrt{\left(\left(1-\vartheta^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\vartheta^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)},\ 1-\sqrt{\left(\left(1-\vartheta^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\vartheta^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right]\\ \preceq\ &\left[1-\sqrt{\left(\left(1-\vartheta^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\vartheta^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)},\ 1-\sqrt{\left(\left(1-\vartheta^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\vartheta^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right] \end{aligned}$$

and

$$\begin{aligned} &\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\vartheta^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\vartheta^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right),\ \sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\vartheta^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\vartheta^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right)\right]\\ \preceq\ &\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\vartheta^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\vartheta^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right),\ \sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\vartheta^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\vartheta^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right)\right]. \end{aligned} \tag{2}$$

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Equations 1 and 2,

$$\frac{\left[\sum_{i=1}^{n}\left(\vartheta^{\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\vartheta^{\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right),\ \sum_{i=1}^{n}\left(\vartheta^{\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\vartheta^{\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\vartheta^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\vartheta^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right),\ \sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\vartheta^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\vartheta^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right)\right]}$$
$$\preceq\frac{\left[\sum_{i=1}^{n}\left(\vartheta^{\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\cdot\vartheta^{\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right),\ \sum_{i=1}^{n}\left(\vartheta^{\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\cdot\vartheta^{\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\vartheta^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\vartheta^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right),\ \sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\vartheta^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\vartheta^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right)\right]}$$

implies

$$\left[\frac{\left[\sum_{i=1}^{n}\left(\vartheta^{\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\vartheta^{\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\vartheta^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\vartheta^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right)\right]},\ \frac{\left[\sum_{i=1}^{n}\left(\vartheta^{\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\vartheta^{\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\vartheta^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\vartheta^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right)\right]}\right]$$
$$\preceq\left[\frac{\left[\sum_{i=1}^{n}\left(\vartheta^{\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\cdot\vartheta^{\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\vartheta^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\vartheta^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right)\right]},\ \frac{\left[\sum_{i=1}^{n}\left(\vartheta^{\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\cdot\vartheta^{\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\vartheta^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\vartheta^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right)\right]}\right].$$

Therefore

$$\widetilde{\mathscr{T}}_1\left(\overbrace{\widetilde{\mathscr{P}}(\varepsilon)(\chi_j)},\overbrace{\widetilde{\mathscr{R}}(\varepsilon)(\chi_j)}\right)\preceq\widetilde{\mathscr{T}}_1\left(\overbrace{\widetilde{\mathscr{Q}}(\varepsilon)(\chi_j)},\overbrace{\widetilde{\mathscr{R}}(\varepsilon)(\chi_j)}\right).\tag{3}$$

Clearly,

$$\left[\left(\varpi^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\varpi^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right),\left(\varpi^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\varpi^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]$$
$$\preceq\left[\left(\varpi^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\cdot\varpi^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right),\left(\varpi^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\cdot\varpi^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]$$

implies

$$\left[\sum_{i=1}^{n}\left(\varpi^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\varpi^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right),\sum_{i=1}^{n}\left(\varpi^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\varpi^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]$$
$$\preceq\left[\sum_{i=1}^{n}\left(\varpi^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\cdot\varpi^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right),\sum_{i=1}^{n}\left(\varpi^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\cdot\varpi^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]\tag{4}$$

for $j=1,2,...,m$.
Clearly,

$$\left[\varpi^{4\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j),\ \varpi^{4\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right]\preceq\left[\varpi^{4\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j),\ \varpi^{4\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right]\preceq\left[\varpi^{4\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j),\ \varpi^{4\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right]$$

implies

$$\left[-\varpi^{4\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j),\ -\varpi^{4\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right]\succeq\left[-\varpi^{4\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j),\ -\varpi^{4\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right]\succeq\left[-\varpi^{4\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j),\ -\varpi^{4\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right]$$

and

$$\left[1-\varpi^{4\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j),\ 1-\varpi^{4\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right]\succeq\left[1-\varpi^{4\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j),\ 1-\varpi^{4\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right]\succeq\left[1-\varpi^{4\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j),\ 1-\varpi^{4\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right]$$

and

$$\left[\left(\left(1-\varpi^{4\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\varpi^{4\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right),\ \left(\left(1-\varpi^{4\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\varpi^{4\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)\right]$$
$$\succeq\left[\left(\left(1-\varpi^{4\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\varpi^{4\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right),\ \left(\left(1-\varpi^{4\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\varpi^{4\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)\right]$$

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and

$$\begin{aligned}&\left[\sqrt{\left(\left(1-\varpi^{4\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\varpi^{4\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)},\ \sqrt{\left(\left(1-\varpi^{4\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\varpi^{4\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right]\\ \succeq\ &\left[\sqrt{\left(\left(1-\varpi^{4\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\varpi^{4\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)},\ \sqrt{\left(\left(1-\varpi^{4\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\varpi^{4\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right]\end{aligned}$$

and

$$\begin{aligned}&1-\left[\sqrt{\left(\left(1-\varpi^{4\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\varpi^{4\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)},\ \sqrt{\left(\left(1-\varpi^{4\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\varpi^{4\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right]\\ \preceq\ &1-\left[\sqrt{\left(\left(1-\varpi^{4\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\varpi^{4\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)},\ \sqrt{\left(\left(1-\varpi^{4\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\varpi^{4\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right]\end{aligned}$$

and

$$\begin{aligned}&\left[1-\sqrt{\left(\left(1-\varpi^{4\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\varpi^{4\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)},\ 1-\sqrt{\left(\left(1-\varpi^{4\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\varpi^{4\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right]\\ \preceq\ &\left[1-\sqrt{\left(\left(1-\varpi^{4\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\varpi^{4\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)},\ 1-\sqrt{\left(\left(1-\varpi^{4\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\varpi^{4\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right]\end{aligned}$$

and

$$\begin{aligned}&\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\varpi^{4\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\varpi^{4\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right),\ \sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\varpi^{4\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\varpi^{4\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right)\right]\\ \preceq\ &\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\varpi^{4\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\varpi^{4\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right),\ \sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\varpi^{4\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\varpi^{4\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right)\right].\end{aligned}\tag{5}$$

Equations 4 and 5,

$$\begin{aligned}&\frac{\left[\sum_{i=1}^{n}\left(\varpi^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\varpi^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right),\ \sum_{i=1}^{n}\left(\varpi^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\varpi^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\varpi^{4\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\varpi^{4\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right),\ \sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\varpi^{4\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\varpi^{4\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right)\right]}\\ \preceq\ &\frac{\left[\sum_{i=1}^{n}\left(\varpi^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\cdot\varpi^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right),\ \sum_{i=1}^{n}\left(\varpi^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\cdot\varpi^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\varpi^{4\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\varpi^{4\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right),\ \sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\varpi^{4\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\varpi^{4\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right)\right]}\end{aligned}$$

implies

$$\begin{aligned}&\left[\frac{\left[\sum_{i=1}^{n}\left(\varpi^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\varpi^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\varpi^{4\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\varpi^{4\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right)\right]},\ \frac{\left[\sum_{i=1}^{n}\left(\varpi^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\varpi^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\varpi^{4\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\varpi^{4\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right)\right]}\right]\\ \preceq\ &\left[\frac{\left[\sum_{i=1}^{n}\left(\varpi^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\cdot\varpi^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\varpi^{4\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\varpi^{4\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right)\right]},\ \frac{\left[\sum_{i=1}^{n}\left(\varpi^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\cdot\varpi^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]}{\left[\sum_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\varpi^{4\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right)\cdot\left(1-\varpi^{4\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right)}\right)\right]}\right].\end{aligned}$$

Therefore

$$\widetilde{\mathscr{T}_2}\left(\overbrace{\widetilde{\mathscr{P}}(\varepsilon)(\chi_j)},\overbrace{\widetilde{\mathscr{R}}(\varepsilon)(\chi_j)}\right)\preceq\widetilde{\mathscr{T}_2}\left(\overbrace{\widetilde{\mathscr{Q}}(\varepsilon)(\chi_j)},\overbrace{\widetilde{\mathscr{R}}(\varepsilon)(\chi_j)}\right).\tag{6}$$

Clearly,

$$\left[\tau^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j),\tau^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right]\succeq\left[\tau^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j),\tau^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right]\succeq\left[\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j),\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right]$$

and

$$\left[\tau^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j),\tau^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right]-\left[\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j),\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right]\succeq\left[\tau^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j),\tau^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right]-\left[\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j),\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right]$$

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imply

$$\left|\left[\tau^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j),\tau^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\right]-\left[\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j),\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right]\right|\succeq\left|\left[\tau^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j),\tau^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\right]-\left[\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j),\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right]\right|$$

and

$$\left|\left[\tau^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)-\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\ ,\ \tau^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)-\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right]\right|\succeq\left|\left[\tau^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)-\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\ ,\ \tau^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)-\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right]\right|.$$

Hence

$$\begin{aligned}&\left|\left[\sum_{i=1}^{n}\left(\tau^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)-\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\ ,\ \sum_{i=1}^{n}\left(\tau^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)-\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]\right|\\ \succeq\ &\left|\left[\sum_{i=1}^{n}\left(\tau^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)-\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\ ,\ \sum_{i=1}^{n}\left(\tau^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)-\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]\right|. \qquad (7)\end{aligned}$$

Also,

$$\begin{aligned}&\left[\left(\tau^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right),\left(\tau^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]\\ \succeq\ &\left[\left(\tau^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\cdot\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right),\left(\tau^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\cdot\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]\end{aligned}$$

implies

$$\begin{aligned}&\left|\left[\left(\tau^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right),\left(\tau^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]\right|\\ \succeq\ &\left|\left[\left(\tau^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\cdot\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right),\left(\tau^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\cdot\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]\right|\end{aligned}$$

and

$$\begin{aligned}&\left|\left[\sum_{i=1}^{n}1+\left(\tau^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right),\sum_{i=1}^{n}1+\left(\tau^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]\right|\\ \succeq\ &\left|\left[\sum_{i=1}^{n}1+\left(\tau^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\cdot\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right),\sum_{i=1}^{n}1+\left(\tau^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\cdot\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]\right| \qquad (8)\end{aligned}$$

for $j=1,2,...,m$.
Equations 7 and 8,

$$\begin{aligned}&\frac{\left|\left[\sum_{i=1}^{n}\left(\tau^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)-\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\ ,\ \sum_{i=1}^{n}\left(\tau^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)-\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]\right|}{\left|\left[\sum_{i=1}^{n}1+\left(\tau^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right),\sum_{i=1}^{n}1+\left(\tau^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]\right|}\\ \succeq\ &\frac{\left|\left[\sum_{i=1}^{n}\left(\tau^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)-\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\ ,\ \sum_{i=1}^{n}\left(\tau^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)-\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]\right|}{\left|\left[\sum_{i=1}^{n}1+\left(\tau^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\cdot\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right),\sum_{i=1}^{n}1+\left(\tau^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\cdot\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)\right]\right|}\end{aligned}$$

implies

$$\begin{aligned}&\left|\left[\frac{\sum_{i=1}^{n}\left(\tau^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)-\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)}{\sum_{i=1}^{n}1+\left(\tau^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)}\ ,\ \frac{\sum_{i=1}^{n}\left(\tau^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)-\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)}{\sum_{i=1}^{n}1+\left(\tau^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)}\right]\right|\\ \succeq\ &\left|\left[\frac{\sum_{i=1}^{n}\left(\tau^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)-\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)}{\sum_{i=1}^{n}1+\left(\tau^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\cdot\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)}\ ,\ \frac{\sum_{i=1}^{n}\left(\tau^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)-\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)}{\sum_{i=1}^{n}1+\left(\tau^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\cdot\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)}\right]\right|\end{aligned}$$

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and

$$\sqrt{\left|\left[\frac{\sum_{i=1}^{n}\left(\tau^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)-\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)}{\sum_{i=1}^{n}1+\left(\tau^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)},\ \frac{\sum_{i=1}^{n}\left(\tau^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)-\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)}{\sum_{i=1}^{n}1+\left(\tau^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)}\right]\right|}$$
$$\succeq \sqrt{\left|\left[\frac{\sum_{i=1}^{n}\left(\tau^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)-\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)}{\sum_{i=1}^{n}1+\left(\tau^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\cdot\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)},\ \frac{\sum_{i=1}^{n}\left(\tau^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)-\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)}{\sum_{i=1}^{n}1+\left(\tau^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\cdot\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)}\right]\right|}$$

and

$$\left[1-\sqrt{\left|\left[\frac{\sum_{i=1}^{n}\left(\tau^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)-\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)}{\sum_{i=1}^{n}1+\left(\tau^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)},\ \frac{\sum_{i=1}^{n}\left(\tau^{2\mathscr{U}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)-\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)}{\sum_{i=1}^{n}1+\left(\tau^{2\mathscr{L}}_{\mathscr{P}(\varepsilon_i)}(\chi_j)\cdot\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)}\right]\right|}\right]$$
$$\preceq \left[1-\sqrt{\left|\left[\frac{\sum_{i=1}^{n}\left(\tau^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)-\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)}{\sum_{i=1}^{n}1+\left(\tau^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\cdot\tau^{2\mathscr{U}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)},\ \frac{\sum_{i=1}^{n}\left(\tau^{2\mathscr{U}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)-\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)}{\sum_{i=1}^{n}1+\left(\tau^{2\mathscr{L}}_{\mathscr{Q}(\varepsilon_i)}(\chi_j)\cdot\tau^{2\mathscr{L}}_{\mathscr{R}(\varepsilon_i)}(\chi_j)\right)}\right]\right|}\right].$$

Therefore

$$\widetilde{S}\left(\overbrace{\widetilde{\mathscr{P}}(\varepsilon)(\chi_j)},\overbrace{\widetilde{\mathscr{R}}(\varepsilon)(\chi_j)}\right)\preceq\widetilde{S}\left(\overbrace{\widetilde{\mathscr{Q}}(\varepsilon)(\chi_j)},\overbrace{\widetilde{\mathscr{R}}(\varepsilon)(\chi_j)}\right). \tag{9}$$

Equations 3, 6 and 9, for each $j = 1, 2, ..., m$,

$$\varphi(\overbrace{\widetilde{\mathscr{P}}},\overbrace{\widetilde{\mathscr{R}}})\preceq\varphi(\overbrace{\widetilde{\mathscr{Q}}},\overbrace{\widetilde{\mathscr{R}}}). \tag{10}$$

Clearly,

$$\left[u^{\mathscr{L}}(\varepsilon_i),u^{\mathscr{U}}(\varepsilon_i)\right]\preceq\left[v^{\mathscr{L}}(\varepsilon_i),v^{\mathscr{U}}(\varepsilon_i)\right]\preceq\left[w^{\mathscr{L}}(\varepsilon_i),w^{\mathscr{U}}(\varepsilon_i)\right]$$

and

$$\left[\min\left\{[u^{\mathscr{L}}(\varepsilon_i),w^{\mathscr{L}}(\varepsilon_i)]\right\},\max\left\{[u^{\mathscr{U}}(\varepsilon_i),w^{\mathscr{U}}(\varepsilon_i)]\right\}\right]\preceq\left[\min\left\{[v^{\mathscr{L}}(\varepsilon_i),w^{\mathscr{L}}(\varepsilon_i)]\right\},\max\left\{[v^{\mathscr{U}}(\varepsilon_i),w^{\mathscr{U}}(\varepsilon_i)]\right\}\right].$$

Hence

$$\begin{aligned}&\Big[\sum_{i=1}^{n}\min\left\{[u^{\mathscr{L}}(\varepsilon_i),w^{\mathscr{L}}(\varepsilon_i)]\right\},\sum_{i=1}^{n}\max\left\{[u^{\mathscr{U}}(\varepsilon_i),w^{\mathscr{U}}(\varepsilon_i)]\right\}\Big]\\ \preceq\ &\Big[\sum_{i=1}^{n}\min\left\{[v^{\mathscr{L}}(\varepsilon_i),w^{\mathscr{L}}(\varepsilon_i)]\right\},\sum_{i=1}^{n}\max\left\{[v^{\mathscr{U}}(\varepsilon_i),w^{\mathscr{U}}(\varepsilon_i)]\right\}\Big].\end{aligned} \tag{11}$$

Also,

$$\Big[\left(u^{\mathscr{L}}(\varepsilon_i)+w^{\mathscr{L}}(\varepsilon_i)\right),\left(u^{\mathscr{U}}(\varepsilon_i)+w^{\mathscr{U}}(\varepsilon_i)\right)\Big]\preceq\Big[\left(v^{\mathscr{L}}(\varepsilon_i)+w^{\mathscr{L}}(\varepsilon_i)\right),\left(v^{\mathscr{U}}(\varepsilon_i)+w^{\mathscr{U}}(\varepsilon_i)\right)\Big]$$

and

$$\begin{aligned}&\Big[\sum_{i=1}^{n}\left(u^{\mathscr{L}}(\varepsilon_i)+w^{\mathscr{L}}(\varepsilon_i)\right),\sum_{i=1}^{n}\left(u^{\mathscr{U}}(\varepsilon_i)+w^{\mathscr{U}}(\varepsilon_i)\right)\Big]\\ \preceq\ &\Big[\sum_{i=1}^{n}\left(v^{\mathscr{L}}(\varepsilon_i)+w^{\mathscr{L}}(\varepsilon_i)\right),\sum_{i=1}^{n}\left(v^{\mathscr{U}}(\varepsilon_i)+w^{\mathscr{U}}(\varepsilon_i)\right)\Big].\end{aligned} \tag{12}$$

Equations 11 and 12,

$$\begin{aligned}&-\frac{\left[\sum_{i=1}^{n}\min\left\{\left[u^{\mathscr{L}}(\varepsilon_i),w^{\mathscr{L}}(\varepsilon_i)\right]\right\},\sum_{i=1}^{n}\max\left\{\left[u^{\mathscr{U}}(\varepsilon_i),w^{\mathscr{U}}(\varepsilon_i)\right]\right\}\right]}{\left[\sum_{i=1}^{n}\left(u^{\mathscr{L}}(\varepsilon_i)+w^{\mathscr{L}}(\varepsilon_i)\right),\sum_{i=1}^{n}\left(u^{\mathscr{U}}(\varepsilon_i)+w^{\mathscr{U}}(\varepsilon_i)\right)\right]}\\ \succeq\ &-\frac{\left[\sum_{i=1}^{n}\min\left\{\left[v^{\mathscr{L}}(\varepsilon_i),w^{\mathscr{L}}(\varepsilon_i)\right]\right\},\sum_{i=1}^{n}\max\left\{\left[v^{\mathscr{U}}(\varepsilon_i),w^{\mathscr{U}}(\varepsilon_i)\right]\right\}\right]}{\left[\sum_{i=1}^{n}\left(v^{\mathscr{L}}(\varepsilon_i)+w^{\mathscr{L}}(\varepsilon_i)\right),\sum_{i=1}^{n}\left(v^{\mathscr{U}}(\varepsilon_i)+w^{\mathscr{U}}(\varepsilon_i)\right)\right]}\end{aligned}$$

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implies

$$-\left[\frac{\sum_{i=1}^{n}\min\left\{\left[u^{\mathscr{L}}(\varepsilon_i),w^{\mathscr{L}}(\varepsilon_i)\right]\right\}}{\sum_{i=1}^{n}\left(u^{\mathscr{U}}(\varepsilon_i)+w^{\mathscr{U}}(\varepsilon_i)\right)},\ \frac{\sum_{i=1}^{n}\max\left\{\left[u^{\mathscr{U}}(\varepsilon_i),w^{\mathscr{U}}(\varepsilon_i)\right]\right\}}{\sum_{i=1}^{n}\left(u^{\mathscr{L}}(\varepsilon_i)+w^{\mathscr{L}}(\varepsilon_i)\right)}\right]$$
$$\succeq\ -\left[\frac{\sum_{i=1}^{n}\min\left\{\left[v^{\mathscr{L}}(\varepsilon_i),w^{\mathscr{L}}(\varepsilon_i)\right]\right\}}{\sum_{i=1}^{n}\left(v^{\mathscr{U}}(\varepsilon_i)+w^{\mathscr{U}}(\varepsilon_i)\right)},\ \frac{\sum_{i=1}^{n}\max\left\{\left[v^{\mathscr{U}}(\varepsilon_i),w^{\mathscr{U}}(\varepsilon_i)\right]\right\}}{\sum_{i=1}^{n}\left(v^{\mathscr{L}}(\varepsilon_i)+w^{\mathscr{L}}(\varepsilon_i)\right)}\right].$$

Thus,

$$1-\left[\frac{\sum_{i=1}^{n}\min\left\{\left[u^{\mathscr{L}}(\varepsilon_i),w^{\mathscr{L}}(\varepsilon_i)\right]\right\}}{\sum_{i=1}^{n}\left(u^{\mathscr{U}}(\varepsilon_i)+w^{\mathscr{U}}(\varepsilon_i)\right)},\ \frac{\sum_{i=1}^{n}\max\left\{\left[u^{\mathscr{U}}(\varepsilon_i),w^{\mathscr{U}}(\varepsilon_i)\right]\right\}}{\sum_{i=1}^{n}\left(u^{\mathscr{L}}(\varepsilon_i)+w^{\mathscr{L}}(\varepsilon_i)\right)}\right]$$
$$\succeq\ 1-\left[\frac{\sum_{i=1}^{n}\min\left\{\left[v^{\mathscr{L}}(\varepsilon_i),w^{\mathscr{L}}(\varepsilon_i)\right]\right\}}{\sum_{i=1}^{n}\left(v^{\mathscr{U}}(\varepsilon_i)+w^{\mathscr{U}}(\varepsilon_i)\right)},\ \frac{\sum_{i=1}^{n}\max\left\{\left[v^{\mathscr{U}}(\varepsilon_i),w^{\mathscr{U}}(\varepsilon_i)\right]\right\}}{\sum_{i=1}^{n}\left(v^{\mathscr{L}}(\varepsilon_i)+w^{\mathscr{L}}(\varepsilon_i)\right)}\right].$$

Hence

$$\psi(\tilde{u},\tilde{w})\succeq\psi(\tilde{v},\tilde{w}). \tag{13}$$

Equations 10 and 13,

$$\varphi(\overbrace{\tilde{\mathscr{P}}},\overbrace{\tilde{\mathscr{R}}})\cdot\psi(\tilde{u},\tilde{w})\preceq\varphi(\overbrace{\tilde{\mathscr{Q}}},\overbrace{\tilde{\mathscr{R}}})\cdot\psi(\tilde{v},\tilde{w}).$$

Hence $\mathrm{Sim}(\overbrace{\tilde{\mathscr{P}}_u},\overbrace{\tilde{\mathscr{R}}_w})\preceq\mathrm{Sim}(\overbrace{\tilde{\mathscr{Q}}_v},\overbrace{\tilde{\mathscr{R}}_w})$. □

Example 4.3. Determine the similarity between the two type-II GPyNSIVS sets, namely $\overbrace{\mathscr{P}_u}$ and $\overbrace{\mathscr{Q}_v}$. We choose $\mathscr{X}=\{\chi_1,\chi_2,\chi_3\}$ and a parameter $\mathscr{E}=\{\varepsilon_1,\varepsilon_2,\varepsilon_3\}$, that can be defined below:

$\overbrace{\mathscr{P}_u(\varepsilon)}$	ε_1	ε_2	ε_3
$\overbrace{\mathscr{P}(\varepsilon)(\chi_1)}$	$[0.45,0.55]$ $[0.25,0.35]$ $[0.55,0.6]$	$[0.65,0.7]$ $[0.15,0.3]$ $[0.45,0.5]$	$[0.55,0.6]$ $[0.35,0.5]$ $[0.5,0.55]$
$\overbrace{\mathscr{P}(\varepsilon)(\chi_2)}$	$[0.35,0.45]$ $[0.2,0.3]$ $[0.6,0.65]$	$[0.55,0.65]$ $[0.1,0.3]$ $[0.5,0.55]$	$[0.25,0.4]$$[0.3,0.5]$ $[0.65,0.7]$
$\overbrace{\mathscr{P}(\varepsilon)(\chi_3)}$	$[0.25,0.45]$ $[0.5,0.55]$ $[0.3,0.5]$	$[0.35,0.45]$ $[0.4,0.45]$ $[0.4,0.55]$	$[0.15,0.3]$ $[0.6,0.65]$ $[0.5,0.6]$
$p(\varepsilon)$	$[0.45,0.55]$ $[0.45,0.65]$ $[0.35,0.45]$	$[0.35,0.5]$ $[0.55,0.6]$ $[0.25,0.3]$	$[0.55,0.6]$ $[0.4,0.45]$ $[0.15,0.3]$

$\overbrace{\mathscr{Q}_v(\varepsilon)}$	ε_1	ε_2	ε_3
$\overbrace{\mathscr{Q}(\varepsilon)(\chi_1)}$	$[0.35,0.45]$ $[0.45,0.55]$ $[0.45,0.5]$	$[0.45,0.5]$ $[0.25,0.45]$ $[0.35,0.4]$	$[0.35,0.4]$ $[0.55,0.6]$ $[0.3,0.4]$
$\overbrace{\mathscr{Q}(\varepsilon)(\chi_2)}$	$[0.3,0.4]$ $[0.4,0.5]$ $[0.4,0.5]$	$[0.4,0.5]$ $[0.6,0.7]$ $[0.3,0.4]$	$[0.3,0.4]$ $[0.5,0.6]$ $[0.5,0.6]$
$\overbrace{\mathscr{Q}(\varepsilon)(\chi_3)}$	$[0.6,0.65]$ $[0.6,0.65]$ $[0.15,0.2]$	$[0.5,0.6]$ $[0.4,0.5]$ $[0.3,0.35]$	$[0.4,0.5]$ $[0.3,0.6]$ $[0.3,0.4]$
$q(\varepsilon)$	$[0.4,0.5]$ $[0.5,0.6]$ $[0.4,0.5]$	$[0.5,0.55]$ $[0.1,0.2]$ $[0.6,0.75]$	$[0.6,0.65]$ $[0.2,0.3]$ $[0.5,0.65]$

Using Definition 4.1 and routine calculation, we get

$\mathscr{T}_1\left(\overbrace{\mathscr{P}(\varepsilon)(\chi_1)},\overbrace{\mathscr{Q}(\varepsilon)(\chi_1)}\right)=\left[\frac{0.6425}{0.702473},\frac{0.8375}{0.902496}\right]=\left[0.914626,0.927982\right]$,

$\mathscr{T}_2\left(\overbrace{\mathscr{P}(\varepsilon)(\chi_1)},\overbrace{\mathscr{Q}(\varepsilon)(\chi_1)}\right)=\left[\frac{0.051119}{0.078869},\frac{0.145281}{0.175394}\right]=\left[0.648146,0.828314\right]$, and

$\mathscr{S}\left(\overbrace{\mathscr{P}(\varepsilon)(\chi_1)},\overbrace{\mathscr{Q}(\varepsilon)(\chi_1)}\right)=1-\sqrt{\left|\left[\frac{0.185}{3.1784},\frac{0.4975}{3.108563}\right]\right|}=1-\left[0.241258,0.400052\right]=\left[0.599948,0.758742\right]$.

Now,

$$\left[\begin{array}{l}\min\left\{\mathscr{T}_1^{\mathscr{L}}\left(\overbrace{\mathscr{P}(\varepsilon)(\chi_1)},\overbrace{\mathscr{Q}(\varepsilon)(\chi_1)}\right),\mathscr{T}_2^{\mathscr{L}}\left(\overbrace{\mathscr{P}(\varepsilon)(\chi_1)},\overbrace{\mathscr{Q}(\varepsilon)(\chi_1)}\right),\mathscr{S}^{\mathscr{L}}\left(\overbrace{\mathscr{P}(\varepsilon)(\chi_1)},\overbrace{\mathscr{Q}(\varepsilon)(\chi_1)}\right)\right\},\\ \max\left\{\mathscr{T}_1^{\mathscr{U}}\left(\overbrace{\mathscr{P}(\varepsilon)(\chi_1)},\overbrace{\mathscr{Q}(\varepsilon)(\chi_1)}\right),\mathscr{T}_2^{\mathscr{U}}\left(\overbrace{\mathscr{P}(\varepsilon)(\chi_1)},\overbrace{\mathscr{Q}(\varepsilon)(\chi_1)}\right),\mathscr{S}^{\mathscr{U}}\left(\overbrace{\mathscr{P}(\varepsilon)(\chi_1)},\overbrace{\mathscr{Q}(\varepsilon)(\chi_1)}\right)\right\}\end{array}\right]$$
$=\left[0.599948,0.927982\right]$.

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Similarly,
$\mathscr{T}_1\left(\overbrace{\mathscr{P}(\varepsilon)(\chi_2)},\overbrace{\mathscr{Q}(\varepsilon)(\chi_2)}\right)=\left[\frac{0.4}{0.417309},\frac{0.665}{0.683403}\right]=\left[0.958522,0.973071\right]$,
$\mathscr{T}_2\left(\overbrace{\mathscr{P}(\varepsilon)(\chi_2)},\overbrace{\mathscr{Q}(\varepsilon)(\chi_2)}\right)=\left[\frac{0.0325}{0.116451},\frac{0.1566}{0.264172}\right]=\left[0.279088,0.592797\right]$, and
$\mathscr{S}\left(\overbrace{\mathscr{P}(\varepsilon)(\chi_2)},\overbrace{\mathscr{Q}(\varepsilon)(\chi_2)}\right)=1-\sqrt{\left|\left[\frac{0.2625}{3.330425},\frac{0.715}{3.185725}\right]\right|}=1-\left[0.280747,0.47375\right]=\left[0.52625,0.719253\right]$.
Now,

$$\begin{bmatrix}\min\left\{\mathscr{T}_1^{\mathscr{L}}\left(\overbrace{\mathscr{P}(\varepsilon)(\chi_2)},\overbrace{\mathscr{Q}(\varepsilon)(\chi_2)}\right),\mathscr{T}_2^{\mathscr{L}}\left(\overbrace{\mathscr{P}(\varepsilon)(\chi_2)},\overbrace{\mathscr{Q}(\varepsilon)(\chi_2)}\right),\mathscr{S}^{\mathscr{L}}\left(\overbrace{\mathscr{P}(\varepsilon)(\chi_2)},\overbrace{\mathscr{Q}(\varepsilon)(\chi_2)}\right)\right\},\\ \max\left\{\mathscr{T}_1^{\mathscr{U}}\left(\overbrace{\mathscr{P}(\varepsilon)(\chi_2)},\overbrace{\mathscr{Q}(\varepsilon)(\chi_2)}\right),\mathscr{T}_2^{\mathscr{U}}\left(\overbrace{\mathscr{P}(\varepsilon)(\chi_2)},\overbrace{\mathscr{Q}(\varepsilon)(\chi_2)}\right),\mathscr{S}^{\mathscr{U}}\left(\overbrace{\mathscr{P}(\varepsilon)(\chi_2)},\overbrace{\mathscr{Q}(\varepsilon)(\chi_2)}\right)\right\}\end{bmatrix}$$

$=\left[0.279088,0.973071\right]$.
Similarly,
$\mathscr{T}_1\left(\overbrace{\mathscr{P}(\varepsilon)(\chi_3)},\overbrace{\mathscr{Q}(\varepsilon)(\chi_3)}\right)=\left[\frac{0.385}{0.508009},\frac{0.7125}{0.780799}\right]=\left[0.757861,0.912527\right]$,
$\mathscr{T}_2\left(\overbrace{\mathscr{P}(\varepsilon)(\chi_3)},\overbrace{\mathscr{Q}(\varepsilon)(\chi_3)}\right)=\left[\frac{0.148}{0.193107},\frac{0.330531}{0.342321}\right]=\left[0.766416,0.965558\right]$, and
$\mathscr{S}\left(\overbrace{\mathscr{P}(\varepsilon)(\chi_3)},\overbrace{\mathscr{Q}(\varepsilon)(\chi_3)}\right)=1-\sqrt{\left|\left[\frac{0.1775}{3.104656},\frac{0.71}{3.038925}\right]\right|}=1-\left[0.239107,0.483358\right]=\left[0.516642,0.760893\right]$.
Now,

$$\begin{bmatrix}\min\left\{\mathscr{T}_1^{\mathscr{L}}\left(\overbrace{\mathscr{P}(\varepsilon)(\chi_3)},\overbrace{\mathscr{Q}(\varepsilon)(\chi_3)}\right),\mathscr{T}_2^{\mathscr{L}}\left(\overbrace{\mathscr{P}(\varepsilon)(\chi_3)},\overbrace{\mathscr{Q}(\varepsilon)(\chi_3)}\right),\mathscr{S}^{\mathscr{L}}\left(\overbrace{\mathscr{P}(\varepsilon)(\chi_3)},\overbrace{\mathscr{Q}(\varepsilon)(\chi_3)}\right)\right\},\\ \max\left\{\mathscr{T}_1^{\mathscr{U}}\left(\overbrace{\mathscr{P}(\varepsilon)(\chi_3)},\overbrace{\mathscr{Q}(\varepsilon)(\chi_3)}\right),\mathscr{T}_2^{\mathscr{U}}\left(\overbrace{\mathscr{P}(\varepsilon)(\chi_3)},\overbrace{\mathscr{Q}(\varepsilon)(\chi_3)}\right),\mathscr{S}^{\mathscr{U}}\left(\overbrace{\mathscr{P}(\varepsilon)(\chi_3)},\overbrace{\mathscr{Q}(\varepsilon)(\chi_3)}\right)\right\}\end{bmatrix}$$

$=\left[0.516642,0.965558\right]$.
Thus, $\varphi(\overbrace{\widetilde{\mathscr{P}}},\overbrace{\widetilde{\mathscr{Q}}})=\left[\frac{0.599948+0.279088+0.516642}{3},\frac{0.927982+0.973071+0.965558}{3}\right]=\left[0.465226,0.955537\right]$.
Also, $\psi(\widetilde{u},\widetilde{v})=1-\left[0.307692,0.732877\right]=\left[0.267123,0.692308\right]$.
Hence, $\mathrm{Sim}(\overbrace{\widetilde{\mathscr{P}}_u},\overbrace{\widetilde{\mathscr{Q}}_v})=\left[0.465226\cdot 0.267123,\ \ 0.955537\cdot 0.692308\right]=\left[0.1242726,0.661526\right]$.

5 Medical diagnosis problem solved by type-II GPyNSIVS set model

Human society as a whole deals with a lot of decision making issues, which are also commonly used in practical domains like economics, management, engineering, and health care. But as science and technology advance, ambiguity increasingly dominates the decision making process at some points. In this application, we provide a technique based on the similarity measure of type-II GPyNSIVS sets for a medical diagnosis problem. This method of comparing two type-II GPyNSIVS sets can be used to determine if a sick person has a particular disease or not. We start by providing the definition below:

Definition 5.1. Let $\overbrace{\mathscr{P}_u}$ and $\overbrace{\mathscr{Q}_v}$ be the type-II GPyNSIVS sets on $(\mathscr{X},\mathscr{E})$. The type-II GPyNSIVS sets to be significantly similar if $\mathrm{Sim}^{\mathscr{U}}(\overbrace{\mathscr{P}_u},\overbrace{\mathscr{Q}_v})>0.70$.

With the assistance of a medical professional, we create a type-II GPyNSIVS set for the illness and one for the patient. The similarity between two type-II GPyNSIVS sets is then determined. If there is a significant similarity between them, we can assume that the person may have a sickness; otherwise, we cannot.

5.1 Algorithms for type-II GPyNSIVS set Model

An algorithm for decision making problems using type-II GPyNSIVS set model is explained.
Step 1. Enter the type-II GPyNSIVS set in table form.

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Step 2. Enter the set of choice parameters $A \sqsubseteq \mathscr{E}$.
Step 3. Compute the $\mathscr{T}_1(\chi_j)$, $\mathscr{T}_2(\chi_j)$ and $\mathscr{S}(\chi_j)$ and $1 \preceq j \preceq m$.
Step 4. Calculate $\varphi = \frac{1}{m}\sum_{j=1}^{m}\left[\min\{\mathscr{T}_1^{\mathscr{L}}(\chi_j), \mathscr{T}_2^{\mathscr{L}}(\chi_j), \mathscr{S}^{\mathscr{L}}(\chi_j)\}, \max\{\mathscr{T}_1^{\mathscr{U}}(\chi_j), \mathscr{T}_2^{\mathscr{U}}(\chi_j), \mathscr{S}^{\mathscr{U}}(\chi_j)\}\right]$.
Step 5. Determine the value $\psi(\widetilde{u},\widetilde{v}) = 1 - \left[\frac{\sum_{i=1}^{n}\min\left\{\left[u^{\mathscr{L}}(\varepsilon_i), v^{\mathscr{L}}(\varepsilon_i)\right]\right\}}{\sum_{i=1}^{n}(u^{\mathscr{U}}(\varepsilon_i)+v^{\mathscr{U}}(\varepsilon_i))}, \frac{\sum_{i=1}^{n}\max\left\{\left[u^{\mathscr{U}}(\varepsilon_i), v^{\mathscr{U}}(\varepsilon_i)\right]\right\}}{\sum_{i=1}^{n}(u^{\mathscr{L}}(\varepsilon_i)+v^{\mathscr{L}}(\varepsilon_i))}\right]$ and $1 \preceq i \preceq n$.
Step 6. Compute the similarity measure = $\varphi \cdot \psi$.
Step 7. When appropriate criteria for considerably similarity exist, choose the similarity measure.
Step 8. Finally, decision to the given real problem.
Step 9. End.

5.2 Data Analysis

Assume that five patients with certain dengue hemorrhagic fever symptoms are present in a hospital: $\mathscr{P}_1$, $\mathscr{P}_2$, $\mathscr{P}_3$, $\mathscr{P}_4$, and $\mathscr{P}_5$. Let there be just three components in the universal set. That is terrible, $\mathscr{X} = \{\chi_1$: severe, χ_2: mild, χ_3 : no$\}$. If the parameter set $\mathscr{E}$ represents the collection of specific dengue hemorrhagic fever symptoms as $\mathscr{E} = \{\varepsilon_1$: severe abdominal pain, ε_2: persistent vomiting, ε_3 : rapid breathing, ε_4 : bleeding gums, ε_5: restlessness and blood in vomit$\}$.

The dengue hemorrhagic fever prepared with a medical professional is shown in Table 1.

Table 1
Pneumonia type-II GPyNSIVS set (dengue hemorrhagic fever)

$\overbrace{\mathscr{L}_{p(\varepsilon)}}$	ε_1	ε_2	ε_3
$\overbrace{\mathscr{L}(\varepsilon)(\chi_1)}$	$[0.45, 0.65]$ $[0.5, 0.55]$ $[0.35, 0.45]$	$[0.35, 0.55]$ $[0.3, 0.35]$ $[0.4, 0.45]$	$[0.4, 0.45]$ $[0.15, 0.25]$ $[0.45, 0.55]$
$\overbrace{\mathscr{L}(\varepsilon)(\chi_2)}$	$[0.4, 0.5]$ $[0.2, 0.4]$ $[0.5, 0.6]$	$[0.5, 0.6]$ $[0.4, 0.5]$ $[0.4, 0.5]$	$[0.6, 0.65]$ $[0.3, 0.4]$ $[0.4, 0.45]$
$\overbrace{\mathscr{L}(\varepsilon)(\chi_3)}$	$[0.5, 0.7]$ $[0.3, 0.45]$ $[0.2, 0.35]$	$[0.4, 0.6]$ $[0.3, 0.55]$ $[0.4, 0.55]$	$[0.35, 0.55]$ $[0.4, 0.6]$ $[0.3, 0.45]$
$\overbrace{p(\varepsilon)}$	$[1, 1, 1]$	$[1, 1, 1]$	$[1, 1, 1]$

$\overbrace{\mathscr{L}_{p(\varepsilon)}}$	ε_4	ε_5
$\overbrace{\mathscr{L}(\varepsilon)(\chi_1)}$	$[0.3, 0.35]$ $[0.4, 0.45]$ $[0.25, 0.35]$	$[0.2, 0.3]$ $[0.45, 0.55]$ $[0.3, 0.45]$
$\overbrace{\mathscr{L}(\varepsilon)(\chi_2)}$	$[0.3, 0.45]$ $[0.6, 0.65]$ $[0.3, 0.45]$	$[0.5, 0.55]$ $[0.4, 0.55]$ $[0.4, 0.55]$
$\overbrace{\mathscr{L}(\varepsilon)(\chi_3)}$	$[0.45, 0.6]$ $[0.3, 0.4]$ $[0.2, 0.35]$	$[0.25, 0.55]$ $[0.6, 0.65]$ $[0.45, 0.5]$
$\overbrace{p(\varepsilon)}$	$[1, 1, 1]$	$[1, 1, 1]$

For the five patients we are considering, we create type-II GPyNSIVS sets as shown in Tables 2, 3, 4, 5, and 6.

Table 2
Type-II GPyNSIVS are configured for the sick person $\mathscr{P}_1$

$\overbrace{\mathscr{P}1_{p_1(\varepsilon)}}$	ε_1	ε_2	ε_3
$\overbrace{\mathscr{P}_1(\varepsilon)(\chi_1)}$	$[0.25, 0.45]$ $[0.25, 0.4]$ $[0.2, 0.25]$	$[0.25, 0.35]$ $[0.45, 0.6]$ $[0.3, 0.35]$	$[0.3, 0.4]$ $[0.25, 0.3]$ $[0.1, 0.3]$
$\overbrace{\mathscr{P}_1(\varepsilon)(\chi_2)}$	$[0.35, 0.45]$ $[0.45, 0.6]$ $[0.4, 0.45]$	$[0.3, 0.35]$ $[0.55, 0.7]$ $[0.2, 0.3]$	$[0.35, 0.4]$ $[0.2, 0.35]$ $[0.3, 0.35]$
$\overbrace{\mathscr{P}_1(\varepsilon)(\chi_3)}$	$[0.3, 0.4]$ $[0.2, 0.6]$ $[0.1, 0.15]$	$[0.15, 0.3]$ $[0.25, 0.7]$ $[0.3, 0.35]$	$[0.5, 0.6]$ $[0.3, 0.35]$ $[0.2, 0.25]$
$\overbrace{p_1(\varepsilon)}$	$[0.4, 0.5]$ $[0.1, 0.5]$ $[0.6, 0.65]$	$[0.5, 0.55]$ $[0.2, 0.4]$ $[0.5, 0.55]$	$[0.2, 0.4]$ $[0.4, 0.5]$ $[0.45, 0.6]$

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$\overbrace{\mathscr{P}1_{p_1(\varepsilon)}}$	ε_4	ε_5
$\overbrace{\mathscr{P}_1(\varepsilon)(\chi_1)}$	$[0.2, 0.35]$ $[0.4, 0.5]$ $[0.15, 0.2]$	$[0.2, 0.25]$ $[0.5, 0.6]$ $[0.2, 0.25]$
$\overbrace{\mathscr{P}_1(\varepsilon)(\chi_2)}$	$[0.45, 0.6]$ $[0.4, 0.5]$ $[0.15, 0.2]$	$[0.4, 0.5]$ $[0.5, 0.65]$ $[0.2, 0.35]$
$\overbrace{\mathscr{P}_1(\varepsilon)(\chi_3)}$	$[0.4, 0.6]$ $[0.6, 0.7]$ $[0.1, 0.15]$	$[0.2, 0.5]$ $[0.5, 0.6]$ $[0.35, 0.4]$
$\overbrace{p_1(\varepsilon)}$	$[0.3, 0.5]$ $[0.3, 0.35]$ $[0.45, 0.7]$	$[0.5, 0.6]$ $[0.2, 0.4]$ $[0.55, 0.65]$

Table 3
Type-II GPyNSIVS are configured for the sick person $\mathscr{P}_2$

$\overbrace{\mathscr{P}2_{p_2(\varepsilon)}}$	ε_1	ε_2	ε_3
$\overbrace{\mathscr{P}_2(\varepsilon)(\chi_1)}$	$[0.15, 0.25]$ $[0.2, 0.4]$ $[0.1, 0.15]$	$[0.15, 0.3]$ $[0.4, 0.6]$ $[0.2, 0.3]$	$[0.2, 0.45]$ $[0.2, 0.3]$ $[0.1, 0.25]$
$\overbrace{\mathscr{P}_2(\varepsilon)(\chi_2)}$	$[0.5, 0.55]$ $[0.4, 0.6]$ $[0.2, 0.3]$	$[0.3, 0.35]$ $[0.5, 0.7]$ $[0.1, 0.2]$	$[0.4, 0.6]$ $[0.2, 0.3]$ $[0.2, 0.35]$
$\overbrace{\mathscr{P}_2(\varepsilon)(\chi_3)}$	$[0.6, 0.7]$ $[0.2, 0.5]$ $[0.1, 0.15]$	$[0.2, 0.3]$ $[0.25, 0.6]$ $[0.25, 0.35]$	$[0.5, 0.6]$ $[0.3, 0.35]$ $[0.1, 0.25]$
$\overbrace{p_2(\varepsilon)}$	$[0.45, 0.5]$ $[0.15, 0.5]$ $[0.1, 0.3]$	$[0.55, 0.6]$ $[0.25, 0.4]$ $[0.3, 0.5]$	$[0.25, 0.4]$ $[0.45, 0.5]$ $[0.2, 0.6]$

$\overbrace{\mathscr{P}2_{p_2(\varepsilon)}}$	ε_4	ε_5
$\overbrace{\mathscr{P}_2(\varepsilon)(\chi_1)}$	$[0.3, 0.35]$ $[0.4, 0.5]$ $[0.15, 0.2]$	$[0.1, 0.2]$ $[0.5, 0.6]$ $[0.15, 0.2]$
$\overbrace{\mathscr{P}_2(\varepsilon)(\chi_2)}$	$[0.5, 0.6]$ $[0.4, 0.5]$ $[0.2, 0.25]$	$[0.5, 0.7]$ $[0.5, 0.6]$ $[0.15, 0.3]$
$\overbrace{\mathscr{P}_2(\varepsilon)(\chi_3)}$	$[0.4, 0.55]$ $[0.6, 0.7]$ $[0.1, 0.15]$	$[0.3, 0.4]$ $[0.5, 0.6]$ $[0.3, 0.4]$
$\overbrace{p_2(\varepsilon)}$	$[0.35, 0.5]$ $[0.35, 0.45]$ $[0.4, 0.7]$	$[0.55, 0.6]$ $[0.25, 0.4]$ $[0.5, 0.6]$

Table 4
Type-II GPyNSIVS are configured for the sick person $\mathscr{P}_3$

$\overbrace{\mathscr{P}3_{p_3(\varepsilon)}}$	ε_1	ε_2	ε_3
$\overbrace{\mathscr{P}_3(\varepsilon)(\chi_1)}$	$[0.15, 0.25]$ $[0.3, 0.4]$ $[0.15, 0.25]$	$[0.25, 0.3]$ $[0.4, 0.6]$ $[0.25, 0.3]$	$[0.2, 0.35]$ $[0.5, 0.6]$ $[0.3, 0.4]$
$\overbrace{\mathscr{P}_3(\varepsilon)(\chi_2)}$	$[0.25, 0.55]$ $[0.4, 0.6]$ $[0.25, 0.4]$	$[0.3, 0.35]$ $[0.5, 0.7]$ $[0.15, 0.3]$	$[0.45, 0.6]$ $[0.2, 0.3]$ $[0.25, 0.35]$
$\overbrace{\mathscr{P}_3(\varepsilon)(\chi_3)}$	$[0.2, 0.7]$ $[0.2, 0.3]$ $[0.1, 0.15]$	$[0.1, 0.3]$ $[0.25, 0.5]$ $[0.25, 0.35]$	$[0.35, 0.6]$ $[0.3, 0.35]$ $[0.2, 0.25]$
$\overbrace{p_3(\varepsilon)}$	$[0.6, 0.65]$ $[0.35, 0.5]$ $[0.5, 0.55]$	$[0.6, 0.7]$ $[0.35, 0.5]$ $[0.35, 0.5]$	$[0.5, 0.6]$ $[0.45, 0.65]$ $[0.25, 0.3]$

$\overbrace{\mathscr{P}3_{p_3(\varepsilon)}}$	ε_4	ε_5
$\overbrace{\mathscr{P}_3(\varepsilon)(\chi_1)}$	$[0.3, 0.35]$ $[0.4, 0.7]$ $[0.15, 0.2]$	$[0.1, 0.4]$ $[0.3, 0.6]$ $[0.2, 0.25]$
$\overbrace{\mathscr{P}_3(\varepsilon)(\chi_2)}$	$[0.5, 0.6]$ $[0.4, 0.5]$ $[0.2, 0.25]$	$[0.5, 0.65]$ $[0.5, 0.6]$ $[0.3, 0.35]$
$\overbrace{\mathscr{P}_3(\varepsilon)(\chi_3)}$	$[0.45, 0.55]$ $[0.6, 0.7]$ $[0.1, 0.15]$	$[0.35, 0.4]$ $[0.5, 0.65]$ $[0.25, 0.4]$
$\overbrace{p_3(\varepsilon)}$	$[0.45, 0.5]$ $[0.35, 0.5]$ $[0.6, 0.7]$	$[0.55, 0.6]$ $[0.25, 0.4]$ $[0.5, 0.6]$

Table 5
Type-II GPyNSIVS are configured for the sick person $\mathscr{P}_4$

$\overbrace{\mathscr{P}4_{p_4(\varepsilon)}}$	ε_1	ε_2	ε_3
$\overbrace{\mathscr{P}_4(\varepsilon)(\chi_1)}$	$[0.2, 0.45]$ $[0.25, 0.4]$ $[0.25, 0.3]$	$[0.25, 0.35]$ $[0.45, 0.6]$ $[0.3, 0.35]$	$[0.3, 0.45]$ $[0.25, 0.3]$ $[0.3, 0.35]$
$\overbrace{\mathscr{P}_4(\varepsilon)(\chi_2)}$	$[0.15, 0.45]$ $[0.45, 0.55]$ $[0.2, 0.25]$	$[0.2, 0.35]$ $[0.55, 0.65]$ $[0.1, 0.35]$	$[0.15, 0.4]$ $[0.2, 0.35]$ $[0.2, 0.3]$
$\overbrace{\mathscr{P}_4(\varepsilon)(\chi_3)}$	$[0.25, 0.35]$ $[0.2, 0.5]$ $[0.1, 0.15]$	$[0.1, 0.3]$ $[0.25, 0.6]$ $[0.2, 0.25]$	$[0.45, 0.6]$ $[0.3, 0.35]$ $[0.15, 0.25]$
$\overbrace{p_4(\varepsilon)}$	$[0.4, 0.5]$ $[0.3, 0.5]$ $[0.65, 0.7]$	$[0.5, 0.55]$ $[0.3, 0.4]$ $[0.55, 0.6]$	$[0.2, 0.4]$ $[0.2, 0.5]$ $[0.6, 0.65]$

$\overbrace{\mathscr{P}4_{p_4(\varepsilon)}}$	ε_4	ε_5
$\overbrace{\mathscr{P}_4(\varepsilon)(\chi_1)}$	$[0.1, 0.55]$ $[0.4, 0.5]$ $[0.15, 0.2]$	$[0.15, 0.65]$ $[0.5, 0.6]$ $[0.2, 0.25]$
$\overbrace{\mathscr{P}_4(\varepsilon)(\chi_2)}$	$[0.4, 0.6]$ $[0.4, 0.45]$ $[0.15, 0.25]$	$[0.4, 0.5]$ $[0.5, 0.55]$ $[0.1, 0.25]$
$\overbrace{\mathscr{P}_4(\varepsilon)(\chi_3)}$	$[0.35, 0.45]$ $[0.6, 0.65]$ $[0.1, 0.15]$	$[0.1, 0.4]$ $[0.5, 0.55]$ $[0.3, 0.4]$
$\overbrace{p_4(\varepsilon)}$	$[0.3, 0.5]$ $[0.1, 0.35]$ $[0.7, 0.75]$	$[0.4, 0.6]$ $[0.2, 0.4]$ $[0.65, 0.7]$

Table 6
Type-II GPyNSIVS are configured for the sick person $\mathscr{P}_5$

$\overbrace{\mathscr{P}5_{p_5(\varepsilon)}}$	ε_1	ε_2	ε_3
$\overbrace{\mathscr{P}_5(\varepsilon)(\chi_1)}$	$[0.25, 0.75]$ $[0.2, 0.4]$ $[0.3, 0.33]$	$[0.25, 0.65]$ $[0.2, 0.6]$ $[0.35, 0.37]$	$[0.1, 0.55]$ $[0.3, 0.6]$ $[0.4, 0.43]$
$\overbrace{\mathscr{P}_5(\varepsilon)(\chi_2)}$	$[0.25, 0.65]$ $[0.45, 0.5]$ $[0.45, 0.47]$	$[0.3, 0.5]$ $[0.55, 0.7]$ $[0.3, 0.35]$	$[0.45, 0.6]$ $[0.25, 0.3]$ $[0.35, 0.38]$
$\overbrace{\mathscr{P}_5(\varepsilon)(\chi_3)}$	$[0.1, 0.7]$ $[0.2, 0.35]$ $[0.15, 0.18]$	$[0.15, 0.3]$ $[0.25, 0.5]$ $[0.3, 0.35]$	$[0.45, 0.6]$ $[0.3, 0.35]$ $[0.25, 0.28]$
$\overbrace{p_5(\varepsilon)}$	$[0.1, 0.65]$ $[0.45, 0.5]$ $[0.5, 0.55]$	$[0.2, 0.7]$ $[0.45, 0.5]$ $[0.35, 0.6]$	$[0.55, 0.6]$ $[0.55, 0.6]$ $[0.25, 0.5]$

$\overbrace{\mathscr{P}5_{p_5(\varepsilon)}}$	ε_4	ε_5
$\overbrace{\mathscr{P}_5(\varepsilon)(\chi_1)}$	$[0.2, 0.5]$ $[0.4, 0.7]$ $[0.2, 0.23]$	$[0.1, 0.3]$ $[0.3, 0.6]$ $[0.25, 0.28]$
$\overbrace{\mathscr{P}_5(\varepsilon)(\chi_2)}$	$[0.3, 0.6]$ $[0.45, 0.5]$ $[0.25, 0.27]$	$[0.5, 0.7]$ $[0.55, 0.6]$ $[0.35, 0.38]$
$\overbrace{\mathscr{P}_5(\varepsilon)(\chi_3)}$	$[0.2, 0.55]$ $[0.6, 0.65]$ $[0.15, 0.18]$	$[0.15, 0.4]$ $[0.5, 0.65]$ $[0.35, 0.4]$
$\overbrace{p_5(\varepsilon)}$	$[0.3, 0.5]$ $[0.35, 0.4]$ $[0.6, 0.7]$	$[0.5, 0.6]$ $[0.25, 0.4]$ $[0.5, 0.65]$

Based on their evaluation of the options in comparison to the criteria being taken into consideration, the experts offered the type-II generalized Pythagorean neutrosophic interval valued values in Table 2-Table 6. In this example, using Definition 4.1, we should determine how similar the type-II GPyNSIVS sets in Table 2 through Table 1 are to each other. Below the table is a formula for calculating how similar $\mathscr{P}_1$ to $\mathscr{P}_5$ sick peoples.

	$\mathscr{T}_1(\chi_1)$	$\mathscr{T}_2(\chi_1)$	$\mathscr{S}(\chi_1)$	$\mathscr{T}_1(\chi_2)$	$\mathscr{T}_2(\chi_2)$
$\overbrace{(\mathscr{L}, \mathscr{P}_1)}$	$[0.914317, 0.938117]$	$[0.808065, 0.853766]$	$[0.593638, 0.773818]$	$[0.907314, 0.923287]$	$[0.755334, 0.842895]$
$\overbrace{(\mathscr{L}, \mathscr{P}_2)}$	$[0.759727, 0.830579]$	$[0.793875, 0.853766]$	$[0.569865, 0.724914]$	$[0.92455, 0.951179]$	$[0.793329, 0.843195]$
$\overbrace{(\mathscr{L}, \mathscr{P}_3)}$	$[0.807605, 0.827498]$	$[0.640276, 0.703714]$	$[0.602679, 0.791063]$	$[0.923793, 0.956832]$	$[0.793329, 0.843195]$
$\overbrace{(\mathscr{L}, \mathscr{P}_4)}$	$[0.839512, 0.870977]$	$[0.808065, 0.853766]$	$[0.620271, 0.801925]$	$[0.724866, 0.923287]$	$[0.755334, 0.864217]$
$\overbrace{(\mathscr{L}, \mathscr{P}_5)}$	$[0.786524, 0.972696]$	$[0.646290, 0.703714]$	$[0.668435, 0.878662]$	$[0.942952, 0.965173]$	$[0.790137, 0.869350]$

	$\mathscr{S}(\chi_2)$	$\mathscr{T}_1(\chi_3)$	$\mathscr{T}_2(\chi_3)$	$\mathscr{S}(\chi_3)$
$\overbrace{(\mathscr{L}, \mathscr{P}_1)}$	$[0.563692, 0.783327]$	$[0.892194, 0.918809]$	$[0.747863, 0.795089]$	$[0.620208, 0.832766]$
$\overbrace{(\mathscr{L}, \mathscr{P}_2)}$	$[0.518511, 0.714939]$	$[0.946471, 0.953762]$	$[0.747863, 0.818239]$	$[0.596853, 0.832766]$
$\overbrace{(\mathscr{L}, \mathscr{P}_3)}$	$[0.546068, 0.774334]$	$[0.840244, 0.953762]$	$[0.747863, 0.799470]$	$[0.597355, 0.832766]$
$\overbrace{(\mathscr{L}, \mathscr{P}_4)}$	$[0.512178, 0.713110]$	$[0.823720, 0.876835]$	$[0.747863, 0.834689]$	$[0.594295, 0.800287]$
$\overbrace{(\mathscr{L}, \mathscr{P}_5)}$	$[0.625280, 0.851321]$	$[0.701164, 0.953762]$	$[0.747863, 0.843519]$	$[0.632926, 0.855302]$

	φ	ψ	Similarity
$\overbrace{(\mathscr{L}, \mathscr{P}_1)}$	$[0.592513, 0.926738]$	$[0.273608, 0.752735]$	$[0.162116, 0.697588]$
$\overbrace{(\mathscr{L}, \mathscr{P}_2)}$	$[0.561743, 0.919569]$	$[0.253731, 0.757143]$	$[0.142532, 0.696245]$
$\overbrace{(\mathscr{L}, \mathscr{P}_3)}$	$[0.582034, 0.912697]$	$[0.307159, 0.713978]$	$[0.178777, 0.651646]$
$\overbrace{(\mathscr{L}, \mathscr{P}_4)}$	$[0.575581, 0.890367]$	$[0.287411, 0.738095]$	$[0.165428, 0.657175]$
$\overbrace{(\mathscr{L}, \mathscr{P}_5)}$	$[0.634832, 0.963877]$	$[0.282297, 0.748401]$	$[\mathbf{0.179211}, \mathbf{0.721366}]$

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According to the aforementioned findings, the upper similarity measure for the first four patients is 0.70, however it is $\overbrace{(\mathscr{L}, \mathscr{P}_5)} = 0.721366 > 0.70$ for the fifth patient $\mathscr{P}_5$. These two type-II GPyNSIVS sets are hence remarkably comparable. Thus, we draw the conclusion that the patient $\mathscr{P}_5$ has dengue hemorrhagic fever.

6 Conclusion

This work's primary objective is to propose a type-II GPyNSIVS set and examine some of its characteristics. It is described how to apply the similarity measure of two type-II GPyNSIVS sets to medical diagnosis. The theory of generalized PyNS cubic soft sets and generalized bipolar Pythagorean neutrosophic soft sets will be applied in the future.

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