A Study of Neutrosophic Property Functions

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Abstract

This paper is dedicated to neutrosophic property functions its generalizations, especially neutrosophic Gamma function, neutrosophic Beta function, neutrosophic Zeta function. Also, this work gives the interested reader a background in the study of neutrosophic polynomial orthogonality.

Keywords: Neutrosophic Property Functions; Neutrosophic Gamma Function; Neutrosophic Beta Function; Neutrosophic Zeta Function**.**

1.Introduction

Neutrosophy is a new branch of philosophy founded by Smarandache [6,36], It is studying the indeterminacy in the real world problems and science. It has a huge effect in many aspects such as topology [7,27,29], decision making [8], and number theory [35].

Neutrosophic algebra began with the definitions of neutrosophic groups [9,17], and rings [13]. The neutrosophic rings and their generalizations such as refined neutrosophic rings [19,13,14,15], and n-refined neutrosophic rings [11,12], were very useful in the study of linear structures.

Neutrosophic linear structures were defined as new generalizations of classical ones based on neutrosophic rings and fields, where we find many concepts from linear algebra were generalized into neutrosophic systems such as neutrosophic matrices and spaces over neutrosophic fields [1, 2,42], refined neutrosophic spaces and matrices over refined neutrosophic fields [24], n-refined neutrosophic spaces over n-refined neutrosophic fields [21,32], linear modules and ideals [4,5,20,22,32].

Through this paper, we give the interested reader a good background to deal with analytic neutrosophic linear structures such as neutrosophic Gamma function, neutrosophic Beta function, neutrosophic Zeta function**.**

2 Definitions.

Definition 2.1. Neutrosophic Real Number:

Suppose that w is a neutrosophic number, then it takes the following standard form: $w = a + bI$ where a, b are real coefficients, and I represents the indeterminacy, where $0.I = 0$ and $I^n = I$ for all positive integers n.

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For example:

$w = 1 + 2I, w = 3 = 3 + 0I.$

Definition 2.2. **Division of neutrosophic real numbers**:

Suppose that w_1, w_2 are two neutrosophic number,where

$$w_1 = a_1 + b_1 I, w_2 = a_2 + b_2 I$$

Then:

$$\frac{w_1}{w_2} = \frac{a_1 + b_1 I}{a_2 + b_2 I} = \frac{a_1}{a_2} + \frac{a_2 b_1 - a_1 b_2}{a_2(a_2 + b_2)} I$$

3 Neutrosophic Gamma function.

Definition 3.1

Let $R(I) = \{a + bI\ ; a, b \in R\}$ where $I^2 = I$be the neutrosophic field of reals. The one-dimensional isometry (AH-Isometry) is defined as follows: [49]

$$T: R(I) \rightarrow R \times R$$
$$T(a + bI) = (a, a + b)$$

Remark 3.2.

T is an algebraic isomorphism between two rings, it has the following properties:

1) T is bijective.
2) T preserves addition and multiplication, i.e.:
$$T[(a + bI) + (c + dI)] = T(a + bI) + T(c + dI)$$
And
$$T[(a + bI) \cdot (c + dI)] = T(a + bI) \cdot T(c + dI)$$
3) Since T is bijective, then it is invertible by:
$$T^{-1}: R \times R \rightarrow R(I)$$
$$T^{-1}(a, b) = a + (b - a)I$$
4) T preserves distances, i.e.:

The distance on $R(I)$ can be defined as follows:

Let $A = a + bI, B = \ c + dI$ be two neutrosophic real numbers, then $L = \|\overrightarrow{AB}\| = d[(a + bI, c + dI)] = |a + bI - (c + dI)| = |(a - c) + I(b - d)| = |a - c| + I[|a + b - c - d| - |a - c|]$.

On the other hand, we have:

$T(\|\overrightarrow{AB}\|) = (|a - c|, |(a + b) - (c + d)|) = \big(d(a, c), d(a + b, c + d)\big) = d[(a, a + b), (c, c + d)]$=$d\big(T(a + bI), T(c + dI)\big)$

=$\|T(\overrightarrow{AB})\|$.

This implies that the distance is preserved up to isometry. i.e.$\|T(AB)\| = T(\|AB\|)$

Definition 3.3.

Let $f: R(I) \rightarrow R(I); f = f(X)$ and $X = x + yI \in R(I)$ the f is called a neutrosophic real function with one neutrosophic variable.

a neutrosophic real function $f(X)$ written as follows:

$$f(X) = f(x + yI) = f(x) + I[f(x + y) - f(x)]$$

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Theorem3.4. any neutrosophic real function into two classical real functions, i.e., to the classical Euclidean plane $R \times R$.

Proof.

Let $f(X) = f(x + yI) = f(x) + I[f(x + y) - f(x)]$ a neutrosophic real function.

Now, Using the one-dimensional AH-isometry, we have.

$T\big(f(X)\big) = T(f(x) + I[f(x + y) - f(x)])$, then.

$(f_1, f_2) = \big(f(x), f(x + y)\big)$, then, we have.

$\begin{cases} f_1 = f(x) \\ f_2 = f(x + y) \end{cases}$

the functions $f(x), f(x + y)$ are a real functions.

Definition 3.5.

Let $a + bI$ be the neutrosophic real number, then. we define:

$$(a + bI)! = a! + I[(a + b)! - a!]$$

Now, we have:

$$(a + bI)! = \int_0^1 (-\ln(t_1 + t_2I))^{a+bI} d(t_1 + t_2I)$$

$a! + I[(a + b)! - a!] = \int_0^1 (-\ln(t_1))^a d(t_1) + I\left[\int_0^1 (-\ln(t_1 + t_2))^{a+b} d(t_1 + t_2) - \int_0^1 (-\ln(t_1))^a d(t_1)\right]$.

Remark 3.6.

1. $(a + bI)! = (a + bI)(a - 1 + bI)!$

Proof.

$l_2 = (a + bI)(a - 1 + bI)! = (a + bI)\big[(a - 1)! + I[(a + b - 1)! - (a - 1)!]\big]$
$l_2 = a(a - 1)! + I[b(a - 1)! + a(a + b - 1)! - a(a - 1)! + b(a + b - 1)! - b(a - 1)!]$
$l_2 = a! + I[a(a + b - 1)! + b(a + b - 1)! - a!]$
$l_2 = a! + I[(a + b)(a + b - 1)! - a!] = a! + I[(a + b)! - a!] = (a + bI)! = l_1$.

2. $(0)! = 1$

Proof.

$(0)! = (0 + 0I)! = 0! + I[(0 + 0)! - 0!] = 0! + I[0! - 0!] = 1 + I[1 - 1] = 1$

3. $I! = 1$

Proof.

$I! = (0 + I)! = 0! + I[(0 + 1)! - 0!] = 0! + I[1! - 0!] = 1 + I[1 - 1] = 1$

Now, if We define the function as follows:

$$F(a + bI) = (a - 1 + bI)!$$

Then, we have.

$F(1) = (0)! = 1$,

$F(a + 1 + bI) = (a + bI)! = (a - 1 + bI)! = (a + bI)F(a + bI)$

Definition 3.7. Neutrosophic Gamma function.

We define a neutrosophic Gamma function as follows:

$$\Gamma(z_1 + z_2I) = \int_0^\infty (t_1 + t_2I)^{(z_1+z_2I)-1}\, e^{-(t_1+t_2I)}\, d(t_1 + t_2I)$$

Remark 3.8.

1. $\Gamma(1) = 1$

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Proof.

$$\Gamma(1)=\int_0^\infty (t_1+t_2I)^{1-1}\,e^{-(t_1+t_2I)}\,d(t_1+t_2I)=\int_0^\infty (t_1+t_2I)^{0}\,e^{-(t_1+t_2I)}\,d(t_1+t_2I)$$

$$=\int_0^\infty (t_1)^0e^{-t_1}d(t_1)+I\left[\int_0^\infty (t_1+t_2)^0e^{-(t_1+t_2)}d(t_1+t_2)-\int_0^\infty (t_1)^0e^{-t_1}d(t_1)\right]$$

$$\Gamma(1)=[-e^{-t_1}]_0^\infty+I\left(\left[-e^{-(t_1+t_2)}\right]_0^\infty-[-e^{-t_1}]_0^\infty\right)$$

$$\Gamma(1)=1+I(1-1)1+0I=1$$

2. $\Gamma(z_1+z_2I+1)=(z_1+z_2I)\Gamma(z_1+z_2I)$

Proof.

We have.

$$\Gamma(z_1+z_2I)=\int_0^\infty (t_1+t_2I)^{(z_1+z_2I)-1}\,e^{-(t_1+t_2I)}\,d(t_1+t_2I)$$

$$=\int_0^\infty (t_1)^{z_1}e^{-t_1}d(t_1)+I\left[\int_0^\infty (t_1+t_2)^{(z_1+z_2)}e^{-(t_1+t_2)}d(t_1+t_2)-\int_0^\infty (t_1)^{z_1}e^{-t_1}d(t_1)\right]$$

$$\Gamma(z_1+z_2I)=\left[\frac{(t_1)^{z_1}}{z_1}e^{-t_1}\right]_0^\infty+\int_0^\infty \frac{(t_1)^{z_1}}{z_1}e^{-t_1}d(t_1)$$

$$+I\left(\left[\frac{(t_1+t_2)^{(z_1+z_2)}}{(z_1+z_2)}e^{-(t_1+t_2)}\right]_0^\infty+\int_0^\infty \frac{(t_1+t_2)^{(z_1+z_2)}}{(z_1+z_2)}e^{-(t_1+t_2)}d(t_1+t_2)\right.$$

$$\left.-\left(\left[\frac{(t_1)^{z_1}}{z_1}e^{-t_1}\right]_0^\infty+\int_0^\infty \frac{(t_1)^{z_1}}{z_1}e^{-t_1}d(t_1)\right)\right)$$

$$\Gamma(z_1+z_2I)=0+\int_0^\infty \frac{(t_1)^{z_1}}{z_1}e^{-t_1}d(t_1)$$

$$+I\left(0+\int_0^\infty \frac{(t_1+t_2)^{(z_1+z_2)}}{(z_1+z_2)}e^{-(t_1+t_2)}d(t_1+t_2)-\left(0+\int_0^\infty \frac{(t_1)^{z_1}}{z_1}e^{-t_1}d(t_1)\right)\right)$$

$$\Gamma(z_1+z_2I)=\int_0^\infty \frac{(t_1)^{z_1}}{z_1}e^{-t_1}d(t_1)+I\left(\int_0^\infty \frac{(t_1+t_2)^{(z_1+z_2)}}{(z_1+z_2)}e^{-(t_1+t_2)}d(t_1+t_2)-\left(\int_0^\infty \frac{(t_1)^{z_1}}{z_1}e^{-t_1}d(t_1)\right)\right)$$

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$$\Gamma(z_1+z_2I)=\frac{1}{z_1}\int_0^\infty (t_1)^{z_1}e^{-t_1}d(t_1)$$
$$+I\left(\frac{1}{z_1+z_2}\int_0^\infty (t_1+t_2)^{(z_1+z_2)}e^{-(t_1+t_2)}d(t_1+t_2)-\left(\frac{1}{z_1}\int_0^\infty (t_1)^{z_1}e^{-t_1}d(t_1)\right)\right)$$

$$\Gamma(z_1+z_2I)=\frac{1}{z_1}\Gamma(z_1+1)+I\left(\frac{1}{z_1+z_2}\Gamma(z_1+z_2+1)-\left(\frac{1}{z_1}\Gamma(z_1+1)\right)\right)$$

$\Gamma(z_1+z_2I)=\frac{1}{(z_1+z_2I)}\Gamma\big((z_1+z_2I)\big)$, so that.

$\Gamma(z_1+z_2I+1)=(z_1+z_2I)\Gamma(z_1+z_2I)$.

By (2), we have.

$$\Gamma(z_1+z_2I)=\frac{\Gamma(z_1+z_2I+1)}{(z_1+z_2I)}$$

Remark 3.9.

1. $\Gamma(z_1+z_2I)=\frac{1}{(z_1+z_2I)}\frac{1}{(z_1+z_2I+1)}\,\Gamma(z_1+z_2I+2)$
2. $\Gamma(z_1+z_2I)=\frac{1}{(z_1+z_2I)}\frac{1}{(z_1+z_2I+1)}\frac{1}{(z_1+z_2I+2)}\,\Gamma(z_1+z_2I+3)$
3. $\Gamma(z_1+z_2I)=\ \dots\dots\dots\dots\dots\dots.etc.$

Theorem 3.10. **(neutrosophic residues).**

The function (z_1+z_2I) in the neutrosophic cpmplex plane has simple poles in points $(z_1+z_2I)=-(n_1+n_2I)$,where

$(n_1+n_2I)=0,-1,-2,-3,\dots\dots$ and the value of the residue in it is:

$$Res[\Gamma(z_1+z_2I)-(n_1+n_2I)]=\frac{(-1)^{(n_1+n_2I)}}{(n_1+n_2I)!}\ ;(n_1+n_2I)=0,1,2,\dots\dots$$

Proof:

Let function $\Gamma(z_1+z_2I)$ by form:

$$\Gamma(z_1+z_2I)=\int_0^\infty (t_1+t_2I)^{(z_1+z_2I)-1}\,e^{-(t_1+t_2I)}\,d(t_1+t_2I)$$
$$=\int_0^1 (t_1+t_2I)^{(z_1+z_2I)-1}\,e^{-(t_1+t_2I)}\,d(t_1+t_2I)+\int_1^\infty (t_1+t_2I)^{(z_1+z_2I)-1}\,e^{-(t_1+t_2I)}\,d(t_1+t_2I)$$
$$=\Gamma_1(z_1+z_2I)+\Gamma_2(z_1+z_2I)$$

As $t^{(z_1+z_2I)-1}=e^{[(z_1+z_2I)-1]\ln(t_1+t_2I)}$ and $\ln(t_1+t_2I)$ is a real number where $(t_1+t_2I)>0$, then $\Gamma_2(z_1+z_2I)$ is analytical function in the neutrosophic cpmplex plane, hence.

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$$\Gamma_1(z_1+z_2I)=\int_0^1 (t_1+t_2I)^{(z_1+z_2I)-1}\, e^{-(t_1+t_2I)}\, d(t_1+t_2I)$$

As $e^{-(t_1+t_2I)}=\sum_{n=0}^{\infty}\frac{(-1)^{(n_1+n_2I)}}{(n_1+n_2I)!}(t_1+t_2I)^{(n_1+n_2I)}$, this series is convergen uniformaly then sum and integral can be exchanged, then.

$$\Gamma_1(z_1+z_2I)=\int_0^1 (t_1+t_2I)^{(z_1+z_2I)-1}\sum_{n=0}^{\infty}\frac{(-1)^{(n_1+n_2I)}}{(n_1+n_2I)!}(t_1+t_2I)^{(n_1+n_2I)}\, d(t_1+t_2I)$$
$$=\sum_{n=0}^{\infty}\frac{(-1)^{(n_1+n_2I)}}{(n_1+n_2I)!}\int_0^1 (t_1+t_2I)^{(z_1+z_2I)-1+(n_1+n_2I)}\,(t_1+t_2I)$$
$$=\sum_{n=0}^{\infty}\frac{(-1)^{(n_1+n_2I)}}{(n_1+n_2I)!}\frac{1}{(z_1+z_2I)+(n_1+n_2I)}$$

Now we have.

$$\Gamma(z_1+z_2I)=\sum_{n=0}^{\infty}\frac{(-1)^{(n_1+n_2I)}}{(n_1+n_2I)!}\frac{1}{(z_1+z_2I)+(n_1+n_2I)}+\Gamma_2(z_1+z_2I)$$

The series $\sum_{n=0}^{\infty}\frac{(-1)^{(n_1+n_2I)}}{(n_1+n_2I)!}\frac{1}{(z_1+z_2I)+(n_1+n_2I)}$ converges absolutely and uniformaly in every finite arena of neutrosophic complex plane when some of its first limits are neglected, as there are simple poles at points $(z_1+z_2I)=-(n_1+n_2I)$, and thus the required is fulfilled.

Remark 3.11.

For $(z_1+z_2I)\neq 0,\mp1,\mp2,\ldots\ldots$,be.

$$\Gamma(z_1+z_2I)\Gamma\big(1-(z_1+z_2I)\big)=\frac{\pi}{sin[\pi(z_1+z_2I)]}$$

4. Neutrosophic Beta function.

Definition 4.1.

Let $(p_1+p_2I),(q_1+q_2I)$ be any two Neutrosophic complex numbers, We define a neutrosophic Beta function as follows:

$$\mathrm{B}(p_1+p_2I,q_1+q_2I)=\int_0^1 (t_1+t_2I)^{(p_1+p_2I)-1}\big(1-(t_1+t_2I)\big)^{(q_1+q_2I)-1}\, d(t_1+t_2I)$$

Properites 4.2.

1. $\mathbf{B}(\boldsymbol{p_1+p_2I,q_1+q_2I})=\mathbf{B}(\boldsymbol{q_1+q_2I,p_1+p_2I})$.
2. $\mathbf{B}(\boldsymbol{p_1+p_2I,q_1+q_2I})=\mathbf{B}\big((\boldsymbol{p_1+p_2I})+\mathbf{1},\boldsymbol{q_1+q_2I}\big)$.
3. $(\boldsymbol{p_1+p_2I}).\mathbf{B}(\boldsymbol{p_1+p_2I},(\boldsymbol{q_1+q_2I})+\mathbf{1})=(\boldsymbol{q_1+q_2I}).\mathbf{B}\big((\boldsymbol{p_1+p_2I})+\mathbf{1},\boldsymbol{q_1+q_2I}\big)$

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Theorem 4.3:

A neutrosophic Beta function can be written as follows**:**

$$\mathrm{B}(p_1+p_2I,q_1+q_2I)=\int_0^{\infty}\frac{(t_1+t_2I)^{(p_1+p_2I)-1}}{(1-(t_1+t_2I))^{(p_1+p_2I)+(q_1+q_2I)}}\,d(t_1+t_2I)$$

Proof.

Let $(\boldsymbol{t_1}+\boldsymbol{t_2I})=\frac{(\tau_1+\tau_2I)}{1+(\tau_1+\tau_2I)}$ then.

$$\mathbf{B}(p_1+p_2I,q_1+q_2I)=\int_0^1(t_1+t_2I)^{(p_1+p_2I)-1}\left(1-(t_1+t_2I)\right)^{(q_1+q_2I)-1}d(t_1+t_2I)$$

$$=\int_0^{\infty}\left(\frac{\tau_1+\tau_2I}{1+\tau_1+\tau_2I}\right)^{(p_1+p_2I)-1}\left(1-\frac{\tau_1+\tau_2I}{1+\tau_1+\tau_2I}\right)^{(q_1+q_2I)-1}\frac{d(\tau_1+\tau_2I)}{(1+\tau_1+\tau_2I)^2}$$

$$\mathbf{B}(p_1+p_2I,q_1+q_2I)=\int_0^{\infty}\frac{(\tau_1+\tau_2I)^{(p_1+p_2I)-1}}{(1+\tau_1+\tau_2I)^{(p_1+p_2I)+(q_1+q_2I)}}d(\tau_1+\tau_2I)$$

Remark 4.4:

For $(\boldsymbol{q_1}+\boldsymbol{q_2I})=\mathbf{1}-(\boldsymbol{p_1}+\boldsymbol{p_2I})$, we have.

$$\mathrm{B}(p_1+p_2I,q_1+q_2I)=\int_0^{\infty}\frac{(t_1+t_2I)^{(p_1+p_2I)-1}}{1+(t_1+t_2I)}\,d(t_1+t_2I)=\frac{\pi}{\sin[\pi(p_1+p_2I)]}$$

Theorem 4.5: (relation between a neutrosophic Gamma function and neutrosophic Beta function).

$$\mathrm{B}(p_1+p_2I,q_1+q_2I)=\frac{\Gamma(p_1+p_2I).\Gamma(q_1+q_2I)}{\Gamma\left((p_1+p_2I)+(q_1+q_2I)\right)}$$

By deginition of a neutrosophic Gamma function, we have.

$$\boldsymbol{\Gamma}(z_1+z_2I)=\int_0^{\infty}(t_1+t_2I)^{(z_1+z_2I)-1}\,e^{-(t_1+t_2I)}\,d(t_1+t_2I)$$

Now let $\boldsymbol{t_1}+\boldsymbol{t_2I}=(\boldsymbol{a_1}+\boldsymbol{a_2I})(\boldsymbol{\tau_1}+\boldsymbol{\tau_2I})$, where $\boldsymbol{a_1}+\boldsymbol{a_2I}>\mathbf{0}$, then.

$$\boldsymbol{\Gamma}(z_1+z_2I)=\int_0^{\infty}\left((a_1+a_2I)(\tau_1+\tau_2I)\right)^{(z_1+z_2I)-1}e^{-(a_1+a_2I)(\tau_1+\tau_2I)}\,(a_1+a_2I)\,d(\tau_1+\tau_2I)$$

$$=(a_1+a_2I)^{(z_1+z_2I)}\int_0^{\infty}\tau^{(z_1+z_2I)-1}\,e^{-(a_1+a_2I)(\tau_1+\tau_2I)}\,d(\tau_1+\tau_2I)$$

Let $(\boldsymbol{z_1}+\boldsymbol{z_2I})=(\boldsymbol{p_1}+\boldsymbol{p_2I})+(\boldsymbol{q_1}+\boldsymbol{q_2I})$, and $(\boldsymbol{a_1}+\boldsymbol{a_2I})=\mathbf{1}+(\boldsymbol{t_1}+\boldsymbol{t_2I})$, then.

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$$\frac{\Gamma(p_1+p_2I).\Gamma(q_1+q_2I)}{\left(1+(\tau_1+\tau_2I)\right)^{(p_1+p_2I)+(q_1+q_2I)}}=\int_0^\infty (\tau_1+\tau_2I)^{(p_1+p_2I)+(q_1+q_2I)-1}\, e^{-(1+(t_1+t_2I))(\tau_1+\tau_2I)}\ d(\tau_1+\tau_2I)$$

We multiply both sides by $(t_1+t_2I)^{(p_1+p_2I)-1}$, and integrate between 0 and ∞ to find.

$$\Gamma\left((p_1+p_2I)+(q_1+q_2I)\right).\int_0^\infty \frac{(t_1+t_2I)^{(p_1+p_2I)-1}}{(1+t_1+t_2I)^{(p_1+p_2I)+(q_1+q_2I)}}\ d(t_1+t_2I)=$$

$$\int_0^\infty (t_1+t_2I)^{(p_1+p_2I)-1}\left[\int_0^\infty (\tau_1+\tau_2I)^{(p_1+p_2I)+(q_1+q_2I)}\, e^{-(1+t_1+t_2I)(\tau_1+\tau_2I)}\ d(\tau_1+\tau_2I)\right]d(t_1+t_2I)=$$

$$\int_0^\infty (\tau_1+\tau_2I)^{(p_1+p_2I)+(q_1+q_2I)-1}e^{-(\tau_1+\tau_2I)}\left[\int_0^\infty (\tau_1+\tau_2I)^{(p_1+p_2I)-1}e^{-(t_1+t_2I)(\tau_1+\tau_2I)}\ d(t_1+t_2I)\right]d(\tau_1+\tau_2I)$$

We have.

$$\Gamma\left((p_1+p_2I)+(q_1+q_2I)\right).\mathrm{B}(p_1+p_2I,q_1+q_2I)=$$

$$\int_0^\infty (\tau_1+\tau_2I)^{(p_1+p_2I)+(q_1+q_2I)-1}e^{-(\tau_1+\tau_2I)}(\tau_1+\tau_2I)^{(p+I)}\Gamma(p_1+p_2I)d(\tau_1+\tau_2I)=$$

$$\Gamma(p_1+p_2I)\int_0^\infty (\tau_1+\tau_2I)^{(q_1+q_2I)-1}e^{-(\tau_1+\tau_2I)}d(\tau_1+\tau_2I)=\Gamma(p_1+p_2I).\Gamma(q_1+q_2I)$$

5 . Neutrosophic Zeta function.

Definition 5.1. Let (z_1+z_2I) be any Neutrosophic complex number, We define a neutrosophic Zeta function as follows:

$$\zeta(z_1+z_2I)=\sum_{n=1}^{\infty}\frac{1}{(n_1+n_2I)^{(z_1+z_2I)}}$$

Remark 5.2. A neutrosophic Beta function can be written as follows**:**

$$\zeta(z_1+z_2I)=\frac{1}{\Gamma(z_1+z_2I)}\int_0^\infty \frac{(t_1+t_2I)^{(z_1+z_2I)-1}}{e^{(t_1+t_2I)}-1}dt\ ;Re(z_1+z_2I)>1$$

6. Conclusion

In this work, we have given a study for many special functions, such as neutrosophic Gamma function, neutrosophic Beta function, neutrosophic Zeta function.

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This review will be very helpful to the interested reader to continuo the research efforts in this area of study 0sn the other hand, many related concepts were shown and discussed such as neutrosophic Chebyshev polynomials, neutrosophic Jacobi polynomials, neutrosophic Legendre polynomials, neutrosophic Hermite polynomials, and neutrosophic polynomials orthogonality.

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