Analyses the least cost using Travelling Salesman problem through Neutrosophic Fuzzy system

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Abstract

The following paper introduces a methodology to calculate the least cost for a directed network through Travelling salesman problem. Dynamic programming method is used to find the minimum Cost. The recursion formula is used. The edge weights of the networks are being taken in terms of Triangular, Trapezoidal and Pentagonal Neutrosophic set. Score function for the Triangular, Trapezoidal and Pentagonal Neutrosophic sets are being defined for deneutrosophication. The least cost is estimated using all the above said Neutrosophic sets and the result is compared.

Keywords: Triangular Neutrosophic Number (TNN); Trapezoidal Neutrosophic Number (TRNN); Pentagonal Neutrosophic number (PNN); Least Cost; Travelling salesman; Deneutrosophication; Recursion formula; Dynamic Programming

1. Introduction

Neutrosophic Set comprises of a membership degree, indeterminate membership degree and non–membership degree holding a condition less than or equal to three. [1] The pentagonal Neutrosophic numbness was urbanized by Said Broumi and it came into existence in 2019. Neutrosophic sets has been developed from Intuitionistic Fuzzy Sets (IFS). Florentine Smarandache introduced Neutrosophic Sets in 1995. It includes vague information. It also comprises unpredictable facts. Fuzzy sets (FS) and fuzzy logic (FL) have been used earlier for data with vagueness. Single valued neutrosophic number is an outcome of a fuzzy number (FN) and Intuitionistic fuzzy number (IFN). Traveling Salesman Problem (TSP) is a common premeditated problem which involves more of combinatorial optimization techniques. It doesn't involve applications which are in direct usage. It offers an ultimate stage for learning of common procedures which are useful in discrete optimization problems. Really, many direct applications of TSP problem bring life to conclusion of best results which makes our work more effective. The paper envisages to find the minimal path for a salesman who travels from a city visiting all other cities and comes back to the starting city.

This paper contains of 7 segments. Segment I picturise about the introductory of the method suggested. Segment II briefs about the meaning of Neutrosophic set (NS) & Single valued Neutrosophic set (SVNS), Triangular NN, Trapezoidal NN and Pentagonal NN. The score function

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for Triangular, Trapezoidal and Pentagonal Neutrosophic number are defined. Segment III gives the algorithm of the method proposed. In segments IV, V and VI, the least cost for the triangular, trapezoidal and the pentagonal neutrosophic network are estimated. Segment 7 contains the conclusion with result.

2. Discussion

Step 1
Consider a directed graph. The Triangular, Trapezoidal and Pentagonal Neutrosophic numbers are used to take the cost of the edge weight.
Step 2
The corresponding cost adjacency matrix has taken using the edge weights of the directed graph. The formula for deneutrosophication is used to convert corresponding neutrosophic number NN.
Step 3
The Travelling salesman problem is to denote a vertex as a starting vertex and find out the path such that it is going through all the vertices and returning back to the starting vertex. Dynamic programming can be used for solving this problem.
Step 4
First let us start with vertex A. Then from A we can go to the remaining vertices B, C, D. This is the second level.
Step 5
Then we have to move on to the third level. Here there are 3 possibilities.

- From the vertex B we can go to either the vertex C or D
- From the vertex C we can go to either the vertex B or D
- From the vertex D we can go to either the vertex B or C

Step 6
Then return back to the starting vertex. The recursion tree is expanded.
Step 7
The Recursion formula involved in finding the least cost is given below

$$g(j,t) = \min_{s \in t} \{C_{js}, h(s,t) - \{s\}\}$$

Where j = Beginning vertex, t = Other Vertices, s = Any other vertex, h = Cost function
Step 8
Then find the minimum cost of starting vertex A. To find the cost of vertex A we need to find the cost of all other vertices. To find the cost of A we have to start from the last level. Find the cost of the edges which are in the third level.
Step 9
Then moving upward to the second level. Find the cost of the edges in the second level.
Step 10
Finally go to the first level and find the minimum cost of the first vertex which is the required one.

3. Mathematical Model of Linear Programming Having Neutrosophic Values in Its Coefficients

3.1 Single valued Triangular Neutrosophic number [8,9,10,11]

A single valued triangular neutrosophic number on the real number R is $A_{svtnn} = \{(p_1, q_1, r_1); (T_1, I_1, F_1)\}$ where T_1 is a truth membership, I_1 is an indeterminate membership and F_1 falsity membership.

$$T_1(x) = \{\frac{(x_1 - p_1)T_1}{(q_1 - p_1)}, \quad p_1 \le x_1 \le q_1\ T_1, \qquad x_1 = q_1\ \frac{(r_1 - x_1)T_1}{(r_1 - q_1)}, \quad q_1 \le x_1 \le r_1\ 0 \qquad otherwise$$

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$$I_1(x) = \{\frac{(q_1 - x_1) + I_1(x_1 - p_1)}{(q_1 - p_1)},$$

$$p_1 \le x_1 \le q_1 \; I_1, \qquad x_1 = q_1 \; \frac{(x_1 - q_1)+I_1(r_1 - x_1)}{(r_1 - q_1)},$$

$$q_1 \le x_1 \le r_1 \; 1 \qquad otherwise$$

$$F_1(x) = \{\frac{(q_1 - x_1) + F_1(x_1 - p_1)}{(b_1 - a_1)},$$

$$p_1 \le x_1 \le q_1 \; F_1, \qquad x_1 = q_1 \; \frac{(x_1 - r_1)+F_1(r_1 - x_1)}{(r_1 - q_1)},$$

$$q_1 \le x_1 \le r_1 \; 1 \qquad otherwise$$

Where $0 \le T_1 \le 1$; $0 \le I_1 \le 1$; $0 \le F_1 \le 1$ and $0 \le T_1 + I_1 + F_1 \le 3$

3.2 Single Valued Trapezoidal Neutrosophic number:

A single valued trapezoidal Neutrosophic number on the real number R is $B_{TrNN} = \{(m_1, n_1, o_1, p_1); (T_2, I_2, F_2)\}$ where T_1 is a truth membership, I_1 is an indeterminate membership and F_1 falsity membership.

$$T_2(x) = \{\frac{(x_1 - m_1)T_2}{(n_1 - m_1)}, \quad m_1 \le x_1 \le n_1 \; T_2, \qquad n_1 \le x_1 \le o_1 \; \frac{(p_1 - x_1)T_2}{(p_1 - o_1)},$$

$$o_1 \le x_1 \le p_1 \; 0 \qquad otherwise$$

$$I_2(x) = \{\frac{(n_1 - x_1) + I_2(x_1 - m_1)}{(b_1 - a_1)},$$

$$m_1 \le x_1 \le n_1 \; I_2, \qquad n_1 \le x_1 \le o_1 \; \frac{(x_1 - o_1)+I_2(p_1 - x_1)}{(p_1 - o_1)},$$

$$o_1 \le x_1 \le p_1 \; 1 \qquad otherwise$$

Score Function for Triangular, Trapezoidal, pentagonal Neutrosophic set [6,7]

3.3 The formula for Deneutrosophication of Triangular Neutrosophic set

$$s(\widetilde{TNN}) = \frac{1}{3}\left(2 + \frac{(a_1 + b_1 + c_1)}{3} - \frac{(d_1 + e_1 + f_1)}{3} - \frac{(l_1 + m_1 + n_1)}{3}\right)$$

3.4 The formula for Deneutrosophication of Trapezoidal Neutrosophic set

$$s(\widetilde{TrNN}) = \frac{1}{3}\left(2 + \frac{a_1 + b_1 + c_1 + d_1}{4} - \frac{e_1 + f_1 + g_1 + h_1}{4} - \frac{l_1 + m_1 + n_1 + p_1}{4}\right)$$

4. Practical Example

A. The Neutrosophic Context of the Problem

In this directed network the cost of the edges is considered as Triangular Neutrosophic number

THE COST ADJACENCY MATRIX FOR TRIANGULAR NEUTROSCOPIC NETWORK

	A	**B**	**C**	**D**
A	0	< (0.3,0.4,0.5); (0.6,0.7,0.8); (0.8,0.9,0.9) >	< (0.3,0.4,0.5); (0.1,0.2,0.3); (0.2,0.3,0.4) >	< (0.3,0.4,0.5); (0.4,0.5,0.6); (0.5,0.6,0.8) >
B	< (0.3,0.4,0.5);	0	< (0.1,0.2,0.3):	< (0.7,0.8,0.9);

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	(0.6,0.7,0.8); (0.8,0.9,0.9) >		(0.4,0.5,0.6); (0.3,0.5,0.6) >	(0.2,0.3,0.4); (0.1,0.3,0.5) >
C	< (0.1,0.2,0.3); (0.2,0.3,0.4); (0.6,0.7,0.8) >	< (0.2,0.3,0.4); (0.3,0.4,0.5); (0.7,0.8,0.9) >	0	< (0.2,0.3,0.4) (0.5,0.6,0.7); (0.8,0.9,0.9) >
D	< (0.2,0.5,0.7); (0.4,0.6,0.7); (0.6,0.7,0.8) >	< (0.3,0.4,0.5); (0.7,0.8,0.9); (0.8,0.9,0.9 >	< (0.5,0.6,0.7); (0.8,0.9,0.9); (0.1,0.2,0.3) >	0

The Cost Adjacency Matrix for Triangular Neutrosophic Network After De-Neutrosophication

$A\ B\ C\ D$ (0 0.8333 0.6333 0.4222 0.2776 0 0.5111 0.7333 0.4 0.3667 0 0.4777 0.4 0.7333 0.5111 0)

The least cost is estimated using the above algorithm. The recursive Tree is given below.

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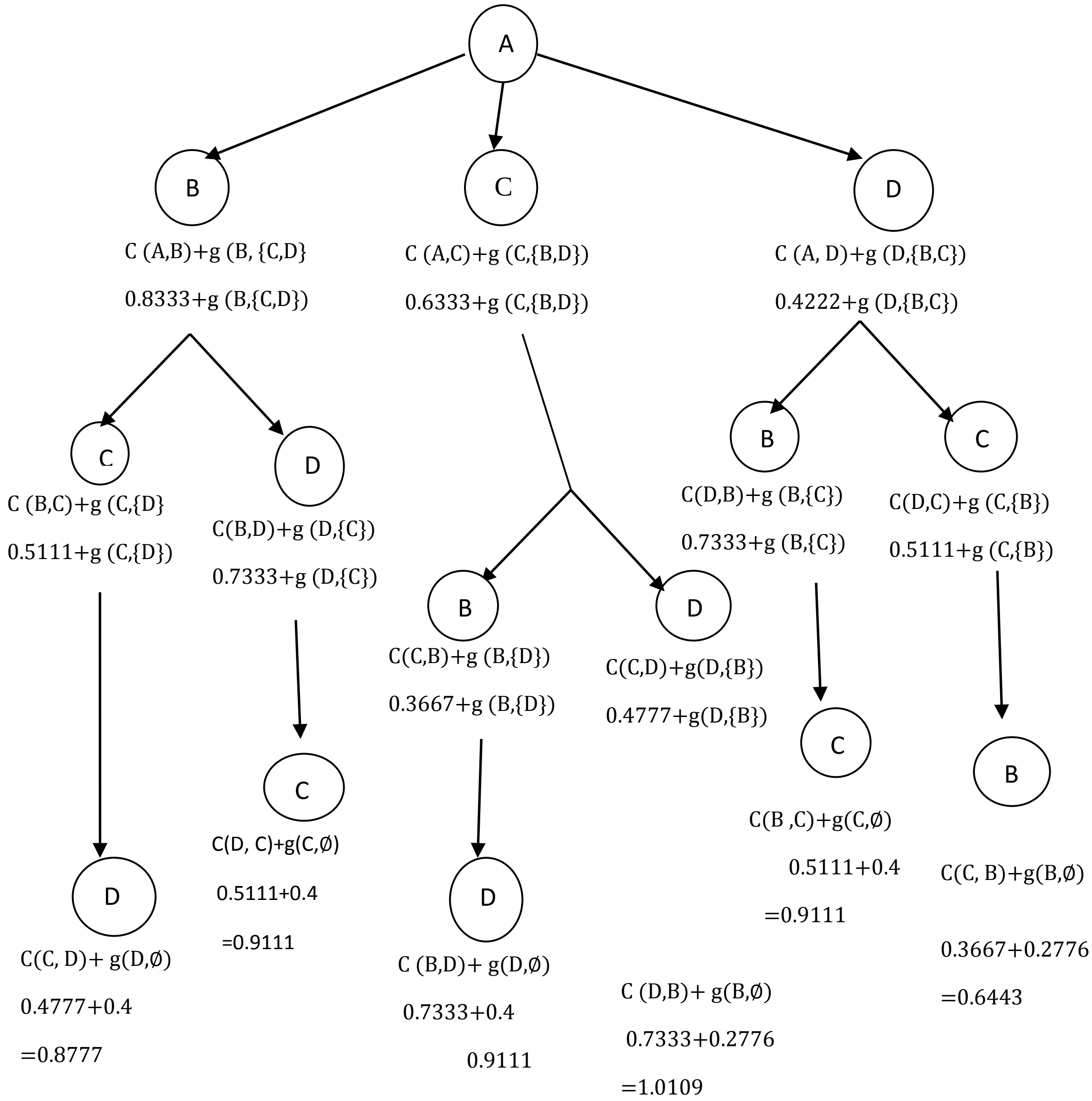

Figure 1: Recursive Tree 1

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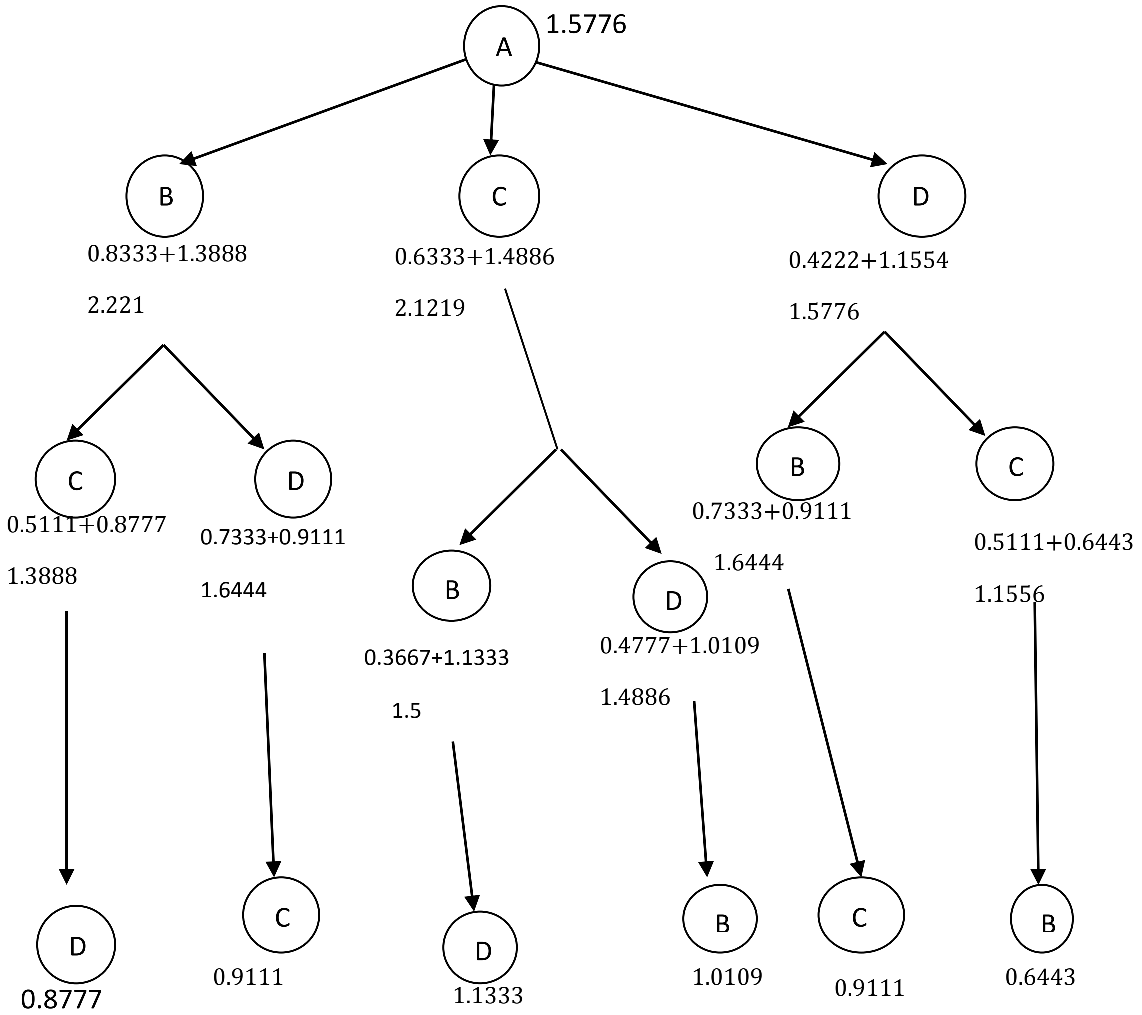

Figure 2: Recursive Tree

From the Recursive Tree the obtained minimum cost for the triangular Neutrosophic network is 1.5776

5.Estimation of least cost for the Trapezoidal Neutrosophic Network in this directed network the cost of the edges is considered as Trapezoidal Neutrosophic number

THE COST ADJACENCY MATRIX FOR TRAPEZOIDAL NEUTROSCOPIC NETWORK

	A	B	C	D
A	0	<(0.3,0.4,0.5,0.6); (0.6,0.7,0.8,0.9); (0.8,0.9,0.9,0.9)>	<(0.3,0.4,0.5,0.6); (0.1,0.2,0.3,0.4); (0.2,0.3,0.4,0.5)>	<(0.3,0.4,0.5,0.6); (0.4,0.5,0.6,0.7); (0.5,0.6,0.8,0.9)>

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B	<(0.3,0.4,0.5,0.6); (0.6,0.7,0.8,0.9); (0.8,0.9,0.9,0.9)>	0	<(0.1,0.2,0.3,0.4); (0.4,0.5,0.6,0.7); (0.3,0.5,0.6,0.7);	<(0.7,0.8,0.9,0.9); (0.2,0.3,0.4,0.5); (0.1,0.3,0.5,0.6)>
C	<(0.1,0.2,0.3,0.4); (0.2, 0.3, 0.4, 0.5); (06,0.7,0.8,0.9)>	<(0.2,0.3,0.4,0.5) (0.3,0.4,0.5,0.6) (0.7,0.8,0.9,0.9)>	0	<(0.2,0.3,0.4,0.5) (0.5,0.6,0.7,0.8) (0.8,0.8,0.9,0.9)>
D	<(0.2,0.5,0.7,0.8) (0.4,0.6,0.7,0.8) (0.6,0.7,0.8,0.9)>	<(0.3,0.4,0.5,0.6) (0.7,0.8,0.8,0.9) (0.8,0.8,0.9,0.9)>	<(0.5,0.6,0.7,0.8) (0.7,0.8,0.8,0.9) (0.1,0.2,0.3,0.4)>	0

The Cost Adjacency Matrix for Trapezoidal Neutrosophic Network After Deneutrosophication

$A\ B\ C\ D$ (0 0.275 0.6167 0.4 0.2063 0 0.3917 0.7 0.3833 0.3583 0 0.275 0.3917 0.25 0.5083 0)

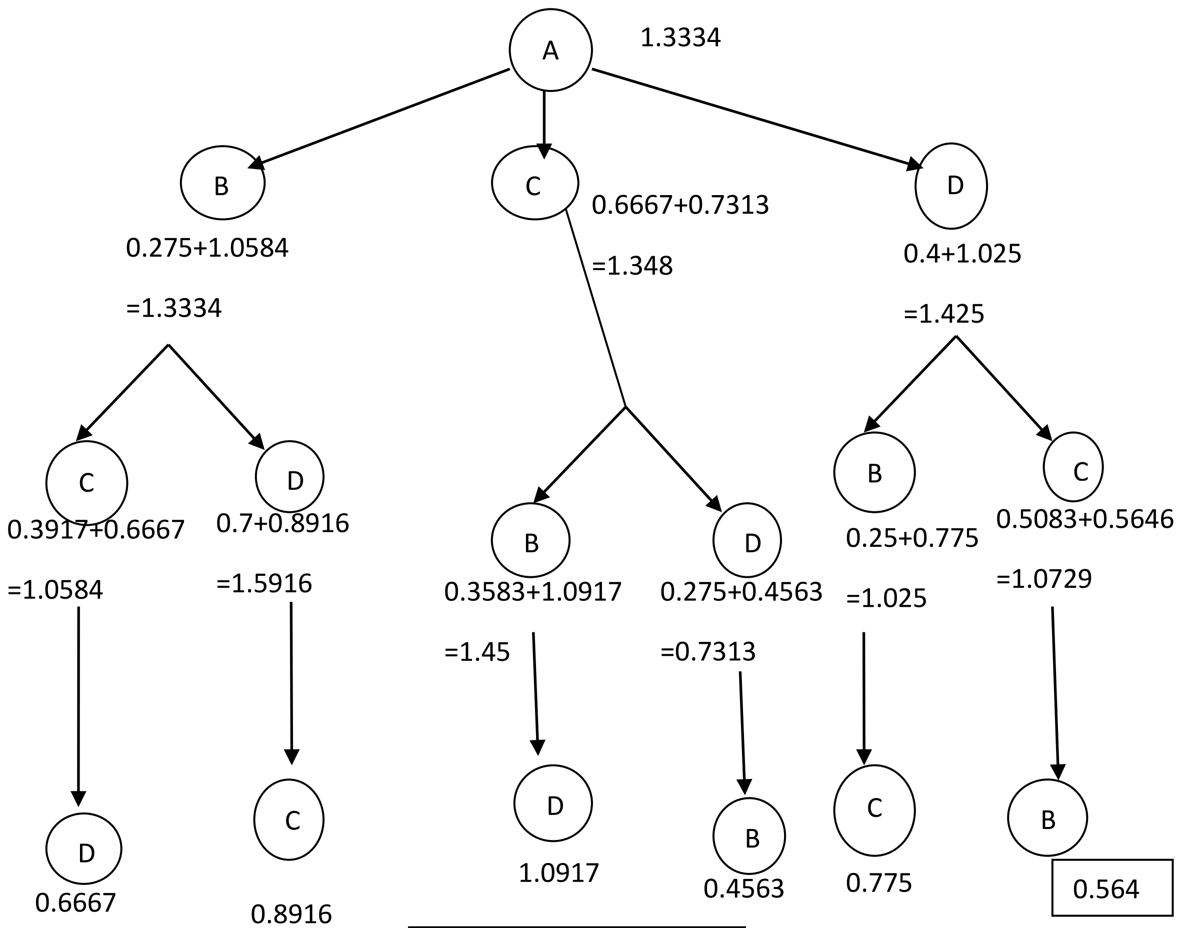

Figure 3 Recursive Tree

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From the recursive tree the obtained minimum cost for Trapezoidal Neutrosophic network is 1.3334.

6.Estimation of least cost for the Pentagonal Neutrosophic Network in this directed network the cost of the edges is considered as Pentagonal Neutrosophic number
THE COST ADJACENCY MATRIX FOR PENTAGONAL NEUTROSCOPIC NETWORK

	A	B	C	D
A	0	< (0.3,0.4,0.5,0.6,0.7); (0.6,0.7,0.8,0.9,0.9); (0.6,0.9,0.9,0.9,0.9) >	< (0.3,0.4,0.5,0.6,0.7); (0.1,0.2,0.3,0.4,0.5); (0.2,0.3,0.4,0.5,0.6) >;	< (0.3,0.4,0.5,0.6,0.7); (0.4,0.5,0.6,0.7,0.8); (0.5,0.6,0.8,0.9,0.9) >
B	< (0.3,0.4,0.5,0.6,0.7); (0.6,0.7,0.8,0.9,0.9); (0.8,0.9,0.9,0.9,0.9) >;	0	< (0.1,0.2,0.3,0.4,0.5); (0.4,0.5,0.6,0.7,0.8); (0.3,0.5,0.6,0.7,0.8) >	< (0.7,0.8,0.9,0.9,0.9); (0.2,0.3,0.4,0.5,0.6); (0.1,0.3,0.5,0.5,0.6) >
C	< (0.1,0.2,0.3,0.4,0.5); (0.2,0.3,0.4,0.5,0.6); (0.6,0.7,0.8,0.9,0.9) >	< (0.2,0.3,0.4,0.5,0.6) (0.3,0.4,0.5,0.6,0.7); (0.7,0.8,0.9,0.9,0.9) >	0	< (0.2,0.3,0.4,0.5,0.6); (0.5,0.6,0.7,0.8,0.9); (0.8,0.8,0.9,0.9,0.9) >
D	< (0.2,0.5,0.7,0.8,0.9); (0.4,0.6,0.7,0.8,0.9); (0.6,0.7,0.8,0.9,0.9) >	< (0.3,0.4,0.5,0.6,0.7); (0.7,0.8,0.8,0.9,0.9); (0.8,0.8,0.8,0.9,0.9) >	< (0.5,0.6,0.7,0.8,0.9); (0.7,0.8,0.8,0.9,0.9); (0.1,0.2,0.3,0.4,0.5) >	0

The Cost Adjacency Matrix for Pentagonal Neutrosophic Network After Deneutrosophication

A B C D (0 0.28 0. 0.3867 0.28 0 0.3733 0.6667 0.3733 0.3533 0 0.28 0.3867 1.173 0.5267 0)
From the recursive tree the obtained minimum cost for the Pentagonal Neutrosophic network is 1.3203

6. Conclusion

The proposed method providing the least cost of the directed network using travelling salesman problem through dynamic programming. In this paper least cost is calculated using Triangular, Trapezoidal and Pentagonal Neutrosophic numbers and the result is compared. From the result it is concluded that the cost obtained for the pentagonal Neutrosophic set is minimum for the directed network. Neutrosophic Travelling salesman problem is very useful to find the least cost for the directed network where uncertainty occurs. The work can be extended for the Triskaidecagonal Neutrosophic set, the Tetra decagonal Neutrosophic set and the Penta decagonal Neutrosophic set and the results can be analysed

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