



New approach towards (ζ_1, ζ_2) -interval valued Q_1 neutrosophic subbisemirings of bisemirings and its extension

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Abstract

We introduce the notions of (τ_1, τ_2) -interval valued Q_1 neutrosophic subbisemirings (IV Q_1 NSBSs), level sets of a (τ_1, τ_2) -IV Q_1 NSBS, and (τ_1, τ_2) -interval valued Q_1 neutrosophic normal subbisemirings ((τ_1, τ_2)-IV Q_1 NNSBS) of a bisemiring. Let \widehat{Z}_1 be a (τ_1, τ_2) -IV Q_1 NSBS of a bisemiring \mathbb{M} and \widehat{V} be the strongest (τ_1, τ_2) -interval valued Q_1 neutrosophic relation of \mathbb{M} . To illustrate \widehat{Z}_1 is a (τ_1, τ_2) -IV Q_1 NSBS of \mathbb{M} if and only if \widehat{V} is a (τ_1, τ_2) -IV Q_1 NSBS of $\mathbb{M} * \mathbb{M}$. We show that homomorphic image of (τ_1, τ_2) -IV Q_1 NSBS is again a (τ_1, τ_2) -IV Q_1 NSBS. To determine homomorphic pre-image of (τ_1, τ_2) -IV Q_1 NSBS is also a (τ_1, τ_2) -IV Q_1 NSBS. Examples are given to strengthen our results.

Keywords: bisemiring; (τ_1, τ_2) -IV Q_1 NSBS; (τ_1, τ_2) -IV Q_1 NNSBS; SBS; homomorphism.

1 Introduction

Golan claims that Vandiver first proposed the concept of semirings, which is a generalization of rings, in 1935.⁷ Semirings are a natural topic to study in algebra because they are the algebraic structure of the set of natural numbers. Semirings are naturally present in many fields of mathematics. Semirings have been useful for resolving challenges in a range of domains of applied mathematics and information science because its structure provides an algebraic framework for modeling and researching the factors necessary. For instance, semirings can be utilized to address issues in the fields of theoretical computer science, graph theory, and optimization. One of the structures that includes semigroups and rings is the semiring. However, since 1950, the idea of semirings has been developing. A FS is a set with degrees of belongingness ranging from 0 to 1; the value of an elements membership in the particular set is represented by these grades. The important work known as Zadeh²⁰ was published in 1965 by the fuzzy set (FS) theory. This definition states that FS is a function that has a membership value that can be used to describe it. Numerous uncertain theories, such as the FS,²⁰ intuitionistic FS (IFS),⁴ Pythagorean FS (PFS),¹⁹ picture FS,⁵ and spherical FS³ have been proposed to address the uncertainties. Later, Atanassov put up the idea of an IFS logic, which is classified by the sum of its MD with an NMD value of no more than 1.⁴ When using a decision-making strategy, we occasionally convey

a single concern when the sum of the grades for MD and NMD is larger than 1. In order to broaden IFS, Yager developed the innovative concept of PFS logic, which is defined by the square sum of its MD and NMD and whose value is not larger than 1. These hypotheses, however, are unable to demonstrate the neutral condition (neither favour nor disfavor). Cuong and Kreinovich, the creators of image FS logic, used three pointers with a combined maximum grade of 1: positive MD, neutral MD, and negative MD. Finally, it delivers more benefits than IFS and PFS for a select applications. The neutrosophic set (NS), in which the truth MD, indeterminacy MD, and falsity MD are all independently stored, is a generalization of the FS and IFS.

Neutrosophy was created by Smarandache¹⁷ to address the issues of ambiguous and inconsistent information because this collection presents several difficulties in terms of applicability. A brand-new theory known as NS has recently been put out. Neutrosophy is the study of neutral cognition, and this neutral is the main distinction between IFS and FS. The invention of NS is attributed to Smarandache.¹⁷ It is a logic that establishes the degrees of veracity, ambiguity, and untruth of each proposition. Each aspect of the universe has an amount of certainty, ambiguity, and untruth that falls between $[0, 1]$ in the NS set. A NS's ability to generalize a classical set, such as FS, IVFS, etc. has been proven. A semiring $(S, +, \cdot)$ is a nonempty set in which S are semigroups both the operations “+” and “ \cdot ”, and “ \cdot ” is distributive over “+”. In 1993, Ahsan et al.² introduced the notion of fuzzy semirings. In 2001, Sen et al. introduced in the concept of bisemirings.^{6,16} In their discussion of numerous ideal structures and their applications, Palanikumar et al.^{1,8-15} Recently, numerous authors have discussed the applicability of neutrosophic structures.^{1,8,18}

This research aims to examine many aspects of the SBS theory to (τ_1, τ_2) -IVNQ₁SBS idea and provide findings. The following five guiding principles make up the article. Section 1 has the introduction. Section 2 contains details on how to prepare for semiring and SBS. Section 3, of (τ_1, τ_2) -IVNQ₁SBS presents its properties. The idea of (τ_1, τ_2) -IVNQ₁NSBS homomorphism is presented in Section 4. Give some numerical examples to illustrate how to evaluate the (τ_1, τ_2) -IVNQ₁SBS and (τ_1, τ_2) -IVNQ₁NSBS.

2 Preliminaries

We will go over the ideas of semirings and bisemirings in this section to make the presentation as full as possible and to make the next talks more convenient.

Let X and Q_1 denote nonempty sets. A mapping $\Lambda : X * Q_1 \rightarrow [0, 1]$ is called a Q_1 fuzzy set in X .

Let $(\mathbb{M}, +, \cdot)$ be semiring. A Q_1 fuzzy set $\Lambda : \mathbb{M} * Q_1 \rightarrow [0, 1]$ is called a Q_1 fuzzy subsemiring of \mathbb{M} if $\Lambda_{Z_1}(\vartheta + \eta, q^*) \geq \min\{\Lambda_{Z_1}(\vartheta, q^*), \Lambda_{Z_1}(\eta, q^*)\}$ and $\Lambda_{Z_1}(\vartheta \cdot \eta, q^*) \geq \min\{\Lambda_{Z_1}(\vartheta, q^*), \Lambda_{Z_1}(\eta, q^*)\}$ for all $\vartheta, \eta \in \mathbb{M}$ and $q^* \in Q_1$.

An intuitionistic Q_1 fuzzy set defined on X and Q_1 is of the form

$$Z_1 = \{ \langle \vartheta, \Lambda_{Z_1}(\vartheta, q^*), \Phi_{Z_1}(\vartheta, q^*) \rangle \mid \vartheta \in X, q^* \in Q_1 \},$$

where $\Lambda_{Z_1}, \Phi_{Z_1} : X * Q_1 \rightarrow [0, 1]$ define the MD and NMD of the element $\vartheta \in X$, respectively and every $\vartheta \in X$ and $q^* \in Q_1$ satisfying $0 \leq \Lambda_{Z_1}(\vartheta, q^*) + \Phi_{Z_1}(\vartheta, q^*) \leq 1$.

Let Z_1 and Z_2 be any two intuitionistic Q_1 fuzzy sets of X . Then,

$$Z_1 \cap Z_2 = \{ \langle \vartheta, \min\{\Lambda_{Z_1}(\vartheta, q^*), \Lambda_{Z_2}(\vartheta, q^*)\}, \max\{\Phi_{Z_1}(\vartheta, q^*), \Phi_{Z_2}(\vartheta, q^*)\} \rangle \mid \vartheta \in X, q^* \in Q_1 \},$$

$$Z_1 \cup Z_2 = \{ \langle \vartheta, \max\{\Lambda_{Z_1}(\vartheta, q^*), \Lambda_{Z_2}(\vartheta, q^*)\}, \min\{\Phi_{Z_1}(\vartheta, q^*), \Phi_{Z_2}(\vartheta, q^*)\} \rangle \mid \vartheta \in X, q^* \in Q_1 \}.$$

Let Z_1 be an intuitionistic Q_1 fuzzy set of X . For $t_1, t_2 \in [0, 1]$, the level subset of Z_1 is the set $Z_{1(t_1, t_2)} = \{ \vartheta \in X \mid \Lambda_{Z_1}(\vartheta, q^*) \geq t_1, \Lambda_{Z_1}(\vartheta, q^*) \leq t_2, q^* \in Q_1 \}$.

An interval valued neutrosophic set (IVN set) \widehat{Z}_1 in X is of the form

$$\widehat{Z}_1 = \left\{ \left\langle \vartheta, \widehat{\Lambda}_{Z_1}^T(\vartheta), \widehat{\Lambda}_{Z_1}^I(\vartheta), \widehat{\Lambda}_{Z_1}^F(\vartheta) \right\rangle \mid \vartheta \in X \right\},$$

where $\widehat{\Lambda}_{Z_1}^T(\vartheta) = [\Lambda_{Z_1}^{TL}, \Lambda_{Z_1}^{TU}]$, $\widehat{\Lambda}_{Z_1}^I(\vartheta) = [\Lambda_{Z_1}^{IL}, \Lambda_{Z_1}^{IU}]$, $\widehat{\Lambda}_{Z_1}^F(\vartheta) = [\Lambda_{Z_1}^{FL}, \Lambda_{Z_1}^{FU}]$ and $\widehat{\Lambda}_{Z_1}^T(\vartheta), \widehat{\Lambda}_{Z_1}^I(\vartheta), \widehat{\Lambda}_{Z_1}^F(\vartheta) : X \rightarrow D[0, 1]$ represent the truth, indeterminacy, and falsity membership function respectively. Simplicity the symbol $\langle \widehat{\Lambda}_{Z_1}^T, \widehat{\Lambda}_{Z_1}^I, \widehat{\Lambda}_{Z_1}^F \rangle$ is IVN set $\widehat{Z}_1 = \{ \langle \vartheta, \widehat{\Lambda}_{Z_1}^T(\vartheta), \widehat{\Lambda}_{Z_1}^I(\vartheta), \widehat{\Lambda}_{Z_1}^F(\vartheta) \rangle \mid \vartheta \in X \}$.

Let $\widehat{Z}_1 = \langle \widehat{\Lambda}_{Z_1}^T, \widehat{\Lambda}_{Z_1}^I, \widehat{\Lambda}_{Z_1}^F \rangle$ and $\widehat{Z}_2 = \langle \widehat{\Lambda}_{Z_2}^T, \widehat{\Lambda}_{Z_2}^I, \widehat{\Lambda}_{Z_2}^F \rangle$ be two IVN sets of X . Then,

$$\widehat{Z}_1 \cap \widehat{Z}_2 = \{ \langle \vartheta, \min\{\widehat{\Lambda}_{Z_1}^T(\vartheta), \widehat{\Lambda}_{Z_2}^T(\vartheta)\}, \min\{\widehat{\Lambda}_{Z_1}^I(\vartheta), \widehat{\Lambda}_{Z_2}^I(\vartheta)\}, \max\{\widehat{\Lambda}_{Z_1}^F(\vartheta), \widehat{\Lambda}_{Z_2}^F(\vartheta)\} \rangle \mid \vartheta \in X \},$$

$$\widehat{Z}_1 \cup \widehat{Z}_2 = \{ \langle \vartheta, \max\{\widehat{\Lambda}_{Z_1}^T(\vartheta), \widehat{\Lambda}_{Z_2}^T(\vartheta)\}, \max\{\widehat{\Lambda}_{Z_1}^I(\vartheta), \widehat{\Lambda}_{Z_2}^I(\vartheta)\}, \min\{\widehat{\Lambda}_{Z_1}^F(\vartheta), \widehat{\Lambda}_{Z_2}^F(\vartheta)\} \rangle \mid \vartheta \in X \}.$$

For any IVN set $\widehat{Z}_1 = \langle \widehat{\Lambda}_{Z_1}^T, \widehat{\Lambda}_{Z_1}^I, \widehat{\Lambda}_{Z_1}^F \rangle$ of a set X , we defined a (τ_1, τ_2) -cut of as the crisp set $\{ \vartheta \in X \mid \widehat{\Lambda}_{Z_1}^T(\vartheta) \geq \widehat{\tau}_1, \widehat{\Lambda}_{Z_1}^I(\vartheta) \geq \widehat{\tau}_1, \widehat{\Lambda}_{Z_1}^F(\vartheta) \leq \widehat{\tau}_2 \}$ of X .

Let \widehat{Z}_1 and \widehat{Z}_2 be the IVN sets of X . The cartesian product of \widehat{Z}_1 and \widehat{Z}_2 is defined as $\widehat{Z}_1 * \widehat{Z}_2 = \{ \langle (\vartheta, \eta), \widehat{\Lambda}_{Z_1 * Z_2}^T(\vartheta, \eta), \widehat{\Lambda}_{Z_1 * Z_2}^I(\vartheta, \eta) \rangle \mid \vartheta, \eta \in X \}$, where $\widehat{\Lambda}_{Z_1 * Z_2}^T(\vartheta, \eta) = \min\{\widehat{\Lambda}_{Z_1}^T(\vartheta), \widehat{\Lambda}_{Z_2}^T(\eta)\}$, $\widehat{\Lambda}_{Z_1 * Z_2}^I(\vartheta, \eta) = \frac{\widehat{\Lambda}_{Z_1}^I(\vartheta) + \widehat{\Lambda}_{Z_2}^I(\eta)}{2}$, $\widehat{\Lambda}_{Z_1 * Z_2}^F(\vartheta, \eta) = \max\{\widehat{\Lambda}_{Z_1}^F(\vartheta), \widehat{\Lambda}_{Z_2}^F(\eta)\}$.

A bisemiring $(\mathbb{M}, +, \circ, *)$ is an algebraic structure in which $(\mathbb{M}, +, \circ)$ and $(\mathbb{M}, \circ, *)$ are semirings, $(\mathbb{M}, +)$, (\mathbb{M}, \circ) and $(\mathbb{M}, *)$ are semigroups and (1) $\vartheta \circ (\eta + \pi) = (\vartheta \circ \eta) + (\vartheta \circ \pi)$, (2) $(\eta + \pi) \circ \vartheta = (\eta \circ \vartheta) + (\pi \circ \vartheta)$, (3) $\vartheta * (\eta \circ \pi) = (\vartheta * \eta) \circ (\vartheta * \pi)$, and (4) $(\eta \circ \pi) * \vartheta = (\eta * \vartheta) \circ (\pi * \vartheta)$ for all $\vartheta, \eta, \pi \in \mathbb{M}$.

A nonempty subset Y of a bisemiring $(\mathbb{M}, +, \circ, *)$ is a subbisemiring if and only if $\vartheta + \eta \in Y, \vartheta \circ \eta \in Y$, and $\vartheta * \eta \in Y$ for all $\vartheta, \eta \in Y$.

A fuzzy set Z_1 of a bisemiring $(\mathbb{M}, \nabla_1, \nabla_2, \nabla_3)$ is said to be a fuzzy subsemiring (SBS) of \mathbb{M} if $\Lambda_{Z_1}(\vartheta \nabla_1 \eta) \geq \min\{\Lambda_{Z_1}(\vartheta), \Lambda_{Z_1}(\eta)\}$, $\Lambda_{Z_1}(\vartheta \nabla_2 \eta) \geq \min\{\Lambda_{Z_1}(\vartheta), \Lambda_{Z_1}(\eta)\}$, and $\Lambda_{Z_1}(\vartheta \nabla_3 \eta) \geq \min\{\Lambda_{Z_1}(\vartheta), \Lambda_{Z_1}(\eta)\}$ for all $\vartheta, \eta \in \mathbb{M}$.

A fuzzy set Z_1 of $(\mathbb{M}, \nabla_1, \nabla_2, \nabla_3)$ is said to be a fuzzy normal SBS of \mathbb{M} if $\Lambda_{Z_1}(\vartheta \nabla_1 \eta) = \Lambda_{Z_1}(\eta \nabla_1 \vartheta)$, $\Lambda_{Z_1}(\vartheta \nabla_2 \eta) = \Lambda_{Z_1}(\eta \nabla_2 \vartheta)$, and $\Lambda_{Z_1}(\vartheta \nabla_3 \eta) = \Lambda_{Z_1}(\eta \nabla_3 \vartheta)$ for all $\vartheta, \eta \in \mathbb{M}$.

Let $(\mathbb{M}, +, \cdot, *)$ and $(\mathbb{N}, \bar{\wedge}, \circ, \otimes)$ be bisemirings. The function $\varphi : \mathbb{M} \rightarrow \mathbb{N}$ is said to be a homomorphism if $\varphi(\vartheta + \eta) = \varphi(\vartheta) \bar{\wedge} \varphi(\eta)$, $\varphi(\vartheta \cdot \eta) = \varphi(\vartheta) \circ \varphi(\eta)$, and $\varphi(\vartheta * \eta) = \varphi(\vartheta) \otimes \varphi(\eta)$ for all $\vartheta, \eta \in \mathbb{M}$.

3 (τ_1, τ_2) -IVQ₁NSBS

In what follows that $(\widehat{\tau}_1, \widehat{\tau}_2) \in D[0, 1]$ be such that $0 \leq \widehat{\tau}_1 < \widehat{\tau}_2 \leq 1$.

An IVN set \widehat{Z}_1 of \mathbb{M} is called a (τ_1, τ_2) -IVQ₁NSBS of \mathbb{M} if

$$\left\{ \begin{array}{l} \max\{\widehat{\Lambda}_{Z_1}^T(\vartheta \nabla_1 \eta, q^*), \widehat{\tau}_1\} \geq \min\{\widehat{\Lambda}_{Z_1}^T(\vartheta, q^*), \widehat{\Lambda}_{Z_1}^T(\eta, q^*), \widehat{\tau}_2\} \\ \max\{\widehat{\Lambda}_{Z_1}^T(\vartheta \nabla_2 \eta, q^*), \widehat{\tau}_1\} \geq \min\{\widehat{\Lambda}_{Z_1}^T(\vartheta, q^*), \widehat{\Lambda}_{Z_1}^T(\eta, q^*), \widehat{\tau}_2\} \\ \max\{\widehat{\Lambda}_{Z_1}^T(\vartheta \nabla_3 \eta, q^*), \widehat{\tau}_1\} \geq \min\{\widehat{\Lambda}_{Z_1}^T(\vartheta, q^*), \widehat{\Lambda}_{Z_1}^T(\eta, q^*), \widehat{\tau}_2\} \end{array} \right\}$$

$$\left\{ \begin{array}{l} \max\{\widehat{\Lambda}_{Z_1}^I(\vartheta \nabla_1 \eta, q^*), \widehat{\tau}_1\} \geq \min\left\{ \frac{\widehat{\Lambda}_{Z_1}^I(\vartheta, q^*) + \widehat{\Lambda}_{Z_1}^I(\eta, q^*)}{2}, \widehat{\tau}_2 \right\} \\ \text{or} \\ \max\{\widehat{\Lambda}_{Z_1}^I(\vartheta \nabla_2 \eta, q^*), \widehat{\tau}_1\} \geq \min\left\{ \frac{\widehat{\Lambda}_{Z_1}^I(\vartheta, q^*) + \widehat{\Lambda}_{Z_1}^I(\eta, q^*)}{2}, \widehat{\tau}_2 \right\} \\ \text{or} \\ \max\{\widehat{\Lambda}_{Z_1}^I(\vartheta \nabla_3 \eta, q^*), \widehat{\tau}_1\} \geq \min\left\{ \frac{\widehat{\Lambda}_{Z_1}^I(\vartheta, q^*) + \widehat{\Lambda}_{Z_1}^I(\eta, q^*)}{2}, \widehat{\tau}_2 \right\} \end{array} \right\}$$

$$\left\{ \begin{aligned} \min\{\widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\vartheta \nabla_1 \eta, q^*), \widehat{\tau}_1\} &\leq \max\{\widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\vartheta, q^*), \widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\eta, q^*), \widehat{\tau}_2\} \\ \min\{\widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\vartheta \nabla_2 \eta, q^*), \widehat{\tau}_1\} &\leq \max\{\widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\vartheta, q^*), \widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\eta, q^*), \widehat{\tau}_2\} \\ \min\{\widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\vartheta \nabla_3 \eta, q^*), \widehat{\tau}_1\} &\leq \max\{\widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\vartheta, q^*), \widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\eta, q^*), \widehat{\tau}_2\} \end{aligned} \right\}$$

for all $\vartheta, \eta \in \mathbb{M}$ and $q^* \in Q_1 \subseteq Z_1$.

Let $\mathbb{M} = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$ be the bisemiring with the following Cayley tables:

∇_1	σ_1	σ_2	σ_3	σ_4	∇_2	σ_1	σ_2	σ_3	σ_4	∇_3	σ_1	σ_2	σ_3	σ_4
σ_1	σ_1	σ_1	σ_1	σ_1	σ_1	σ_1	σ_2	σ_3	σ_4	σ_1	σ_1	σ_1	σ_1	σ_1
σ_2	σ_1	σ_2	σ_1	σ_2	σ_2	σ_2	σ_2	σ_4	σ_4	σ_2	σ_1	σ_2	σ_3	σ_4
σ_3	σ_1	σ_1	σ_3	σ_3	σ_3	σ_3	σ_4	σ_3	σ_4	σ_3	σ_4	σ_4	σ_4	σ_4
σ_4	σ_1	σ_2	σ_3	σ_4	σ_4	σ_4	σ_4	σ_4	σ_4	σ_4	σ_4	σ_4	σ_4	σ_4

$\sigma =$	σ_1	σ_2	σ_3	σ_4
$\widehat{\Lambda}_{Z_1}^{\mathbb{T}}(\sigma)$	[0.75, 0.80]	[0.7, 0.75]	[0.50, 0.55]	[0.65, 0.70]
$\widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\sigma)$	[0.70, 0.75]	[0.65, 0.70]	[0.57, 0.59]	[0.60, 0.65]
$\widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\sigma)$	[0.25, 0.30]	[0.55, 0.60]	[0.70, 0.75]	[0.60, 0.65]

Then \widehat{Z}_1 is a $([0.35, 0.40], [0.50, 0.55])IVQ_1NSBS$ of \mathbb{M} .

The intersection of a family of (τ_1, τ_2) - IVQ_1NSBS s of \mathbb{M} is a (τ_1, τ_2) - IVQ_1NSBS of \mathbb{M} .

Proof. Let $\{\widehat{W}_i \mid i \in I\}$ be a family of (τ_1, τ_2) - IVQ_1NSBS s of \mathbb{M} and $\widehat{Z}_1 = \bigcap_{i \in I} \widehat{W}_i$. Let $\vartheta, \eta \in \mathbb{M}$ and $q^* \in Q_1$. Now,

$$\begin{aligned} \max\{\widehat{\Lambda}_{Z_1}^{\mathbb{T}}(\vartheta \nabla_1 \eta, q^*), \widehat{\tau}_1\} &= \inf_{i \in I} \{ \max\{\widehat{\Lambda}_{W_i}^{\mathbb{T}}(\vartheta \nabla_1 \eta, q^*), \widehat{\tau}_1\} \} \\ &\geq \inf_{i \in I} \{ \min\{\widehat{\Lambda}_{W_i}^{\mathbb{T}}(\vartheta, q^*), \widehat{\Lambda}_{W_i}^{\mathbb{T}}(\eta, q^*), \widehat{\tau}_2\} \} \\ &= \min \left\{ \inf_{i \in I} \{ \widehat{\Lambda}_{W_i}^{\mathbb{T}}(\vartheta, q^*) \}, \inf_{i \in I} \{ \widehat{\Lambda}_{W_i}^{\mathbb{T}}(\eta, q^*) \}, \widehat{\tau}_2 \right\} \\ &= \min\{\widehat{\Lambda}_{Z_1}^{\mathbb{T}}(\vartheta, q^*), \widehat{\Lambda}_{Z_1}^{\mathbb{T}}(\eta, q^*), \widehat{\tau}_2\}. \end{aligned}$$

Similarly, $\max\{\widehat{\Lambda}_{Z_1}^{\mathbb{T}}(\vartheta \nabla_2 \eta, q^*), \widehat{\tau}_1\} \geq \min\{\widehat{\Lambda}_{Z_1}^{\mathbb{T}}(\vartheta, q^*), \widehat{\Lambda}_{Z_1}^{\mathbb{T}}(\eta, q^*), \widehat{\tau}_2\}$ and $\max\{\widehat{\Lambda}_{Z_1}^{\mathbb{T}}(\vartheta \nabla_3 \eta, q^*), \widehat{\tau}_1\} \geq \min\{\widehat{\Lambda}_{Z_1}^{\mathbb{T}}(\vartheta, q^*), \widehat{\Lambda}_{Z_1}^{\mathbb{T}}(\eta, q^*), \widehat{\tau}_2\}$. Now,

$$\begin{aligned} \max\{\widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\vartheta \nabla_1 \eta, q^*), \widehat{\tau}_1\} &= \inf_{i \in I} \{ \max\{\widehat{\Lambda}_{W_i}^{\mathbb{I}}(\vartheta \nabla_1 \eta, q^*), \widehat{\tau}_1\} \} \\ &\geq \inf_{i \in I} \left\{ \min \left\{ \frac{\widehat{\Lambda}_{W_i}^{\mathbb{I}}(\vartheta, q^*) + \widehat{\Lambda}_{W_i}^{\mathbb{I}}(\eta, q^*)}{2}, \widehat{\tau}_2 \right\} \right\} \\ &= \min \left\{ \frac{\inf_{i \in I} \{ \widehat{\Lambda}_{W_i}^{\mathbb{I}}(\vartheta, q^*) \} + \inf_{i \in I} \{ \widehat{\Lambda}_{W_i}^{\mathbb{I}}(\eta, q^*) \}}{2}, \widehat{\tau}_2 \right\} \\ &= \min \left\{ \frac{\widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\vartheta, q^*) + \widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\eta, q^*)}{2}, \widehat{\tau}_2 \right\}. \end{aligned}$$

Similarly, $\max\{\widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\vartheta \nabla_2 \eta, q^*), \widehat{\tau}_1\} \geq \min\left\{\frac{\widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\vartheta, q^*) + \widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\eta, q^*)}{2}, \widehat{\tau}_2\right\}$ and $\max\{\widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\vartheta \nabla_3 \eta, q^*), \widehat{\tau}_1\} \geq \min\left\{\frac{\widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\vartheta, q^*) + \widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\eta, q^*)}{2}, \widehat{\tau}_2\right\}$. Now,

$$\begin{aligned} \min\{\widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\vartheta \nabla_1 \eta, q^*), \widehat{\tau}_1\} &= \sup_{i \in I} \{\min\{\widehat{\Lambda}_{W_i}^{\mathbb{F}}(\vartheta \nabla_1 \eta, q^*), \widehat{\tau}_1\}\} \\ &\leq \sup_{i \in I} \{\max\{\widehat{\Lambda}_{W_i}^{\mathbb{F}}(\vartheta, q^*), \widehat{\Lambda}_{W_i}^{\mathbb{F}}(\eta, q^*), \widehat{\tau}_2\}\} \\ &= \max\left\{\sup_{i \in I} \{\widehat{\Lambda}_{W_i}^{\mathbb{F}}(\vartheta, q^*)\}, \sup_{i \in I} \{\widehat{\Lambda}_{W_i}^{\mathbb{F}}(\eta, q^*)\}, \widehat{\tau}_2\right\} \\ &= \max\{\widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\vartheta, q^*), \widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\eta, q^*), \widehat{\tau}_2\}. \end{aligned}$$

Similarly, $\min\{\widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\vartheta \nabla_2 \eta, q^*), \widehat{\tau}_1\} \leq \max\{\widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\vartheta, q^*), \widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\eta, q^*), \widehat{\tau}_2\}$ and $\min\{\widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\vartheta \nabla_3 \eta, q^*), \widehat{\tau}_1\} \leq \max\{\widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\vartheta, q^*), \widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\eta, q^*), \widehat{\tau}_2\}$. Hence, \widehat{Z}_1 is a (τ_1, τ_2) -IVQ₁NSBS of \mathbb{M} . \square

If \widehat{Z}_1 and \widehat{Z}_2 are any two (τ_1, τ_2) -IVQ₁NSBSs of \mathbb{M}_1 and \mathbb{M}_2 respectively, then $\widehat{Z}_1 * \widehat{Z}_2$ is a (τ_1, τ_2) -IVQ₁NSBS of $\mathbb{M}_1 * \mathbb{M}_2$.

Proof. Let \widehat{Z}_1 and \widehat{Z}_2 be two (τ_1, τ_2) -IVQ₁NSBSs of \mathbb{M}_1 and \mathbb{M}_2 respectively. Let $\vartheta_1, \vartheta_2 \in \mathbb{M}_1$ and $\eta_1, \eta_2 \in \mathbb{M}_2$ and $q^* \in Q_1$. Then (ϑ_1, η_1) and (ϑ_2, η_2) are in $\mathbb{M}_1 * \mathbb{M}_2$. Now,

$$\begin{aligned} &\max\left\{\widehat{\Lambda}_{Z_1 * Z_2}^{\mathbb{T}}[(\vartheta_1, \eta_1) \nabla_1 (\vartheta_2, \eta_2), q^*], \widehat{\tau}_1\right\} \\ &= \max\left\{\widehat{\Lambda}_{Z_1 * Z_2}^{\mathbb{T}}((\vartheta_1 \nabla_1 \vartheta_2, \eta_1 \nabla_1 \eta_2), q^*), \widehat{\tau}_1\right\} \\ &= \min\left\{\max\{\widehat{\Lambda}_{Z_1}^{\mathbb{T}}(\vartheta_1 \nabla_1 \vartheta_2, q^*), \widehat{\tau}_1\}, \max\{\widehat{\Lambda}_{Z_2}^{\mathbb{T}}(\eta_1 \nabla_1 \eta_2, q^*), \widehat{\tau}_1\}\right\} \\ &\geq \min\left\{\min\{\widehat{\Lambda}_{Z_1}^{\mathbb{T}}(\vartheta_1, q^*), \widehat{\Lambda}_{Z_1}^{\mathbb{T}}(\vartheta_2, q^*), \widehat{\tau}_2\}, \min\{\widehat{\Lambda}_{Z_2}^{\mathbb{T}}(\eta_1, q^*), \widehat{\Lambda}_{Z_2}^{\mathbb{T}}(\eta_2, q^*), \widehat{\tau}_2\}\right\} \\ &= \min\left\{\min\{\widehat{\Lambda}_{Z_1}^{\mathbb{T}}(\vartheta_1, q^*), \widehat{\Lambda}_{Z_2}^{\mathbb{T}}(\eta_1, q^*)\}, \min\{\widehat{\Lambda}_{Z_1}^{\mathbb{T}}(\vartheta_2, q^*), \widehat{\Lambda}_{Z_2}^{\mathbb{T}}(\eta_2, q^*)\}, \widehat{\tau}_2\right\} \\ &= \min\left\{\widehat{\Lambda}_{Z_1 * Z_2}^{\mathbb{T}}((\vartheta_1, \eta_1), q^*), \widehat{\Lambda}_{Z_1 * Z_2}^{\mathbb{T}}((\vartheta_2, \eta_2), q^*), \widehat{\tau}_2\right\}. \end{aligned}$$

Also, $\max\left\{\widehat{\Lambda}_{Z_1 * Z_2}^{\mathbb{T}}[(\vartheta_1, \eta_1) \nabla_2 (\vartheta_2, \eta_2), q^*], \widehat{\tau}_1\right\} \geq \min\left\{\widehat{\Lambda}_{Z_1 * Z_2}^{\mathbb{T}}((\vartheta_1, \eta_1), q^*), \widehat{\Lambda}_{Z_1 * Z_2}^{\mathbb{T}}((\vartheta_2, \eta_2), q^*), \widehat{\tau}_2\right\}$ and $\max\left\{\widehat{\Lambda}_{Z_1 * Z_2}^{\mathbb{T}}[(\vartheta_1, \eta_1) \nabla_3 (\vartheta_2, \eta_2), q^*], \widehat{\tau}_1\right\} \geq \min\left\{\widehat{\Lambda}_{Z_1 * Z_2}^{\mathbb{T}}((\vartheta_1, \eta_1), q^*), \widehat{\Lambda}_{Z_1 * Z_2}^{\mathbb{T}}((\vartheta_2, \eta_2), q^*), \widehat{\tau}_2\right\}$. Now,

$$\begin{aligned} &\max\left\{\widehat{\Lambda}_{Z_1 * Z_2}^{\mathbb{I}}[(\vartheta_1, \eta_1) \nabla_1 (\vartheta_2, \eta_2), q^*], \widehat{\tau}_1\right\} \\ &= \max\left\{\widehat{\Lambda}_{Z_1 * Z_2}^{\mathbb{I}}((\vartheta_1 \nabla_1 \vartheta_2, \eta_1 \nabla_1 \eta_2), q^*), \widehat{\tau}_1\right\} \\ &= \min\left\{\frac{1}{2}\left[\max\left\{\widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\vartheta_1 \nabla_1 \vartheta_2, q^*), \widehat{\tau}_1\right\} + \max\left\{\widehat{\Lambda}_{Z_2}^{\mathbb{I}}(\eta_1 \nabla_1 \eta_2, q^*), \widehat{\tau}_1\right\}\right]\right\} \\ &\geq \min\left\{\frac{1}{2}\left[\min\left\{\frac{\widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\vartheta_1, q^*) + \widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\vartheta_2, q^*)}{2}, \widehat{\tau}_2\right\} + \min\left\{\frac{\widehat{\Lambda}_{Z_2}^{\mathbb{I}}(\eta_1, q^*) + \widehat{\Lambda}_{Z_2}^{\mathbb{I}}(\eta_2, q^*)}{2}, \widehat{\tau}_2\right\}\right]\right\} \\ &= \min\left\{\frac{1}{2}\left[\frac{\widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\vartheta_1, q^*) + \widehat{\Lambda}_{Z_2}^{\mathbb{I}}(\eta_1, q^*)}{2} + \frac{\widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\vartheta_2, q^*) + \widehat{\Lambda}_{Z_2}^{\mathbb{I}}(\eta_2, q^*)}{2}\right], \widehat{\tau}_2\right\} \\ &= \min\left\{\frac{\widehat{\Lambda}_{Z_1 * Z_2}^{\mathbb{I}}((\vartheta_1, \eta_1), q^*) + \widehat{\Lambda}_{Z_1 * Z_2}^{\mathbb{I}}((\vartheta_2, \eta_2), q^*)}{2}, \widehat{\tau}_2\right\}. \end{aligned}$$

Also, $\max \left\{ \widehat{\Lambda_{Z_1 * Z_2}^{\mathbb{I}}}[(\vartheta_1, \eta_1) \nabla_2 (\vartheta_2, \eta_2), q^*], \widehat{\tau}_1 \right\} \geq \min \left\{ \frac{\widehat{\Lambda_{Z_1 * Z_2}^{\mathbb{I}}}((\vartheta_1, \eta_1), q^*) + \widehat{\Lambda_{Z_1 * Z_2}^{\mathbb{I}}}((\vartheta_2, \eta_2), q^*)}{2}, \widehat{\tau}_2 \right\}$ and $\max \left\{ \widehat{\Lambda_{Z_1 * Z_2}^{\mathbb{I}}}[(\vartheta_1, \eta_1) \nabla_3 (\vartheta_2, \eta_2), q^*], \widehat{\tau}_1 \right\} \geq \min \left\{ \frac{\widehat{\Lambda_{Z_1 * Z_2}^{\mathbb{I}}}((\vartheta_1, \eta_1), q^*) + \widehat{\Lambda_{Z_1 * Z_2}^{\mathbb{I}}}((\vartheta_2, \eta_2), q^*)}{2}, \widehat{\tau}_2 \right\}$. Similarly,

$$\begin{aligned} & \min \left\{ \widehat{\Lambda_{Z_1 * Z_2}^{\mathbb{F}}}[(\vartheta_1, \eta_1) \nabla_1 (\vartheta_2, \eta_2), q^*], \widehat{\tau}_1 \right\} \\ &= \min \left\{ \widehat{\Lambda_{Z_1 * Z_2}^{\mathbb{F}}}((\vartheta_1 \nabla_1 \vartheta_2, \eta_1 \nabla_1 \eta_2), q^*), \widehat{\tau}_1 \right\} \\ &= \max \left\{ \min \left\{ \widehat{\Lambda_{Z_1}^{\mathbb{F}}}(\vartheta_1 \nabla_1 \vartheta_2, q^*), \widehat{\tau}_1 \right\}, \min \left\{ \widehat{\Lambda_{Z_2}^{\mathbb{F}}}(\eta_1 \nabla_1 \eta_2, q^*), \widehat{\tau}_1 \right\} \right\} \\ &\leq \max \left\{ \max \left\{ \widehat{\Lambda_{Z_1}^{\mathbb{F}}}(\vartheta_1, q^*), \widehat{\Lambda_{Z_1}^{\mathbb{F}}}(\vartheta_2, q^*), \widehat{\tau}_2 \right\}, \max \left\{ \widehat{\Lambda_{Z_2}^{\mathbb{F}}}(\eta_1, q^*), \widehat{\Lambda_{Z_2}^{\mathbb{F}}}(\eta_2, q^*), \widehat{\tau}_2 \right\} \right\} \\ &= \max \left\{ \left\{ \max \left\{ \widehat{\Lambda_{Z_1}^{\mathbb{F}}}(\vartheta_1, q^*), \widehat{\Lambda_{Z_2}^{\mathbb{F}}}(\eta_1, q^*) \right\}, \max \left\{ \widehat{\Lambda_{Z_1}^{\mathbb{F}}}(\vartheta_2, q^*), \widehat{\Lambda_{Z_2}^{\mathbb{F}}}(\eta_2, q^*) \right\} \right\}, \widehat{\tau}_2 \right\} \\ &= \max \left\{ \widehat{\Lambda_{Z_1 * Z_2}^{\mathbb{F}}}((\vartheta_1, \eta_1), q^*), \widehat{\Lambda_{Z_1 * Z_2}^{\mathbb{F}}}((\vartheta_2, \eta_2), q^*), \widehat{\tau}_2 \right\}. \end{aligned}$$

Also, $\min \left\{ \widehat{\Lambda_{Z_1 * Z_2}^{\mathbb{F}}}[(\vartheta_1, \eta_1) \nabla_2 (\vartheta_2, \eta_2), q^*], \widehat{\tau}_1 \right\} \leq \max \left\{ \widehat{\Lambda_{Z_1 * Z_2}^{\mathbb{F}}}((\vartheta_1, \eta_1), q^*), \widehat{\Lambda_{Z_1 * Z_2}^{\mathbb{F}}}((\vartheta_2, \eta_2), q^*), \widehat{\tau}_2 \right\}$ and $\min \left\{ \widehat{\Lambda_{Z_1 * Z_2}^{\mathbb{F}}}[(\vartheta_1, \eta_1) \nabla_3 (\vartheta_2, \eta_2), q^*], \widehat{\tau}_1 \right\} \leq \max \left\{ \widehat{\Lambda_{Z_1 * Z_2}^{\mathbb{F}}}((\vartheta_1, \eta_1), q^*), \widehat{\Lambda_{Z_1 * Z_2}^{\mathbb{F}}}((\vartheta_2, \eta_2), q^*), \widehat{\tau}_2 \right\}$. Hence, $\widehat{Z_1} * \widehat{Z_2}$ is a (τ_1, τ_2) -IVQ₁NSBSs of \mathbb{M} . □

If $\widehat{Z_1}, \widehat{Z_2}, \dots, \widehat{Z_n}$ are (τ_1, τ_2) -IVQ₁NSBSs of $\mathbb{M}_1, \mathbb{M}_2, \dots, \mathbb{M}_n$ respectively, then $\widehat{Z_1} * \widehat{Z_2} * \dots * \widehat{Z_n}$ is a (τ_1, τ_2) -IVQ₁NSBS of $\mathbb{M}_1 * \mathbb{M}_2 * \dots * \mathbb{M}_n$.

Let $\widehat{Z_1}$ be a (τ_1, τ_2) -IVN set in \mathbb{M} , the strongest (τ_1, τ_2) -IVN relation on \mathbb{M} , that is a (τ_1, τ_2) -IVN relation on $\widehat{Z_1}$ is \widehat{V} given by

$$\begin{aligned} \max \left\{ \widehat{\Lambda_W^{\mathbb{T}}}((\vartheta, \eta), q^*), \widehat{\tau}_1 \right\} &= \min \left\{ \widehat{\Lambda_{Z_1}^{\mathbb{T}}}(\vartheta, q^*), \widehat{\Lambda_{Z_1}^{\mathbb{T}}}(\eta, q^*), \widehat{\tau}_2 \right\}, \\ \max \left\{ \widehat{\Lambda_W^{\mathbb{I}}}((\vartheta, \eta), q^*), \widehat{\tau}_1 \right\} &= \min \left\{ \widehat{\Lambda_{Z_1}^{\mathbb{I}}}(\vartheta, q^*), \widehat{\Lambda_{Z_1}^{\mathbb{I}}}(\eta, q^*), \widehat{\tau}_2 \right\}, \\ \min \left\{ \widehat{\Lambda_W^{\mathbb{F}}}((\vartheta, \eta), q^*), \widehat{\tau}_1 \right\} &= \max \left\{ \widehat{\Lambda_{Z_1}^{\mathbb{F}}}(\vartheta, q^*), \widehat{\Lambda_{Z_1}^{\mathbb{F}}}(\eta, q^*), \widehat{\tau}_2 \right\} \end{aligned}$$

for all $\vartheta, \eta \in \mathbb{M}$ and $q^* \in Q_1 \subseteq Z_1$.

Let $\widehat{Z_1}$ be a (τ_1, τ_2) -IVQ₁NSBS of \mathbb{M} and \widehat{V} be the strongest (τ_1, τ_2) -IVN relation of \mathbb{M} . Then $\widehat{Z_1}$ is a (τ_1, τ_2) -IVQ₁NSBS of \mathbb{M} if and only if \widehat{V} is a (τ_1, τ_2) -IVQ₁NSBS of $\mathbb{M} * \mathbb{M}$.

Let $(\mathbb{M}_1, \bar{\Lambda}_1, \bar{\Lambda}_2, \bar{\Lambda}_3)$ and $(\mathbb{M}_2, \bar{\Lambda}_1, \bar{\Lambda}_2, \bar{\Lambda}_3)$ be any two bisemirings. The homomorphic image of (τ_1, τ_2) -IVQ₁NSBS of \mathbb{M}_1 is a (τ_1, τ_2) -IVQ₁NSBS of \mathbb{M}_2 .

Proof. Let $\Theta : \mathbb{M}_1 \rightarrow \mathbb{M}_2$ be any homomorphism. Then $\Theta(\vartheta \bar{\Lambda}_1 \eta, q^*) = \Theta(\vartheta, q^*) \bar{\Lambda}_1 \Theta(\eta, q^*)$, $\Theta(\vartheta \bar{\Lambda}_2 \eta, q^*) = \Theta(\vartheta, q^*) \bar{\Lambda}_2 \Theta(\eta, q^*)$, and $\Theta(\vartheta \bar{\Lambda}_3 \eta, q^*) = \Theta(\vartheta, q^*) \bar{\Lambda}_3 \Theta(\eta, q^*)$ for all $\vartheta, \eta \in \mathbb{M}_1$. Let $\widehat{V} = \Theta(\widehat{Z_1})$, where $\widehat{Z_1}$ is any (τ_1, τ_2) -IVQ₁NSBS of \mathbb{M}_1 . Let $\Theta(\vartheta, q^*), \Theta(\eta, q^*) \in \mathbb{M}_2$. Let $(\vartheta, q^*) \in \Theta^{-1}(\Theta(\vartheta, q^*))$ and $(\eta, q^*) \in \Theta^{-1}(\Theta(\eta, q^*))$ be such that $\widehat{\Lambda_{Z_1}^{\mathbb{T}}}(\vartheta, q^*) = \sup_{(\pi, q^*) \in \Theta^{-1}(\Theta(\vartheta, q^*))} \{ \widehat{\Lambda_{Z_1}^{\mathbb{T}}}(\pi, q^*) \}$ and $\widehat{\Lambda_{Z_1}^{\mathbb{I}}}(\eta, q^*) = \sup_{(\pi, q^*) \in \Theta^{-1}(\Theta(\eta, q^*))} \{ \widehat{\Lambda_{Z_1}^{\mathbb{I}}}(\pi, q^*) \}$. Now,

$$\begin{aligned} \max \left\{ \widehat{\Lambda_W^{\mathbb{T}}}(\Theta(\vartheta, q^*) \bar{\Lambda}_1 \Theta(\eta, q^*)), \widehat{\tau}_1 \right\} &= \max \left\{ \sup_{(\pi', q^*) \in \Theta^{-1}(\Theta(\vartheta, q^*) \bar{\Lambda}_1 \Theta(\eta, q^*))} \{ \widehat{\Lambda_{Z_1}^{\mathbb{T}}}(\pi', q^*) \}, \widehat{\tau}_1 \right\} \\ &= \max \left\{ \sup_{(\pi', q^*) \in \Theta^{-1}(\Theta(\vartheta \bar{\Lambda}_1 \eta, q^*))} \{ \widehat{\Lambda_{Z_1}^{\mathbb{T}}}(\pi', q^*) \}, \widehat{\tau}_1 \right\} \\ &= \max \left\{ \widehat{\Lambda_{Z_1}^{\mathbb{T}}}(\vartheta \bar{\Lambda}_1 \eta, q^*), \widehat{\tau}_1 \right\} \\ &\geq \min \left\{ \widehat{\Lambda_{Z_1}^{\mathbb{T}}}(\vartheta, q^*), \widehat{\Lambda_{Z_1}^{\mathbb{T}}}(\eta, q^*), \widehat{\tau}_2 \right\} \\ &= \min \left\{ \widehat{\Lambda_W^{\mathbb{T}}}\Theta(\vartheta, q^*), \widehat{\Lambda_W^{\mathbb{T}}}\Theta(\eta, q^*), \widehat{\tau}_2 \right\}. \end{aligned}$$

Thus, $\max \left\{ \widehat{\Lambda}_W^{\mathbb{T}}(\Theta(\vartheta, q^*) \bar{\lambda}_1 \Theta(\eta, q^*)), \widehat{\tau}_1 \right\} \geq \min \left\{ \widehat{\Lambda}_W^{\mathbb{T}}\Theta(\vartheta, q^*), \widehat{\Lambda}_W^{\mathbb{T}}\Theta(\eta, q^*), \widehat{\tau}_2 \right\}$. Similarly,

$$\max \left\{ \widehat{\Lambda}_W^{\mathbb{T}}(\Theta(\vartheta, q^*) \bar{\lambda}_2 \Theta(\eta, q^*)), \widehat{\tau}_1 \right\} \geq \min \left\{ \widehat{\Lambda}_W^{\mathbb{T}}\Theta(\vartheta, q^*), \widehat{\Lambda}_W^{\mathbb{T}}\Theta(\eta, q^*), \widehat{\tau}_2 \right\}$$

and

$$\max \left\{ \widehat{\Lambda}_W^{\mathbb{T}}(\Theta(\vartheta, q^*) \bar{\lambda}_3 \Theta(\eta, q^*)), \widehat{\tau}_1 \right\} \geq \min \left\{ \widehat{\Lambda}_W^{\mathbb{T}}\Theta(\vartheta, q^*), \widehat{\Lambda}_W^{\mathbb{T}}\Theta(\eta, q^*), \widehat{\tau}_2 \right\}.$$

Let $\Theta(\vartheta, q^*), \Theta(\eta, q^*) \in \mathbb{M}_2$. Let $(\vartheta, q^*) \in \Theta^{-1}(\Theta(\vartheta, q^*))$ and $(\eta, q^*) \in \Theta^{-1}(\Theta(\eta, q^*))$ be such that $\widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\vartheta, q^*) = \sup_{(\pi, q^*) \in \Theta^{-1}(\Theta(\vartheta, q^*))} \left\{ \widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\pi, q^*) \right\}$ and $\widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\eta, q^*) = \sup_{(\pi, q^*) \in \Theta^{-1}(\Theta(\eta, q^*))} \left\{ \widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\pi, q^*) \right\}$. Now,

$$\begin{aligned} \max \left\{ \widehat{\Lambda}_W^{\mathbb{I}}(\Theta(\vartheta, q^*) \bar{\lambda}_1 \Theta(\eta, q^*)), \widehat{\tau}_1 \right\} &= \max \left\{ \sup_{(\pi', q^*) \in \Theta^{-1}(\Theta(\vartheta, q^*) \bar{\lambda}_1 \Theta(\eta, q^*))} \left\{ \widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\pi', q^*) \right\}, \widehat{\tau}_1 \right\} \\ &= \max \left\{ \sup_{(\pi', q^*) \in \Theta^{-1}(\Theta(\vartheta \bar{\lambda}_1 \eta, q^*))} \left\{ \widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\pi', q^*) \right\}, \widehat{\tau}_1 \right\} \\ &= \max \left\{ \widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\vartheta \bar{\lambda}_1 \eta, q^*), \widehat{\tau}_1 \right\} \\ &\geq \min \left\{ \frac{\widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\vartheta, q^*) + \widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\eta, q^*)}{2}, \widehat{\tau}_2 \right\} \\ &= \min \left\{ \frac{\widehat{\Lambda}_W^{\mathbb{I}}\Theta(\vartheta, q^*) + \widehat{\Lambda}_W^{\mathbb{I}}\Theta(\eta, q^*)}{2}, \widehat{\tau}_2 \right\}. \end{aligned}$$

Thus, $\max \left\{ \widehat{\Lambda}_W^{\mathbb{I}}(\Theta(\vartheta, q^*) \bar{\lambda}_1 \Theta(\eta, q^*)), \widehat{\tau}_1 \right\} \geq \min \left\{ \frac{\widehat{\Lambda}_W^{\mathbb{I}}\Theta(\vartheta, q^*) + \widehat{\Lambda}_W^{\mathbb{I}}\Theta(\eta, q^*)}{2}, \widehat{\tau}_2 \right\}$. Similarly,

$$\max \left\{ \widehat{\Lambda}_W^{\mathbb{I}}(\Theta(\vartheta, q^*) \bar{\lambda}_2 \Theta(\eta, q^*)), \widehat{\tau}_1 \right\} \geq \min \left\{ \frac{\widehat{\Lambda}_W^{\mathbb{I}}\Theta(\vartheta, q^*) + \widehat{\Lambda}_W^{\mathbb{I}}\Theta(\eta, q^*)}{2}, \widehat{\tau}_2 \right\}$$

and

$$\max \left\{ \widehat{\Lambda}_W^{\mathbb{I}}(\Theta(\vartheta, q^*) \bar{\lambda}_3 \Theta(\eta, q^*)), \widehat{\tau}_1 \right\} \geq \min \left\{ \frac{\widehat{\Lambda}_W^{\mathbb{I}}\Theta(\vartheta, q^*) + \widehat{\Lambda}_W^{\mathbb{I}}\Theta(\eta, q^*)}{2}, \widehat{\tau}_2 \right\}.$$

Let $(\vartheta, q^*) \in \Theta^{-1}(\Theta(\vartheta, q^*))$ and $(\eta, q^*) \in \Theta^{-1}(\Theta(\eta, q^*))$ be such that

$\widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\vartheta, q^*) = \inf_{(\pi, q^*) \in \Theta^{-1}(\Theta(\vartheta, q^*))} \left\{ \widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\pi, q^*) \right\}$ and $\widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\eta, q^*) = \inf_{(\pi, q^*) \in \Theta^{-1}(\Theta(\eta, q^*))} \left\{ \widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\pi, q^*) \right\}$. Now,

$$\begin{aligned} \min \left\{ \widehat{\Lambda}_W^{\mathbb{F}}(\Theta(\vartheta, q^*) \bar{\lambda}_1 \Theta(\eta, q^*)), \widehat{\tau}_1 \right\} &= \min \left\{ \inf_{(\pi', q^*) \in \Theta^{-1}(\Theta(\vartheta, q^*) \bar{\lambda}_1 \Theta(\eta, q^*))} \left\{ \widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\pi', q^*) \right\}, \widehat{\tau}_1 \right\} \\ &= \min \left\{ \inf_{(\pi', q^*) \in \Theta^{-1}(\Theta(\vartheta \bar{\lambda}_1 \eta, q^*))} \left\{ \widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\pi', q^*) \right\}, \widehat{\tau}_1 \right\} \\ &= \min \left\{ \widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\vartheta \bar{\lambda}_1 \eta, q^*), \widehat{\tau}_1 \right\} \\ &\leq \max \left\{ \widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\vartheta, q^*), \widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\eta, q^*), \widehat{\tau}_2 \right\} \\ &= \max \left\{ \widehat{\Lambda}_W^{\mathbb{F}}\Theta(\vartheta, q^*), \widehat{\Lambda}_W^{\mathbb{F}}\Theta(\eta, q^*), \widehat{\tau}_2 \right\}. \end{aligned}$$

Thus, $\min \left\{ \widehat{\Lambda}_W^{\mathbb{F}}(\Theta(\vartheta, q^*) \bar{\lambda}_1 \Theta(\eta, q^*)), \widehat{\tau}_1 \right\} \leq \max \left\{ \widehat{\Lambda}_W^{\mathbb{F}}\Theta(\vartheta, q^*), \widehat{\Lambda}_W^{\mathbb{F}}\Theta(\eta, q^*), \widehat{\tau}_2 \right\}$. Similarly,

$$\min \left\{ \widehat{\Lambda}_W^{\mathbb{F}}(\Theta(\vartheta, q^*) \bar{\lambda}_2 \Theta(\eta, q^*)), \widehat{\tau}_1 \right\} \leq \max \left\{ \widehat{\Lambda}_W^{\mathbb{F}}\Theta(\vartheta, q^*), \widehat{\Lambda}_W^{\mathbb{F}}\Theta(\eta, q^*), \widehat{\tau}_2 \right\}$$

and

$$\min \left\{ \widehat{\Lambda}_W^{\mathbb{F}}(\Theta(\vartheta, q^*) \bar{\lambda}_3 \Theta(\eta, q^*)), \widehat{\tau}_1 \right\} \leq \max \left\{ \widehat{\Lambda}_W^{\mathbb{F}}\Theta(\vartheta, q^*), \widehat{\Lambda}_W^{\mathbb{F}}\Theta(\eta, q^*), \widehat{\tau}_2 \right\}.$$

Hence \widehat{V} is a (τ_1, τ_2) -IVQ₁NSBS of \mathbb{M}_2 . □

Let $(\mathbb{M}_1, \bar{\lambda}_1, \bar{\lambda}_2, \bar{\lambda}_3)$ and $(\mathbb{M}_2, \bar{\lambda}_1, \bar{\lambda}_2, \bar{\lambda}_3)$ be any two bisemirings. The homomorphic pre-image of (τ_1, τ_2) -IVQ₁NSBS of \mathbb{M}_2 is a (τ_1, τ_2) -IVQ₁NSBS of \mathbb{M}_1 .

Proof. Let $\Theta : \mathbb{M}_1 \rightarrow \mathbb{M}_2$ be any homomorphism. Then $\Theta(\vartheta \bar{\lambda}_1 \eta, q^*) = \Theta(\vartheta, q^*) \bar{\lambda}_1 \Theta(\eta, q^*)$, $\Theta(\vartheta \bar{\lambda}_2 \eta, q^*) = \Theta(\vartheta, q^*) \bar{\lambda}_2 \Theta(\eta, q^*)$, and $\Theta(\vartheta \bar{\lambda}_3 \eta, q^*) = \Theta(\vartheta, q^*) \bar{\lambda}_3 \Theta(\eta, q^*)$ for all $\vartheta, \eta \in \mathbb{M}_1$. Let $\widehat{V} = \Theta(\widehat{Z}_1)$, where \widehat{V} is any (τ_1, τ_2) -IVQ₁NSBS of \mathbb{M}_2 . Let $\vartheta, \eta \in \mathbb{M}_1$. Then $\max\{\widehat{\Lambda}_{Z_1}^{\mathbb{T}}(\vartheta \bar{\lambda}_1 \eta, q^*), \widehat{\tau}_1\} = \max\{\widehat{\Lambda}_W^{\mathbb{T}}(\Theta(\vartheta \bar{\lambda}_1 \eta, q^*)), \widehat{\tau}_1\} = \max\{\widehat{\Lambda}_W^{\mathbb{T}}(\Theta(\vartheta, q^*) \bar{\lambda}_1 \Theta(\eta, q^*)), \widehat{\tau}_1\} \geq \min\{\widehat{\Lambda}_W^{\mathbb{T}}(\Theta(\vartheta, q^*)), \widehat{\Lambda}_W^{\mathbb{T}}(\Theta(\eta, q^*)), \widehat{\tau}_2\} = \min\{\widehat{\Lambda}_{Z_1}^{\mathbb{T}}(\vartheta, q^*), \widehat{\Lambda}_{Z_1}^{\mathbb{T}}(\eta, q^*), \widehat{\tau}_2\}$. Thus,

$$\max\{\widehat{\Lambda}_{Z_1}^{\mathbb{T}}(\vartheta \bar{\lambda}_1 \eta, q^*), \widehat{\tau}_1\} \geq \min\{\widehat{\Lambda}_{Z_1}^{\mathbb{T}}(\vartheta, q^*), \widehat{\Lambda}_{Z_1}^{\mathbb{T}}(\eta, q^*), \widehat{\tau}_2\}.$$

Now, $\max\{\widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\vartheta \bar{\lambda}_1 \eta, q^*), \widehat{\tau}_1\} = \max\{\widehat{\Lambda}_W^{\mathbb{I}}(\Theta(\vartheta \bar{\lambda}_1 \eta, q^*)), \widehat{\tau}_1\} = \max\{\widehat{\Lambda}_W^{\mathbb{I}}(\Theta(\vartheta, q^*) \bar{\lambda}_1 \Theta(\eta, q^*)), \widehat{\tau}_1\} \geq \min\{\widehat{\Lambda}_W^{\mathbb{I}}(\Theta(\vartheta, q^*)), \widehat{\Lambda}_W^{\mathbb{I}}(\Theta(\eta, q^*)), \widehat{\tau}_2\} = \min\{\widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\vartheta, q^*), \widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\eta, q^*), \widehat{\tau}_2\}$. Thus,

$$\max\{\widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\vartheta \bar{\lambda}_1 \eta, q^*), \widehat{\tau}_1\} \geq \min\{\widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\vartheta, q^*), \widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\eta, q^*), \widehat{\tau}_2\}.$$

Now, $\min\{\widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\vartheta \bar{\lambda}_1 \eta, q^*), \widehat{\tau}_1\} = \min\{\widehat{\Lambda}_W^{\mathbb{F}}(\Theta(\vartheta \bar{\lambda}_1 \eta, q^*)), \widehat{\tau}_1\} = \min\{\widehat{\Lambda}_W^{\mathbb{F}}(\Theta(\vartheta, q^*) \bar{\lambda}_1 \Theta(\eta, q^*)), \widehat{\tau}_1\} \leq \max\{\widehat{\Lambda}_W^{\mathbb{F}}(\Theta(\vartheta, q^*)), \widehat{\Lambda}_W^{\mathbb{F}}(\Theta(\eta, q^*)), \widehat{\tau}_2\} = \max\{\widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\vartheta, q^*), \widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\eta, q^*), \widehat{\tau}_2\}$. Thus,

$$\min\{\widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\vartheta \bar{\lambda}_1 \eta, q^*), \widehat{\tau}_1\} \leq \max\{\widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\vartheta, q^*), \widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\eta, q^*), \widehat{\tau}_2\}.$$

Hence \widehat{Z}_1 is a (τ_1, τ_2) -IVQ₁NSBS of \mathbb{M}_1 . □

4 (τ_1, τ_2) -IVQ₁NNSBS

An IVN set \widehat{Z}_1 of \mathbb{M} is said to be a IVQ₁NNSBS of \mathbb{M} if

$$\left\{ \begin{array}{l} \widehat{\Lambda}_{Z_1}^{\mathbb{T}}(\vartheta \nabla_1 \eta, q^*) = \widehat{\Lambda}_{Z_1}^{\mathbb{T}}(\eta \nabla_1 \vartheta, q^*) \\ \widehat{\Lambda}_{Z_1}^{\mathbb{T}}(\vartheta \nabla_2 \eta, q^*) = \widehat{\Lambda}_{Z_1}^{\mathbb{T}}(\eta \nabla_2 \vartheta, q^*) \\ \widehat{\Lambda}_{Z_1}^{\mathbb{T}}(\vartheta \nabla_3 \eta, q^*) = \widehat{\Lambda}_{Z_1}^{\mathbb{T}}(\eta \nabla_3 \vartheta, q^*) \end{array} \right\} \left\{ \begin{array}{l} \widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\vartheta \nabla_1 \eta, q^*) = \widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\eta \nabla_1 \vartheta, q^*) \\ \text{OR} \\ \widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\vartheta \nabla_2 \eta, q^*) = \widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\eta \nabla_2 \vartheta, q^*) \\ \text{OR} \\ \widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\vartheta \nabla_3 \eta, q^*) = \widehat{\Lambda}_{Z_1}^{\mathbb{I}}(\eta \nabla_3 \vartheta, q^*) \end{array} \right\}$$

$$\left\{ \begin{array}{l} \widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\vartheta \nabla_1 \eta, q^*) = \widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\eta \nabla_1 \vartheta, q^*) \\ \widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\vartheta \nabla_2 \eta, q^*) = \widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\eta \nabla_2 \vartheta, q^*) \\ \widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\vartheta \nabla_3 \eta, q^*) = \widehat{\Lambda}_{Z_1}^{\mathbb{F}}(\eta \nabla_3 \vartheta, q^*) \end{array} \right\}$$

for all $\vartheta, \eta \in \mathbb{M}$ and $q^* \in Q_1 \subseteq Z_1$.

- (1) The intersection of a family of IVQ₁NNSBSs of \mathbb{M} is a IVQ₁NNSBS of \mathbb{M} .
- (2) The intersection of a family of (τ_1, τ_2) -IVQ₁NNSBSs of \mathbb{M} is a (τ_1, τ_2) -IVQ₁NNSBS of \mathbb{M} .
- (1) If $\widehat{Z}_1, \widehat{Z}_2, \dots, \widehat{Z}_n$ are IVQ₁NNSBSs of $\mathbb{M}_1, \mathbb{M}_2, \dots, \mathbb{M}_n$ respectively, then $\widehat{Z}_1 * \widehat{Z}_2 * \dots * \widehat{Z}_n$ is a IVQ₁NNSBS of $\mathbb{M}_1 * \mathbb{M}_2 * \dots * \mathbb{M}_n$.
- (2) If $\widehat{Z}_1, \widehat{Z}_2, \dots, \widehat{Z}_n$ are (τ_1, τ_2) -IVQ₁NNSBSs of $\mathbb{M}_1, \mathbb{M}_2, \dots, \mathbb{M}_n$ respectively, then $\widehat{Z}_1 * \widehat{Z}_2 * \dots * \widehat{Z}_n$ is a (τ_1, τ_2) -IVQ₁NNSBS of $\mathbb{M}_1 * \mathbb{M}_2 * \dots * \mathbb{M}_n$.

- (1) Let \widehat{Z}_1 be any IVQ_1 NNSBS of \mathbb{M} and \widehat{V} be the strongest IVN relation of \mathbb{M} . Then \widehat{Z}_1 is a IVQ_1 NNSBS of \mathbb{M} if and only if \widehat{V} is a IVQ_1 NNSBS of $\mathbb{M} * \mathbb{M}$.
- (2) Let \widehat{Z}_1 be any (τ_1, τ_2) - IVQ_1 NNSBS of \mathbb{M} and \widehat{V} be the strongest (τ_1, τ_2) -IVN relation of \mathbb{M} . Then \widehat{Z}_1 is a (τ_1, τ_2) - IVQ_1 NNSBS of \mathbb{M} if and only if \widehat{V} is a (τ_1, τ_2) - IVQ_1 NNSBS of $\mathbb{M} * \mathbb{M}$.

Let $(\mathbb{M}_1, \bar{\lambda}_1, \bar{\lambda}_2, \bar{\lambda}_3)$ and $(\mathbb{M}_2, \bar{\bar{\lambda}}_1, \bar{\bar{\lambda}}_2, \bar{\bar{\lambda}}_3)$ be any two bisemirings.

- (1) The homomorphic image of IVQ_1 NNSBS of \mathbb{M}_1 is a IVQ_1 NNSBS of \mathbb{M}_2 .
- (2) The homomorphic image of (τ_1, τ_2) - IVQ_1 NNSBS of \mathbb{M}_1 is a (τ_1, τ_2) - IVQ_1 NNSBS of \mathbb{M}_2 .

Let $(\mathbb{M}_1, \bar{\lambda}_1, \bar{\lambda}_2, \bar{\lambda}_3)$ and $(\mathbb{M}_2, \bar{\bar{\lambda}}_1, \bar{\bar{\lambda}}_2, \bar{\bar{\lambda}}_3)$ be any two bisemirings.

- (1) The homomorphic pre-image of IVQ_1 NNSBS of \mathbb{M}_2 is a IVQ_1 NNSBS of \mathbb{M}_1 .
- (2) The homomorphic pre-image of (τ_1, τ_2) - IVQ_1 NNSBS of \mathbb{M}_2 is a (τ_1, τ_2) - IVQ_1 NNSBS of \mathbb{M}_1 .

5 Conclusion

We created the theories behind level sets of a (τ_1, τ_2) - IVQ_1 NSBS, a (τ_1, τ_2) - IVQ_1 NNSBS, and a (τ_1, τ_2) - IVQ_1 NSBS. For (τ_1, τ_2) - IVQ_1 NSBS and (τ_1, τ_2) - IVQ_1 NNSBS of bisemiring. The main goal of this study is to convert (τ_1, τ_2) -fuzzy SBS of SBS to (τ_1, τ_2) -interval valued neutrosophic SBS of SBS. We should thus consider the applications of (τ_1, τ_2) -cubic SBS and (τ_1, τ_2) -interval valued soft neutrosophic SBS in the future.

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References

- [1] A. B. AL-Nafee, S. Broumi, L. A. Swidi, n -Valued refined neutrosophic crisp sets, International Journal of Neutrosophic Science, vol. 17, no. 2, pp. 87-95, 2021.
- [2] J. Ahsan, K. Saifullah, F. Khan, Fuzzy semirings, Fuzzy Sets and systems, vol. 60, pp. 309-320, 1993.
- [3] S. Ashraf, S. Abdullah, T. Mahmood, F. Ghani, T. Mahmood, Spherical fuzzy sets and their applications in multi-attribute decision making problems, Journal of Intelligent and Fuzzy Systems, vol. 36, pp. 2829-2844, 2019.
- [4] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, vol. 20, no. 1, pp. 87-96, 1986.
- [5] B. C. Cuong, V. Kreinovich, Picture fuzzy sets a new concept for computational intelligence problems, Proceedings of 2013 Third World Congress on Information and Communication Technologies (WICT 2013), IEEE, pp. 1-6, 2013.
- [6] F. Hussian, R. M. Hashism, A. Khan, M. Naeem, Generalization of bisemirings, International Journal of Computer Science and Information Security, vol. 14, no. 9, pp. 275-289, 2016.
- [7] S. J. Golan, Semirings and their applications, Kluwer Academic Publishers, London, 1999.
- [8] M. Lathamaheswari, S. Broumi, F. Smarandache, S. Sudha, Neutrosophic perspective of neutrosophic probability distributions and its application, International Journal of Neutrosophic Science, vol. 17, no. 2, pp. 96-109, 2021.

- [9] M. Palanikumar, K. Arulmozhi, On various ideals and its applications of bisemirings, Gedrag and Organisatie Review, vol. 33, no. 2, pp. 522-533, 2020.
- [10] M. Palanikumar, K. Arulmozhi, On intuitionistic fuzzy normal subbisemirings of bisemirings, Nonlinear Studies, vol. 28, no. 3, pp. 717-721, 2021.
- [11] M. Palanikumar, K. Arulmozhi, On new ways of various ideals in ternary semigroups, Matrix Science Mathematic, vol. 4, no. 1, pp. 6-9, 2020.
- [12] M. Palanikumar, K. Arulmozhi, (α, β) -Neutrosophic subbisemiring of bisemiring, Neutrosophic Sets and Systems, vol. 48, pp. 368-385, 2022.
- [13] M. Palanikumar, K. Arulmozhi, On various tri-ideals in ternary semirings, Bulletin of the International Mathematical Virtual Institute, vol. 11, no. 1, pp. 79-90, 2021.
- [14] M. Palanikumar, K. Arulmozhi, On Pythagorean normal subbisemiring of bisemiring, Annals of Communications in Mathematics, vol. 4, no. 1, pp. 63-72, 2021.
- [15] M. Palanikumar, K. Arulmozhi, On various almost ideals of semirings, Annals of Communications in Mathematics, vol. 4, no. 1, pp. 17-25, 2021.
- [16] M. K. Sen, S. Ghosh, An introduction to bisemirings, Asian Bulletin of Mathematics, vol. 28, no. 3, pp. 547-559, 2001.
- [17] F. Smarandache, A unifying field in logics. Neutrosophy: neutrosophic probability, set and logic, American Research Press, Rehoboth, 1999.
- [18] S. Dhouib, S. Broumi, M. Lathamaheswari, Single valued trapezoidal neutrosophic travelling salesman problem with novel Greedy method: the Dhouib-Matrix-TSP1 (DM-TSP1), International Journal of Neutrosophic Science, vol. 17, no. 2, pp. 144-157, 2021.
- [19] R. R. Yager, Pythagorean membership grades in multi criteria decision-making, IEEE Transactions on Fuzzy Systems, vol. 22, no. 4, pp. 958-965, 2014.
- [20] L. A. Zadeh, Fuzzy sets, Information and Control, vol. 8, no. 3, pp. 338-353, 1965.