



Mapping on Interval Complex Neutrosophic Soft Sets

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Abstract

The neutrosophic idea is a fertile environment adaptable to different mathematical tools. This paper aims to introduce the mapping of complex interval neutrosophic soft sets (MI-CNSSs). Further, the images and inverse images of complex interval neutrosophic soft sets and their properties are explored. These will be supported by concrete examples, and this paper will present some theorems about complex interval neutrosophic soft images and inverse images.

Keywords: Soft set; Neutrosophic set; Complex neutrosophic set; Interval complex neutrosophic soft set;

1. Introduction

Smarandache [1] defined a neutrosophic set (NS) as a generalization of [2,3] to be more suitable in dealing with various mathematical models such as topology, graphs, and algebra, so several researchers investigated the generalisation of the concept of a neutrosophic set with these mathematical models. This theory can cope with ambiguous, indeterminate, and discordant information when the indeterminacy is quantified clearly and truth, indeterminacy, and falsity membership are all entirely independent. So it has the ability to solve many real-life problems in the fields of medicine, economics, engineering, the environment, and so on. In 1999, Molodtsov [4] introduced the concept of the soft set (SS) as a mathematical tool for dealing with uncertainties. The notion of the neutrosophic soft set has been proposed by Maji [5] to blend the forces of both neutrosophic sets and soft sets. The researchers [6-8] made numerous contributions based on ideas NS and SS that contributed to the development of a mechanism for dealing with uncertain data in a variety of life applications. After Ramot [9] presented a complex fuzzy set, many concepts were based on this concept [10-12]. Recently, Al-Sharqi et al. [13-15] developed the above concepts and presented some of the works on interval complex neutrosophic soft sets (ICNSS). The concept of mapping has been widely applied to fuzzy theory and its extensions, beginning with the SS proposed by Kharal and Ahmad. Dependent on it, other concepts emerged based on this idea [16,17], and as a follow-up to these studies in this work, we will apply the mapping idea on I-CNSS by define the images and inverse images of interval complex neutrosophic soft sets (ICNSSs) and their fundamental operations, which will then lead to the concept of mapping on classes of interval complex neutrosophic soft sets. We also present the fulfilment of several of the concept's basic functions and features.

This article is organized as follows: In Section 2, we have briefly redefined each of NS, I-CNSS, and some properties of I-CNSS. In Section 3, we presented in detail each of the images and inverse images of complex interval neutrosophic soft sets, and their properties are explored. We also gave some numerical

examples to illustrate the work of these concepts, as well as some theories with their proofs. Section 4 contains this work's conclusion and recommendations for future studies.

2.Preliminaries

We will retrieve some fundamental ideas of the NS, I-CNSS, and some of their properties.

Definition 1.[1]. A N is a neutrosophic set (NS) on universe of a non-empty universe U and defined as $N = \{ \langle u, T_N(u), I_N(u), F_N(u) \rangle \}$, where $T_N(u), I_N(u), F_N(u)$ are truth, indeterminacy and falsity memberships respectively, such that $0 \leq T_N(u) + I_N(u) + F_N(u) \leq 3$.

Definition 2. [13] Let U be a non-empty universe with generic elements denoted u and $A \subseteq E$ when E be a set of all parameters. Thus an interval-complex neutrosophic soft set (shortly I-CNSS) (N, A) in U is defined by the following membership functions which are an interval-membership function $T_{N_a}(u)$, indeterminacy interval-membership function $I_{N_a}(u)$, and an interval non membership function $F_{N_a}(u)$ as a follows:

$$(N, A) = \left\{ \left(a, \frac{T_{N_a}(u) = t_{N_a}(u).e^{j\mu_{N_a}(u)}, I_{N_a}(u) = i_{N_a}(u).e^{j\omega_{N_a}(u)}, F_{N_a}(u) = f_{N_a}(u).e^{j\varphi_{N_a}(u)}}}{u} \right); u \in U, a \in A \right\}$$

where the interval-amplitude terms $t_{N_a}(u), i_{N_a}(u), f_{N_a}(u)$ can be written as $t_{N_a}(u) = [t_{N_a}^L(u), t_{N_a}^U(u)]$, $i_{N_a}(u) = [i_{N_a}^L(u), i_{N_a}^U(u)]$, and $f_{N_a}(u) = [f_{N_a}^L(u), f_{N_a}^U(u)]$ where $t_{N_a}^L(u), i_{N_a}^L(u), f_{N_a}^L(u)$ denote the lower-bounds, while $t_{N_a}^U(u), i_{N_a}^U(u), f_{N_a}^U(u)$ indicate the upper-bounds. And in like manner, for the interval-terms phases $\mu_{N_a}(u) = [\mu_{N_a}^L(u), \mu_{N_a}^U(u)]$, $\omega_{N_a}(u) = [\omega_{N_a}^L(u), \omega_{N_a}^U(u)]$ and $\varphi_{N_a}(u) = [\varphi_{N_a}^L(u), \varphi_{N_a}^U(u)]$.

Definition 3. [14] Let (N, A) and (M, B) two I-CNSSs over a non-empty universe U . Then we present a basic operations complement, intersection, and union as follows:

i. A complement of an I-CNSS (N, A) is indicate by $(N, A)^c$ and is defined as

$$(N, A)^c = \left\{ \left(a, \frac{T_{N_a^c}(u) = t_{N_a^c}(u).e^{j\mu_{N_a^c}(u)}, I_{N_a^c}(u) = i_{N_a^c}(u).e^{j\omega_{N_a^c}(u)}, F_{N_a^c}(u) = f_{N_a^c}(u).e^{j\varphi_{N_a^c}(u)}}}{u} \right); u \in U, a \in A \right\}$$

where

$T_{N_a^c}(u) = F_{N_a}(u)$ and $\mu_{N_a^c}(u) = 2\pi - \mu_{N_a}(u)$. Similarly, $i_{N_a^c}(u) = (i_{N_a^c}^L(u), i_{N_a^c}^U(u))$ where $i_{N_a^c}^L(u) = 1 - i_{N_a}^U(u)$ and $i_{N_a^c}^U(u) = 1 - i_{N_a}^L(u)$ with the phase term $\omega_{N_a^c}(u) = 2\pi - \omega_{N_a}(u)$. Also $f_{N_a^c}(u) = t_{N_a}(u)$, while the phase term $\varphi_{N_a^c}(u) = 2\pi - \varphi_{N_a}(u)$.

ii. For union and intersection of two I-CNSSs (M, A) and (N, B) are an I-CNSSs indicate as $(M, A) \cup (N, B)$ and all interval-membership functions of the I-CNSSs defined as

$$T_{M_a \cup (N) N_b}(u) = [t_{M_a \cup (N) N_b}^L(u), t_{M_a \cup (N) N_b}^U(u)]. e^{j2\pi\mu_{M_a \cup (N) N_b}(u)}$$

$$I_{M_a \cup (N) N_b}(u) = [i_{M_a \cup (N) N_b}^L(u), i_{M_a \cup (N) N_b}^U(u)]. e^{j2\pi\omega_{M_a \cup (N) N_b}(u)}$$

$$F_{M_a \cup (N) N_b}(u) = [f_{M_a \cup (N) N_b}^L(u), f_{M_a \cup (N) N_b}^U(u)]. e^{j2\pi\varphi_{M_a \cup (N) N_b}(u)}$$

where

$$t_{M_a \cup (N) N_b}^L(u) = \vee (\wedge) (t_{M_a}^L(u), t_{N_b}^L(u)), t_{M_a \cup (N) N_b}^U(u) = \vee (\wedge) (t_{M_a}^U(u), t_{N_b}^U(u))$$

$$i_{M_a \cup (N) N_b}^L(u) = \wedge (\vee) (i_{M_a}^L(u), i_{N_b}^L(u)), i_{M_a \cup (N) N_b}^U(u) = \wedge (\vee) (i_{M_a}^U(u), i_{N_b}^U(u))$$

$$f_{M_a \cup (N) N_b}^L(u) = \wedge (\vee) (f_{M_a}^L(u), f_{N_b}^L(u)), f_{M_a \cup (N) N_b}^U(u) = \wedge (\vee) (f_{M_a}^U(u), f_{N_b}^U(u)).$$

For phase interval-valued terms union and intersection are the same as defined for the amplitude interval-valued terms union and intersection and two symbols \vee, \wedge indicate respectively max, min operators.

3. Mapping on Interval-Complex Neutrosophic Soft Sets (MI-CNSS)

Definition 4. Let $(\widetilde{U}, \widetilde{E})$ and $(\widetilde{V}, \widetilde{E}')$ be an ICNS-classes. Let $r: U \rightarrow V$ and $s: E \rightarrow E'$ be mappings. Then a mapping $F: (\widetilde{U}, \widetilde{E}) \rightarrow (\widetilde{V}, \widetilde{E}')$ is defined as:

For an ICNSS (H, A) in $(\widetilde{U}, \widetilde{E})$ and $(F(H, A), M)$ where $M = s(E) \subseteq E'$ is an ICNSS in $(\widetilde{V}, \widetilde{E}')$, then we get the following:

$$F(H, A)(\beta)(v) = \begin{cases} \bigcup_{x \in r^{-1}(v)} (\bigcup_{\alpha \in s^{-1}(\beta) \cap A} H(\alpha)) & \text{if } r^{-1}(y) \text{ and } s^{-1}(\beta) \cap A \neq \emptyset \\ ([0,0], [1,1], [1,1]) & \text{other wise} \end{cases}$$

For $x \in r^{-1}(v)$, $\beta \in M \subseteq E'$, $v \in V$ and $\forall \alpha \in s^{-1}(\beta) \cap A$. Then $(F(H, A), M)$ is called an ICNS-image of the ICNSS (H, A) . If $E' = M$ then we will write $(F(H, A), M)$ as $F(H, A)$.

Definition 5. Let $(\widetilde{U}, \widetilde{E})$ and $(\widetilde{V}, \widetilde{E}')$ be an interval complex neutrosophic soft classes in short (ICNSS-Classes). Let $r: U \rightarrow Y$ and $s: E \rightarrow E'$ be mappings. Then a mapping $F^{-1}: (\widetilde{V}, \widetilde{E}') \rightarrow (\widetilde{U}, \widetilde{E})$ is defined as follows:

For an ICNSS (G, A) in $(\widetilde{V}, \widetilde{E}')$, $(F^{-1}(G, A), N)$, where $N = s^{-1}(A)$, is an ICNSS in $(\widetilde{U}, \widetilde{E})$, and given by:

$$F^{-1}(G, A)(\alpha)(u) = \begin{cases} G(s(\alpha))(r(u)) & \text{if } s(\alpha) \in A \\ ([0,0], [1,1], [1,1]) & \text{other wise} \end{cases}$$

for $\alpha \in N \subseteq E'$ and $u \in U$ then $(F^{-1}(G, A), N)$ is called an ICNS-inverse image of the ICNSS (G, A) . If $E' = N$ then we will write $(F^{-1}(G, A), M)$ as $F^{-1}(G, A)$.

The two Definitions above are illustrated as follows in the next example.

Example 1: Let $U = \{u_1, u_2, u_3\}$, $V = \{v_1, v_2, v_3\}$ and let $A \subseteq E = \{e_1, e_2, e_3\}$ and $A' \subseteq E' = \{e'_1, e'_2, e'_3\}$.

Suppose that $(\widetilde{U}, \widetilde{A})$ and $(\widetilde{V}, \widetilde{A}')$ are interval complex neutrosophic soft classes defined as: $r: U \rightarrow V, s: A \rightarrow A'$ as follows:

$$r(u_1) = y_1, r(u_2) = y_3, r(u_3) = y_2 \text{ and } s(e_1) = e'_2, s(e_2) = e'_1, s(e_3) = e'_1.$$

Let (H, A) and (G, A') be two I-CNSSs over U and V respectively such that.

$$(H, A) = \left\{ \left\{ e_1, \left(\frac{\left(\left[[0.3, 0.4] \cdot e^{j2\pi \left[\frac{2}{12}, \frac{3}{12} \right]}, [0.2, 0.3] \cdot e^{j2\pi \left[\frac{3}{12}, \frac{5}{12} \right]}, [0, 6, 0.7] \cdot e^{j2\pi \left[\frac{9}{12}, \frac{11}{12} \right]} \right)}{u_1} \right) \right\} \right\}, \left\{ \left(\frac{\left([0.6, 0.9] \cdot e^{j2\pi \left[\frac{7}{12}, \frac{9}{12} \right]}, [0.5, 0.6] \cdot e^{j2\pi \left[\frac{5}{12}, \frac{7}{12} \right]}, [0, 3, 0.4] \cdot e^{j2\pi \left[0, \frac{1}{12} \right]} \right)}{u_2} \right) \right\}, \left\{ \left(\frac{\left([0.5, 0.9] \cdot e^{j2\pi \left[\frac{7}{12}, \frac{10}{12} \right]}, [0.2, 0.2] \cdot e^{j2\pi \left[\frac{1}{12}, \frac{2}{12} \right]}, [0.2, 0.3] \cdot e^{j2\pi \left[0, \frac{1}{12} \right]} \right)}{u_3} \right) \right\} \right\}, \left\{ e_2, \left(\frac{\left([0.1, 0.3] \cdot e^{j2\pi \left[\frac{1}{12}, \frac{3}{12} \right]}, [0.4, 0.8] \cdot e^{j2\pi \left[\frac{7}{12}, \frac{11}{12} \right]}, [0.8, 0.9] \cdot e^{j2\pi \left[\frac{8}{12}, \frac{12}{12} \right]} \right)}{u_1} \right) \right\} \right\}$$

$$\left(\frac{\left([0.4,0.7]. e^{j2\pi[\frac{7}{12},\frac{9}{12}]}, [0.3,0.7]. e^{j2\pi[\frac{4}{12},\frac{9}{12}]}, [0.2,0.6]. e^{j2\pi[\frac{6}{12},\frac{9}{12}]} \right)}{u_2} \right)$$

$$\cdot \left(\frac{\left([0.3,0.6]. e^{j2\pi[\frac{5}{12},\frac{10}{12}]}, [0.5,0.8]. e^{j2\pi[\frac{7}{12},\frac{9}{12}]}, [0.1,0.3]. e^{j2\pi[\frac{1}{12},\frac{4}{12}]} \right)}{u_3} \right) \Bigg\},$$

$$\left\{ e_3, \left(\frac{\left([0.2,0.3]. e^{j2\pi[\frac{2}{12},\frac{5}{12}]}, [0.2,0.7]. e^{j2\pi[\frac{7}{12},\frac{10}{12}]}, [0,0.1]. e^{j2\pi[\frac{9}{12},\frac{11}{12}]} \right)}{u_1} \right) \right\},$$

$$\left(\frac{\left([0.3,0.8]. e^{j2\pi[\frac{4}{12},\frac{7}{12}]}, [0.2,0.6]. e^{j2\pi[\frac{5}{12},\frac{7}{12}]}, [0.2,0.7]. e^{j2\pi[\frac{2}{12},\frac{4}{12}]} \right)}{u_2} \right)$$

$$\cdot \left(\frac{\left([0.2,0.3]. e^{j2\pi[\frac{3}{12},\frac{4}{12}]}, [0.5,0.8]. e^{j2\pi[\frac{4}{12},\frac{6}{12}]}, [0.3,0.9]. e^{j2\pi[\frac{7}{12},\frac{11}{12}]} \right)}{u_3} \right) \Bigg\}$$

$$(G, A) =$$

$$\left\{ \dot{e}_1, \left(\frac{\left([0.6,0.8]. e^{j2\pi[\frac{6}{12},\frac{10}{12}]}, [0,0.1]. e^{j2\pi[\frac{2}{12},\frac{3}{12}]}, [0.2,0.4]. e^{j2\pi[\frac{1}{12},\frac{2}{12}]} \right)}{v_1} \right) \right\},$$

$$\left(\frac{\left([0.5,0.7]. e^{j2\pi[\frac{2}{12},\frac{4}{12}]}, [0.3,0.4]. e^{j2\pi[\frac{2}{12},\frac{4}{12}]}, [0.1,0.2]. e^{j2\pi[\frac{3}{12},\frac{6}{12}]} \right)}{v_2} \right)$$

$$\cdot \left(\frac{\left([0.3,0.6]. e^{j2\pi[\frac{5}{12},\frac{7}{12}]}, [0.2,0.4]. e^{j2\pi[\frac{7}{12},\frac{9}{12}]}, [0.5,0.9]. e^{j2\pi[\frac{2}{12},\frac{6}{12}]} \right)}{v_3} \right) \Bigg\},$$

$$\left\{ \dot{e}_2, \left(\frac{\left([0.4,0.7]. e^{j2\pi[\frac{3}{12},\frac{7}{12}]}, [0.2,0.5]. e^{j2\pi[\frac{4}{12},\frac{5}{12}]}, [0.1,0.2]. e^{j2\pi[\frac{2}{12},\frac{4}{12}]} \right)}{v_1} \right) \right\},$$

$$\left(\frac{\left([0.2,0.4]. e^{j2\pi[\frac{3}{12},\frac{7}{12}]}, [0.1,0.3]. e^{j2\pi[\frac{6}{12},\frac{8}{12}]}, [0.3,0.5]. e^{j2\pi[\frac{4}{12},\frac{6}{12}]} \right)}{v_2} \right)$$

$$\cdot \left(\frac{\left([0.3,0.8]. e^{j2\pi[\frac{6}{12},\frac{9}{12}]}, [0.2,0.6]. e^{j2\pi[\frac{1}{12},\frac{4}{12}]}, [0.3,0.5]. e^{j2\pi[0,\frac{2}{12}]} \right)}{v_3} \right) \Bigg\},$$

$$\left\{ \dot{e}_3, \left(\frac{\left([0,0.3]. e^{j2\pi[\frac{1}{12},\frac{3}{12}]}, [0.4,0.6]. e^{j2\pi[\frac{6}{12},\frac{8}{12}]}, [0.5,0.9]. e^{j2\pi[\frac{3}{12},\frac{7}{12}]} \right)}{v_1} \right) \right\},$$

$$\left(\frac{\left([0.2,0.4]. e^{j2\pi[\frac{5}{12},\frac{7}{12}]}, [0.3,0.7]. e^{j2\pi[\frac{3}{12},\frac{5}{12}]}, [0.3,0.8]. e^{j2\pi[\frac{3}{12},\frac{7}{12}]} \right)}{v_2} \right)$$

$$\left(\left(\frac{\langle [0.5,0.9]. e^{j2\pi[\frac{7}{12},\frac{10}{12}]}, [0.2,0.2]. e^{j2\pi[\frac{1}{12},\frac{2}{12}]}, [0.2,0.3]. e^{j2\pi[0,\frac{1}{12}]} \rangle}{v_3} \right) \right)$$

By above definition, we have the mapping $F: (\widetilde{U}, E) \rightarrow (\widetilde{V}, E)$.

For an I-CNSS (H, A) in (U, E) , $(F(H, A), M)$ is an I-CNSS in (V, E) such that $M = s(E) = \{e_1, e_2, e_3\}$ and is obtained as follows:

$$\begin{aligned} F(H, A)(e_1)(v_1) &= \bigvee_{u \in r^{-1}(v_1)} (\bigvee_{\alpha \in s^{-1}(e_1) \cap A} H(\alpha)) = \bigvee_{u \in \{u_1\}} (\bigvee_{\alpha \in \{e_2, e_3\}} H(\alpha)) \\ &= \langle [0.1,0.3]. e^{j2\pi[\frac{1}{12},\frac{3}{12}]}, [0.4,0.8]. e^{j2\pi[\frac{7}{12},\frac{11}{12}]}, [0.8,0.9]. e^{j2\pi[\frac{8}{12},\frac{12}{12}]} \rangle \cup \\ &\langle [0.2,0.3]. e^{j2\pi[\frac{2}{12},\frac{5}{12}]}, [0.2,0.7]. e^{j2\pi[\frac{7}{12},\frac{10}{12}]}, [0,0.1]. e^{j2\pi[\frac{9}{12},\frac{11}{12}]} \rangle \\ &= \langle [0.2,0.3]. e^{j2\pi[\frac{2}{12},\frac{5}{12}]}, [0.2,0.7]. e^{j2\pi[\frac{2}{12},\frac{5}{12}]}, [0,0.1]. e^{j2\pi[\frac{8}{12},\frac{11}{12}]} \rangle. \end{aligned}$$

$$\begin{aligned} F(H, A)(e_1)(v_2) &= \bigvee_{u \in r^{-1}(v_2)} (\bigvee_{\alpha \in s^{-1}(e_1) \cap A} H(\alpha)) = \bigvee_{u \in \{u_3\}} (\bigvee_{\alpha \in \{e_2, e_3\}} H(\alpha)) \\ &= \langle [0.3,0.6]. e^{j2\pi[\frac{5}{12},\frac{10}{12}]}, [0.5,0.8]. e^{j2\pi[\frac{7}{12},\frac{9}{12}]}, [0.1,0.3]. e^{j2\pi[\frac{1}{12},\frac{4}{12}]} \rangle \cup \\ &\langle [0.2,0.3]. e^{j2\pi[\frac{3}{12},\frac{4}{12}]}, [0.5,0.8]. e^{j2\pi[\frac{4}{12},\frac{6}{12}]}, [0.3,0.9]. e^{j2\pi[\frac{7}{12},\frac{11}{12}]} \rangle \\ &= \langle [0.3,0.6]. e^{j2\pi[\frac{5}{12},\frac{10}{12}]}, [0.5,0.8]. e^{j2\pi[\frac{4}{12},\frac{6}{12}]}, [0.1,0.3]. e^{j2\pi[\frac{8}{12},\frac{11}{12}]} \rangle. \end{aligned}$$

$$\begin{aligned} F(H, A)(e_1)(v_3) &= \bigvee_{u \in r^{-1}(v_3)} (\bigvee_{\alpha \in s^{-1}(e_1) \cap A} H(\alpha)) = \bigvee_{u \in \{u_2\}} (\bigvee_{\alpha \in \{e_2, e_3\}} H(\alpha)) \\ &= \langle [0.4,0.7]. e^{j2\pi[\frac{7}{12},\frac{9}{12}]}, [0.3,0.7]. e^{j2\pi[\frac{4}{12},\frac{9}{12}]}, [0.2,0.6]. e^{j2\pi[\frac{6}{12},\frac{9}{12}]} \rangle \cup \\ &\langle [0.3,0.8]. e^{j2\pi[\frac{4}{12},\frac{7}{12}]}, [0.2,0.6]. e^{j2\pi[\frac{5}{12},\frac{7}{12}]}, [0.2,0.7]. e^{j2\pi[\frac{2}{12},\frac{4}{12}]} \rangle \\ &= \langle [0.4,0.8]. e^{j2\pi[\frac{7}{12},\frac{9}{12}]}, [0.2,0.6]. e^{j2\pi[\frac{4}{12},\frac{7}{12}]}, [0.2,0.6]. e^{j2\pi[\frac{2}{12},\frac{4}{12}]} \rangle. \end{aligned}$$

Then,

$$\begin{aligned} F(H, A)(e_1) &= \\ &\left(\left(\frac{\langle [0.2,0.3]. e^{j2\pi[\frac{2}{12},\frac{5}{12}]}, [0.2,0.7]. e^{j2\pi[\frac{2}{12},\frac{5}{12}]}, [0,0.1]. e^{j2\pi[\frac{8}{12},\frac{11}{12}]} \rangle}{v_1} \right), \right. \\ &\left(\frac{\langle [0.3,0.6]. e^{j2\pi[\frac{5}{12},\frac{10}{12}]}, [0.5,0.8]. e^{j2\pi[\frac{4}{12},\frac{6}{12}]}, [0.1,0.3]. e^{j2\pi[\frac{8}{12},\frac{11}{12}]} \rangle}{v_2} \right) \\ &\left. , \left(\frac{\langle [0.4,0.8]. e^{j2\pi[\frac{7}{12},\frac{9}{12}]}, [0.2,0.6]. e^{j2\pi[\frac{4}{12},\frac{7}{12}]}, [0.2,0.6]. e^{j2\pi[\frac{2}{12},\frac{4}{12}]} \rangle}{v_3} \right) \right) \end{aligned}$$

Using the same arithmetic method as above, we get

$$\begin{aligned} F(H, A)(e_2) &= \\ &\left(\left(\frac{\langle [0.3,0.4]. e^{j2\pi[\frac{2}{12},\frac{3}{12}]}, [0.2,0.3]. e^{j2\pi[\frac{3}{12},\frac{5}{12}]}, [0.6,0.7]. e^{j2\pi[\frac{9}{12},\frac{11}{12}]} \rangle}{v_1} \right), \right. \\ &\left(\frac{\langle [0.5,0.9]. e^{j2\pi[\frac{7}{12},\frac{10}{12}]}, [0.2,0.2]. e^{j2\pi[\frac{1}{12},\frac{2}{12}]}, [0.2,0.3]. e^{j2\pi[0,\frac{1}{12}]} \rangle}{v_2} \right) \\ &\left. , \left(\frac{\langle [0.6,0.9]. e^{j2\pi[\frac{7}{12},\frac{9}{12}]}, [0.5,0.6]. e^{j2\pi[\frac{5}{12},\frac{7}{12}]}, [0.3,0.4]. e^{j2\pi[0,\frac{1}{12}]} \rangle}{v_3} \right) \right) \end{aligned}$$

$$F(H, A)(e_3) = \left(\left(\frac{\langle [0.2, 0.3]. e^{j2\pi[\frac{2}{12} \cdot \frac{5}{12}]}, [0.2, 0.7]. e^{j2\pi[\frac{7}{12} \cdot \frac{10}{12}]}, [0, 0.1]. e^{j2\pi[\frac{9}{12} \cdot \frac{11}{12}]} \rangle}{v_1} \right), \left(\frac{\langle [0.2, 0.3]. e^{j2\pi[\frac{3}{12} \cdot \frac{4}{12}]}, [0.5, 0.8]. e^{j2\pi[\frac{4}{12} \cdot \frac{6}{12}]}, [0.3, 0.9]. e^{j2\pi[\frac{7}{12} \cdot \frac{11}{12}]} \rangle}{v_2} \right), \left(\frac{\langle [0.3, 0.8]. e^{j2\pi[\frac{4}{12} \cdot \frac{7}{12}]}, [0.2, 0.6]. e^{j2\pi[\frac{5}{12} \cdot \frac{7}{12}]}, [0.2, 0.7]. e^{j2\pi[\frac{2}{12} \cdot \frac{4}{12}]} \rangle}{v_3} \right) \right)$$

Hence,

$$(F(H, A), M) = \left(\left\{ \dot{e}_1, \left(\frac{\langle [0.6, 0.8]. e^{j2\pi[\frac{6}{12} \cdot \frac{10}{12}]}, [0, 0.1]. e^{j2\pi[\frac{2}{12} \cdot \frac{3}{12}]}, [0.2, 0.4]. e^{j2\pi[\frac{1}{12} \cdot \frac{2}{12}]} \rangle}{v_1} \right), \left(\frac{\langle [0.5, 0.7]. e^{j2\pi[\frac{2}{12} \cdot \frac{4}{12}]}, [0.3, 0.4]. e^{j2\pi[\frac{2}{12} \cdot \frac{4}{12}]}, [0.1, 0.2]. e^{j2\pi[\frac{3}{12} \cdot \frac{6}{12}]} \rangle}{v_2} \right), \left(\frac{\langle [0.3, 0.6]. e^{j2\pi[\frac{5}{12} \cdot \frac{7}{12}]}, [0.2, 0.4]. e^{j2\pi[\frac{7}{12} \cdot \frac{9}{12}]}, [0.5, 0.9]. e^{j2\pi[\frac{2}{12} \cdot \frac{6}{12}]} \rangle}{v_3} \right) \right\}, \left\{ \dot{e}_2, \left(\frac{\langle [0.4, 0.7]. e^{j2\pi[\frac{3}{12} \cdot \frac{7}{12}]}, [0.2, 0.5]. e^{j2\pi[\frac{4}{12} \cdot \frac{5}{12}]}, [0.1, 0.2]. e^{j2\pi[\frac{2}{12} \cdot \frac{4}{12}]} \rangle}{v_1} \right), \left(\frac{\langle [0.2, 0.4]. e^{j2\pi[\frac{3}{12} \cdot \frac{7}{12}]}, [0.1, 0.3]. e^{j2\pi[\frac{6}{12} \cdot \frac{8}{12}]}, [0.3, 0.5]. e^{j2\pi[\frac{4}{12} \cdot \frac{6}{12}]} \rangle}{v_2} \right), \left(\frac{\langle [0.3, 0.8]. e^{j2\pi[\frac{6}{12} \cdot \frac{9}{12}]}, [0.2, 0.6]. e^{j2\pi[\frac{1}{12} \cdot \frac{4}{12}]}, [0.3, 0.5]. e^{j2\pi[0 \cdot \frac{2}{12}]} \rangle}{v_3} \right) \right\}, \left\{ \dot{e}_3, \left(\frac{\langle [0, 0.3]. e^{j2\pi[\frac{1}{12} \cdot \frac{3}{12}]}, [0.4, 0.6]. e^{j2\pi[\frac{6}{12} \cdot \frac{8}{12}]}, [0.5, 0.9]. e^{j2\pi[\frac{3}{12} \cdot \frac{7}{12}]} \rangle}{v_1} \right), \left(\frac{\langle [0.2, 0.4]. e^{j2\pi[\frac{5}{12} \cdot \frac{7}{12}]}, [0.3, 0.7]. e^{j2\pi[\frac{3}{12} \cdot \frac{5}{12}]}, [0.3, 0.8]. e^{j2\pi[\frac{3}{12} \cdot \frac{7}{12}]} \rangle}{v_2} \right), \left(\frac{\langle [0.5, 0.9]. e^{j2\pi[\frac{7}{12} \cdot \frac{10}{12}]}, [0.2, 0.2]. e^{j2\pi[\frac{1}{12} \cdot \frac{2}{12}]}, [0.2, 0.3]. e^{j2\pi[0 \cdot \frac{1}{12}]} \rangle}{v_3} \right) \right\} \right)$$

Definition 6. Let $f: (\widetilde{U}, E) \rightarrow (\widetilde{V}, E)$ be a mapping and $(H, A), (G, A)$ I-CNSSs in (\widetilde{U}, E) . Then for $\beta \in E, v \in V$, the union and intersection of I-CNS-images (H, A) and (G, A) are defined as follows:

1. $(F((H, A) \vee F(G, A)))(\beta)(v) = F(H, A)(\beta)(v) \vee F(G, A)(\beta)(v)$.
2. $(F(H, A) \wedge F(G, A))(\beta)(v) = F(H, A)(\beta)(v) \wedge F(G, A)(\beta)(v)$.

Definition 7. Let $f: (\widetilde{U}, \widetilde{E}) \rightarrow (\widetilde{V}, \widetilde{E'})$ be a mapping and $(H, A), (G, A')$ I-CNSSs in $(\widetilde{U}, \widetilde{E})$ then for $\alpha \in E, u \in U$, the union and intersection of I-CNS-images (H, A) and (G, A') are defined as follows:

1. $(F^{-1}(H, A) \vee F^{-1}(G, A'))(\alpha)(u) = F^{-1}(H, A)(\alpha)(u) \vee F^{-1}(G, A')(\alpha)(u)$.
2. $(F^{-1}(H, A) \wedge F^{-1}(G, A'))(\alpha)(u) = F^{-1}(H, A)(\alpha)(u) \wedge F^{-1}(G, A')(\alpha)(u)$.

Theorem 1. Let $f: (\widetilde{U}, \widetilde{E}) \rightarrow (\widetilde{V}, \widetilde{E'})$ be a mapping. Then for two I-CNSSs (H, A) and (G, A') in the I-CNS-class $(\widetilde{U}, \widetilde{E})$, we have:

1. $F(\emptyset) = \emptyset$
2. $F(\Omega) = \Omega$
3. $F((H, A) \vee (G, A')) = F(H, A) \vee F(G, A')$
4. $F((H, A) \wedge (G, A')) = F(H, A) \wedge F(G, A')$
5. If $(H, A) \subseteq (G, A')$, then $F(H, A) \subseteq F(G, A')$

Proof: For (1), (2), and (3) the proof is trivial by from the above definitions. So, we just prove (3) and (4).

For $\beta \in M \subseteq \dot{E}$ and $v \in V$, we try to prove that $F((H, A) \vee (G, A')) = F(H, A) \vee F(G, A')$
 For left side consider $F((H, A) \vee (G, A'))(\beta)(v) = F(K, A \cup B)(\beta)(v)$.

Then,

$$F(K, A \cup B)(\beta)(v) = \begin{cases} \bigcup_{x \in r^{-1}(y)} (\bigcup_{\alpha} K(\alpha)) \text{ if } r^{-1}(y) \text{ and } v^{-1}(\beta) \cap (A \cup B) \neq \emptyset \\ ([0,0], [1,1], [1,1]) & \text{other wise} \end{cases} \quad (1)$$

such that $K(\alpha) = F(\alpha) \cup G(\alpha)$.

Considering only the non-trivial case. Then the equation (1) becomes:

$$F(K, A \cup B)(\beta)(v) = \bigcup_{u \in r^{-1}(v)} (\bigcup (F(\alpha) \cup G(\alpha))) \quad (2)$$

For the right-hand side and by means of definition 3.3, we have

$$\begin{aligned} F((H, A) \cup (G, B))(\beta)(v) &= F(H, A)(\beta)(v) \cup F(G, B)(\beta)(v) \\ &= (\bigcup_{u \in r^{-1}(v)} (\bigcup_{\alpha \in v^{-1}(\beta) \cap A} H(\alpha))(u)) \cup (\bigcup_{u \in r^{-1}(v)} (\bigcup_{\alpha \in v^{-1}(\beta) \cap B} G(\alpha))(u)) \\ &= (\bigcup_{u \in r^{-1}(v)} \bigcup_{\alpha \in r^{-1}(\beta) \cap (A \cup B)} (H(\alpha) \cup G(\alpha))(u)) \\ &= \bigcup_{u \in r^{-1}(v)} (H(\alpha) \cup G(\alpha)). \end{aligned} \quad (3)$$

From above three equations, we get

$$F((H, A) \vee (G, A')) = F(H, A) \vee F(G, A').$$

Proof (5): Let $\beta \in M \subseteq \dot{E}$ and $v \in V$. By Definition 7, we have

$$\begin{aligned} F((H, A) \wedge (G, A'))(\beta)(v) &= f(K, A \cup B)(\beta)(v) \\ &= \bigcup_{u \in r^{-1}(v)} (\bigcup_{u \in v^{-1}(\beta) \cap A \cup B} K(\alpha))(u) \\ &= \bigcup_{u \in r^{-1}(v)} (\bigcup_{u \in v^{-1}(\beta) \cap A \cup B} H(\alpha) \cup G(\alpha))(u) \\ &\subseteq (\bigcup_{u \in r^{-1}(v)} (\bigcup_{\alpha \in v^{-1}(\beta) \cap A} H(\alpha))) \cap (\bigcup_{u \in r^{-1}(v)} (\bigcup_{\alpha \in v^{-1}(\beta) \cap B} G(\alpha))) \\ &= (F(H, A)(\beta)(v) \wedge F(G, A')(\beta)(v)) \end{aligned}$$

Its completes proof (5).

Theorem 2. Let $f: (\widetilde{U}, \widetilde{E}) \rightarrow (\widetilde{V}, \widetilde{E'})$ be a mapping. Then for two I-CNSSs $H, A)$ and (G, A') in the I-CNS-class $(\widetilde{Y}, \widetilde{E'})$, we have:

1. $F^{-1}(\emptyset) = \emptyset$

2. $F^{-1}(\Omega) \subseteq \Omega$
3. $F^{-1}((H, A) \vee (G, A)) = F^{-1}(G, A) \vee F^{-1}(H, A)$
4. $F^{-1}((H, A) \wedge (G, A)) = F^{-1}(H, A) \wedge F^{-1}(G, A)$
5. If $(H, A) \subseteq (G, A)$, then $F^{-1}(H, A) \subseteq F^{-1}(G, A)$

Proof: We can prove using the same method as in the previous proof.

4. CONCLUSION

In this work, we studied mappings on I-CNSSs and their basic properties. We also give some illustrative examples of mapping on I-CNSS. We hope by these essential results will help several researchers to enhance the research on ICNSS theory and its application in our daily life. Moreover, in next future work we planning and recurrent to apply this idea with another some mathematical structures like hypersoft set [18].

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