Algebraic Structure for (λ, μ)-Diophantine Neutrosophic Bisemirings

M. Palanikumar[1], **Aiyared Iampan**[2*]
[1]Department of Advanced Mathematical Science, Saveetha School of Engineering, Saveetha University,
Saveetha Institute of Medical and Technical Sciences, Chennai-602105, India
[2]Fuzzy Algebras and Decision-Making Problems Research Unit, Department of Mathematics,
School of Science, University of Phayao, Mae Ka, Mueang, Phayao 56000, Thailand
[*] Corresponding author: palanimaths86@gmail.com[1]; aiyared.ia@up.ac.th[2]

Abstract

We introduce the notion of Diophantine neutrosophic subbisemiring (DioNSBS), level sets of DioNSBS of a bisemiring. The concept of DioNSBS is a generalization of fuzzy subbisemiring over bisemiring. We interact the theory for (λ, μ)-DioNSBS over bisemiring. Let α be the Diophantine neutrosophic subset in $\mathscr{S}$, we show that $\alpha = \langle(\Xi_{\alpha}^{\mathscr{T}}, \Xi_{\alpha}^{\mathscr{I}}, \Xi_{\alpha}^{\mathscr{F}}), (\Gamma_{\alpha}, \Delta_{\alpha}, \Theta_{\alpha})\rangle$ is a DioNSBS of $\mathscr{S}$ if and only if all non empty level set $\alpha^{(t,s)}$ is a subbisemiring of $\mathscr{S}$ for $t, s \in [0, 1]$. Let α be the DioNSBS of a bisemiring $\mathscr{S}$ and W be the strongest Diophantine neutrosophic relation of $\mathscr{S}$, we observe that α is a DioNSBS of $\mathscr{S}$ if and only if W is a DioNSBS of $\mathscr{S} \times \mathscr{S}$. Let $\alpha_1, \alpha_2, ..., \alpha_n$ be the family of $DioNSBS^s$ of $\mathscr{S}_1, \mathscr{S}_2, ..., \mathscr{S}_n$ respectively. We show that $\alpha_1 \times \alpha_2 \times ... \times \alpha_n$ is a DioNSBS of $\mathscr{S}_1 \times \mathscr{S}_2 \times ... \times \mathscr{S}_n$. The homomorphic image of DioNSBS is a DioNSBS. The homomorphic preimage of DioNSBS is a DioNSBS. Examples are provided to illustrate our results.

Keywords: fuzzy subbisemiring; neutrosophic subbisemiring; Diophantine neutrosophic bisemiring; (λ, μ)-Diophantine neutrosophic subbisemiring; homomorphism

1 Introduction

The study of semirings was opened by the Dedekind in interaction with ideals of commutative rings. Vandiver inaugurates the proposal of semirings as a part of the generalization of rings.[21] It was basically the generalization of rings and distributive lattices. In 1950, However the developments of the theory in semirings had been taking place. Iséki[6,7] developed an ideal theory for semirings that is not necessarily commutative under either operation. Iséki[8] used this abstraction for semirings in the absence of zero and proved many important results based on semirings. Several authors and researchers have characterized the many different ideals based on semirings.[5] Several authors have studied aspects of ordered algebraic structures such as semigroups, semirings, hypersemigroups. The classic article of 1965, Zadeh proposed fuzzy set theory.[22] According to this definition, a fuzzy set is a function described by a membership value. It takes degrees in real unit interval. But, later it has been seen that this definition is inadequate by considering not only the degree of membership but also the degree of non-membership. Neutrosophic set is a generalization of the fuzzy set and intuitionistic fuzzy set, where the truth-membership, indeterminacy-membership, and falsity-membership are represented independently. Atanassov[3] described a set that is called an intuitionistic fuzzy set to handle mentioned ambiguity. Since this set has some problems in applications, Smarandache[20] introduced neutrosophy to deal with the problems that involves indeterminate and inconsistent information. Riaz et al. discussed[18] linear Diophantine fuzzy set (LDFS) with the addition of reference parameters. The LDFS is more efficient and flexible rather than other approaches due to the use of reference parameters. LDFS also categorize the data in MADM problems by changing the physical sense of reference parameters.

https://doi.org/10.54216/IJNS.190403
Received: May 05, 2022 Accepted: November 10, 2022

Arulmozhi interact the theory for various algebraic structures.[2] A semiring $(S, +, \cdot)$ is a non-empty set in which $(S, +)$ and $(S, \cdot)$ are semigroups such that " $\cdot$ " is distributive over " $+$ ".[5] In 1993, Ahsan et al.[1] introduced the notion of fuzzy semirings. In 2001, Sen and Ghosh were introduced in bisemirings. A bisemiring $(\mathscr{S}, +, \circ, \times)$ is an algebraic structure in which $(\mathscr{S}, +, \circ)$ and $(\mathscr{S}, \circ, \times)$ are semirings in which $(\mathscr{S}, +), (\mathscr{S}, \circ)$ and $(\mathscr{S}, \times)$ are semigroups such that (i) $x \circ (y + z) = (x \circ y) + (x \circ z)$, (ii) $(y + z) \circ x = (y \circ x) + (z \circ x)$ (iii) $x \times (y \circ z) = (x \times y) \circ (x \times z)$ and (iv) $(y \circ z) \times x = (y \times x) \circ (z \times x), \forall x, y, z \in \mathscr{S}$.[19] A non-empty subset Z of a bisemiring $(\mathscr{S}, +, \circ, \times)$ is a subbisemiring if and only if $x + y \in Z,\ x \circ y \in Z$ and $x \times y \in Z, \forall x, y \in Z$.[4] Palanikumar et al. discussed various ideal structure of subbisemiring theory and its applications.[9–17]

2 Preliminaries

Definition 2.1. [20] A neutrosophic set α in the universe $\mathbb{U}$ is of the following form : $\alpha = \{u, \Xi^{\mathscr{T}}_{\alpha}(u), \Xi^{\mathscr{I}}_{\alpha}(u), \Xi^{\mathscr{F}}_{\alpha}(u) \mid u \in \mathbb{U}\}$, where $\Xi^{\mathscr{T}}_{\alpha}(u), \Xi^{\mathscr{I}}_{\alpha}(u)\ \Xi^{\mathscr{F}}_{\alpha}(u)$ represents the degree of truth-membership, degree of indeterminacy membership and degree of falsity-membership of α respectively. The mapping $\Xi^{\mathscr{T}}_{\alpha}, \Xi^{\mathscr{I}}_{\alpha}, \Xi^{\mathscr{F}}_{\alpha} : \mathbb{U} \to [0, 1]$ and $0 \preceq \Xi^{\mathscr{T}}_{\alpha}(u) + \Xi^{\mathscr{I}}_{\alpha}(u) + \Xi^{\mathscr{F}}_{\alpha}(u) \preceq 3$.

Definition 2.2. [20] Let $\alpha_1 = \langle \Xi^{\mathscr{T}}_{\alpha_1}, \Xi^{\mathscr{I}}_{\alpha_1}, \Xi^{\mathscr{F}}_{\alpha_1}\rangle$, $\alpha_2 = \langle \Xi^{\mathscr{T}}_{\alpha_2}, \Xi^{\mathscr{I}}_{\alpha_2}, \Xi^{\mathscr{F}}_{\alpha_2}\rangle$ and $\alpha_3 = \langle \Xi^{\mathscr{T}}_{\alpha_3}, \Xi^{\mathscr{I}}_{\alpha_3}, \Xi^{\mathscr{F}}_{\alpha_3}\rangle$ be the three neutrosophic numbers over $\mathbb{U}$. Then
(i) ${\alpha_1}^c = \langle \Xi^{\mathscr{F}}_{\alpha_1}, \Xi^{\mathscr{I}}_{\alpha_1}, \Xi^{\mathscr{T}}_{\alpha_1}\rangle$
(ii) $\alpha_2 \bigvee \alpha_3 = \Big\langle \max(\Xi^{\mathscr{T}}_{\alpha_2}, \Xi^{\mathscr{T}}_{\alpha_3}), \min(\Xi^{\mathscr{I}}_{\alpha_2}, \Xi^{\mathscr{I}}_{\alpha_3}), \min(\Xi^{\mathscr{F}}_{\alpha_2}, \Xi^{\mathscr{F}}_{\alpha_3})\Big\rangle$
(iii) $\alpha_2 \bigwedge \alpha_3 = \Big\langle \min(\Xi^{\mathscr{T}}_{\alpha_2}, \Xi^{\mathscr{T}}_{\alpha_3}), \max(\Xi^{\mathscr{I}}_{\alpha_2}, \Xi^{\mathscr{I}}_{\alpha_3}), \max(\Xi^{\mathscr{F}}_{\alpha_2}, \Xi^{\mathscr{F}}_{\alpha_3})\Big\rangle$
(iv) $\alpha_2 \succeq \alpha_3$ iff $\Xi^{\mathscr{T}}_{\alpha_2} \succeq \Xi^{\mathscr{T}}_{\alpha_3}$ and $\Xi^{\mathscr{I}}_{\alpha_2} \preceq \Xi^{\mathscr{I}}_{\alpha_3}$ and $\Xi^{\mathscr{F}}_{\alpha_2} \preceq \Xi^{\mathscr{F}}_{\alpha_3}$
(v) $\alpha_2 = \alpha_3$ iff $\Xi^{\mathscr{T}}_{\alpha_2} = \Xi^{\mathscr{T}}_{\alpha_3}$ and $\Xi^{\mathscr{I}}_{\alpha_2} = \Xi^{\mathscr{I}}_{\alpha_3}$ and $\Xi^{\mathscr{F}}_{\alpha_2} = \Xi^{\mathscr{F}}_{\alpha_3}$.

Definition 2.3. [20] For any neutrosophic set $\alpha = \{z, \Xi^{\mathscr{T}}_{\alpha}(z), \Xi^{\mathscr{I}}_{\alpha}(z), \Xi^{\mathscr{F}}_{\alpha}(z)\}$ of a set U, we defined a (λ, μ)-cut of as the crisp subset $\{z \in U \mid \Xi^{\mathscr{T}}_{\alpha}(z) \succeq \lambda, \Xi^{\mathscr{I}}_{\alpha}(z) \succeq \lambda, \Xi^{\mathscr{F}}_{\alpha}(z) \preceq \mu\}$ of U.

Definition 2.4. [20] Let α and β be be two neutrosophic subsets of $\mathscr{S}$. The Cartesian product of α and β denoted by $\alpha \times \beta$ is defined as $\alpha \times \beta = \{\Xi^{\mathcal{T}}_{\alpha\times\beta}(z, y), \Xi^{\mathscr{I}}_{\alpha\times\beta}(z, y), \Xi^{\mathscr{F}}_{\alpha\times\beta}(z, y) \mid \mathit{for\ all}\ z, y \in \mathscr{S}\}$, where $\Xi^{\mathcal{T}}_{\alpha\times\beta}(z, y) = \min\{\Xi^{\mathcal{T}}_{\alpha}(z), \Xi^{\mathcal{T}}_{\beta}(y)\}, \Xi^{\mathscr{I}}_{\alpha\times\beta}(z, y) = \frac{\Xi^{\mathscr{I}}_{\alpha}(z)+\Xi^{\mathscr{I}}_{\beta}(y)}{2}, \Xi^{\mathscr{F}}_{\alpha\times\beta}(z, y) = \max\{\Xi^{\mathscr{F}}_{\alpha}(z), \Xi^{\mathscr{F}}_{\beta}(y)\}$.

Definition 2.5. [17] A fuzzy subset α of a bisemiring $(\mathscr{S}, \bigcirc_1, \bigcirc_2, \bigcirc_3)$ is said to be a fuzzy subbisemiring of $\mathscr{S}$ if $\Xi_{\alpha}(z \bigcirc_1 y) \succeq \min\{\Xi_{\alpha}(z), \Xi_{\alpha}(y)\}, \Xi_{\alpha}(z \bigcirc_2 y) \succeq \min\{\Xi_{\alpha}(z), \Xi_{\alpha}(y)\}, \Xi_{\alpha}(z \bigcirc_3 y) \succeq \min\{\Xi_{\alpha}(z), \Xi_{\alpha}(y)\}$, for all $z, y \in \mathscr{S}$.

Definition 2.6. [17] A fuzzy subset α of a bisemiring $(\mathscr{S}, \bigcirc_1, \bigcirc_2, \bigcirc_3)$ is said to be a fuzzy normal subbisemiring of $\mathscr{S}$ if $\Xi_{\alpha}(z \bigcirc_1 y) = \Xi_{\alpha}(y \bigcirc_1 z), \Xi_{\alpha}(z \bigcirc_2 y) = \Xi_{\alpha}(y \bigcirc_2 z), \Xi_{\alpha}(z \bigcirc_3 y) = \Xi_{\alpha}(y \bigcirc_3 z)$, for all $z, y \in \mathscr{S}$.

Definition 2.7. [4] Let $(\mathscr{S}, +, \cdot, \times)$ and $(\mathcal{T}, \oplus, \circ, \bullet)$ be two bisemirings. A function $\phi : \mathscr{S} \to \mathcal{T}$ is said to be a homomorphism if $\phi(z + y) = \phi(z) \oplus \phi(y), \phi(z \cdot y) = \phi(z) \circ \phi(y), \phi(z \times y) = \phi(z) \bullet \phi(y)$, for all $z, y \in \mathscr{S}$.

Definition 2.8. A Diophantine neutrosophic set α in $\mathbb{U}$ is of the form : $\alpha = \Big\{u, \Big(\Xi^{\mathscr{T}}_{\alpha}(u), \Xi^{\mathscr{I}}_{\alpha}(u), \Xi^{\mathscr{F}}_{\alpha}(u)\Big), \Big(\Gamma^{\mathscr{T}}_{\alpha}(u), \Delta^{\mathscr{I}}_{\alpha}(u), \Theta^{\mathscr{F}}_{\alpha}(u)\Big) \mid u \in \mathbb{U}\Big\}$, where $\Xi^{\mathscr{T}}_{\alpha}(u), \Xi^{\mathscr{I}}_{\alpha}(u)\ \Xi^{\mathscr{F}}_{\alpha}(u)$ represents the degree of truth-membership, degree of indeterminacy membership and degree of falsity-membership of α respectively and $\Gamma_{\alpha}(u) + \Delta_{\alpha}(u) + \Theta_{\alpha}(u) \preceq 1$. The mapping $\Xi^{\mathscr{T}}_{\alpha}, \Xi^{\mathscr{I}}_{\alpha}, \Xi^{\mathscr{F}}_{\alpha} : \mathbb{U} \to [0, 1]$ and $0 \preceq \Gamma^{\mathscr{T}}_{\alpha}(u) \cdot \Xi^{\mathscr{T}}_{\alpha}(u) + \Delta^{\mathscr{I}}_{\alpha}(u) \cdot \Xi^{\mathscr{I}}_{\alpha}(u) + \Theta^{\mathscr{F}}_{\alpha}(u) \cdot \Xi^{\mathscr{F}}_{\alpha}(u) \preceq 2$. Since $\alpha = \langle(\Xi^{\mathscr{T}}_{\alpha}, \Xi^{\mathscr{I}}_{\alpha}, \Xi^{\mathscr{F}}_{\alpha}), (\Gamma_{\alpha}, \Delta_{\alpha}, \Theta_{\alpha})\rangle$ is called a Diophantine neutrosophic number.

Definition 2.9. A neutrosophic subset α of $\mathscr{S}$ is said to be a NSBS of $\mathscr{S}$ if it satisfies the following conditions:

$$\left\{\begin{array}{c} \Xi^{\mathscr{T}}_{\alpha}(z \bigcirc_1 y) \succeq \min\{\Xi^{\mathscr{T}}_{\alpha}(z), \Xi^{\mathscr{T}}_{\alpha}(y)\} \\ \Xi^{\mathscr{T}}_{\alpha}(z \bigcirc_2 y) \succeq \min\{\Xi^{\mathscr{T}}_{\alpha}(z), \Xi^{\mathscr{T}}_{\alpha}(y)\} \\ \Xi^{\mathscr{T}}_{\alpha}(z \bigcirc_3 y) \succeq \min\{\Xi^{\mathscr{T}}_{\alpha}(z), \Xi^{\mathscr{T}}_{\alpha}(y)\} \end{array}\right\} \quad \left\{\begin{array}{c} \Xi^{\mathscr{I}}_{\alpha}(z \bigcirc_1 y) \succeq \frac{\Xi^{\mathscr{I}}_{\alpha}(z)+\Xi^{\mathscr{I}}_{\alpha}(y)}{2} \\ \text{OR} \\ \Xi^{\mathscr{I}}_{\alpha}(z \bigcirc_2 y) \succeq \frac{\Xi^{\mathscr{I}}_{\alpha}(z)+\Xi^{\mathscr{I}}_{\alpha}(y)}{2} \\ \text{OR} \\ \Xi^{\mathscr{I}}_{\alpha}(z \bigcirc_3 y) \succeq \frac{\Xi^{\mathscr{I}}_{\alpha}(z)+\Xi^{\mathscr{I}}_{\alpha}(y)}{2} \end{array}\right\}$$

$$\left\{\begin{array}{l} \Xi_\alpha^{\mathscr{F}}(z \bigcirc_1 y) \preceq \max\{\Xi_\alpha^{\mathscr{F}}(z), \Xi_\alpha^{\mathscr{F}}(y)\} \\ \Xi_\alpha^{\mathscr{F}}(z \bigcirc_2 y) \preceq \max\{\Xi_\alpha^{\mathscr{F}}(z), \Xi_\alpha^{\mathscr{F}}(y)\} \\ \Xi_\alpha^{\mathscr{F}}(z \bigcirc_3 y) \preceq \max\{\Xi_\alpha^{\mathscr{F}}(z), \Xi_\alpha^{\mathscr{F}}(y)\} \end{array}\right\}$$

for all $z, y \in \mathscr{S}$.

3 (λ, μ)-Diophantine neutrosophic subbisemiring

In what follows, let $\mathscr{S}$ denote a bisemiring unless otherwise stated. Here DioNSBS stands for Diophantine neutrosophic subbisemiring. In this section, we discuss about (λ, μ)-Diophantine neutrosophic subbisemiring. In what follows that, $(\lambda, \mu) \in [0, 1]$ be such that $0 \preceq \lambda < \mu \preceq 1$.

Definition 3.1. Let α be any Diophantine neutrosophic subset of $\mathscr{S}$ is called a (λ, μ)-DioNSBS of $\mathscr{S}$ if it satisfies the following conditions:

$$\left\{\begin{array}{l} \max\{\Xi_\alpha^{\mathscr{T}}(z \bigcirc_1 y), \lambda\} \succeq \min\{\Xi_\alpha^{\mathscr{T}}(z), \Xi_\alpha^{\mathscr{T}}(y), \mu\} \\ \max\{\Xi_\alpha^{\mathscr{T}}(z \bigcirc_2 y), \lambda\} \succeq \min\{\Xi_\alpha^{\mathscr{T}}(z), \Xi_\alpha^{\mathscr{T}}(y), \mu\} \\ \max\{\Xi_\alpha^{\mathscr{T}}(z \bigcirc_3 y), \lambda\} \succeq \min\{\Xi_\alpha^{\mathscr{T}}(z), \Xi_\alpha^{\mathscr{T}}(y), \mu\} \end{array}\right\}$$

$$\left\{\begin{array}{c} \max\{\Xi_\alpha^{\mathscr{I}}(z \bigcirc_1 y), \lambda\} \succeq \min\left\{\frac{\Xi_\alpha^{\mathscr{I}}(z)+\Xi_\alpha^{\mathscr{I}}(y)}{2}, \mu\right\} \\ \text{OR} \\ \max\{\Xi_\alpha^{\mathscr{I}}(z \bigcirc_2 y), \lambda\} \succeq \min\left\{\frac{\Xi_\alpha^{\mathscr{I}}(z)+\Xi_\alpha^{\mathscr{I}}(y)}{2}, \mu\right\} \\ \text{OR} \\ \max\{\Xi_\alpha^{\mathscr{I}}(z \bigcirc_3 y), \lambda\} \succeq \min\left\{\frac{\Xi_\alpha^{\mathscr{I}}(z)+\Xi_\alpha^{\mathscr{I}}(y)}{2}, \mu\right\} \end{array}\right\}$$

$$\left\{\begin{array}{l} \min\{\Xi_\alpha^{\mathscr{F}}(z \bigcirc_1 y), \lambda\} \preceq \max\{\Xi_\alpha^{\mathscr{F}}(z), \Xi_\alpha^{\mathscr{F}}(y), \mu\} \\ \min\{\Xi_\alpha^{\mathscr{F}}(z \bigcirc_2 y), \lambda\} \preceq \max\{\Xi_\alpha^{\mathscr{F}}(z), \Xi_\alpha^{\mathscr{F}}(y), \mu\} \\ \min\{\Xi_\alpha^{\mathscr{F}}(z \bigcirc_3 y), \lambda\} \preceq \max\{\Xi_\alpha^{\mathscr{F}}(z), \Xi_\alpha^{\mathscr{F}}(y), \mu\} \end{array}\right\}$$

$$\left\{\begin{array}{l} \max\{\Gamma_\alpha(z \bigcirc_1 y), \lambda\} \succeq \min\{\Gamma_\alpha(z), \Gamma_\alpha(y), \mu\} \\ \max\{\Gamma_\alpha(z \bigcirc_2 y), \lambda\} \succeq \min\{\Gamma_\alpha(z), \Gamma_\alpha(y), \mu\} \\ \max\{\Gamma_\alpha(z \bigcirc_3 y), \lambda\} \succeq \min\{\Gamma_\alpha(z), \Gamma_\alpha(y), \mu\} \end{array}\right\}$$

$$\left\{\begin{array}{c} \max\{\Delta_\alpha(z \bigcirc_1 y), \lambda\} \succeq \min\left\{\frac{\Delta_\alpha(z)+\Delta_\alpha(y)}{2}, \mu\right\} \\ \text{OR} \\ \max\{\Delta_\alpha(z \bigcirc_2 y), \lambda\} \succeq \min\left\{\frac{\Delta_\alpha(z)+\Delta_\alpha(y)}{2}, \mu\right\} \\ \text{OR} \\ \max\{\Delta_\alpha(z \bigcirc_3 y), \lambda\} \succeq \min\left\{\frac{\Delta_\alpha(z)+\Delta_\alpha(y)}{2}, \mu\right\} \end{array}\right\}$$

$$\left\{\begin{array}{l} \min\{\Theta_\alpha(z \bigcirc_1 y), \lambda\} \preceq \max\{\Theta_\alpha(z), \Theta_\alpha(y), \mu\} \\ \min\{\Theta_\alpha(z \bigcirc_2 y), \lambda\} \preceq \max\{\Theta_\alpha(z), \Theta_\alpha(y), \mu\} \\ \min\{\Theta_\alpha(z \bigcirc_3 y), \lambda\} \preceq \max\{\Theta_\alpha(z), \Theta_\alpha(y), \mu\} \end{array}\right\}$$

for all $z, y \in \mathscr{S}$.

Example 3.2. Let $\mathscr{S} = \{o_1, o_2, o_3, o_4\}$ be the bisemiring with the following Cayley table:

$\bigcirc_1$	o_1	o_2	o_3	o_4
o_1	o_1	o_1	o_1	o_1
o_2	o_1	o_2	o_1	o_2
o_3	o_1	o_1	o_3	o_3
o_4	o_1	o_2	o_3	o_4

$\bigcirc_2$	o_1	o_2	o_3	o_4
o_1	o_1	o_2	o_3	o_4
o_2	o_2	o_2	o_4	o_4
o_3	o_3	o_4	o_3	o_4
o_4	o_4	o_4	o_4	o_4

$\bigcirc_3$	o_1	o_2	o_3	o_4
o_1	o_1	o_1	o_1	o_1
o_2	o_1	o_2	o_3	o_4
o_3	o_4	o_4	o_4	o_4
o_4	o_4	o_4	o_4	o_4

	$o = o_1$	$o = o_2$	$o = o_3$	$o = o_4$
$\big(\Xi^{\mathscr{T}}_{\alpha}(o), \Gamma_{\alpha}(o)\big)$	$(0.85, 0.35)$	$(0.80, 0.30)$	$(0.60, 0.20)$	$(0.75, 0.25)$
$\big(\Xi^{\mathscr{I}}_{\alpha}(o), \Delta_{\alpha}(o)\big)$	$(0.80, 0.20)$	$(0.75, 0.15)$	$(0.67, 0.10)$	$(0.70, 0.10)$
$\big(\Xi^{\mathscr{F}}_{\alpha}(o), \Theta_{\alpha}(o)\big)$	$(0.40, 0.25)$	$(0.70, 0.30)$	$(0.85, 0.40)$	$(0.75, 0.35)$

Clearly, α is a $\langle(0.50, 0.65), (0.26, 0.28)\rangle$ DioNSBS of $\mathscr{S}$.

Theorem 3.3. *The intersection of a family of* (λ, μ)*-*$DioNSBS^s$ *of* $\mathscr{S}$ *is a* (λ, μ)*-DioNSBS of* $\mathscr{S}$*.*

Proof. Let $\{W_i : i \in I\}$ be a family of (λ, μ)-$DioNSBS^s$ of $\mathscr{S}$ and $\alpha = \bigcap\limits_{i \in \mathscr{I}} W_i$.

Let z and y in $\mathscr{S}$. Now,

$$\begin{aligned}
\max\{\Xi^{\mathscr{T}}_{\alpha}(z \bigcirc_1 y), \lambda\} &= \inf_{i \in \mathscr{I}} \ \max\{\Xi^{\mathscr{T}}_{W_i}(z \bigcirc_1 y), \lambda\} \\
&\succeq \inf_{i \in \mathscr{I}} \ \min\{\Xi^{\mathscr{T}}_{W_i}(z), \Xi^{\mathscr{T}}_{W_i}(y), \mu\} \\
&= \min\left\{\inf_{i \in \mathscr{I}} \ \Xi^{\mathscr{T}}_{W_i}(z), \inf_{i \in \mathscr{I}} \ \Xi^{\mathscr{T}}_{W_i}(y), \mu\right\} \\
&= \min\{\Xi^{\mathscr{T}}_{\alpha}(z), \Xi^{\mathscr{T}}_{\alpha}(y), \mu\}.
\end{aligned}$$

Similarly, $\max\{\Xi^{\mathscr{T}}_{\alpha}(z \bigcirc_2 y), \lambda\} \succeq \min\{\Xi^{\mathscr{T}}_{\alpha}(z), \Xi^{\mathscr{T}}_{\alpha}(y), \mu\}$ and $\max\{\Xi^{\mathscr{T}}_{\alpha}(z \bigcirc_3 y), \lambda\} \succeq \min\{\Xi^{\mathscr{T}}_{\alpha}(z), \Xi^{\mathscr{T}}_{\alpha}(y), \mu\}$. Now,

$$\begin{aligned}
\max\{\Xi^{\mathscr{I}}_{\alpha}(z \bigcirc_1 y), \lambda\} &= \inf_{i \in \mathscr{I}} \ \max\{\Xi^{\mathscr{I}}_{W_i}(z \bigcirc_1 y), \lambda\} \\
&\succeq \inf_{i \in \mathscr{I}} \ \min\left\{\frac{\Xi^{\mathscr{I}}_{W_i}(z) + \Xi^{\mathscr{I}}_{W_i}(y)}{2}, \mu\right\} \\
&= \min\left\{\frac{\inf\limits_{i \in \mathscr{I}} \ \Xi^{\mathscr{I}}_{W_i}(z) + \inf\limits_{i \in \mathscr{I}} \ \Xi^{\mathscr{I}}_{W_i}(y)}{2}, \mu\right\} \\
&= \min\left\{\frac{\Xi^{\mathscr{I}}_{\alpha}(z) + \Xi^{\mathscr{I}}_{\alpha}(y)}{2}, \mu\right\}.
\end{aligned}$$

Similarly, $\max\{\Xi^{\mathscr{I}}_{\alpha}(z \bigcirc_2 y), \lambda\} \succeq \min\left\{\frac{\Xi^{\mathscr{I}}_{\alpha}(z) + \Xi^{\mathscr{I}}_{\alpha}(y)}{2}, \mu\right\}$ and $\max\{\Xi^{\mathscr{I}}_{\alpha}(z \bigcirc_3 y), \lambda\} \succeq \min\left\{\frac{\Xi^{\mathscr{I}}_{\alpha}(z) + \Xi^{\mathscr{I}}_{\alpha}(y)}{2}, \mu\right\}$. Now,

$$\begin{aligned}
\min\{\Xi^{\mathscr{F}}_{\alpha}(z \bigcirc_1 y), \lambda\} &= \sup_{i \in \mathscr{I}} \ \min\{\Xi^{\mathscr{F}}_{W_i}(z \bigcirc_1 y), \lambda\} \\
&\preceq \sup_{i \in \mathscr{I}} \ \max\{\Xi^{\mathscr{F}}_{W_i}(z), \Xi^{\mathscr{F}}_{W_i}(y), \mu\} \\
&= \max\left\{\sup_{i \in \mathscr{I}} \ \Xi^{\mathscr{F}}_{W_i}(z), \sup_{i \in \mathscr{I}} \ \Xi^{\mathscr{F}}_{W_i}(y), \mu\right\} \\
&= \max\{\Xi^{\mathscr{F}}_{\alpha}(z), \Xi^{\mathscr{F}}_{\alpha}(y), \mu\}.
\end{aligned}$$

Similarly, $\min\{\Xi^{\mathscr{F}}_{\alpha}(z \bigcirc_2 y), \lambda\} \preceq \max\{\Xi^{\mathscr{F}}_{\alpha}(z), \Xi^{\mathscr{F}}_{\alpha}(y), \mu\}$ and $\min\{\Xi^{\mathscr{F}}_{\alpha}(z \bigcirc_3 y), \lambda\} \preceq \max\{\Xi^{\mathscr{F}}_{\alpha}(z), \Xi^{\mathscr{F}}_{\alpha}(y), \mu\}$. □

$$\begin{aligned}\max\{\Gamma_\alpha(z \bigcirc_1 y), \lambda\} &= \inf_{i\in\mathscr{I}} \ \max\{\Gamma_{W_i}(z \bigcirc_1 y), \lambda\}\\ &\succeq \inf_{i\in\mathscr{I}} \ \min\{\Gamma_{W_i}(z), \Gamma_{W_i}(y), \mu\}\\ &= \min\left\{\inf_{i\in\mathscr{I}} \ \Gamma_{W_i}(z), \inf_{i\in\mathscr{I}} \ \Gamma_{W_i}(y), \mu\right\}\\ &= \min\{\Gamma_\alpha(z), \Gamma_\alpha(y), \mu\}.\end{aligned}$$

Similarly, $\max\{\Gamma_\alpha(z \bigcirc_2 y), \lambda\} \succeq \min\{\Gamma_\alpha(z), \Gamma_\alpha(y), \mu\}$ and $\max\{\Gamma_\alpha(z \bigcirc_3 y), \lambda\} \succeq \min\{\Gamma_\alpha(z), \Gamma_\alpha(y), \mu\}$. Now,

$$\begin{aligned}\max\{\Delta_\alpha(z \bigcirc_1 y), \lambda\} &= \inf_{i\in\mathscr{I}} \ \max\{\Delta_{W_i}(z \bigcirc_1 y), \lambda\}\\ &\succeq \inf_{i\in\mathscr{I}} \ \min\left\{\frac{\Delta_{W_i}(z) + \Delta_{W_i}(y)}{2}, \mu\right\}\\ &= \min\left\{\frac{\inf_{i\in\mathscr{I}} \ \Delta_{W_i}(z) + \inf_{i\in\mathscr{I}} \ \Delta_{W_i}(y)}{2}, \mu\right\}\\ &= \min\left\{\frac{\Delta_\alpha(z) + \Delta_\alpha(y)}{2}, \mu\right\}.\end{aligned}$$

Similarly, $\max\{\Delta_\alpha(z\bigcirc_2 y), \lambda\} \succeq \min\left\{\frac{\Delta_\alpha(z)+\Delta_\alpha(y)}{2}, \mu\right\}$ and $\max\{\Delta_\alpha(z\bigcirc_3 y), \lambda\} \succeq \min\left\{\frac{\Delta_\alpha(z)+\Delta_\alpha(y)}{2}, \mu\right\}$. Now,

$$\begin{aligned}\min\{\Theta_\alpha(z \bigcirc_1 y), \lambda\} &= \sup_{i\in\mathscr{I}} \ \min\{\Theta_{W_i}(z \bigcirc_1 y), \lambda\}\\ &\preceq \sup_{i\in\mathscr{I}} \ \max\{\Theta_{W_i}(z), \Theta_{W_i}(y), \mu\}\\ &= \max\left\{\sup_{i\in\mathscr{I}} \ \Theta_{W_i}(z), \sup_{i\in\mathscr{I}} \ \Theta_{W_i}(y), \mu\right\}\\ &= \max\{\Theta_\alpha(z), \Theta_\alpha(y), \mu\}.\end{aligned}$$

Similarly, $\min\{\Theta_\alpha(z\bigcirc_2 y), \lambda\} \preceq \max\{\Theta_\alpha(z), \Theta_\alpha(y), \mu\}$ and $\min\{\Theta_\alpha(z\bigcirc_3 y), \lambda\} \preceq \max\{\Theta_\alpha(z), \Theta_\alpha(y), \mu\}$.

Hence, α is a (λ, μ)-DioNSBS of $\mathscr{S}$.

Theorem 3.4. *If α and β are any two (λ, μ)-$DioNSBS^s$ of $\mathscr{S}_1$ and $\mathscr{S}_2$ respectively, then $\alpha \times \beta$ is a (λ, μ)-DioNSBS of $\mathscr{S}_1 \times \mathscr{S}_2$.*

Proof. Let α and β be two (λ, μ)-$DioNSBS^s$ of $\mathscr{S}_1$ and $\mathscr{S}_2$ respectively. Let $z_1, z_2 \in \mathscr{S}_1$ and $y_1, y_2 \in \mathscr{S}_2$. Then (z_1, y_1) and (z_2, y_2) are in $\mathscr{S}_1 \times \mathscr{S}_2$. Now

$$\begin{aligned}\max\left\{\Xi^{\mathscr{T}}_{\alpha\times\beta}[(z_1, y_1) \bigcirc_1 (z_2, y_2)], \lambda\right\} &= \max\left\{\Xi^{\mathscr{T}}_{\alpha\times\beta}(z_1 \bigcirc_1 z_2, y_1 \bigcirc_1 y_2), \lambda\right\}\\ &= \min\left\{\max\{\Xi^{\mathscr{T}}_{\alpha}(z_1 \bigcirc_1 z_2), \lambda\}, \max\{\Xi^{\mathscr{T}}_{\beta}(y_1 \bigcirc_1 y_2), \lambda\}\right\}\\ &\succeq \min\left\{\min\{\Xi^{\mathscr{T}}_{\alpha}(z_1), \Xi^{\mathscr{T}}_{\alpha}(z_2), \mu\}, \min\{\Xi^{\mathscr{T}}_{\beta}(y_1), \Xi^{\mathscr{T}}_{\beta}(y_2), \mu\}\right\}\\ &= \min\left\{\{\min\{\Xi^{\mathscr{T}}_{\alpha}(z_1), \Xi^{\mathscr{T}}_{\beta}(y_1)\}, \min\{\Xi^{\mathscr{T}}_{\alpha}(z_2), \Xi^{\mathscr{T}}_{\beta}(y_2)\}\}, \mu\right\}\\ &= \min\left\{\Xi^{\mathscr{T}}_{\alpha\times\beta}(z_1, y_1), \Xi^{\mathscr{T}}_{\alpha\times\beta}(z_2, y_2), \mu\right\}.\end{aligned}$$

Also, $\max\left\{\Xi^{\mathscr{T}}_{\alpha\times\beta}[(z_1, y_1) \bigcirc_2 (z_2, y_2)], \lambda\right\} \succeq \min\left\{\Xi^{\mathscr{T}}_{\alpha\times\beta}(z_1, y_1), \Xi^{\mathscr{T}}_{\alpha\times\beta}(z_2, y_2), \mu\right\}$ and $\max\left\{\Xi^{\mathscr{T}}_{\alpha\times\beta}[(z_1, y_1) \bigcirc_3 (z_2, y_2)], \lambda\right\} \succeq \min\left\{\Xi^{\mathscr{T}}_{\alpha\times\beta}(z_1, y_1), \Xi^{\mathscr{T}}_{\alpha\times\beta}(z_2, y_2), \mu\right\}$.

Now, $\max\Big\{\Xi^{\mathscr{I}}_{\alpha\times\beta}[(z_1,y_1)\bigcirc_1(z_2,y_2)],\lambda\Big\}$

$$
\begin{aligned}
&=\max\Big\{\Xi^{\mathscr{I}}_{\alpha\times\beta}(z_1\bigcirc_1 z_2, y_1\bigcirc_1 y_2),\lambda\Big\}\\
&=\min\left\{\frac{1}{2}\left[\max\left\{\Xi^{\mathscr{I}}_{\alpha}(z_1\bigcirc_1 z_2),\lambda\right\}+\max\left\{\Xi^{\mathscr{I}}_{\beta}(y_1\bigcirc_1 y_2),\lambda\right\}\right]\right\}\\
&\succeq\min\left\{\frac{1}{2}\left[\min\left\{\frac{\Xi^{\mathscr{I}}_{\alpha}(z_1)+\Xi^{\mathscr{I}}_{\alpha}(z_2)}{2},\mu\right\}+\min\left\{\frac{\Xi^{\mathscr{I}}_{\beta}(y_1)+\Xi^{\mathscr{I}}_{\beta}(y_2)}{2},\mu\right\}\right]\right\}\\
&=\min\left\{\frac{1}{2}\left[\frac{\Xi^{\mathscr{I}}_{\alpha}(z_1)+\Xi^{\mathscr{I}}_{\beta}(y_1)}{2}+\frac{\Xi^{\mathscr{I}}_{\alpha}(z_2)+\Xi^{\mathscr{I}}_{\beta}(y_2)}{2}\right],\mu\right\}\\
&=\min\left\{\frac{\Xi^{\mathscr{I}}_{\alpha\times\beta}(z_1,y_1)+\Xi^{\mathscr{I}}_{\alpha\times\beta}(z_2,y_2)}{2},\mu\right\}.
\end{aligned}
$$

Also, $\max\Big\{\Xi^{\mathscr{I}}_{\alpha\times\beta}[(z_1,y_1)\bigcirc_2(z_2,y_2)],\lambda\Big\}\succeq\min\left\{\frac{\Xi^{\mathscr{I}}_{\alpha\times\beta}(z_1,y_1)+\Xi^{\mathscr{I}}_{\alpha\times\beta}(z_2,y_2)}{2},\mu\right\}$ and $\max\Big\{\Xi^{\mathscr{I}}_{\alpha\times\beta}[(z_1,y_1)\bigcirc_3(z_2,y_2)],\lambda\Big\}\succeq\min\left\{\frac{\Xi^{\mathscr{I}}_{\alpha\times\beta}(z_1,y_1)+\Xi^{\mathscr{I}}_{\alpha\times\beta}(z_2,y_2)}{2},\mu\right\}$.
Similarly,

$$
\begin{aligned}
\min\Big\{\Xi^{\mathscr{F}}_{\alpha\times\beta}[(z_1,y_1)\bigcirc_1(z_2,y_2)],\lambda\Big\}&=\min\Big\{\Xi^{\mathscr{F}}_{\alpha\times\beta}(z_1\bigcirc_1 z_2,y_1\bigcirc_1 y_2),\lambda\Big\}\\
&=\max\Big\{\min\{\Xi^{\mathscr{F}}_{\alpha}(z_1\bigcirc_1 z_2),\lambda\},\min\{\Xi^{\mathscr{F}}_{\beta}(y_1\bigcirc_1 y_2),\lambda\}\Big\}\\
&\preceq\max\Big\{\max\{\Xi^{\mathscr{F}}_{\alpha}(z_1),\Xi^{\mathscr{F}}_{\alpha}(z_2),\mu\},\max\{\Xi^{\mathscr{F}}_{\beta}(y_1),\Xi^{\mathscr{F}}_{\beta}(y_2),\mu\}\Big\}\\
&=\max\Big\{\{\max\{\Xi^{\mathscr{F}}_{\alpha}(z_1),\Xi^{\mathscr{F}}_{\beta}(y_1)\},\max\{\Xi^{\mathscr{F}}_{\alpha}(z_2),\Xi^{\mathscr{F}}_{\beta}(y_2)\}\},\mu\Big\}\\
&=\max\Big\{\Xi^{\mathscr{F}}_{\alpha\times\beta}(z_1,y_1),\Xi^{\mathscr{F}}_{\alpha\times\beta}(z_2,y_2),\mu\Big\}.
\end{aligned}
$$

Also, $\min\Big\{\Xi^{\mathscr{F}}_{\alpha\times\beta}[(z_1,y_1)\bigcirc_2(z_2,y_2)],\lambda\Big\}\preceq\max\Big\{\Xi^{\mathscr{F}}_{\alpha\times\beta}(z_1,y_1),\Xi^{\mathscr{F}}_{\alpha\times\beta}(z_2,y_2),\mu\Big\}$, $\min\Big\{\Xi^{\mathscr{F}}_{\alpha\times\beta}[(z_1,y_1)\bigcirc_3(z_2,y_2)],\lambda\Big\}\preceq\max\Big\{\Xi^{\mathscr{F}}_{\alpha\times\beta}(z_1,y_1),\Xi^{\mathscr{F}}_{\alpha\times\beta}(z_2,y_2),\mu\Big\}$. Similarly,

$$
\begin{aligned}
\max\Big\{\Gamma_{\alpha\times\beta}[(z_1,y_1)\bigcirc_1(z_2,y_2)],\lambda\Big\}&=\max\Big\{\Gamma_{\alpha\times\beta}(z_1\bigcirc_1 z_2,y_1\bigcirc_1 y_2),\lambda\Big\}\\
&=\min\Big\{\max\{\Gamma_{\alpha}(z_1\bigcirc_1 z_2),\lambda\},\max\{\Gamma_{\beta}(y_1\bigcirc_1 y_2),\lambda\}\Big\}\\
&\succeq\min\Big\{\min\{\Gamma_{\alpha}(z_1),\Gamma_{\alpha}(z_2),\mu\},\min\{\Gamma_{\beta}(y_1),\Gamma_{\beta}(y_2),\mu\}\Big\}\\
&=\min\Big\{\{\min\{\Gamma_{\alpha}(z_1),\Gamma_{\beta}(y_1)\},\min\{\Gamma_{\alpha}(z_2),\Gamma_{\beta}(y_2)\}\},\mu\Big\}\\
&=\min\Big\{\Gamma_{\alpha\times\beta}(z_1,y_1),\Gamma_{\alpha\times\beta}(z_2,y_2),\mu\Big\}.
\end{aligned}
$$

Also, $\max\Big\{\Gamma_{\alpha\times\beta}[(z_1,y_1)\bigcirc_2(z_2,y_2)],\lambda\Big\}\succeq\min\Big\{\Gamma_{\alpha\times\beta}(z_1,y_1),\Gamma_{\alpha\times\beta}(z_2,y_2),\mu\Big\}$ and $\max\Big\{\Gamma_{\alpha\times\beta}[(z_1,y_1)\bigcirc_3(z_2,y_2)],\lambda\Big\}\succeq\min\Big\{\Gamma_{\alpha\times\beta}(z_1,y_1),\Gamma_{\alpha\times\beta}(z_2,y_2),\mu\Big\}$.

Now, $\max\Big\{\Delta_{\alpha\times\beta}[(z_1,y_1)\bigcirc_1(z_2,y_2)],\lambda\Big\}$

$$
\begin{aligned}
&= \max\Big\{\Delta_{\alpha\times\beta}(z_1\bigcirc_1 z_2, y_1\bigcirc_1 y_2),\lambda\Big\}\\
&= \min\left\{\frac{1}{2}\left[\max\left\{\Delta_\alpha(z_1\bigcirc_1 z_2),\lambda\right\}+\max\left\{\Delta_\beta(y_1\bigcirc_1 y_2),\lambda\right\}\right]\right\}\\
&\succeq \min\left\{\frac{1}{2}\left[\min\left\{\frac{\Delta_\alpha(z_1)+\Delta_\alpha(z_2)}{2},\mu\right\}+\min\left\{\frac{\Delta_\beta(y_1)+\Delta_\beta(y_2)}{2},\mu\right\}\right]\right\}\\
&= \min\left\{\frac{1}{2}\left[\frac{\Delta_\alpha(z_1)+\Delta_\beta(y_1)}{2}+\frac{\Delta_\alpha(z_2)+\Delta_\beta(y_2)}{2}\right],\mu\right\}\\
&= \min\left\{\frac{\Delta_{\alpha\times\beta}(z_1,y_1)+\Delta_{\alpha\times\beta}(z_2,y_2)}{2},\mu\right\}.
\end{aligned}
$$

Also, $\max\Big\{\Delta_{\alpha\times\beta}[(z_1,y_1)\bigcirc_2(z_2,y_2)],\lambda\Big\}\succeq\min\left\{\frac{\Delta_{\alpha\times\beta}(z_1,y_1)+\Delta_{\alpha\times\beta}(z_2,y_2)}{2},\mu\right\}$ and

$\max\Big\{\Delta_{\alpha\times\beta}[(z_1,y_1)\bigcirc_3(z_2,y_2)],\lambda\Big\}\succeq\min\left\{\frac{\Delta_{\alpha\times\beta}(z_1,y_1)+\Delta_{\alpha\times\beta}(z_2,y_2)}{2},\mu\right\}$.

Similarly,

$$
\begin{aligned}
\min\Big\{\Theta_{\alpha\times\beta}[(z_1,y_1)\bigcirc_1(z_2,y_2)],\lambda\Big\} &= \min\Big\{\Theta_{\alpha\times\beta}(z_1\bigcirc_1 z_2,y_1\bigcirc_1 y_2),\lambda\Big\}\\
&= \max\Big\{\min\{\Theta_\alpha(z_1\bigcirc_1 z_2),\lambda\},\min\{\Theta_\beta(y_1\bigcirc_1 y_2),\lambda\}\Big\}\\
&\preceq \max\Big\{\max\{\Theta_\alpha(z_1),\Theta_\alpha(z_2),\mu\},\max\{\Theta_\beta(y_1),\Theta_\beta(y_2),\mu\}\Big\}\\
&= \max\Big\{\{\max\{\Theta_\alpha(z_1),\Theta_\beta(y_1)\},\max\{\Theta_\alpha(z_2),\Theta_\beta(y_2)\}\},\mu\Big\}\\
&= \max\Big\{\Theta_{\alpha\times\beta}(z_1,y_1),\Theta_{\alpha\times\beta}(z_2,y_2),\mu\Big\}.
\end{aligned}
$$

Also, $\min\Big\{\Theta_{\alpha\times\beta}[(z_1,y_1)\bigcirc_2(z_2,y_2)],\lambda\Big\}\preceq\max\Big\{\Theta_{\alpha\times\beta}(z_1,y_1),\Theta_{\alpha\times\beta}(z_2,y_2),\mu\Big\}$,
$\min\Big\{\Theta_{\alpha\times\beta}[(z_1,y_1)\bigcirc_3(z_2,y_2)],\lambda\Big\}\preceq\max\Big\{\Theta_{\alpha\times\beta}(z_1,y_1),\Theta_{\alpha\times\beta}(z_2,y_2),\mu\Big\}$.
Hence $\alpha\times\beta$ is a (λ,μ)-DioNSBS of $\mathscr{S}_1\times\mathscr{S}_2$. ☐

Corollary 3.5. *If $\alpha_1,\alpha_2,...,\alpha_n$ are the family of (λ,μ)-$DioNSBS^s$ of $\mathscr{S}_1,\mathscr{S}_2,...,\mathscr{S}_n$ respectively, then $\alpha_1\times\alpha_2\times...\times\alpha_n$ is a (λ,μ)-DioNSBS of $\mathscr{S}_1\times\mathscr{S}_2\times...\times\mathscr{S}_n$.*

Definition 3.6. Let α be a (λ,μ)-Diophantine neutrosophic subset in $\mathscr{S}$, the strongest (λ,μ)-Diophantine neutrosophic relation on $\mathscr{S}$, that is a (λ,μ)-Diophantine neutrosophic relation on α is W given by

$$
\left\{\begin{array}{l}
\max\{\Xi_W^{\mathscr{T}}(z,y),\lambda\}=\min\{\Xi_\alpha^{\mathscr{T}}(z),\Xi_\alpha^{\mathscr{T}}(y),\mu\}\\
\max\{\Xi_W^{\mathscr{I}}(z,y),\lambda\}=\min\{\Xi_\alpha^{\mathscr{I}}(z),\Xi_\alpha^{\mathscr{I}}(y),\mu\}\\
\min\{\Xi_W^{\mathscr{F}}(z,y),\lambda\}=\max\{\Xi_\alpha^{\mathscr{F}}(z),\Xi_\alpha^{\mathscr{F}}(y),\mu\}
\end{array}\right\}
\left\{\begin{array}{l}
\max\{\Gamma_W(z,y),\lambda\}=\min\{\Gamma_\alpha(z),\Gamma_\alpha(y),\mu\}\\
\max\{\Delta_W(z,y),\lambda\}=\min\{\Delta_\alpha(z),\Delta_\alpha(y),\mu\}\\
\min\{\Theta_W(z,y),\lambda\}=\max\{\Theta_\alpha(z),\Theta_\alpha(y),\mu\}
\end{array}\right\}.
$$

Theorem 3.7. *Let α be a (λ,μ)-DioNSBS of $\mathscr{S}$ and W be the strongest (λ,μ)-Diophantine neutrosophic relation of $\mathscr{S}$. Then α is a (λ,μ)-DioNSBS of $\mathscr{S}$ if and only if W is a (λ,μ)-DioNSBS of $\mathscr{S}\times\mathscr{S}$.*

Definition 3.8. Let $(\mathscr{S}_1,\star_1,\star_2,\star_3)$ and $(\mathscr{S}_2,\bullet_1,\bullet_2,\bullet_3)$ be any two bisemirings. Let $\Phi:\mathscr{S}_1\to\mathscr{S}_2$ be any function and α be any DioNSBS in $\mathscr{S}_1$, W be any DioNSBS in $\Phi(\mathscr{S}_1)=\mathscr{S}_2$. If $\Xi_\alpha=\langle(\Xi_\alpha^{\mathscr{T}},\Xi_\alpha^{\mathscr{I}},\Xi_\alpha^{\mathscr{F}}),(\Gamma_\alpha,\Delta_\alpha,\Theta_\alpha)\rangle$ is a Diophantine neutrosophic set in $\mathscr{S}_1$, then Ξ_W is a Diophantine neutrosophic set in $\mathscr{S}_2$, defined by

$$
\Xi_W^{\mathscr{T}}(y)=\begin{cases}\sup\Xi_\alpha^{\mathscr{T}}(z) & if\ \ z\in\Phi^{-1}y\\ 0 & \text{otherwise}\end{cases}
\qquad
\Xi_W^{\mathscr{I}}(y)=\begin{cases}\sup\Xi_\alpha^{\mathscr{I}}(z) & if\ \ z\in\Phi^{-1}y\\ 0 & \text{otherwise}\end{cases}
$$

$$
\Xi_W^{\mathscr{F}}(y)=\begin{cases}\inf\Xi_\alpha^{\mathscr{F}}(z) & if\ \ z\in\Phi^{-1}y\\ 1 & \text{otherwise}\end{cases}
$$

$$\Gamma_W(y)=\begin{cases}\sup\Gamma_\alpha(z) & if\ \ z\in\Phi^{-1}y\\ 0 & \text{otherwise}\end{cases}\qquad \Delta_W(y)=\begin{cases}\sup\Delta_\alpha(z) & if\ \ z\in\Phi^{-1}y\\ 0 & \text{otherwise}\end{cases}$$

$$\Theta_W(y)=\begin{cases}\inf\Theta_\alpha(z) & if\ \ z\in\Phi^{-1}y\\ 1 & \text{otherwise}\end{cases}$$

for all $z\in\mathscr{S}_1$ and $y\in\mathscr{S}_2$ is called the image of Ξ_α under Φ. Similarly, if $\Xi_W=\langle(\Xi_W^{\mathscr{T}},\Xi_W^{\mathscr{I}},\Xi_W^{\mathscr{F}}),(\Gamma_W,\Delta_W,\Theta_W)\rangle$ is a Diophantine neutrosophic set in $\mathscr{S}_2$, then Diophantine neutrosophic set $\Xi_\alpha=\Phi\circ\Xi_W$ in $\mathscr{S}_1$ [i.e., the Diophantine neutrosophic set defined by $\Xi_\alpha(z)=\Xi_W(\Phi(z))$] is called the preimage of Ξ_W under Φ.

Theorem 3.9. *Let $(\mathscr{S}_1,\star_1,\star_2,\star_3)$ and $(\mathscr{S}_2,\bullet_1,\bullet_2,\bullet_3)$ be any two bisemirings. The homomorphic image of (λ,μ)-DioNSBS of $\mathscr{S}_1$ is a (λ,μ)-DioNSBS of $\mathscr{S}_2$.*

Proof. Let $\Phi:\mathscr{S}_1\to\mathscr{S}_2$ be any homomorphism. Then $\Phi(z\star_1 y)=\Phi(z)\bullet_1\Phi(y),\Phi(z\star_2 y)=\Phi(z)\bullet_2\Phi(y)$ and $\Phi(z\star_3 y)=\Phi(z)\bullet_3\Phi(y)$ for all $z,y\in\mathscr{S}_1$. Let $W=\Phi(\alpha)$, α is any (λ,μ)-DioNSBS of $\mathscr{S}_1$. Let $\Phi(z),\Phi(y)\in\mathscr{S}_2$. Let $z\in\Phi^{-1}(\Phi(z))$ and $y\in\Phi^{-1}(\Phi(y))$ be such that $\Xi_\alpha^{\mathcal{T}}(z)=\sup\limits_{z'\in\Phi^{-1}(\Phi(z))}\Xi_\alpha^{\mathcal{T}}(z')$ and $\Xi_\alpha^{\mathcal{T}}(y)=\sup\limits_{z'\in\Phi^{-1}(\Phi(y))}\Xi_\alpha^{\mathcal{T}}(z')$. Now,

$$\begin{aligned}\max\left[\Xi_W^{\mathcal{T}}(\Phi(z)\bullet_1\Phi(y))\ ,\lambda\right]&=\max\left[\sup_{z''\in\Phi^{-1}(\Phi(z)\bullet_1\Phi(y))}\Xi_\alpha^{\mathcal{T}}(z'')\ ,\lambda\right]\\&=\max\left[\sup_{z''\in\Phi^{-1}(\Phi(z\star_1 y)}\Xi_\alpha^{\mathcal{T}}(z'')\ ,\lambda\right]\\&=\max\left[\Xi_\alpha^{\mathcal{T}}(z\star_1 y),\lambda\right]\\&\succeq\min\left\{\Xi_\alpha^{\mathcal{T}}(z),\Xi_\alpha^{\mathcal{T}}(y),\mu\right\}\\&=\min\left\{\Xi_W^{\mathcal{T}}\Phi(z),\Xi_W^{\mathcal{T}}\Phi(y),\mu\right\}.\end{aligned}$$

Thus, $\max\left[\Xi_W^{\mathcal{T}}(\Phi(z)\bullet_1\Phi(y))\ ,\lambda\right]\succeq\min\left\{\Xi_W^{\mathcal{T}}\Phi(z),\Xi_W^{\mathcal{T}}\Phi(y),\mu\right\}$.
Similarly, $\max\left[\Xi_W^{\mathcal{T}}(\Phi(z)\bullet_2\Phi(y))\ ,\lambda\right]\succeq\min\left\{\Xi_W^{\mathcal{T}}\Phi(z),\Xi_W^{\mathcal{T}}\Phi(y),\mu\right\}$ and
$\max\left[\Xi_W^{\mathcal{T}}(\Phi(z)\bullet_3\Phi(y))\ ,\lambda\right]\succeq\min\left\{\Xi_W^{\mathcal{T}}\Phi(z),\Xi_W^{\mathcal{T}}\Phi(y),\mu\right\}$.
Let $\Phi(z),\Phi(y)\in\mathscr{S}_2$. Let $z\in\Phi^{-1}(\Phi(z))$ and $y\in\Phi^{-1}(\Phi(y))$ be such that $\Xi_\alpha^{\mathcal{I}}(z)=\sup\limits_{z'\in\Phi^{-1}(\Phi(z))}\Xi_\alpha^{\mathcal{I}}(z')$ and $\Xi_\alpha^{\mathcal{I}}(y)=\sup\limits_{z'\in\Phi^{-1}(\Phi(y))}\Xi_\alpha^{\mathcal{I}}(z')$. Now,

$$\begin{aligned}\max\left[\Xi_W^{\mathcal{I}}(\Phi(z)\bullet_1\Phi(y))\ ,\lambda\right]&=\max\left[\sup_{z''\in\Phi^{-1}(\Phi(z)\bullet_1\Phi(y))}\Xi_\alpha^{\mathcal{I}}(z'')\ ,\lambda\right]\\&=\max\left[\sup_{z''\in\Phi^{-1}(\Phi(z\star_1 y)}\Xi_\alpha^{\mathcal{I}}(z'')\ ,\lambda\right]\\&=\max\left[\Xi_\alpha^{\mathcal{I}}(z\star_1 y),\lambda\right]\\&\succeq\min\left\{\frac{\Xi_\alpha^{\mathcal{I}}(z)+\Xi_\alpha^{\mathcal{I}}(y)}{2},\mu\right\}\\&=\min\left\{\frac{\Xi_W^{\mathcal{I}}\Phi(z)+\Xi_W^{\mathcal{I}}\Phi(y)}{2},\mu\right\}\end{aligned}$$

Thus, $\max\left[\Xi_W^{\mathcal{I}}(\Phi(z)\bullet_1\Phi(y))\ ,\lambda\right]\succeq\min\left\{\frac{\Xi_W^{\mathcal{I}}\Phi(z)+\Xi_W^{\mathcal{I}}\Phi(y)}{2},\mu\right\}$.
Similarly, $\max\left[\Xi_W^{\mathcal{I}}(\Phi(z)\bullet_2\Phi(y))\ ,\lambda\right]\succeq\min\left\{\frac{\Xi_W^{\mathcal{I}}\Phi(z)+\Xi_W^{\mathcal{I}}\Phi(y)}{2},\mu\right\}$ and $\max\left[\Xi_W^{\mathcal{I}}(\Phi(z)\bullet_3\Phi(y))\ ,\lambda\right]\succeq\min\left\{\frac{\Xi_W^{\mathcal{I}}\Phi(z)+\Xi_W^{\mathcal{I}}\Phi(y)}{2},\mu\right\}$.

Received: May 05, 2022 Accepted: November 10, 2022

Let $z \in \Phi^{-1}(\Phi(z))$ and $y \in \Phi^{-1}(\Phi(y))$ be such that $\Xi^{\mathcal{F}}_{\alpha}(z) = \inf\limits_{z' \in \Phi^{-1}(\Phi(z))} \Xi^{\mathcal{F}}_{\alpha}(z')$ and $\Xi^{\mathcal{F}}_{\alpha}(y) = \inf\limits_{z' \in \Phi^{-1}(\Phi(y))} \Xi^{\mathcal{F}}_{\alpha}(z')$. Now,

$$\begin{aligned}
\min\left[\Xi^{\mathcal{F}}_{W}(\Phi(z) \bullet_1 \Phi(y)) \ , \lambda\right] &= \min\left[\inf_{z'' \in \Phi^{-1}(\Phi(z) \bullet_1 \Phi(y))} \Xi^{\mathcal{F}}_{\alpha}(z'') \ , \lambda\right] \\
&= \min\left[\inf_{z'' \in \Phi^{-1}(\Phi(z \star_1 y)} \Xi^{\mathcal{F}}_{\alpha}(z'') \ , \lambda\right] \\
&= \min\left[\Xi^{\mathcal{F}}_{\alpha}(z \star_1 y), \lambda\right] \\
&\preceq \max\left\{\Xi^{\mathcal{F}}_{\alpha}(z), \Xi^{\mathcal{F}}_{\alpha}(y), \mu\right\} \\
&= \max\left\{\Xi^{\mathcal{F}}_{W}\Phi(z), \Xi^{\mathcal{F}}_{W}\Phi(y), \mu\right\}.
\end{aligned}$$

Thus, $\min\left[\Xi^{\mathcal{F}}_{W}(\Phi(z) \bullet_1 \Phi(y)) \ , \lambda\right] \preceq \max\left\{\Xi^{\mathcal{F}}_{W}\Phi(z), \Xi^{\mathcal{F}}_{W}\Phi(y), \mu\right\}$.
Similarly, $\min\left[\Xi^{\mathcal{F}}_{W}(\Phi(z) \bullet_2 \Phi(y)) \ , \lambda\right] \preceq \max\left\{\Xi^{\mathcal{F}}_{W}\Phi(z), \Xi^{\mathcal{F}}_{W}\Phi(y), \mu\right\}$ and
$\min\left[\Xi^{\mathcal{F}}_{W}(\Phi(z) \bullet_3 \Phi(y)) \ , \lambda\right] \preceq \max\left\{\Xi^{\mathcal{F}}_{W}\Phi(z), \Xi^{\mathcal{F}}_{W}\Phi(y), \mu\right\}$.
Let $W = \Phi(\alpha)$, α is any (λ, μ)-DioNSBS of $\mathscr{S}_1$. Let $\Phi(z), \Phi(y) \in \mathscr{S}_2$. Let $z \in \Phi^{-1}(\Phi(z))$ and $y \in \Phi^{-1}(\Phi(y))$ be such that $\Gamma_{\alpha}(z) = \sup\limits_{z' \in \Phi^{-1}(\Phi(z))} \Gamma_{\alpha}(z')$ and $\Gamma_{\alpha}(y) = \sup\limits_{z' \in \Phi^{-1}(\Phi(y))} \Gamma_{\alpha}(z')$. Now,

$$\begin{aligned}
\max\left[\Gamma_{W}(\Phi(z) \bullet_1 \Phi(y)) \ , \lambda\right] &= \max\left[\sup_{z'' \in \Phi^{-1}(\Phi(z) \bullet_1 \Phi(y))} \Gamma_{\alpha}(z'') \ , \lambda\right] \\
&= \max\left[\sup_{z'' \in \Phi^{-1}(\Phi(z \star_1 y)} \Gamma_{\alpha}(z'') \ , \lambda\right] \\
&= \max\left[\Gamma_{\alpha}(z \star_1 y), \lambda\right] \\
&\succeq \min\left\{\Gamma_{\alpha}(z), \Gamma_{\alpha}(y), \mu\right\} \\
&= \min\left\{\Gamma_{W}\Phi(z), \Gamma_{W}\Phi(y), \mu\right\}.
\end{aligned}$$

Thus, $\max\left[\Gamma_{W}(\Phi(z) \bullet_1 \Phi(y)) \ , \lambda\right] \succeq \min\left\{\Gamma_{W}\Phi(z), \Gamma_{W}\Phi(y), \mu\right\}$.
Similarly, $\max\left[\Gamma_{W}(\Phi(z) \bullet_2 \Phi(y)) \ , \lambda\right] \succeq \min\left\{\Gamma_{W}\Phi(z), \Gamma_{\alpha W}\Phi(y), \mu\right\}$ and
$\max\left[\Gamma_{W}(\Phi(z) \bullet_3 \Phi(y)) \ , \lambda\right] \succeq \min\left\{\Gamma_{W}\Phi(z), \Gamma_{\alpha W}\Phi(y), \mu\right\}$.
Let $\Phi(z), \Phi(y) \in \mathscr{S}_2$. Let $z \in \Phi^{-1}(\Phi(z))$ and $y \in \Phi^{-1}(\Phi(y))$ be such that $\Delta_{\alpha}(z) = \sup\limits_{z' \in \Phi^{-1}(\Phi(z))} \Delta_{\alpha}(z')$ and $\Delta_{\alpha}(y) = \sup\limits_{z' \in \Phi^{-1}(\Phi(y))} \Delta_{\alpha}(z')$. Now,

$$\begin{aligned}
\max\left[\Delta_{W}(\Phi(z) \bullet_1 \Phi(y)) \ , \lambda\right] &= \max\left[\sup_{z'' \in \Phi^{-1}(\Phi(z) \bullet_1 \Phi(y))} \Delta_{\alpha}(z'') \ , \lambda\right] \\
&= \max\left[\sup_{z'' \in \Phi^{-1}(\Phi(z \star_1 y)} \Delta_{\alpha}(z'') \ , \lambda\right] \\
&= \max\left[\Delta_{\alpha}(z \star_1 y), \lambda\right] \\
&\succeq \min\left\{\frac{\Delta_{\alpha}(z) + \Delta_{\alpha}(y)}{2}, \mu\right\} \\
&= \min\left\{\frac{\Delta_{W}\Phi(z) + \Delta_{W}\Phi(y)}{2}, \mu\right\}
\end{aligned}$$

Thus, $\max\left[\Delta_{W}(\Phi(z) \bullet_1 \Phi(y)) \ , \lambda\right] \succeq \min\left\{\frac{\Delta_{W}\Phi(z) + \Delta_{W}\Phi(y)}{2}, \mu\right\}$.
Similarly, $\max\left[\Delta_{W}(\Phi(z) \bullet_2 \Phi(y)) \ , \lambda\right] \succeq \min\left\{\frac{\Delta_{W}\Phi(z) + \Delta_{W}\Phi(y)}{2}, \mu\right\}$ and

Received: May 05, 2022 Accepted: November 10, 2022

$\max\Big[\Delta_W(\Phi(z) \bullet_3 \Phi(y))\ , \lambda\Big] \succeq \min\Big\{\frac{\Delta_W\Phi(z)+\Delta_W\Phi(y)}{2}, \mu\Big\}$.
Let $z \in \Phi^{-1}(\Phi(z))$ and $y \in \Phi^{-1}(\Phi(y))$ be such that $\Theta_\alpha(z) = \inf\limits_{z' \in \Phi^{-1}(\Phi(z))} \Theta_\alpha(z')$ and $\Theta_\alpha(y) = \inf\limits_{z' \in \Phi^{-1}(\Phi(y))} \Theta_\alpha(z')$. Now,

$$\begin{aligned}
\min\Big[\Theta_W(\Phi(z) \bullet_1 \Phi(y))\ , \lambda\Big] &= \min\left[\inf_{z'' \in \Phi^{-1}(\Phi(z)\bullet_1\Phi(y))} \Theta_\alpha(z'')\ , \lambda\right] \\
&= \min\left[\inf_{z'' \in \Phi^{-1}(\Phi(z\star_1 y)} \Theta_\alpha(z'')\ , \lambda\right] \\
&= \min\Big[\Theta_\alpha(z \star_1 y), \lambda\Big] \\
&\preceq \max\Big\{\Theta_\alpha(z), \Theta_\alpha(y), \mu\Big\} \\
&= \max\Big\{\Theta_W\Phi(z), \Theta_W\Phi(y), \mu\Big\}.
\end{aligned}$$

Thus, $\min\Big[\Theta_W(\Phi(z) \bullet_1 \Phi(y))\ , \lambda\Big] \preceq \max\Big\{\Theta_W\Phi(z), \Theta_W\Phi(y), \mu\Big\}$.
Similarly, $\min\Big[\Theta_W(\Phi(z) \bullet_2 \Phi(y))\ , \lambda\Big] \preceq \max\Big\{\Theta_W\Phi(z), \Theta_W\Phi(y), \mu\Big\}$ and
$\min\Big[\Theta_W(\Phi(z) \bullet_3 \Phi(y))\ , \lambda\Big] \preceq \max\Big\{\Theta_W\Phi(z), \Theta_W\Phi(y), \mu\Big\}$.
Hence W is a (λ, μ)-DioNSBS of $\mathscr{S}_2$. □

Theorem 3.10. *Let $(\mathscr{S}_1, \star_1, \star_2, \star_3)$ and $(\mathscr{S}_2, \bullet_1, \bullet_2, \bullet_3)$ be any two bisemirings. The homomorphic preimage of (λ, μ)-DioNSBS of $\mathscr{S}_2$ is a (λ, μ)-DioNSBS of $\mathscr{S}_1$.*

Proof. Let $\Phi : \mathscr{S}_1 \to \mathscr{S}_2$ be any homomorphism. Then $\Phi(z \star_1 y) = \Phi(z) \bullet_1 \Phi(y), \Phi(z \star_2 y) = \Phi(z) \bullet_2 \Phi(y)$ and $\Phi(z \star_3 y) = \Phi(z) \bullet_3 \Phi(y)$ for all $z, y \in \mathscr{S}_1$. Let $W = \Phi(\alpha)$, where W is any (λ, μ)-DioNSBS of $\mathscr{S}_2$. Let $z, y \in \mathscr{S}_1$. Then $\max\{\Xi^{\mathscr{T}}_\alpha(z \star_1 y), \lambda\} = \max\{\Xi^{\mathscr{T}}_W(\Phi(z \star_1 y)), \lambda\} = \max\{\Xi^{\mathscr{T}}_W(\Phi(z) \bullet_1 \Phi(y)), \lambda\} \succeq \min\{\Xi^{\mathscr{T}}_W\Phi(z), \Xi^{\mathscr{T}}_W\Phi(y), \mu\} = \min\{\Xi^{\mathscr{T}}_\alpha(z), \Xi^{\mathscr{T}}_\alpha(y), \mu\}$.
Thus, $\max\{\Xi^{\mathscr{T}}_\alpha(z \star_1 y), \lambda\} \succeq \min\{\Xi^{\mathscr{T}}_\alpha(z), \Xi^{\mathscr{T}}_\alpha(y), \mu\}$. Now, $\max\{\Xi^{\mathscr{I}}_\alpha(z \star_1 y), \lambda\} = \max\{\Xi^{\mathscr{I}}_W(\Phi(z \star_1 y)), \lambda\} = \max\{\Xi^{\mathscr{I}}_W(\Phi(z) \bullet_1 \Phi(y)), \lambda\} \succeq \min\{\Xi^{\mathscr{I}}_W\Phi(z), \Xi^{\mathscr{I}}_W\Phi(y), \mu\} = \min\{\Xi^{\mathscr{I}}_\alpha(z), \Xi^{\mathscr{I}}_\alpha(y), \mu\}$. Thus, $\max\{\Xi^{\mathscr{I}}_\alpha(z \star_1 y), \lambda\} \succeq \min\{\Xi^{\mathscr{I}}_\alpha(z), \Xi^{\mathscr{I}}_\alpha(y), \mu\}$. Now, $\min\{\Xi^{\mathscr{F}}_\alpha(z \star_1 y), \lambda\} = \min\{\Xi^{\mathscr{F}}_W(\Phi(z \star_1 y)), \lambda\} = \min\{\Xi^{\mathscr{F}}_W(\Phi(z) \bullet_1 \Phi(y)), \lambda\} \preceq \max\{\Xi^{\mathscr{F}}_W\Phi(z), \Xi^{\mathscr{F}}_W\Phi(y), \mu\} = \max\{\Xi^{\mathscr{F}}_\alpha(z), \Xi^{\mathscr{F}}_\alpha(y), \mu\}$.
Thus, $\min\{\Xi^{\mathscr{F}}_\alpha(z \star_1 y), \lambda\} \preceq \max\{\Xi^{\mathscr{F}}_\alpha(z), \Xi^{\mathscr{F}}_\alpha(y), \mu\}$. Similarly, $\max\{\Gamma_\alpha(z \star_1 y), \lambda\} = \max\{\Gamma_W(\Phi(z \star_1 y)), \lambda\} = \max\{\Gamma_W(\Phi(z) \bullet_1 \Phi(y)), \lambda\} \succeq \min\{\Gamma_W\Phi(z), \Gamma_W\Phi(y), \mu\} = \min\{\Gamma_\alpha(z), \Gamma_\alpha(y), \mu\}$.
Thus, $\max\{\Gamma_\alpha(z \star_1 y), \lambda\} \succeq \min\{\Gamma_\alpha(z), \Gamma_\alpha(y), \mu\}$. Now, $\max\{\Delta_\alpha(z \star_1 y), \lambda\} = \max\{\Delta_W(\Phi(z \star_1 y)), \lambda\} = \max\{\Delta_W(\Phi(z) \bullet_1 \Phi(y)), \lambda\} \succeq \min\{\Delta_W\Phi(z), \Delta_W\Phi(y), \mu\} = \min\{\Delta_\alpha(z), \Delta_\alpha(y), \mu\}$. Thus, $\max\{\Delta_\alpha(z \star_1 y), \lambda\} \succeq \min\{\Delta_\alpha(z), \Delta_\alpha(y), \mu\}$. Now, $\min\{\Theta_\alpha(z \star_1 y), \lambda\} = \min\{\Theta_W(\Phi(z \star_1 y)), \lambda\} = \min\{\Theta_W(\Phi(z) \bullet_1 \Phi(y)), \lambda\} \preceq \max\{\Theta_W\Phi(z), \Theta_W\Phi(y), \mu\} = \max\{\Theta_\alpha(z), \Theta_\alpha(y), \mu\}$. Thus, $\min\{\Theta_\alpha(z \star_1 y), \lambda\} \preceq \max\{\Theta_\alpha(z), \Theta_\alpha(y), \mu\}$. Similarly to prove other two operations, hence α is a (λ, μ)-DioNSBS of $\mathscr{S}_1$. □

4 (λ, μ)-Diophantine neutrosophic Normal Subbisemiring

In this section, we interact the theory for (λ, μ)-Diophantine neutrosophic normal subbisemiring. Here DioNNSBS stands for Diophantine neutrosophic normal subbisemiring.

Definition 4.1. Let α be any Diophantine neutrosophic subset of $\mathscr{S}$ is said to be a DioNNSBS of $\mathscr{S}$ if it satisfies the following conditions:

$$\left\{\begin{matrix} \Xi^{\mathscr{T}}_\alpha(z \bigcirc_1 y) = \Xi^{\mathscr{T}}_\alpha(y \bigcirc_1 z) \\ \Xi^{\mathscr{T}}_\alpha(z \bigcirc_2 y) = \Xi^{\mathscr{T}}_\alpha(y \bigcirc_2 z) \\ \Xi^{\mathscr{T}}_\alpha(z \bigcirc_3 y) = \Xi^{\mathscr{T}}_\alpha(y \bigcirc_3 z) \end{matrix}\right\} \quad \left\{\begin{matrix} \Xi^{\mathscr{I}}_\alpha(z \bigcirc_1 y) = \Xi^{\mathscr{I}}_\alpha(y \bigcirc_1 z) \\ \text{OR} \\ \Xi^{\mathscr{I}}_\alpha(z \bigcirc_2 y) = \Xi^{\mathscr{I}}_\alpha(y \bigcirc_2 z) \\ \text{OR} \\ \Xi^{\mathscr{I}}_\alpha(z \bigcirc_3 y) = \Xi^{\mathscr{I}}_\alpha(y \bigcirc_3 z) \end{matrix}\right\} \quad \left\{\begin{matrix} \Xi^{\mathscr{F}}_\alpha(z \bigcirc_1 y) = \Xi^{\mathscr{F}}_\alpha(y \bigcirc_1 z) \\ \Xi^{\mathscr{F}}_\alpha(z \bigcirc_2 y) = \Xi^{\mathscr{F}}_\alpha(y \bigcirc_2 z) \\ \Xi^{\mathscr{F}}_\alpha(z \bigcirc_3 y) = \Xi^{\mathscr{F}}_\alpha(y \bigcirc_3 z) \end{matrix}\right\}$$

https://doi.org/10.54216/IJNS.190403
Received: May 05, 2022 Accepted: November 10, 2022

$$\begin{Bmatrix} \Gamma_\alpha(z \bigcirc_1 y) = \Gamma_\alpha(y \bigcirc_1 z) \\ \Gamma_\alpha(z \bigcirc_2 y) = \Gamma_\alpha(y \bigcirc_2 z) \\ \Gamma_\alpha(z \bigcirc_3 y) = \Gamma_\alpha(y \bigcirc_3 z) \end{Bmatrix} \quad \begin{Bmatrix} \Delta_\alpha(z \bigcirc_1 y) = \Delta_\alpha(y \bigcirc_1 z) \\ \text{OR} \\ \Delta_\alpha(z \bigcirc_2 y) = \Delta_\alpha(y \bigcirc_2 z) \\ \text{OR} \\ \Delta_\alpha(z \bigcirc_3 y) = \Delta_\alpha(y \bigcirc_3 z) \end{Bmatrix} \quad \begin{Bmatrix} \Theta_\alpha(z \bigcirc_1 y) = \Theta_\alpha(y \bigcirc_1 z) \\ \Theta_\alpha(z \bigcirc_2 y) = \Theta_\alpha(y \bigcirc_2 z) \\ \Theta_\alpha(z \bigcirc_3 y) = \Theta_\alpha(y \bigcirc_3 z) \end{Bmatrix}$$

for all $z, y \in \mathscr{S}$.

Corollary 4.2. *(i) The intersection of a family of (λ,μ)-DioNNSBS of $\mathscr{S}$ is a (λ,μ)-DioNNSBS s of $\mathscr{S}$.*
(ii) If $\alpha_1, \alpha_2, ..., \alpha_n$ are the family of (λ,μ)-$DioNNSBS^s$ of $\mathscr{S}_1, \mathscr{S}_2, ..., \mathscr{S}_n$ respectively, then $\alpha_1 \times \alpha_2 \times ... \times \alpha_n$ is a (λ,μ)-DioNNSBS of $\mathscr{S}_1 \times \mathscr{S}_2 \times ... \times \mathscr{S}_n$.
(iii) The homomorphic image of any (λ,μ)-DioNNSBS of $\mathscr{S}_1$ is a (λ,μ)-DioNNSBS of $\mathscr{S}_2$.
(iv) The homomorphic preimage of any (λ,μ)-DioNNSBS of $\mathscr{S}_2$ is a (λ,μ)-DioNNSBS of $\mathscr{S}_1$.

Acknowledgments: The authors would like to thank the editor-in-chief and the anonymous referees for their various suggestions and helpful comments that have led to the improved in the quality and clarity version of the paper.

Conflicts of Interest: The authors declare no conflict of interest.

References

[1] J. Ahsan, K. Saifullah, F. Khan, Fuzzy semirings, Fuzzy Sets and Systems, vol. 60, pp. 309–320, 1993.

[2] K. Arulmozhi, The algebraic theory of semigroups and semirings, Lap Lambert Academic Publishing, Mauritius, 2019.

[3] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, vol. 20, no. 1, pp. 87–96, 1986.

[4] F. Hussian, R. M. Hashism, A. Khan, M. Naeem, Generalization of bisemirings, International Journal of Computer Science and Information Security, vol. 14, no. 9, pp. 275–289, 2016.

[5] S. J. Golan, Semirings and their applications, Kluwer Academic Publishers, London, 1999.

[6] K. Iséki, Ideal theory of semiring, Proceedings of the Japan Academy, vol. 32, pp. 554–559, 1956.

[7] K. Iséki, Ideals in semirings, Proceedings of the Japan Academy, vol. 34, no. 1, pp. 29–31, 1958.

[8] K. Iséki, Quasi-ideals in semirings without zero, Proceedings of the Japan Academy, vol. 34, no. 2, pp. 79–81, 1958.

[9] M. Palanikumar, K. Arulmozhi, On vague subbisemirings of bisemirings, Bulletin of the International Mathematical Virtual Institute, vol. 11, no. 3, pp. 419–428, 2022.

[10] M. Palanikumar, A. Iampan, Spherical Fermatean interval valued fuzzy soft set based on multi criteria group decision making, International Journal of Innovative Computing, Information and Control, vol. 18, no. 2, pp. 607–619, 2022.

[11] M. Palanikumar, A. Iampan, Novel approach to decision making based on type-II generalized Fermatean bipolar fuzzy soft sets, International Journal of Innovative Computing, Information and Control, vol. 18, no. 3, pp. 769–781, 2022.

[12] M. Palanikumar, A. Iampan, L. J. Manavalan, M-Bi-base generator of ordered gamma-semigroups, ICIC Express Letters, Part B: Applications, vol. 13, no. 8, pp. 795–802, 2022.

[13] M. Palanikumar, K. Arulmozhi, (α, β)-Neutrosophic subbisemiring of bisemiring, Neutrosophic Sets and Systems, vol. 48, pp. 368–385, 2022.

[14] M. Palanikumar, K. Arulmozhi, On new ways of various ideals in ternary semigroups, Matrix Science Mathematic, vol. 4, no. 1, pp. 6–9, 2020.

[15] M. Palanikumar, K. Arulmozhi, On various tri-ideals in ternary semirings, Bulletin of the International Mathematical Virtual Institute, vol. 11, no. 1, pp. 79–90, 2021.

[16] M. Palanikumar, K. Arulmozhi, On intuitionistic fuzzy normal subbisemiring of bisemiring, Nonlinear Studies, vol. 28, no. 3, pp. 717–721, 2021.

[17] M. Palanikumar, K. Arulmozhi, C. Jana, Multiple attribute decision-making approach for Pythagorean neutrosophic normal interval-valued aggregation operators, Computational and Applied Mathematics, vol. 41, Article number: 90, 2022.

[18] M. Riaz, M. R. Hashmi, Linear Diophantine fuzzy set and its applications towards multi attribute decision making problems, Journal of Intelligent and Fuzzy Systems, vol. 37, no. 4, pp. 5417–5439, 2019.

[19] M. K. Sen, S. Ghosh, An introduction to bisemirings, Southeast Asian Bulletin of Mathematics, vol. 28, no. 3, pp. 547–559, 2001.

[20] F. Smarandache, A unifying field in logics. Neutrosophy: neutrosophic probability, set and logic, American Research Press, Rehoboth, 1999.

[21] H. S. Vandiver, Note on a simple type of algebra in which the cancellation law of addition does not hold, Bulletin of the American Mathematical Society, vol. 40, no. 12, pp. 914–920, 1934.

[22] L. A. Zadeh, Fuzzy sets, Information and Control, vol. 8, no. 3, pp. 338–353, 1965.

https://doi.org/10.54216/IJNS.190403
Received: May 05, 2022 Accepted: November 10, 2022