Multiple attribute decision making for square root diophantine neutrosophic interval-valued sets and their aggregated operators

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Abstract

Square root Diophantine neutrosophic interval-valued set (SRDioNIVS) approaches to multiple attribute decision-making (MADM) problems. The square root neutrosophic sets, interval-valued Diophantine neutrosophic sets are both extensions of square root Diophantine neutrosophic sets. In this section, we discuss aggregating operations and how those interprtautions have evolved over time. The paper is focused on a novel idea known as square root neutrosophic interval-valued weighted averaging (SRDioNIVWA), square root neutrosophic interval-valued weighted geometric (SRDioNIVWG), generalized square root neutrosophic interval-valued weighted averaging (GSRDioNIVWA), and generalized square root neutrosophic interval-valued weighted geometric (GSRDioNIVWG). We also begin an algorithm using these operators. The use of the euclidean and hamming distances is described, and examples from real-world problems are inserted. As a result, the defined models are more accurate and closely tied to Ξ. In order to show the reliability and usefulness of the models under examination, we also compare a few of the proposed and current models. The study's results are also fascinating and intriguing.

Keywords: SRDioNIVWA; SRDioNIVWG; GSRDioNIVWA; GSRDioNIVWG

1 Introduction

In real life, systems are becoming more complex on a daily basis, making it challenging for decision-makers to select the best option from a range of alternatives. Condensing into a single goal is difficult, but it is not impossibly difficult. Setting restrictions on motivation, goals, and opinions was difficult for many organizations. As a result, when making decisions, a person or a committee must consider multiple objectives at once. This reflection suggests that each decision maker is prevented from choosing the best course of action, the one that would satisfy all of the criteria present in the practical problem. As a result, the decision maker is more dedicated to developing more practical and reliable methods to identify the best option. It is not always possible to handle the ambiguity and uncertainty data in decision-making problems using the classical or crisp methods. To deal with the uncertainties, a number of uncertain theories have been proposed, including fuzzy set (FS) introduced by Zadeh,[38] the intuitionistic fuzzy set (IFS) introduced by Atanassov,[5] the Pythagorean fuzzy set (PFS) introduced by Yager,[35] and the neutrosophic set (NS) introduced by Smarandache.[32] A FS where each component of the universal has a level of belongingness that ranges from 0 to 1, with the grades corresponding to these levels being referred to as the membership value of each element in the set. Applications of

https://doi.org/10.54216/IJNS.190401
Received: April 28, 2022 Accepted: November 15, 2022

FSs, such regression prediction for fuzzy time series[34] and fuzzy c-numbers, require clustering techniques.[37] Applications that require imprecise data, including natural language processing, artificial intelligence, handwriting and speech recognition, etc., are perfect candidates for this gradation approach. Later, Atanassov[5] adds the idea of an IFS logic, which is characterized by the requirement that the total of its membership degree (MD) and non-membership degree (NMD) value is ≤ 1. We might have trouble decisions-making (DM) when the MD and NMD sum is ≥ 1. In order to generalize IFS, Yager[35] created the new concept of PFS logic, which is characterized by the requirement that the square total of its MD and NMD is ≤ 1. Akram et al.[1–3] discussed the many applications based on the PFS. We extend Rahman et al.[28] discussion of an interval-valued Pythagorean fuzzy set (IVPFS) logic for geometric aggregation operators to a group DM framework. Pythagorean fuzzy aggregation operator with interval values suggested by Peng et al.[26] A few methods for MADGDM based on the interval-valued Pythagorean fuzzy Einstein aggregation operator were proposed by Rahman et al.[29] Yang et al. developed the idea of IVPFS with aggregation operations for MADM.[36] The square root fuzzy set (SRFS) and its weighted aggregated operators were studied in the context of DM by Shami et al. To eradicate these restrictions, the theory of the linear Diophantine fuzzy set (LDFS) with the addition of reference parameters was explored by Riaz and Hashmi. The LDFS is more efficient and flexible rather than other approaches due to the use of reference parameters. LDFS also categorize the data in MADM problems by changing the physical sense of reference parameters. If we choose the information in the form of $(0.9, 0.8, 0.6)$, then by using the condition of SFSs, that is, the sum complexity of both terms is limited to unit interval, but $0.9^2 + 0.8^2 + 0.6^2 > 1$ the theory of SFS has failed to deal with these kinds of issues, and the theory of Diophantine SFS (DSFS) is very comfortable to deal with the issues mentioned above. In order to do this, we select the reference parameters $(0.3, 0.5, 0.2)$. Then, by applying the DSFS condition, $(0.9^2) \cdot (0.3) + (0.8^2) \cdot (0.5) + (0.6^2) \cdot (0.2) = 0.635 < 1$.

Smarandache recently developed a novel theory, the neutrosophic set (NS).[32] The understanding of neutral mind is referred to as "neutosophy" and this neutrality is the main distinction between FS and IFS. Each statement is given a truth degree (TD), an indeterminacy degree (ID), and a false degree (FD). Each component of the cosmos has a level of TD, ID, FD that falls between $[0, 1]$ in the NS set. Philosophically, it has been shown that an NS generalizes a classical set, an FS, an IFS, and so on. By Smarandache et al.,[16] the Pythagorean neutrosophic interval-valued set (PNIVS) was first introduced. The single-valued NS is applied for medical diagnostics and context analysis.[30] Ejegwa[6] extended distance measures for IFSs, including hamming distance (HD), euclidean distance (ED), ized euclidean distance (NED), as well as their resemblances to PFSs, and applied them to MCDM and MADM problems. Palanikumar et al.[17] addressed the reasoning behind MADM for Pythagorean NIV aggregation operators. As a generalization of the PNIVS, we see that the majority of the distance functions for PNIVSs are presented. Palanikumar et al. discussed various ideal structure of subbisemiring theory and its applications.[18–25]

Peng and Dai[27] addressed the idea of neutrosophic MADM under the MABAC and TOPSIS, whereas Zhang and Xu[39] advocated generalizing PFS based on TOPSIS to incorporate MCDM. Hwang et al.[7] discussed a number of practical MADM uses. According to Jana et al. looked at a brand-new generalization of the bipolar fuzzy soft set logical.[8] Jana discussed the extended bipolar fuzzy MABAC based MAGDM approach.[15] Jana et al. presented a novel approach for robust single valued neutrosophic soft aggregating operators under MCDM[10] with bipolar fuzzy soft.[12] Jana et al. introduced Pythagorean fuzzy dombi aggregation operations.[11] Ullah et al.[33] dealt with distance measuring for complex PFS with practical pattern recognition applications. According to Jana et al.[14] engaged with the new aggregating operators based on the trapezoidal neutrosophic MADM logic. Utilizing a novel approach method for neutrosophic dombi power aggregating operators, Jana et al. cooperated on MCDM.[9] In recent years, Jana et al. presented the MCDM approach under single valued trigonometric number (SVTrN) dombi aggregation mappings.[13] The definition of SRDioNIVS data is to be expanded in this work. Aggregation operators are used to obtain SRDioNIVS data. By way of illustration, we will develop a ranking based on these operators and use it with DM problems.

1. A novel ED and HD measure is introduced for SRDioNIVSs.

2. Use of the newly introduced definition for MADM, SRDioNIVN aggregation operators, and a practical illustration.

3. Based on SRDioNIVWA, SRDioNIVWG, GSRDioNIVWA, and GSRDioNIVWG, determine positive and negative ideal values.

4. Making a decision based on Ξ to arrive at a result.

The paper is divided into the seven sections listed below. Section 1 denotes the introduction. A brief explanation of the linked ideas is given in the section 2. Section 3 discusses MADM based on square root NIV number

https://doi.org/10.54216/IJNS.190401
Received: April 28, 2022 Accepted: November 15, 2022

(SRDioNIVN) and its procedures. Section 4 uses the SRDioNIVNs separation distance to communicate with MADM. Section 5 discusses MADM for SRDioNIVN based on a few aggregation operations. Section 6 discusses MADM using SRDioNIV data, an algorithm with a numerical example, analysis, and discussion. The conclusion is found in section 7.

2 Preliminaries

In this section, we will quickly go over some of the fundamental terms, we will need for our future studies.

Definition 2.1. [35] Let Δ be the universe. The PFS Θ in Δ is $\Theta = \Big\{\tau, \langle \Upsilon^{\mathcal{T}}_{\Theta}(\tau), \Upsilon^{\mathcal{F}}_{\Theta}(\tau)\rangle | \tau \in \Delta\Big\}$, where $\Upsilon^{\mathcal{T}}_{\Theta} : \Delta \to [0,1]$ and $\Upsilon^{\mathcal{F}}_{\Theta} : \Delta \to [0,1]$ are denotes the MD and NMD of $\tau \in \Delta$ to Θ, respectively and $0 \leq (\Upsilon^{\mathcal{T}}_{\Theta}(\tau))^2 + (\Upsilon^{\mathcal{F}}_{\Theta}(\tau))^2 \leq 1$. For the sake of convenience, $\Theta = \langle \Upsilon^{\mathcal{T}}_{\Theta}, \Upsilon^{\mathcal{F}}_{\Theta}\rangle$ is represent a Pythagorean fuzzy number (PFN).

Definition 2.2. [4] The square root fuzzy set (SRFS) Θ in Δ is $\Theta = \Big\{\tau, \langle \Upsilon^{\mathcal{T}}_{\Theta}(\tau), \Upsilon^{\mathcal{F}}_{\Theta}(\tau)\rangle | \tau \in \Delta\Big\}$, where $\Upsilon^{\mathcal{T}}_{\Theta} : \Delta \to [0,1]$ and $\Upsilon^{\mathcal{F}}_{\Theta} : \Delta \to [0,1]$ are denotes the MD and NMD of $\tau \in \Delta$ to Θ, respectively and $0 \leq (\Upsilon^{\mathcal{T}}_{\Theta}(\tau))^2 + \sqrt{\Upsilon^{\mathcal{F}}_{\Theta}(\tau)} \leq 1$. For the sake of convenience, $\Theta = \langle \Upsilon^{\mathcal{T}}_{\Theta}, \Upsilon^{\mathcal{F}}_{\Theta}\rangle$ is represent a square root fuzzy number (SRFN).

Definition 2.3. [26] The Pythagorean interval-valued fuzzy set (PIVFS) Θ in Δ is $\overline{\Theta} = \Big\{\tau, \Big\langle \overline{\Upsilon^{\mathcal{T}}_{\Theta}}(\tau), \overline{\Upsilon^{\mathcal{F}}_{\Theta}}(\tau)\Big\rangle \Big| \tau \in \Delta\Big\}$, where $\overline{\Upsilon^{\mathcal{T}}_{\Theta}} : \Delta \to Int([0,1])$ and $\overline{\Upsilon^{\mathcal{F}}_{\Theta}} : \Delta \to Int([0,1])$ are denotes the MD and NMD of $\tau \in \Delta$ to Θ, respectively, and $0 \leq (\Upsilon^{\mathcal{T}\mathcal{U}}_{\Theta}(\tau))^2 + (\Upsilon^{\mathcal{F}\mathcal{U}}_{\Theta}(\tau))^2 \leq 1$. For the sake of convenience, $\overline{\Theta} = \Big\langle \Big[\Upsilon^{\mathcal{T}\mathcal{L}}_{\Theta}, \Upsilon^{\mathcal{T}\mathcal{U}}_{\Theta}\Big], \Big[\Upsilon^{\mathcal{F}\mathcal{L}}_{\Theta}, \Upsilon^{\mathcal{F}\mathcal{U}}_{\Theta}\Big]\Big\rangle$ is represent a Pythagorean interval-valued fuzzy number (PIVFN).

Definition 2.4. [32] The NS Θ in Δ is $\Theta = \Big\{\tau, \langle \Upsilon^{\mathcal{T}}_{\Theta}(\tau), \Upsilon^{\mathcal{I}}_{\Theta}(\tau), \Upsilon^{\mathcal{F}}_{\Theta}(\tau)\rangle | \tau \in \Delta\Big\}$, $\Upsilon^{\mathcal{T}}_{\Theta} : \Delta \to [0,1]$, $\Upsilon^{\mathcal{I}}_{\Theta} : \Delta \to [0,1]$ and $\Upsilon^{\mathcal{F}}_{\Theta} : \Delta \to [0,1]$ are denotes the TD, ID and FD of $\tau \in \Delta$ to Θ, respectively and $0 \leq \Upsilon^{\mathcal{T}}_{\Theta}(\tau) + \Upsilon^{\mathcal{I}}_{\Theta}(\tau) + \Upsilon^{\mathcal{F}}_{\Theta}(\tau) \leq 3$. For the sake of convenience, $\Theta = \langle \Upsilon^{\mathcal{T}}_{\Theta}, \Upsilon^{\mathcal{I}}_{\Theta}, \Upsilon^{\mathcal{F}}_{\Theta}\rangle$ is represent a neutrosophic number (NN).

Definition 2.5. [16] The Pythagorean neutrosophic set (PNS) Θ in Δ is $\Theta = \Big\{\tau, \langle \Upsilon^{\mathcal{T}}_{\Theta}(\tau), \Upsilon^{\mathcal{I}}_{\Theta}(\tau), \Upsilon^{\mathcal{F}}_{\Theta}(\tau)\rangle | \tau \in \Delta\Big\}$, $\Upsilon^{\mathcal{T}}_{\Theta} : \Delta \to [0,1]$, $\Upsilon^{\mathcal{I}}_{\Theta} : \Delta \to [0,1]$ and $\Upsilon^{\mathcal{F}}_{\Theta} : \Delta \to [0,1]$ are denotes the TD, ID and FD of $\tau \in \Delta$ to Θ, respectively and $0 \leq (\Upsilon^{\mathcal{T}}_{\Theta}(\tau))^2 + (\Upsilon^{\mathcal{I}}_{\Theta}(\tau))^2 + (\Upsilon^{\mathcal{F}}_{\Theta}(\tau))^2 \leq 2$. For the sake of convenience, $\Theta = \langle \Upsilon^{\mathcal{T}}_{\Theta}, \Upsilon^{\mathcal{I}}_{\Theta}, \Upsilon^{\mathcal{F}}_{\Theta}\rangle$ is represent a Pythagorean neutrosophic number (PNN).

Definition 2.6. [26] Let $\overline{\Theta} = \Big\langle [\Upsilon^{\mathcal{T}\mathcal{L}}, \Upsilon^{\mathcal{T}\mathcal{U}}], [\Upsilon^{\mathcal{F}\mathcal{L}}, \Upsilon^{\mathcal{F}\mathcal{U}}]\Big\rangle$, $\overline{\Theta}_1 = \Big\langle [\Upsilon^{\mathcal{T}\mathcal{L}}_1, \Upsilon^{\mathcal{T}\mathcal{U}}_1], [\Upsilon^{\mathcal{F}\mathcal{L}}_1, \Upsilon^{\mathcal{F}\mathcal{U}}_1]\Big\rangle$ and $\overline{\Theta}_2 = \Big\langle [\Upsilon^{\mathcal{T}\mathcal{L}}_2, \Upsilon^{\mathcal{T}\mathcal{U}}_2], [\Upsilon^{\mathcal{F}\mathcal{L}}_2, \Upsilon^{\mathcal{F}\mathcal{U}}_2]\Big\rangle$ be the PIVFNs, and $\Xi > 0$. Then,

1. $\overline{\Theta}_1 \oplus \Theta_2 =$
$$\begin{bmatrix} \Big[\sqrt{(\Upsilon^{\mathcal{T}\mathcal{L}}_1)^2 + (\Upsilon^{\mathcal{T}\mathcal{L}}_2)^2 - (\Upsilon^{\mathcal{T}\mathcal{L}}_1)^2 \cdot (\Upsilon^{\mathcal{T}\mathcal{L}}_2)^2}, \sqrt{(\Upsilon^{\mathcal{T}\mathcal{U}}_1)^2 + (\Upsilon^{\mathcal{T}\mathcal{U}}_2)^2 - (\Upsilon^{\mathcal{T}\mathcal{U}}_1)^2 \cdot (\Upsilon^{\mathcal{T}\mathcal{U}}_2)^2}\Big], \\ \Big[\Upsilon^{\mathcal{F}\mathcal{L}}_1 \cdot \Upsilon^{\mathcal{F}\mathcal{L}}_2, \Upsilon^{\mathcal{F}\mathcal{U}}_1 \cdot \Upsilon^{\mathcal{F}\mathcal{U}}_2\Big] \end{bmatrix},$$
2. $\overline{\Theta}_1 \otimes \overline{\Theta}_2 =$
$$\begin{bmatrix} \Big[\Upsilon^{\mathcal{T}\mathcal{L}}_1 \cdot \Upsilon^{\mathcal{T}\mathcal{L}}_2, \Upsilon^{\mathcal{T}\mathcal{U}}_1 \cdot \Upsilon^{\mathcal{T}\mathcal{U}}_2\Big], \\ \Big[\sqrt{(\Upsilon^{\mathcal{F}\mathcal{L}}_1)^2 + (\Upsilon^{\mathcal{F}\mathcal{L}}_2)^2 - (\Upsilon^{\mathcal{F}\mathcal{L}}_1)^2 \cdot (\Upsilon^{\mathcal{F}\mathcal{L}}_2)^2}, \sqrt{(\Upsilon^{\mathcal{F}\mathcal{U}}_1)^2 + (\Upsilon^{\mathcal{F}\mathcal{U}}_2)^2 - (\Upsilon^{\mathcal{F}\mathcal{U}}_1)^2 \cdot (\Upsilon^{\mathcal{F}\mathcal{U}}_2)^2}\Big] \end{bmatrix},$$
3. $\Xi \cdot \overline{\Theta} = \Big[\Big[\sqrt{1 - \big(1 - (\Upsilon^{\mathcal{T}\mathcal{L}})^2\big)^{\Xi}}, \sqrt{1 - \big(1 - (\Upsilon^{\mathcal{T}\mathcal{U}})^2\big)^{\Xi}}\Big], \Big[(\Upsilon^{\mathcal{F}\mathcal{L}})^{\Xi}, (\Upsilon^{\mathcal{F}\mathcal{U}})^{\Xi}\Big]\Big],$
4. $\overline{\Theta}^{\Xi} = \Big[\Big[(\Upsilon^{\mathcal{T}\mathcal{L}})^{\Xi}, (\Upsilon^{\mathcal{T}\mathcal{U}})^{\Xi}\Big], \Big[\sqrt{1 - \big(1 - (\Upsilon^{\mathcal{F}\mathcal{L}})^2\big)^{\Xi}}, \sqrt{1 - \big(1 - (\Upsilon^{\mathcal{F}\mathcal{U}})^2\big)^{\Xi}}\Big]\Big].$

https://doi.org/10.54216/IJNS.190401
Received: April 28, 2022 Accepted: November 15, 2022

Definition 2.7. [26] For any PIVFN $\overline{\Theta} = \Big\langle [\Upsilon^{\mathcal{TL}}, \Upsilon^{\mathcal{TU}}], [\Upsilon^{\mathcal{FL}}, \Upsilon^{\mathcal{FU}}] \Big\rangle$ and score function $S(\overline{\Theta})$ is defined as $S(\overline{\Theta}) = \frac{1}{2}\Big((\Upsilon^{\mathcal{TL}})^2 + (\Upsilon^{\mathcal{TU}})^2 - (\Upsilon^{\mathcal{FL}})^2 - (\Upsilon^{\mathcal{FU}})^2\Big),\ S(\overline{\Theta}) \in [-1,1]$, and the accuracy function $H(\overline{\Theta})$ is defined as $H(\overline{\Theta}) = \frac{1}{2}\Big((\Upsilon^{\mathcal{TL}})^2 + (\Upsilon^{\mathcal{TU}})^2 + (\Upsilon^{\mathcal{FL}})^2 + (\Upsilon^{\mathcal{FU}})^2\Big),\ H(\overline{\Theta}) \in [0,1]$.

Definition 2.8. For any SRIVFN $\overline{\Theta} = \Big\langle [\Upsilon^{\mathcal{TL}}, \Upsilon^{\mathcal{TU}}], [\Upsilon^{\mathcal{FL}}, \Upsilon^{\mathcal{FU}}] \Big\rangle$ and score function $S(\overline{\Theta})$ is defined as $S(\overline{\Theta}) = \frac{1}{2}\left((\Upsilon^{\mathcal{TL}})^2 + (\Upsilon^{\mathcal{TU}})^2 - \sqrt{\Upsilon^{\mathcal{FL}}} - \sqrt{\Upsilon^{\mathcal{FU}}}\right)$, $S(\overline{\Theta}) \in [-1,1]$, and the accuracy function $H(\overline{\Theta})$ is defined as

$$H(\overline{\Theta}) = \frac{1}{2}\left((\Upsilon^{\mathcal{TL}})^2 + (\Upsilon^{\mathcal{TU}})^2 + \sqrt{\Upsilon^{\mathcal{FL}}} + \sqrt{\Upsilon^{\mathcal{FU}}}\right),\ H(\overline{\Theta}) \in [0,1].$$

3 Basic operations for SRDioNIVN approach

We defined the SRDioNIVN and its operations in relation to the concepts of square root NIVN (SRDioNIVN) and NFN.

Definition 3.1. Let Δ be the universe. The SRDioNIV set $\overline{\Theta}$ in Δ is

$$\overline{\Theta} = \Big\{\tau, \Big\langle \Big(\overline{\Upsilon^{\mathcal{T}}_{\Theta}}(\tau), \overline{\Upsilon^{\mathcal{I}}_{\Theta}}(\tau), \overline{\Upsilon^{\mathcal{F}}_{\Theta}}(\tau)\Big), (\alpha_1, \alpha_2, \alpha_3) \Big\rangle \Big| \tau \in \Delta\Big\},$$

where $\overline{\Upsilon^{T}_{\Theta}} : \Delta \to Int([0,1])$, $\overline{\Upsilon^{I}_{\Theta}} : \Delta \to Int([0,1])$ and $\overline{\Upsilon^{F}_{\Theta}} : \Delta \to INT([0,1])$ are denotes the TD, ID and FD of $\tau \in \Delta$ to $\overline{\Theta}$, respectively and $0 \leq (\overline{\Upsilon^{\mathcal{T}}_{\Theta}}(\tau))^2 + \sqrt{\overline{\Upsilon^{\mathcal{I}}_{\Theta}}(\tau)} + \sqrt{\overline{\Upsilon^{\mathcal{F}}_{\Theta}}(\tau)} \leq 2$, implies $0 \leq (\alpha_1 \Upsilon^{\mathcal{TU}}_{\Theta}(\tau))^2 + \sqrt{\alpha_2 \Upsilon^{\mathcal{IU}}_{\Theta}(\tau)} + \sqrt{\Upsilon^{\mathcal{FU}}_{\Theta}(\tau)} \leq 2$, where $\alpha_1, \alpha_2, \alpha_3 \in [0,1]$ and $\alpha_1 + \alpha_2 + \alpha_3 \leq 1$. For the sake of convenience, $\overline{\Theta} = \Big\langle \Big[\alpha_1 \Upsilon^{\mathcal{TL}}_{\Theta}, \alpha_1 \Upsilon^{\mathcal{TU}}_{\Theta}\Big], \Big[\alpha_2 \Upsilon^{\mathcal{IL}}_{\Theta}, \alpha_2 \Upsilon^{\mathcal{IU}}_{\Theta}\Big], \Big[\Upsilon^{\mathcal{FL}}_{\Theta}, \Upsilon^{\mathcal{FU}}_{\Theta}\Big] \Big\rangle$ is called a square root Diophantine neutrosophic interval-valued number(SRDioNIVN).

Definition 3.2. For $\overline{\Theta} = \Big\langle \Big[\alpha_1 \Upsilon^{\mathcal{TL}}_{\Theta}, \alpha_1 \Upsilon^{\mathcal{TU}}_{\Theta}\Big], \Big[\alpha_2 \Upsilon^{\mathcal{IL}}_{\Theta}, \alpha_2 \Upsilon^{\mathcal{IU}}_{\Theta}\Big], \Big[\Upsilon^{\mathcal{FL}}_{\Theta}, \Upsilon^{\mathcal{FU}}_{\Theta}\Big] \Big\rangle$, the score function

$$S(\overline{\Theta}) = \frac{(\alpha_1 \Upsilon^{\mathcal{TL}})^2 + (\alpha_1 \Upsilon^{\mathcal{TU}})^2}{2} - \frac{\sqrt{\alpha_2 \Upsilon^{\mathcal{IL}}} + \sqrt{\alpha_2 \Upsilon^{\mathcal{IU}}}}{2} + 1 - \frac{\sqrt{\alpha_3 \Upsilon^{\mathcal{FL}}} + \sqrt{\alpha_3 \Upsilon^{\mathcal{FU}}}}{2},$$

where $S(\overline{\Theta}) \in [-1,1]$.

Definition 3.3. Let $\overline{\Theta} = \Big\langle [\alpha \Upsilon^{\mathcal{TL}}, \Upsilon^{\mathcal{TU}}], [\alpha \Upsilon^{\mathcal{IL}}, \alpha \Upsilon^{\mathcal{IU}}], [\alpha \Upsilon^{\mathcal{FL}}, \alpha \Upsilon^{\mathcal{FU}}] \Big\rangle, \overline{\Theta}_1 = \Big\langle [\alpha_1 \Upsilon^{\mathcal{TL}}_1, \alpha_1 \Upsilon^{\mathcal{TU}}_1], [\alpha_2 \Upsilon^{\mathcal{IL}}_1, \alpha_2 \Upsilon^{\mathcal{IU}}_1], [\alpha_3 \Upsilon$ and
$\overline{\Theta}_2 = \Big\langle [\alpha_1 \Upsilon^{\mathcal{TL}}_2, \alpha_1 \Upsilon^{\mathcal{TU}}_2], [\alpha_2 \Upsilon^{\mathcal{IL}}_2, \alpha_2 \Upsilon^{\mathcal{IU}}_2], [\alpha_3 \Upsilon^{\mathcal{FL}}_2, \alpha_3 \Upsilon^{\mathcal{FU}}_2] \Big\rangle$ be the any three SRDioNIVNs, and $\Xi > 0$. Then,

1. $$\overline{\Theta}_1 \oplus \overline{\Theta}_2 = \left[\begin{array}{c} \left[\begin{array}{c} \left(\sqrt[2\Xi]{\alpha_1 \Upsilon^{\mathcal{TL}}_1} + \sqrt[2\Xi]{\alpha_1 \Upsilon^{\mathcal{TL}}_2} - \sqrt[2\Xi]{\alpha_1 \Upsilon^{\mathcal{TL}}_1} \cdot \sqrt[2\Xi]{\alpha_1 \Upsilon^{\mathcal{TL}}_2}\right)^{2\Xi}, \\ \left(\sqrt[2\Xi]{\alpha_1 \Upsilon^{\mathcal{TU}}_1} + \sqrt[2\Xi]{\alpha_1 \Upsilon^{\mathcal{TU}}_2} - \sqrt[2\Xi]{\alpha_1 \Upsilon^{\mathcal{TU}}_1} \cdot \sqrt[2\Xi]{\alpha_1 \Upsilon^{\mathcal{TU}}_2}\right)^{2\Xi} \end{array}\right], \\ \left[\begin{array}{c} \left(\sqrt[\Xi]{\alpha_2 \Upsilon^{\mathcal{IL}}_1} + \sqrt[\Xi]{\alpha_2 \Upsilon^{\mathcal{IL}}_2} - \sqrt[\Xi]{\alpha_2 \Upsilon^{\mathcal{IL}}_1} \cdot \sqrt[\Xi]{\alpha_2 \Upsilon^{\mathcal{IL}}_2}\right)^{\Xi}, \\ \left(\sqrt[\Xi]{\alpha_2 \Upsilon^{\mathcal{IU}}_1} + \sqrt[\Xi]{\alpha_2 \Upsilon^{\mathcal{IU}}_2} - \sqrt[\Xi]{\alpha_2 \Upsilon^{\mathcal{IU}}_1} \cdot \sqrt[\Xi]{\alpha_2 \Upsilon^{\mathcal{IU}}_2}\right)^{\Xi} \end{array}\right], \\ \Big[\alpha_3 \Upsilon^{\mathcal{FL}}_1 \cdot \alpha_3 \Upsilon^{\mathcal{FL}}_2, \alpha_3 \Upsilon^{\mathcal{FU}}_1 \cdot \alpha_3 \Upsilon^{\mathcal{FU}}_2\Big] \end{array}\right],$$

2. $$\overline{\Theta}_1 \otimes \overline{\Theta}_2 = \left[\begin{array}{c} \Big[\alpha_1 \Upsilon^{\mathcal{TL}}_1 \cdot \alpha_1 \Upsilon^{\mathcal{TL}}_2, \alpha_1 \Upsilon^{\mathcal{TU}}_1 \cdot \alpha_1 \Upsilon^{\mathcal{TU}}_2\Big], \\ \left[\begin{array}{c} \left(\sqrt[\Xi]{\alpha_2 \Upsilon^{\mathcal{IL}}_1} + \sqrt[\Xi]{\alpha_2 \Upsilon^{\mathcal{IL}}_2} - \sqrt[\Xi]{\alpha_2 \Upsilon^{\mathcal{IL}}_1} \cdot \sqrt[\Xi]{\alpha_2 \Upsilon^{\mathcal{IL}}_2}\right)^{\Xi}, \\ \left(\sqrt[\Xi]{\alpha_2 \Upsilon^{\mathcal{IU}}_1} + \sqrt[\Xi]{\alpha_2 \Upsilon^{\mathcal{IU}}_2} - \sqrt[\Xi]{\alpha_2 \Upsilon^{\mathcal{IU}}_1} \cdot \sqrt[\Xi]{\alpha_2 \Upsilon^{\mathcal{IU}}_2}\right)^{\Xi} \end{array}\right], \\ \left[\begin{array}{c} \left(\sqrt[2\Xi]{\alpha_3 \Upsilon^{\mathcal{FL}}_1} + \sqrt[2\Xi]{\alpha_3 \Upsilon^{\mathcal{FL}}_2} - \sqrt[2\Xi]{\alpha_3 \Upsilon^{\mathcal{FL}}_1} \cdot \sqrt[2\Xi]{\alpha_3 \Upsilon^{\mathcal{FL}}_2}\right)^{2\Xi}, \\ \left(\sqrt[2\Xi]{\alpha_3 \Upsilon^{\mathcal{FU}}_1} + \sqrt[2\Xi]{\alpha_3 \Upsilon^{\mathcal{FU}}_2} - \sqrt[2\Xi]{\alpha_3 \Upsilon^{\mathcal{FU}}_1} \cdot \sqrt[2\Xi]{\alpha_3 \Upsilon^{\mathcal{FU}}_2}\right)^{2\Xi} \end{array}\right] \end{array}\right],$$

https://doi.org/10.54216/IJNS.190401
Received: April 28, 2022 Accepted: November 15, 2022

3. $\Xi \cdot \overline{\Theta} = \left[\begin{array}{c}\left[\left(1-\left(1-\sqrt[2\Xi]{\alpha_1 \Upsilon^{\mathcal{TL}}}\right)^{\Xi}\right)^{2\Xi}, \left(1-\left(1-\sqrt[2\Xi]{\alpha_1 \Upsilon^{\mathcal{TU}}}\right)^{\Xi}\right)^{2\Xi}\right], \\ \left[\sqrt[\Xi]{\alpha_2 \Upsilon_1^{\mathcal{IL}}}, \sqrt[\Xi]{\alpha_2 \Upsilon_1^{\mathcal{IU}}}\right], \left[(\alpha_3 \Upsilon^{\mathcal{FL}})^{\Xi}, (\alpha_3 \Upsilon^{\mathcal{FU}})^{\Xi}\right]\end{array}\right]$,

4. $\overline{\Theta}^{\Xi} = \left[\begin{array}{c}\left[(\alpha_1 \Upsilon^{\mathcal{TL}})^{\Xi}, (\alpha_1 \Upsilon^{\mathcal{TU}})^{\Xi}\right], \left[\sqrt[\Xi]{\alpha_2 \Upsilon_1^{\mathcal{IL}}}, \sqrt[\Xi]{\alpha_2 \Upsilon_1^{\mathcal{IU}}}\right], \\ \left[\left(1-\left(1-\sqrt[2\Xi]{\alpha_3 \Upsilon^{\mathcal{FL}}}\right)^{\Xi}\right)^{2\Xi}, \left(1-\left(1-\sqrt[2\Xi]{\alpha_3 \Upsilon^{\mathcal{FU}}}\right)^{\Xi}\right)^{2\Xi}\right]\end{array}\right]$.

4 Various distance measure for SRDioNIVN approach

We discuss some of its mathematical features and introduce the ED and HD measures for SRDioNIVNs.

Definition 4.1. For SRDioNIVNs $\overline{\Theta}_1 = \left\langle [\alpha_1 \Upsilon_1^{\mathcal{TL}}, \alpha_1 \Upsilon_1^{\mathcal{TU}}], [\alpha_2 \Upsilon_1^{\mathcal{IL}}, \alpha_2 \Upsilon_1^{\mathcal{IU}}], [\alpha_3 \Upsilon_1^{\mathcal{FL}}, \alpha_3 \Upsilon_1^{\mathcal{FU}}] \right\rangle$ and $\overline{\Theta}_2 = \left\langle [\alpha_1 \Upsilon_2^{\mathcal{TL}}, \alpha_1 \Upsilon_2^{\mathcal{TU}}], [\alpha_2 \Upsilon_2^{\mathcal{IL}}, \alpha_2 \Upsilon_2^{\mathcal{IU}}], [\alpha_3 \Upsilon_2^{\mathcal{FL}}, \alpha_3 \Upsilon_2^{\mathcal{FU}}] \right\rangle$. Then

$$\mathbb{D}_E\left(\overline{\Theta}_1, \overline{\Theta}_2\right) = \frac{1}{2}\sqrt{\begin{array}{c}\left[\begin{array}{c}\frac{1+(\alpha_1\Upsilon_1^{\mathcal{TL}})^2-\sqrt{\alpha_2\Upsilon_1^{\mathcal{IL}}}-\sqrt{\alpha_3\Upsilon_1^{\mathcal{FL}}}+1+(\alpha_1\Upsilon_1^{\mathcal{TU}})^2-\sqrt{\alpha_2\Upsilon_1^{\mathcal{IU}}}-\sqrt{\alpha_3\Upsilon_1^{\mathcal{FU}}}}{4} \\ -\frac{1+(\alpha_1\Upsilon_2^{\mathcal{TL}})^2-\sqrt{\alpha_2\Upsilon_2^{\mathcal{IL}}}-\sqrt{\alpha_3\Upsilon_2^{\mathcal{FL}}}+1+(\alpha_1\Upsilon_2^{\mathcal{TU}})^2-\sqrt{\alpha_2\Upsilon_2^{\mathcal{IU}}}-\sqrt{\alpha_3\Upsilon_2^{\mathcal{FU}}}}{4}\end{array}\right]^2 \\ +\frac{1}{2}\left[\begin{array}{c}\frac{1+(\alpha_1\Upsilon_1^{\mathcal{TL}})^2-\sqrt{\alpha_2\Upsilon_1^{\mathcal{IL}}}-\sqrt{\alpha_3\Upsilon_1^{\mathcal{FL}}}+1+(\alpha_1\Upsilon_1^{\mathcal{TU}})^2-\sqrt{\alpha_2\Upsilon_1^{\mathcal{IU}}}-\sqrt{\alpha_3\Upsilon_1^{\mathcal{FU}}}}{4} \\ -\frac{1+(\alpha_1\Upsilon_2^{\mathcal{TL}})^2-\sqrt{\alpha_2\Upsilon_2^{\mathcal{IL}}}-\sqrt{\alpha_3\Upsilon_2^{\mathcal{FL}}}+1+(\alpha_1\Upsilon_2^{\mathcal{TU}})^2-\sqrt{\alpha_2\Upsilon_2^{\mathcal{IU}}}-\sqrt{\alpha_3\Upsilon_2^{\mathcal{FU}}}}{4}\end{array}\right]^2\end{array}}$$

where $\mathbb{D}_E\left(\overline{\Theta}_1, \overline{\Theta}_2\right)$ is denote the euclidean distance between $\overline{\Theta}_1$ and $\overline{\Theta}_2$.
Also, the hamming distance $\mathbb{D}_H\left(\overline{\Theta}_1, \overline{\Theta}_2\right)$=

$$\frac{1}{2}\left[\begin{array}{c}\left|\begin{array}{c}\frac{1+(\alpha_1\Upsilon_1^{\mathcal{TL}})^2-\sqrt{\alpha_2\Upsilon_1^{\mathcal{IL}}}-\sqrt{\alpha_3\Upsilon_1^{\mathcal{FL}}}+1+(\alpha_1\Upsilon_1^{\mathcal{TU}})^2-\sqrt{\alpha_2\Upsilon_1^{\mathcal{IU}}}-\sqrt{\alpha_3\Upsilon_1^{\mathcal{FU}}}}{4} \\ -\frac{1+(\alpha_1\Upsilon_2^{\mathcal{TL}})^2-\sqrt{\alpha_2\Upsilon_2^{\mathcal{IL}}}-\sqrt{\alpha_3\Upsilon_2^{\mathcal{FL}}}+1+(\alpha_1\Upsilon_2^{\mathcal{TU}})^2-\sqrt{\alpha_2\Upsilon_2^{\mathcal{IU}}}-\sqrt{\alpha_3\Upsilon_2^{\mathcal{FU}}}}{4}\end{array}\right| \\ +\frac{1}{2}\left|\begin{array}{c}\frac{1+(\alpha_1\Upsilon_1^{\mathcal{TL}})^2-\sqrt{\alpha_2\Upsilon_1^{\mathcal{IL}}}-\sqrt{\alpha_3\Upsilon_1^{\mathcal{FL}}}+1+(\alpha_1\Upsilon_1^{\mathcal{TU}})^2-\sqrt{\alpha_2\Upsilon_1^{\mathcal{IU}}}-\sqrt{\alpha_3\Upsilon_1^{\mathcal{FU}}}}{4} \\ -\frac{1+(\alpha_1\Upsilon_2^{\mathcal{TL}})^2-\sqrt{\alpha_2\Upsilon_2^{\mathcal{IL}}}-\sqrt{\alpha_3\Upsilon_2^{\mathcal{FL}}}+1+(\alpha_1\Upsilon_2^{\mathcal{TU}})^2-\sqrt{\alpha_2\Upsilon_2^{\mathcal{IU}}}-\sqrt{\alpha_3\Upsilon_2^{\mathcal{FU}}}}{4}\end{array}\right|\end{array}\right]$$

where $\mathbb{D}_H\left(\overline{\Theta}_1, \overline{\Theta}_2\right)$ is denote the hamming distance between $\overline{\Theta}_1$ and $\overline{\Theta}_2$.

Theorem 4.2. *If any three SRDioNIVNs* $\overline{\Theta}_1 = \left\langle [\alpha_1 \Upsilon_1^{\mathcal{TL}}, \alpha_1 \Upsilon_1^{\mathcal{TU}}], [\alpha_2 \Upsilon_1^{\mathcal{IL}}, \alpha_2 \Upsilon_1^{\mathcal{IU}}], [\alpha_3 \Upsilon_1^{\mathcal{FL}}, \alpha_3 \Upsilon_1^{\mathcal{FU}}] \right\rangle, \overline{\Theta}_2 = \left\langle [\alpha_1 \Upsilon_2^{\mathcal{TL}}, \alpha_1 \Upsilon_2^{\mathcal{TU}}], [\alpha_2 \Upsilon_2^{\mathcal{IL}}, \alpha_2 \Upsilon_2^{\mathcal{IU}}], [\alpha_3 \Upsilon_2^{\mathcal{FL}}, \alpha_3 \Upsilon_2^{\mathcal{FU}}] \right\rangle$, $\overline{\Theta_3} = \left\langle [\alpha_1 \Upsilon_3^{\mathcal{TL}}, \alpha_1 \Upsilon_3^{\mathcal{TU}}], [\alpha_2 \Upsilon_3^{\mathcal{IL}}, \alpha_2 \Upsilon_3^{\mathcal{IU}}], [\alpha_3 \Upsilon_3^{\mathcal{FL}}, \alpha_3 \Upsilon_3^{\mathcal{FU}}] \right\rangle$, *then* $\mathbb{D}_E(\overline{\Theta}_1, \overline{\Theta}_2$ *satisfies the following properties are holds.*

1. $\mathbb{D}_E(\overline{\Theta}_1, \overline{\Theta}_2$ *is zero, whenever* $\overline{\Theta}_1 = \overline{\Theta}_2$ *and vice versa.*
2. $\mathbb{D}_E(\overline{\Theta}_1, \overline{\Theta}_2$ *and* $\mathbb{D}_E(\overline{\Theta}_2, \overline{\Theta}_1$ *are co-occur.*
3. $\mathbb{D}_E(\overline{\Theta}_1, \overline{\Theta_3}) \leq \mathbb{D}_E(\overline{\Theta}_1, \overline{\Theta}_2 + \mathbb{D}_E(\overline{\Theta}_2, \overline{\Theta_3})$.

https://doi.org/10.54216/IJNS.190401
Received: April 28, 2022 Accepted: November 15, 2022

Proof. Now, $\Big(\mathbb{D}_E(\Theta_1, \Theta_2 + \mathbb{D}_E(\Theta_2, \Theta_3)\Big)^2 =$

$$\left[\begin{array}{l} \frac{1}{2}\sqrt{\begin{array}{l}\left[\begin{array}{l}\frac{1+(\alpha_1\Upsilon_1^{\mathcal{TL}})^2-\sqrt{\alpha_2\Upsilon_1^{\mathcal{IL}}}-\sqrt{\alpha_3\Upsilon_1^{\mathcal{FL}}}+1+(\alpha_1\Upsilon_1^{\mathcal{TU}})^2-\sqrt{\alpha_2\Upsilon_1^{\mathcal{IU}}}-\sqrt{\alpha_3\Upsilon_1^{\mathcal{FU}}}}{4}\\ -\frac{1+(\alpha_1\Upsilon_2^{\mathcal{TL}})^2-\sqrt{\alpha_2\Upsilon_2^{\mathcal{IL}}}-\sqrt{\alpha_3\Upsilon_2^{\mathcal{FL}}}+1+(\alpha_1\Upsilon_2^{\mathcal{TU}})^2-\sqrt{\alpha_2\Upsilon_2^{\mathcal{IU}}}-\sqrt{\alpha_3\Upsilon_2^{\mathcal{FU}}}}{4}\end{array}\right]^2\\ +\frac{1}{2}\left[\begin{array}{l}\frac{1+(\alpha_1\Upsilon_1^{\mathcal{TL}})^2-\sqrt{\alpha_2\Upsilon_1^{\mathcal{IL}}}-\sqrt{\alpha_3\Upsilon_1^{\mathcal{FL}}}+1+(\alpha_1\Upsilon_1^{\mathcal{TU}})^2-\sqrt{\alpha_2\Upsilon_1^{\mathcal{IU}}}-\sqrt{\alpha_3\Upsilon_1^{\mathcal{FU}}}}{4}\\ -\frac{1+(\alpha_1\Upsilon_2^{\mathcal{TL}})^2-\sqrt{\alpha_2\Upsilon_2^{\mathcal{IL}}}-\sqrt{\alpha_3\Upsilon_2^{\mathcal{FL}}}+1+(\alpha_1\Upsilon_2^{\mathcal{TU}})^2-\sqrt{\alpha_2\Upsilon_2^{\mathcal{IU}}}-\sqrt{\alpha_3\Upsilon_2^{\mathcal{FU}}}}{4}\end{array}\right]^2\end{array}} \\ +\frac{1}{2}\sqrt{\begin{array}{l}\left[\begin{array}{l}\frac{1+(\alpha_1\Upsilon_2^{\mathcal{TL}})^2-\sqrt{\alpha_2\Upsilon_2^{\mathcal{IL}}}-\sqrt{\alpha_3\Upsilon_2^{\mathcal{FL}}}+1+(\alpha_1\Upsilon_2^{\mathcal{TU}})^2-\sqrt{\alpha_2\Upsilon_2^{\mathcal{IU}}}-\sqrt{\alpha_3\Upsilon_2^{\mathcal{FU}}}}{4}\\ -\frac{1+(\alpha_1\Upsilon_3^{\mathcal{TL}})^2-\sqrt{\alpha_2\Upsilon_3^{\mathcal{IL}}}-\sqrt{\alpha_3\Upsilon_3^{\mathcal{FL}}}+1+(\alpha_1\Upsilon_3^{\mathcal{TU}})^2-\sqrt{\alpha_2\Upsilon_3^{\mathcal{IU}}}-\sqrt{\alpha_3\Upsilon_3^{\mathcal{FU}}}}{4}\end{array}\right]^2\\ +\frac{1}{2}\left[\begin{array}{l}\frac{1+(\alpha_1\Upsilon_2^{\mathcal{TL}})^2-\sqrt{\alpha_2\Upsilon_2^{\mathcal{IL}}}-\sqrt{\alpha_3\Upsilon_2^{\mathcal{FL}}}+1+(\alpha_1\Upsilon_2^{\mathcal{TU}})^2-\sqrt{\alpha_2\Upsilon_2^{\mathcal{IU}}}-\sqrt{\alpha_3\Upsilon_2^{\mathcal{FU}}}}{4}\\ -\frac{1+(\alpha_1\Upsilon_3^{\mathcal{TL}})^2-\sqrt{\alpha_2\Upsilon_3^{\mathcal{IL}}}-\sqrt{\alpha_3\Upsilon_3^{\mathcal{FL}}}+1+(\alpha_1\Upsilon_3^{\mathcal{TU}})^2-\sqrt{\alpha_2\Upsilon_3^{\mathcal{IU}}}-\sqrt{\alpha_3\Upsilon_3^{\mathcal{FU}}}}{4}\end{array}\right]^2\end{array}} \end{array}\right]^2$$

implies

$$\begin{aligned} &\frac{1}{4}\Big((\Lambda_1-\Lambda_2)^2+\frac{1}{2}(\Lambda_1-\Lambda_2)^2\Big)+\frac{1}{4}\Big((\Lambda_2-\Lambda_3)^2+\frac{1}{2}(\Lambda_2-\Lambda_3)^2\Big)\\ &+\frac{1}{2}\left(\sqrt{(\Lambda_1-\Lambda_2)^2+\frac{1}{2}(\Lambda_1-\Lambda_2)^2}\times\sqrt{(\Lambda_2-\Lambda_3)^2+\frac{1}{2}(\Lambda_2-\Lambda_3)^2}\right), \end{aligned}$$

Since,

$$\Lambda_1=\frac{1+(\alpha_1\Upsilon_1^{\mathcal{TL}})^2-\sqrt{\alpha_2\Upsilon_1^{\mathcal{IL}}}-\sqrt{\alpha_3\Upsilon_1^{\mathcal{FL}}}+1+(\alpha_1\Upsilon_1^{\mathcal{TU}})^2-\sqrt{\alpha_2\Upsilon_1^{\mathcal{IU}}}-\sqrt{\alpha_3\Upsilon_1^{\mathcal{FU}}}}{4},$$

$$\Lambda_2=\frac{1+(\alpha_1\Upsilon_2^{\mathcal{TL}})^2-\sqrt{\alpha_2\Upsilon_2^{\mathcal{IL}}}-\sqrt{\alpha_3\Upsilon_2^{\mathcal{FL}}}+1+(\alpha_1\Upsilon_2^{\mathcal{TU}})^2-\sqrt{\alpha_2\Upsilon_2^{\mathcal{IU}}}-\sqrt{\alpha_3\Upsilon_2^{\mathcal{FU}}}}{4},$$

$$\Lambda_3=\frac{1+(\alpha_1\Upsilon_3^{\mathcal{TL}})^2-\sqrt{\alpha_2\Upsilon_3^{\mathcal{IL}}}-\sqrt{\alpha_3\Upsilon_3^{\mathcal{FL}}}+1+(\alpha_1\Upsilon_3^{\mathcal{TU}})^2-\sqrt{\alpha_2\Upsilon_3^{\mathcal{IU}}}-\sqrt{\alpha_3\Upsilon_3^{\mathcal{FU}}}}{4}.$$

Hence, $\Big(\mathbb{D}_E(\Theta_1, \Theta_2 + \mathbb{D}_E(\Theta_2, \Theta_3)\Big)^2$

$$\begin{aligned} &\geq \frac{1}{4}\Big((\Lambda_1-\Lambda_2)^2+\frac{1}{2}(\Lambda_1-\Lambda_2)^2\Big)+\frac{1}{4}\Big((\Lambda_2-\Lambda_3)^2+\frac{1}{2}(\Lambda_2-\Lambda_3)^2\Big)\\ &\quad+\frac{1}{2}\Big((\Lambda_1-\Lambda_2)\times(\Lambda_2-\Lambda_3)+\frac{1}{2}\Big((\Lambda_1-\Lambda_2)\times(\Lambda_2-\Lambda_3)\Big)\\ &= \frac{1}{4}(\Lambda_1-\Lambda_2+\Lambda_2-\Lambda_3)^2+\frac{1}{8}(\Lambda_1-\Lambda_2+\Lambda_2-\Lambda_3)^2\\ &= \frac{1}{4}(\Lambda_1-\Lambda_3)^2+\frac{1}{8}(\Lambda_1-\Lambda_3)^2\\ &= \frac{1}{4}\Big[(\Lambda_1-\Lambda_3)^2+\frac{1}{2}(\Lambda_1-\Lambda_3)^2\Big]\\ &= \mathbb{D}_E(\Theta_1,\Theta_3)^2. \end{aligned}$$

Corollary 4.3. *If any three SRDioNIVNs* $\overline{\Theta}_1=\Big\langle[\alpha_1\Upsilon_1^{\mathcal{TL}},\alpha_1\Upsilon_1^{\mathcal{TU}}],[\alpha_2\Upsilon_1^{\mathcal{IL}},\alpha_2\Upsilon_1^{\mathcal{IU}}],$ $[\alpha_3\Upsilon_1^{\mathcal{FL}},\alpha_3\Upsilon_1^{\mathcal{FU}}]\Big\rangle, \overline{\Theta}_2=\Big\langle[\alpha_1\Upsilon_2^{\mathcal{TL}},\alpha_1\Upsilon_2^{\mathcal{TU}}],[\alpha_2\Upsilon_2^{\mathcal{IL}},\alpha_2\Upsilon_2^{\mathcal{IU}}],[\alpha_3\Upsilon_2^{\mathcal{FL}},\alpha_3\Upsilon_2^{\mathcal{FU}}]\Big\rangle,$ $\overline{\Theta_3}=\Big\langle[\alpha_1\Upsilon_3^{\mathcal{TL}},\alpha_1\Upsilon_3^{\mathcal{TU}}],[\alpha_2\Upsilon_3^{\mathcal{IL}},\alpha_2\Upsilon_3^{\mathcal{IU}}],[\alpha_3\Upsilon_3^{\mathcal{FL}},\alpha_3\Upsilon_3^{\mathcal{FU}}]\Big\rangle$, *then* $\mathbb{D}_H(\Theta_1,\Theta_2$ *satisfies the following properties are holds.*

1. $\mathbb{D}_H(\overline{\Theta}_1,\overline{\Theta}_2$ *is zero, whenever* $\overline{\Theta}_1=\overline{\Theta}_2$ *and vice versa.*
2. $\mathbb{D}_H(\overline{\Theta}_1,\overline{\Theta}_2$ *and* $\mathbb{D}_H(\overline{\Theta}_2,\overline{\Theta}_1$ *are co-occur.*
3. $\mathbb{D}_H(\overline{\Theta}_1,\overline{\Theta_3})\leq\mathbb{D}_H(\overline{\Theta}_1,\overline{\Theta}_2+\mathbb{D}_H(\overline{\Theta}_2,\overline{\Theta_3})$.

https://doi.org/10.54216/IJNS.190401
Received: April 28, 2022 Accepted: November 15, 2022

5 Various aggregation operators for SRDioNIVN

The new operators for SRDioNIVWA, SRDioNIVWG, GSRDioNIVWA, and GSRDioNIVWG are introduced in this section.

5.1 SRDioNIV weighted averaging(SRDioNIVWA) operator

Definition 5.1. Let $\overline{\Theta}_i = \left\langle [\alpha_1\Upsilon_i^{\mathcal{TL}}, \alpha_1\Upsilon_i^{\mathcal{TU}}], [\alpha_2\Upsilon_i^{\mathcal{IL}}, \alpha_2\Upsilon_i^{\mathcal{IU}}], [\alpha_3\Upsilon_i^{\mathcal{FL}}, \alpha_3\Upsilon_i^{\mathcal{FU}}] \right\rangle$ be the family of SRDioNIVNs, $W = (\chi_1, \chi_2, ..., \chi_n)$ be the weight of $\overline{\Theta}_i$, $\chi_i \geq 0$ and $\bigcap_{i=1}^{n} \chi_i = 1$. Then SRDioNIVWA $(\overline{\Theta}_1, \overline{\Theta}_2, ..., \overline{\Theta}_n) = \bigcap_{i=1}^{n} \chi_i \overline{\Theta}_i$ $(i = 1, 2, ..., n)$.

Theorem 5.2. *Let* $\overline{\Theta}_i = \left\langle [\alpha_1\Upsilon_i^{\mathcal{TL}}, \alpha_1\Upsilon_i^{\mathcal{TU}}], [\alpha_2\Upsilon_i^{\mathcal{IL}}, \alpha_2\Upsilon_i^{\mathcal{IU}}], [\alpha_3\Upsilon_i^{\mathcal{FL}}, \alpha_3\Upsilon_i^{\mathcal{FU}}] \right\rangle$ *be the family of SRDioNIVNs. Then* $SRDioNIVWA(\overline{\Theta}_1, \overline{\Theta}_2, ..., \overline{\Theta}_n)$=

$$\left[\begin{array}{c} \left[\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_i^{\mathcal{TL}})}\right)^{\chi_i}\right)^{2\Xi}, \left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_i^{\mathcal{TU}})}\right)^{\chi_i}\right)^{2\Xi}\right], \\ \left[\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_i^{\mathcal{IL}})}\right)^{\chi_i}\right)^{\Xi}, \left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_i^{\mathcal{IU}})}\right)^{\chi_i}\right)^{\Xi}\right], \\ \left[\bigcup_{i=1}^{n}(\alpha_3\Upsilon_i^{\mathcal{FL}})^{\chi_i}, \bigcup_{i=1}^{n}(\alpha_3\Upsilon_i^{\mathcal{FU}})^{\chi_i}\right] \end{array}\right].$$

Proof. The mathematical induction method served as the basis for the proof.
Put $n = 2$, SRDioNIVWA$(\overline{\Theta}_1, \overline{\Theta}_2) = \chi_1\overline{\Theta}_1 \oplus \chi_2\overline{\Theta}_2$, where

$$\chi_1\overline{\Theta}_1 = \left[\begin{array}{c} \left[\left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_1^{\mathcal{TL}})}\right)^{\chi_1}\right)^{2\Xi}, \left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_1^{\mathcal{TU}})}\right)^{\chi_1}\right)^{2\Xi}\right], \\ \left[\left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_1^{\mathcal{IL}})}\right)^{\chi_1}\right)^{\Xi}, \left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_1^{\mathcal{IU}})}\right)^{\chi_1}\right)^{\Xi}\right], \\ \left[(\alpha_3\Upsilon_1^{\mathcal{FL}})^{\chi_1}, (\alpha_3\Upsilon_1^{\mathcal{FU}})^{\chi_1}\right] \end{array}\right]$$

$$\chi_2\overline{\Theta}_2 = \left[\begin{array}{c} \left[\left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_2^{\mathcal{TL}})}\right)^{\chi_2}\right)^{2\Xi}, \left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_2^{\mathcal{TU}})}\right)^{\chi_2}\right)^{2\Xi}\right], \\ \left[\left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_2^{\mathcal{IL}})}\right)^{\chi_2}\right)^{\Xi}, \left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_2^{\mathcal{IU}})}\right)^{\chi_2}\right)^{\Xi}\right], \\ \left[(\alpha_3\Upsilon_2^{\mathcal{FL}})^{\chi_2}, (\alpha_3\Upsilon_2^{\mathcal{FU}})^{\chi_2}\right] \end{array}\right].$$

https://doi.org/10.54216/IJNS.190401
Received: April 28, 2022 Accepted: November 15, 2022

Now,

$$= \begin{bmatrix} \begin{bmatrix} \begin{pmatrix} \left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_1^{\mathcal{TL}})}\right)^{\chi_1}\right)+\left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_2^{\mathcal{TL}})}\right)^{\chi_2}\right) \\ -\left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_1^{\mathcal{TL}})}\right)^{\chi_1}\right)\cdot\left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_2^{\mathcal{TL}})}\right)^{\chi_2}\right) \end{pmatrix}^{2\Xi}, \\ \begin{pmatrix} \left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_1^{\mathcal{TU}})}\right)^{\chi_1}\right)+\left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_2^{\mathcal{TU}})}\right)^{\chi_2}\right) \\ -\left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_1^{\mathcal{TU}})}\right)^{\chi_1}\right)\cdot\left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_2^{\mathcal{TU}})}\right)^{\chi_2}\right) \end{pmatrix}^{2\Xi} \end{bmatrix}, \\ \begin{bmatrix} \begin{pmatrix} \left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_1^{\mathcal{IL}})}\right)^{\chi_1}\right)+\left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_2^{\mathcal{IL}})}\right)^{\chi_2}\right) \\ -\left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_1^{\mathcal{IL}})}\right)^{\chi_1}\right)\cdot\left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_2^{\mathcal{IL}})}\right)^{\chi_2}\right) \end{pmatrix}^{\Xi}, \\ \begin{pmatrix} \left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_1^{\mathcal{IU}})}\right)^{\chi_1}\right)+\left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_2^{\mathcal{IU}})}\right)^{\chi_2}\right) \\ -\left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_1^{\mathcal{IU}})}\right)^{\chi_1}\right)\cdot\left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_2^{\mathcal{IU}})}\right)^{\chi_2}\right) \end{pmatrix}^{\Xi} \end{bmatrix}, \\ \left[(\alpha_3\Upsilon_1^{\mathcal{FL}})^{\chi_1}(\alpha_3\Upsilon_2^{\mathcal{FL}})^{\chi_2},(\alpha_3\Upsilon_1^{\mathcal{FU}})^{\chi_1}(\alpha_3\Upsilon_2^{\mathcal{FU}})^{\chi_2}\right] \end{bmatrix}$$

$$= \begin{bmatrix} \begin{bmatrix} \left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_1^{\mathcal{TL}})}\right)^{\chi_1}\cdot\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_2^{\mathcal{TL}})}\right)^{\chi_2}\right)^{2\Xi}, \\ \left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_1^{\mathcal{TU}})}\right)^{\chi_1}\cdot\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_2^{\mathcal{TU}})}\right)^{\chi_2}\right)^{2\Xi} \end{bmatrix}, \\ \begin{bmatrix} \left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_1^{\mathcal{IL}})}\right)^{\chi_1}\cdot\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_2^{\mathcal{IL}})}\right)^{\chi_2}\right)^{\Xi}, \\ \left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_1^{\mathcal{IU}})}\right)^{\chi_1}\cdot\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_2^{\mathcal{IU}})}\right)^{\chi_2}\right)^{\Xi} \end{bmatrix}, \\ \left[(\alpha_3\Upsilon_1^{\mathcal{FL}})^{\chi_1}\cdot(\alpha_3\Upsilon_2^{\mathcal{FL}})^{\chi_2},(\alpha_3\Upsilon_1^{\mathcal{FU}})^{\chi_1}\cdot(\alpha_3\Upsilon_2^{\mathcal{FU}})^{\chi_2}\right] \end{bmatrix}$$

$SRDioNIVWA(\overline{\Theta}_1,\overline{\Theta}_2)=$

$$\begin{bmatrix} \left[\left(1-\bigcup_{i=1}^{2}\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_i^{\mathcal{TL}})}\right)^{\chi_i}\right)^{2\Xi},\left(1-\bigcup_{i=1}^{2}\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_i^{\mathcal{TU}})}\right)^{\chi_i}\right)^{2\Xi}\right], \\ \left[\left(1-\bigcup_{i=1}^{2}\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_i^{\mathcal{IL}})}\right)^{\chi_i}\right)^{\Xi},\left(1-\bigcup_{i=1}^{2}\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_i^{\mathcal{IU}})}\right)^{\chi_i}\right)^{\Xi}\right], \\ \left[\bigcup_{i=1}^{2}(\alpha_3\Upsilon_i^{\mathcal{FL}})^{\chi_i},\bigcup_{i=1}^{2}(\alpha_3\Upsilon_i^{\mathcal{FU}})^{\chi_i}\right] \end{bmatrix}.$$

Also, valid for $n\geq 3$, hence $SRDioNIVWA(\overline{\Theta}_1,\overline{\Theta}_2,...,\overline{\Theta}_l)=$

$$\begin{bmatrix} \left[\left(1-\bigcup_{i=1}^{l}\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_i^{\mathcal{TL}})}\right)^{\chi_i}\right)^{2\Xi},\left(1-\bigcup_{i=1}^{l}\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_i^{\mathcal{TU}})}\right)^{\chi_i}\right)^{2\Xi}\right], \\ \left[\left(1-\bigcup_{i=1}^{l}\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_i^{\mathcal{IL}})}\right)^{\chi_i}\right)^{\Xi},\left(1-\bigcup_{i=1}^{l}\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_i^{\mathcal{IU}})}\right)^{\chi_i}\right)^{\Xi}\right], \\ \left[\bigcup_{i=1}^{l}(\alpha_3\Upsilon_i^{\mathcal{FL}})^{\chi_i},\bigcup_{i=1}^{l}(\alpha_3\Upsilon_i^{\mathcal{FU}})^{\chi_i}\right] \end{bmatrix}.$$

Received: April 28, 2022 Accepted: November 15, 2022

If $n = l+1$, then SRDioNIVWA $(\overline{\Theta}_1, \overline{\Theta}_2, ..., \overline{\Theta}_l, \overline{\Theta}_{l+1}) =$

$$\left[\begin{array}{c} \left[\begin{array}{c} \left(\begin{array}{c} \bigcap_{i=1}^{l}\left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_i^{\mathcal{TL}})}\right)^{\chi_i}\right)+\left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_{l+1}^{\mathcal{TL}})}\right)^{\chi_{l+1}}\right) \\ -\bigcup_{i=1}^{l}\left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_i^{\mathcal{TL}})}\right)^{\chi_i}\right)\cdot\left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_{l+1}^{\mathcal{TL}})}\right)^{\chi_{l+1}}\right) \end{array}\right)^{2\Xi}, \\ \left(\begin{array}{c} \bigcap_{i=1}^{l}\left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_i^{\mathcal{TU}})}\right)^{\chi_i}\right)+\left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_{l+1}^{\mathcal{TU}})}\right)^{\chi_{l+1}}\right) \\ -\bigcup_{i=1}^{l}\left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_i^{\mathcal{TU}})}\right)^{\chi_i}\right)\cdot\left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_{l+1}^{\mathcal{TU}})}\right)^{\chi_{l+1}}\right) \end{array}\right)^{2\Xi} \end{array}\right], \\ \left[\begin{array}{c} \left(\begin{array}{c} \bigcap_{i=1}^{l}\left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_i^{\mathcal{IL}})}\right)^{\chi_i}\right)+\left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_{l+1}^{\mathcal{IL}})}\right)^{\chi_{l+1}}\right) \\ -\bigcup_{i=1}^{l}\left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_i^{\mathcal{IL}})}\right)^{\chi_i}\right)\cdot\left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_{l+1}^{\mathcal{IL}})}\right)^{\chi_{l+1}}\right) \end{array}\right)^{\Xi}, \\ \left(\begin{array}{c} \bigcap_{i=1}^{l}\left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_i^{\mathcal{IU}})}\right)^{\chi_i}\right)+\left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_{l+1}^{\mathcal{IU}})}\right)^{\chi_{l+1}}\right) \\ -\bigcup_{i=1}^{l}\left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_i^{\mathcal{IU}})}\right)^{\chi_i}\right)\cdot\left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_{l+1}^{\mathcal{IU}})}\right)^{\chi_{l+1}}\right) \end{array}\right)^{\Xi} \end{array}\right], \\ \left[\bigcup_{i=1}^{l}(\alpha_3\Upsilon_i^{\mathcal{FL}})^{\chi_i}\cdot(\alpha_3\Upsilon_{l+1}^{\mathcal{FL}})^{\chi_{l+1}}, \bigcup_{i=1}^{l}(\alpha_3\Upsilon_i^{\mathcal{FU}})^{\chi_i}\cdot(\alpha_3\Upsilon_{l+1}^{\mathcal{FU}})^{\chi_{l+1}}\right] \end{array}\right]$$

$$= \left[\begin{array}{c} \left[\left(1-\bigcup_{i=1}^{l+1}\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_i^{\mathcal{TL}})}\right)^{\chi_i}\right)^{2\Xi}, \left(1-\bigcup_{i=1}^{l+1}\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_i^{\mathcal{TU}})}\right)^{\chi_i}\right)^{2\Xi}\right], \\ \left[\left(1-\bigcup_{i=1}^{l+1}\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_i^{\mathcal{IL}})}\right)^{\chi_i}\right)^{\Xi}, \left(1-\bigcup_{i=1}^{l+1}\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_i^{\mathcal{IU}})}\right)^{\chi_i}\right)^{\Xi}\right], \\ \left[\bigcup_{i=1}^{l+1}(\alpha_3\Upsilon_i^{\mathcal{FL}})^{\chi_i}, \bigcup_{i=1}^{l+1}(\alpha_3\Upsilon_i^{\mathcal{FU}})^{\chi_i}\right] \end{array}\right].$$

Theorem 5.3. *If all* $\overline{\Theta}_i = \left\langle [\alpha_1\Upsilon_i^{\mathcal{TL}}, \alpha_1\Upsilon_i^{\mathcal{TU}}], [\alpha_2\Upsilon_i^{\mathcal{IL}}, \alpha_2\Upsilon_i^{\mathcal{IU}}][\alpha_3\Upsilon_i^{\mathcal{FL}}, \alpha_3\Upsilon_i^{\mathcal{FU}}] \right\rangle$ *are equal, then SRDioNIVWA*$(\overline{\Theta}_1, \overline{\Theta}_2, ..., \overline{\Theta}_n)$ $\overline{\Theta}$*(idempotency property).*

Proof. Given that $[\alpha_1\Upsilon_i^{\mathcal{TL}}, \alpha_1\Upsilon_i^{\mathcal{TU}}] = [\alpha_1\Upsilon^{\mathcal{TL}}, \alpha_1\Upsilon^{\mathcal{TU}}]$, $[\alpha_2\Upsilon_i^{\mathcal{IL}}, \alpha_2\Upsilon_i^{\mathcal{IU}}] = [\alpha_2\Upsilon^{\mathcal{IL}}, \alpha_2\Upsilon^{\mathcal{IU}}]$ and $[\alpha_3\Upsilon_i^{\mathcal{FL}}, \alpha_3\Upsilon_i^{\mathcal{FU}}] = [\alpha_3\Upsilon^{\mathcal{FL}}, \alpha_3\Upsilon^{\mathcal{FU}}]$ and $\bigcap_{i=1}^{n}\chi_i = 1$.

Now, $SRDioNIVWA(\overline{\Theta}_1, \overline{\Theta}_2, ..., \overline{\Theta}_n)$

$$= \left[\begin{array}{c} \left[\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_i^{\mathcal{TL}})}\right)^{\chi_i}\right)^{2\Xi}, \left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_i^{\mathcal{TU}})}\right)^{\chi_i}\right)^{2\Xi}\right], \\ \left[\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_i^{\mathcal{IL}})}\right)^{\chi_i}\right)^{\Xi}, \left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_i^{\mathcal{IU}})}\right)^{\chi_i}\right)^{\Xi}\right], \\ \left[\bigcup_{i=1}^{n}(\alpha_3\Upsilon_i^{\mathcal{FL}})^{\chi_i}, \bigcup_{i=1}^{n}(\alpha_3\Upsilon_i^{\mathcal{FU}})^{\chi_i}\right] \end{array}\right]$$

$$= \left[\begin{array}{c} \left[\left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_i^{\mathcal{TL}})}\right)^{\bigcap_{i=1}^{n}\chi_i}\right)^{2\Xi}, \left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_i^{\mathcal{TU}})}\right)^{\bigcap_{i=1}^{n}\chi_i}\right)^{2\Xi}\right], \\ \left[\left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_i^{\mathcal{IL}})}\right)^{\bigcap_{i=1}^{n}\chi_i}\right)^{\Xi}, \left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_i^{\mathcal{IU}})}\right)^{\bigcap_{i=1}^{n}\chi_i}\right)^{\Xi}\right], \\ \left[(\alpha_3\Upsilon_i^{\mathcal{FL}})^{\bigcap_{i=1}^{n}\chi_i}, (\alpha_3\Upsilon_i^{\mathcal{FU}})^{\bigcap_{i=1}^{n}\chi_i}\right] \end{array}\right]$$

$$= \left[\begin{array}{c} \left[\left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_i^{\mathcal{TL}})}\right)\right)^{2\Xi}, \left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_i^{\mathcal{TU}})}\right)\right)^{2\Xi}\right], \\ \left[\left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_i^{\mathcal{IL}})}\right)\right)^{\Xi}, \left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_i^{\mathcal{IU}})}\right)\right)^{\Xi}\right], \\ \left[(\alpha_3\Upsilon_i^{\mathcal{FL}}), (\alpha_3\Upsilon_i^{\mathcal{FU}})\right] \end{array}\right]$$

$$= \overline{\Theta}.$$

Theorem 5.4. *Let* $\overline{\Theta}_i = \left\langle [\alpha_1\Upsilon_{ij}^{\mathcal{TL}}, \alpha_1\Upsilon_{ij}^{\mathcal{TU}}], [\alpha_2\Upsilon_{ij}^{\mathcal{IL}}, \alpha_2\Upsilon_{ij}^{\mathcal{IU}}][\alpha_3\Upsilon_{ij}^{\mathcal{FL}}, \alpha_3\Upsilon_{ij}^{\mathcal{FU}}] \right\rangle (i = 1 \text{ to } n); (j = 1 \text{ to } i_j)$ *be the SRDioNIVWA, where*
$\widehat{\alpha_1\Upsilon^{\mathcal{TL}}} = \inf \alpha_1\Upsilon_{ij}^{\mathcal{TL}}$, $\overleftarrow{\alpha_1\Upsilon^{\mathcal{TL}}} = \sup \alpha_1\Upsilon_{ij}^{\mathcal{TL}}$, $\widehat{\alpha_1\Upsilon^{\mathcal{TU}}} = \inf \alpha_1\Upsilon_{ij}^{\mathcal{TU}}$, $\overleftarrow{\alpha_1\Upsilon^{\mathcal{TU}}} = \sup \alpha_1\Upsilon_{ij}^{\mathcal{TU}}$,
$\widehat{\alpha_2\Upsilon^{\mathcal{IL}}} = \inf \alpha_2\Upsilon_{ij}^{\mathcal{IL}}$, $\overleftarrow{\alpha_2\Upsilon^{\mathcal{IL}}} = \sup \alpha_2\Upsilon_{ij}^{\mathcal{IL}}$, $\widehat{\alpha_2\Upsilon^{\mathcal{IU}}} = \inf \alpha_2\Upsilon_{ij}^{\mathcal{IU}}$, $\overleftarrow{\alpha_2\Upsilon^{\mathcal{IU}}} = \sup \alpha_2\Upsilon_{ij}^{\mathcal{IU}}$,
$\widehat{\alpha_3\Upsilon^{\mathcal{FL}}} = \inf \alpha_3\Upsilon_{ij}^{\mathcal{FL}}$, $\overleftarrow{\alpha_3\Upsilon^{\mathcal{FL}}} = \sup \alpha_3\Upsilon_{ij}^{\mathcal{FL}}$, $\widehat{\alpha_3\Upsilon^{\mathcal{FU}}} = \inf \alpha_3\Upsilon_{ij}^{\mathcal{FU}}$, $\overleftarrow{\alpha_3\Upsilon^{\mathcal{FU}}} = \sup \alpha_3\Upsilon_{ij}^{\mathcal{FU}}$.
Then, $\left\langle [\widehat{\alpha_1\Upsilon^{\mathcal{TL}}}, \widehat{\alpha_1\Upsilon^{\mathcal{TU}}}], [\widehat{\alpha_2\Upsilon^{\mathcal{IL}}}, \widehat{\alpha_2\Upsilon^{\mathcal{IU}}}], [\overleftarrow{\alpha_3\Upsilon^{\mathcal{FL}}}, \overleftarrow{\alpha_3\Upsilon^{\mathcal{FU}}}] \right\rangle$
$\leq SRDioNIVWA(\overline{\Theta}_1, \overline{\Theta}_2, ..., \overline{\Theta}_n)$
$\leq \left\langle [\overleftarrow{\alpha_1\Upsilon^{\mathcal{TL}}}, \overleftarrow{\alpha_1\Upsilon^{\mathcal{TU}}}], [\overleftarrow{\alpha_2\Upsilon^{\mathcal{IL}}}, \overleftarrow{\alpha_2\Upsilon^{\mathcal{IU}}}], [\widehat{\alpha_3\Upsilon^{\mathcal{FL}}}, \widehat{\alpha_3\Upsilon^{\mathcal{FU}}}] \right\rangle$, *where* $1 \leq i \leq n,\ j = 1, 2, ..., i_j$ *(boundedness property).*

Proof. Since, $\widehat{\alpha_1\Upsilon^{\mathcal{TL}}} = \inf \alpha_1\Upsilon_{ij}^{\mathcal{TL}}$, $\overleftarrow{\alpha_1\Upsilon^{\mathcal{TL}}} = \sup \alpha_1\Upsilon_{ij}^{\mathcal{TL}}$
$\widehat{\alpha_1\Upsilon^{\mathcal{TU}}} = \inf \alpha_1\Upsilon_{ij}^{\mathcal{TU}}$, $\overleftarrow{\alpha_1\Upsilon^{\mathcal{TU}}} = \sup \alpha_1\Upsilon_{ij}^{\mathcal{TU}}$ and $\widehat{\alpha_1\Upsilon^{\mathcal{TL}}} \leq \alpha_1\Upsilon_{ij}^{\mathcal{TL}} \leq \overleftarrow{\alpha_1\Upsilon^{\mathcal{TL}}}$ and $\widehat{\alpha_1\Upsilon^{\mathcal{TU}}} \leq \alpha_1\Upsilon_{ij}^{\mathcal{TU}} \leq \overleftarrow{\alpha_1\Upsilon^{\mathcal{TU}}}$. Now,

$$\begin{aligned}
\widehat{\alpha_1\Upsilon^{\mathcal{TL}}} + \widehat{\alpha_1\Upsilon^{\mathcal{TU}}} &= \left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[2\Xi]{(\widehat{\alpha_1\Upsilon^{\mathcal{TL}}})}\right)^{\chi_i}\right)^{2\Xi} + \left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[2\Xi]{(\widehat{\alpha_1\Upsilon^{\mathcal{TU}}})}\right)^{\chi_i}\right)^{2\Xi} \\
&\leq \left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_{ij}^{\mathcal{TL}})}\right)^{\chi_i}\right)^{2\Xi} + \left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_{ij}^{\mathcal{TU}})}\right)^{\chi_i}\right)^{2\Xi} \\
&\leq \left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[2\Xi]{(\overleftarrow{\alpha_1\Upsilon^{\mathcal{TL}}})}\right)^{\chi_i}\right)^{2\Xi} + \left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[2\Xi]{(\overleftarrow{\alpha_1\Upsilon^{\mathcal{TU}}})}\right)^{\chi_i}\right)^{2\Xi} \\
&= \overleftarrow{\alpha_1\Upsilon^{\mathcal{TL}}} + \overleftarrow{\alpha_1\Upsilon^{\mathcal{TU}}}.
\end{aligned}$$

Since, $\widehat{\alpha_2\Upsilon^{\mathcal{IL}}} = \inf \alpha_2\Upsilon_{ij}^{\mathcal{IL}}$, $\overleftarrow{\alpha_2\Upsilon^{\mathcal{IL}}} = \sup \alpha_2\Upsilon_{ij}^{\mathcal{IL}}$
$\widehat{\alpha_2\Upsilon^{\mathcal{IU}}} = \inf \alpha_2\Upsilon_{ij}^{\mathcal{IU}}$, $\overleftarrow{\alpha_2\Upsilon^{\mathcal{IU}}} = \sup \alpha_2\Upsilon_{ij}^{\mathcal{IU}}$ and $\widehat{\alpha_2\Upsilon^{\mathcal{IL}}} \leq \alpha_2\Upsilon_{ij}^{\mathcal{IL}} \leq \overleftarrow{\alpha_2\Upsilon^{\mathcal{IL}}}$ and $\widehat{\alpha_2\Upsilon^{\mathcal{IU}}} \leq \alpha_2\Upsilon_{ij}^{\mathcal{IU}} \leq$

Received: April 28, 2022 Accepted: November 15, 2022

$\overleftarrow{\alpha_2\Upsilon^{\mathcal{IU}}}$. Now,

$$\begin{aligned}
\widehat{\alpha_2\Upsilon^{\mathcal{IL}}}+\widehat{\alpha_2\Upsilon^{\mathcal{IU}}} &= \left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[\Xi]{(\widehat{\alpha_2\Upsilon^{\mathcal{IL}}})}\right)^{\chi_i}\right)^{\Xi}+\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[\Xi]{(\widehat{\alpha_2\Upsilon^{\mathcal{IU}}})}\right)^{\chi_i}\right)^{\Xi}\\
&\leq \left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_{ij}^{\mathcal{IL}})}\right)^{\chi_i}\right)^{\Xi}+\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_{ij}^{\mathcal{IU}})}\right)^{\chi_i}\right)^{\Xi}\\
&\leq \left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[\Xi]{(\overleftarrow{\alpha_2\Upsilon^{\mathcal{IL}}})}\right)^{\chi_i}\right)^{\Xi}+\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[\Xi]{(\overleftarrow{\alpha_2\Upsilon^{\mathcal{IU}}})}\right)^{\chi_i}\right)^{\Xi}\\
&= \overleftarrow{\alpha_2\Upsilon^{\mathcal{IL}}}+\overleftarrow{\alpha_2\Upsilon^{\mathcal{IU}}}.
\end{aligned}$$

Since, $\widehat{\alpha_3\Upsilon^{\mathcal{FL}}}=\inf\alpha_3\Upsilon_{ij}^{\mathcal{FL}}$, $\overleftarrow{\alpha_3\Upsilon^{\mathcal{FL}}}=\sup\alpha_3\Upsilon_{ij}^{\mathcal{FL}}$
$\widehat{\alpha_3\Upsilon^{\mathcal{FU}}}=\inf\alpha_3\Upsilon_{ij}^{\mathcal{FU}}$, $\overleftarrow{\alpha_3\Upsilon^{\mathcal{FU}}}=\sup\alpha_3\Upsilon_{ij}^{\mathcal{FU}}$ and $\widehat{\alpha_3\Upsilon^{\mathcal{FL}}}\leq\alpha_3\Upsilon_{ij}^{\mathcal{FL}}\leq\overleftarrow{\alpha_3\Upsilon^{\mathcal{FL}}}$ and $\widehat{\alpha_3\Upsilon^{\mathcal{FU}}}\leq\alpha_3\Upsilon_{ij}^{\mathcal{FU}}\leq$ $\overleftarrow{\alpha_3\Upsilon^{\mathcal{FU}}}$. Now,

$$\begin{aligned}
\widehat{\alpha_3\Upsilon^{\mathcal{FL}}}+\widehat{\alpha_3\Upsilon^{\mathcal{FU}}} &= \bigcup_{i=1}^{n}(\widehat{\alpha_3\Upsilon^{\mathcal{FL}}})^{\chi_i}+\bigcup_{i=1}^{n}(\widehat{\alpha_3\Upsilon^{\mathcal{FU}}})^{\chi_i}\\
&\leq \bigcup_{i=1}^{n}(\alpha_3\Upsilon_{ij}^{\mathcal{FL}})^{\chi_i}+\bigcup_{i=1}^{n}(\alpha_3\Upsilon_{ij}^{\mathcal{FU}})^{\chi_i}\\
&\leq \bigcup_{i=1}^{n}(\overleftarrow{\alpha_3\Upsilon^{\mathcal{FL}}})^{\chi_i}+\bigcup_{i=1}^{n}(\overleftarrow{\alpha_3\Upsilon^{\mathcal{FU}}})^{\chi_i}\\
&= \overleftarrow{\alpha_3\Upsilon^{\mathcal{FL}}}+\overleftarrow{\alpha_3\Upsilon^{\mathcal{FU}}}.
\end{aligned}$$

Hence,

$$\begin{aligned}
&\left[\begin{array}{c}\dfrac{\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[2\Xi]{(\widehat{\alpha_1\Upsilon^{\mathcal{TL}}})}\right)^{\chi_i}\right)^{2\Xi}+\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[2\Xi]{(\widehat{\alpha_1\Upsilon^{\mathcal{TU}}})}\right)^{\chi_i}\right)^{2\Xi}}{2}\\
-\dfrac{\sqrt{\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[\Xi]{(\widehat{\alpha_2\Upsilon^{\mathcal{IL}}})}\right)^{\chi_i}\right)^{\Xi}}+\sqrt{\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[\Xi]{(\widehat{\alpha_2\Upsilon^{\mathcal{IU}}})}\right)^{\chi_i}\right)^{\Xi}}}{2}\\
+1-\dfrac{\sqrt{\left(\bigcup_{i=1}^{n}(\overleftarrow{\alpha_3\Upsilon^{\mathcal{FL}}})^{\chi_i}\right)}+\sqrt{\left(\bigcup_{i=1}^{n}(\overleftarrow{\alpha_3\Upsilon^{\mathcal{FU}}})^{\chi_i}\right)}}{2}\end{array}\right]\\
\leq &\left[\begin{array}{c}\dfrac{\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_{ij}^{\mathcal{TL}})}\right)^{\chi_i}\right)^{2\Xi}+\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_{ij}^{\mathcal{TU}})}\right)^{\chi_i}\right)^{2\Xi}}{2}\\
-\dfrac{\sqrt{\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_{ij}^{\mathcal{IL}})}\right)^{\chi_i}\right)^{\Xi}}+\sqrt{\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_{ij}^{\mathcal{IU}})}\right)^{\chi_i}\right)^{\Xi}}}{2}\\
+1-\dfrac{\sqrt{\left(\bigcup_{i=1}^{n}(\alpha_3\Upsilon_{ij}^{\mathcal{FL}})^{\chi_i}\right)}+\sqrt{\left(\bigcup_{i=1}^{n}(\alpha_3\Upsilon_{ij}^{\mathcal{FU}})^{\chi_i}\right)}}{2}\end{array}\right]
\end{aligned}$$

Received: April 28, 2022 Accepted: November 15, 2022

$$\leq \left[\begin{array}{c} \dfrac{\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[2\Xi]{\overleftarrow{(\alpha_1\Upsilon^{\mathcal{TL}}_{ij})}}\right)^{\chi_i}\right)^{2\Xi}+\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[2\Xi]{\overleftarrow{(\alpha_1\Upsilon^{\mathcal{TU}}_{ij})}}\right)^{\chi_i}\right)^{2\Xi}}{2} \\ -\dfrac{\sqrt{\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[\Xi]{\overleftarrow{(\alpha_2\Upsilon^{\mathcal{IL}}_{ij})}}\right)^{\chi_i}\right)^{\Xi}}+\sqrt{\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[\Xi]{\overleftarrow{(\alpha_2\Upsilon^{\mathcal{IU}}_{ij})}}\right)^{\chi_i}\right)^{\Xi}}}{2} \\ +1-\dfrac{\sqrt{\left(\bigcup_{i=1}^{n}(\alpha_3\Upsilon^{\mathcal{FL}}_{ij})^{\chi_i}\right)}+\sqrt{\left(\bigcup_{i=1}^{n}(\alpha_3\Upsilon^{\mathcal{FU}}_{ij})^{\chi_i}\right)}}{2} \end{array} \right].$$

Therefore, $\left\langle [\widehat{\alpha_1\Upsilon^{\mathcal{TL}}},\widehat{\alpha_1\Upsilon^{\mathcal{TU}}}],[\widehat{\alpha_2\Upsilon^{\mathcal{IL}}},\widehat{\alpha_2\Upsilon^{\mathcal{IU}}}],[\overleftarrow{\alpha_3\Upsilon^{\mathcal{FL}}},\overleftarrow{\alpha_3\Upsilon^{\mathcal{FU}}}] \right\rangle$
$\leq SRDioNIVWA(\overline{\Theta}_1,\overline{\Theta}_2,...,\overline{\Theta}_n)$
$\leq \left\langle [\overleftarrow{\alpha_1\Upsilon^{\mathcal{TL}}},\overleftarrow{\alpha_1\Upsilon^{\mathcal{TU}}}],[\overleftarrow{\alpha_2\Upsilon^{\mathcal{IL}}},\overleftarrow{\alpha_2\Upsilon^{\mathcal{IU}}}],[\widehat{\alpha_3\Upsilon^{\mathcal{FL}}},\widehat{\alpha_3\Upsilon^{\mathcal{FU}}}] \right\rangle.$

Theorem 5.5. *Let* $\overline{\Theta}_i=\left\langle [\alpha_1\Upsilon^{\mathcal{TL}}_{t_{ij}},\alpha_1\Upsilon^{\mathcal{TU}}_{t_{ij}}],[\alpha_2\Upsilon^{\mathcal{IL}}_{t_{ij}},\alpha_2\Upsilon^{\mathcal{IU}}_{t_{ij}}],[\alpha_3\Upsilon^{\mathcal{FL}}_{t_{ij}},\alpha_3\Upsilon^{\mathcal{FU}}_{t_{ij}}] \right\rangle$ *and* $\overline{W}_i=\left\langle [\alpha_1\Upsilon^{\mathcal{TL}}_{h_{ij}},\alpha_1\Upsilon^{\mathcal{TU}}_{h_{ij}}],[\alpha_2\Upsilon^{\mathcal{IL}}_{h_{ij}},\alpha_2\right.$ *be the two families of SRDioNIVWAs. For any i, if there is* $\sqrt{\left(\alpha_1\Upsilon^{\mathcal{TL}}_{t_{ij}}\right)}+\sqrt{\left(\alpha_1\Upsilon^{\mathcal{TU}}_{t_{ij}}\right)}\leq\sqrt{\left(\alpha_1\Upsilon^{\mathcal{TL}}_{h_{ij}}\right)}+\sqrt{\left(\alpha_1\Upsilon^{\mathcal{TU}}_{h_{ij}}\right)}$ *and* $\sqrt{\left(\alpha_2\Upsilon^{\mathcal{IL}}_{t_{ij}}\right)}+\sqrt{\left(\alpha_2\Upsilon^{\mathcal{IU}}_{t_{ij}}\right)}\leq\sqrt{\left(\alpha_2\Upsilon^{\mathcal{IL}}_{h_{ij}}\right)}+\sqrt{\left(\alpha_2\Upsilon^{\mathcal{IU}}_{h_{ij}}\right)}$ *and* $\left(\alpha_3\Upsilon^{\mathcal{FL}}_{t_{ij}}\right)+\left(\alpha_3\Upsilon^{\mathcal{FU}}_{t_{ij}}\right)\geq\left(\alpha_3\Upsilon^{\mathcal{FL}}_{h_{ij}}\right)+\left(\alpha_3\Upsilon^{\mathcal{FU}}_{h_{ij}}\right)$ *or* $\overline{\Theta}_i\leq\overline{W}_i$*, then* $SRDioNIVWA\left(\overline{\Theta}_1,\overline{\Theta}_2,...,\overline{\Theta}_n\right)\leq SRDioNIVWA\left(\overline{W}_1,\overline{W}_2,...,\overline{W}_n\right)$*, where* $(i=1\ to\ n);(j=1\ to\ i_j)$ *(monotonicity property).*

Proof. For any i, $\sqrt{\left(\alpha_1\Upsilon^{\mathcal{TL}}_{t_{ij}}\right)}+\sqrt{\left(\alpha_1\Upsilon^{\mathcal{TU}}_{t_{ij}}\right)}\leq\sqrt{\left(\alpha_1\Upsilon^{\mathcal{TL}}_{h_{ij}}\right)}+\sqrt{\left(\alpha_1\Upsilon^{\mathcal{TU}}_{h_{ij}}\right)}$.
Therefore, $1-\sqrt[2\Xi]{\left(\alpha_1\Upsilon^{\mathcal{TL}}_{t_i}\right)}+1-\sqrt[2\Xi]{\left(\alpha_1\Upsilon^{\mathcal{TU}}_{t_i}\right)}\geq 1-\sqrt[2\Xi]{\left(\alpha_1\Upsilon^{\mathcal{TL}}_{h_i}\right)}+1-\sqrt[2\Xi]{\left(\alpha_1\Upsilon^{\mathcal{TU}}_{h_i}\right)}$.
Hence,

$$\bigcup_{i=1}^{n}\left(1-\sqrt[2\Xi]{\left(\alpha_1\Upsilon^{\mathcal{TL}}_{t_i}\right)}\right)^{\chi_i}+\bigcup_{i=1}^{n}\left(1-\sqrt[2\Xi]{\left(\alpha_1\Upsilon^{\mathcal{TU}}_{t_i}\right)}\right)^{\chi_i}\geq \bigcup_{i=1}^{n}\left(1-\sqrt[2\Xi]{\left(\alpha_1\Upsilon^{\mathcal{TL}}_{h_i}\right)}\right)^{\chi_i}+\bigcup_{i=1}^{n}\left(1-\sqrt[2\Xi]{\left(\alpha_1\Upsilon^{\mathcal{TU}}_{h_i}\right)}\right)^{\chi_i}$$

and
$$\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[2\Xi]{\left(\alpha_1\Upsilon^{\mathcal{TL}}_{t_i}\right)}\right)^{\chi_i}\right)^{2\Xi}+\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[2\Xi]{\left(\alpha_1\Upsilon^{\mathcal{TU}}_{t_i}\right)}\right)^{\chi_i}\right)^{2\Xi}\leq \left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[2\Xi]{\left(\alpha_1\Upsilon^{\mathcal{TL}}_{h_i}\right)}\right)^{\chi_i}\right)^{2\Xi}+\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[2\Xi]{\left(\alpha_1\Upsilon^{\mathcal{TU}}_{h_i}\right)}\right)^{\chi_i}\right)^{2\Xi}.$$

For any i, $\sqrt{\left(\alpha_2\Upsilon^{\mathcal{IL}}_{t_{ij}}\right)}+\sqrt{\left(\alpha_2\Upsilon^{\mathcal{IU}}_{t_{ij}}\right)}\leq\sqrt{\left(\alpha_2\Upsilon^{\mathcal{IL}}_{h_{ij}}\right)}+\sqrt{\left(\alpha_2\Upsilon^{\mathcal{IU}}_{h_{ij}}\right)}$.
Therefore, $1-\sqrt[\Xi]{\left(\alpha_2\Upsilon^{\mathcal{IL}}_{t_i}\right)}+1-\sqrt[\Xi]{\left(\alpha_2\Upsilon^{\mathcal{IU}}_{t_i}\right)}\geq 1-\sqrt[\Xi]{\left(\alpha_2\Upsilon^{\mathcal{IL}}_{h_i}\right)}+1-\sqrt[\Xi]{\left(\alpha_2\Upsilon^{\mathcal{IU}}_{h_i}\right)}$.
Hence,

$$\bigcup_{i=1}^{n}\left(1-\sqrt[\Xi]{\left(\alpha_2\Upsilon^{\mathcal{IL}}_{t_i}\right)}\right)^{\chi_i}+\bigcup_{i=1}^{n}\left(1-\sqrt[\Xi]{\left(\alpha_2\Upsilon^{\mathcal{IU}}_{t_i}\right)}\right)^{\chi_i}\geq \bigcup_{i=1}^{n}\left(1-\sqrt[\Xi]{\left(\alpha_2\Upsilon^{\mathcal{IL}}_{h_i}\right)}\right)^{\chi_i}+\bigcup_{i=1}^{n}\left(1-\sqrt[\Xi]{\left(\alpha_2\Upsilon^{\mathcal{IU}}_{h_i}\right)2}\right)^{\chi_i}$$

and
$$\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[\Xi]{\left(\alpha_2\Upsilon^{\mathcal{IL}}_{t_i}\right)}\right)^{\chi_i}\right)^{\Xi}+\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[\Xi]{\left(\alpha_2\Upsilon^{\mathcal{IU}}_{t_i}\right)}\right)^{\chi_i}\right)^{\Xi}\leq \left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[\Xi]{\left(\alpha_2\Upsilon^{\mathcal{IL}}_{h_i}\right)}\right)^{\chi_i}\right)^{\Xi}+\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[\Xi]{\left(\alpha_2\Upsilon^{\mathcal{IU}}_{h_i}\right)}\right)^{\chi_i}\right)^{\Xi}.$$

For any i, $\left(\alpha_3\Upsilon^{\mathcal{FL}}_{t_{ij}}\right)+\left(\alpha_3\Upsilon^{\mathcal{FU}}_{t_{ij}}\right)\geq\left(\alpha_3\Upsilon^{\mathcal{FL}}_{h_{ij}}\right)+\left(\alpha_3\Upsilon^{\mathcal{FU}}_{h_{ij}}\right)$.

https://doi.org/10.54216/IJNS.190401
Received: April 28, 2022 Accepted: November 15, 2022

Therefore, $1-\frac{\left(\bigcup_{i=1}^{n}\alpha_3\Upsilon_{t_{ij}}^{\mathcal{FL}}\right)+\left(\bigcup_{i=1}^{n}\alpha_3\Upsilon_{t_{ij}}^{\mathcal{FU}}\right)}{2}\leq 1-\frac{\left(\bigcup_{i=1}^{n}\alpha_3\Upsilon_{h_{ij}}^{\mathcal{FL}}\right)+\left(\bigcup_{i=1}^{n}\alpha_3\Upsilon_{h_{ij}}^{\mathcal{FU}}\right)}{2}$.
Hence,

$$\leq\left[\begin{array}{c}\frac{\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_{hi}^{\mathcal{TL}})}\right)^{\chi_i}\right)^{2\Xi}+\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_{hi}^{\mathcal{TU}})}\right)^{\chi_i}\right)^{2\Xi}}{2}\\ -\frac{\sqrt{\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_{hi}^{\mathcal{IL}})}\right)^{\chi_i}\right)^{2\Xi}}+\sqrt{\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_{hi}^{\mathcal{IU}})}\right)^{\chi_i}\right)^{2\Xi}}}{2}\\ +1-\frac{\sqrt{\left(\bigcup_{i=1}^{n}(\alpha_3\Upsilon_{hij}^{\mathcal{FL}})\right)}+\sqrt{\left(\bigcup_{i=1}^{n}(\alpha_3\Upsilon_{hij}^{\mathcal{FU}})\right)}}{2}\end{array}\right].$$

Hence, $SRDioNIVWA\left(\overline{\Theta}_1,\overline{\Theta}_2,...,\overline{\Theta}_n\right)\leq SRDioNIVWA\left(\overline{W}_1,\overline{W}_2,...,\overline{W}_n\right)$.

5.2 SRDioNIV weighted geometric(SRDioNIVWG) operator

Definition 5.6. Let $\overline{\Theta}_i=\left\langle[\alpha_1\Upsilon_i^{\mathcal{TL}},\alpha_1\Upsilon_i^{\mathcal{TU}}],[\alpha_2\Upsilon_i^{\mathcal{IL}},\alpha_2\Upsilon_i^{\mathcal{IU}}],[\alpha_3\Upsilon_i^{\mathcal{FL}},\alpha_3\Upsilon_i^{\mathcal{FU}}]\right\rangle$ be the family of SRDioNIVNs. Then SRDioNIVWG $(\overline{\Theta}_1,\overline{\Theta}_2,...,\overline{\Theta}_n)=\bigcup_{i=1}^{n}\overline{\Theta}_i^{\chi_i}$ $(i=1\ to\ n)$.

Theorem 5.7. *Let* $\overline{\Theta}_i=\left\langle[\alpha_1\Upsilon_i^{\mathcal{TL}},\alpha_1\Upsilon_i^{\mathcal{TU}}],[\alpha_3\Upsilon_i^{\mathcal{FL}},\alpha_3\Upsilon_i^{\mathcal{FU}}]\right\rangle$ *be the family of SRDioNIVNs. Prove that*

$$\left[\begin{array}{c}\left[\bigcup_{i=1}^{n}(\alpha_1\Upsilon_i^{\mathcal{TL}})^{\chi_i},\bigcup_{i=1}^{n}(\alpha_1\Upsilon_i^{\mathcal{TU}})^{\chi_i}\right],\\ \left[\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_i^{\mathcal{IL}})}\right)^{\chi_i}\right)^{\Xi},\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_i^{\mathcal{IU}})}\right)^{\chi_i}\right)^{\Xi}\right],\\ \left[\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[2\Xi]{(\alpha_3\Upsilon_i^{\mathcal{FL}})}\right)^{\chi_i}\right)^{2\Xi},\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[2\Xi]{(\alpha_3\Upsilon_i^{\mathcal{FU}})}\right)^{\chi_i}\right)^{2\Xi}\right]\end{array}\right].$$

Theorem 5.8. *If all* $\overline{\Theta}_i=\left\langle[\alpha_1\Upsilon_i^{\mathcal{TL}},\alpha_1\Upsilon_i^{\mathcal{TU}}],[\alpha_2\Upsilon_i^{\mathcal{IL}},\alpha_2\Upsilon_i^{\mathcal{IU}}][\alpha_3\Upsilon_i^{\mathcal{FL}},\alpha_3\Upsilon_i^{\mathcal{FU}}]\right\rangle(i=1,2,...,n)$ *are equal, then SRDioNIVWG*$(\overline{\Theta}_1,\overline{\Theta}_2,...,\overline{\Theta}_n)=\overline{\Theta}$.

Using the SRDioNIVWG operator, the boundedness and monotonicity properties are met.

5.3 Generalized SRDioNIVWA (GSRDioNIVWA) operator

Definition 5.9. Let $\overline{\Theta}_i=\left\langle[\alpha_1\Upsilon_i^{\mathcal{TL}},\alpha_1\Upsilon_i^{\mathcal{TU}}],[\alpha_2\Upsilon_i^{\mathcal{IL}},\alpha_2\Upsilon_i^{\mathcal{IU}}],[\alpha_3\Upsilon_i^{\mathcal{FL}},\alpha_3\Upsilon_i^{\mathcal{FU}}]\right\rangle$ be the family of SRDioNIVN. Then GSRDioNIVWA $(\overline{\Theta}_1,\overline{\Theta}_2,...,\overline{\Theta}_n)=\left(\bigcap_{i=1}^{n}\chi_i\overline{\Theta}_i^{\Xi}\right)^{1/\Xi}$.

Theorem 5.10. *Let* $\overline{\Theta}_i=\left\langle[\alpha_1\Upsilon_i^{\mathcal{TL}},\alpha_1\Upsilon_i^{\mathcal{TU}}],[\alpha_2\Upsilon_i^{\mathcal{IL}},\alpha_2\Upsilon_i^{\mathcal{IU}}],[\alpha_3\Upsilon_i^{\mathcal{FL}},\alpha_3\Upsilon_i^{\mathcal{FU}}]\right\rangle$ *be the family of SRDioNIVNs. Then GSRDioNIVWA* $(\overline{\Theta}_1,\overline{\Theta}_2,...,\overline{\Theta}_n)=$

https://doi.org/10.54216/IJNS.190401
Received: April 28, 2022 Accepted: November 15, 2022

$$\left[\begin{array}{c} \left[\left(\left(1-\bigcup_{i=1}^{n}\left(1-\left(\sqrt[2\Xi]{(\alpha_1\Upsilon_i^{\mathcal{TL}})^{\Xi}}\right)\right)^{\chi_i}\right)^{2\Xi}\right)^{\Xi}, \left(\left(1-\bigcup_{i=1}^{n}\left(1-\left(\sqrt[2\Xi]{(\alpha_1\Upsilon_i^{\mathcal{TU}})^{\Xi}}\right)\right)^{\chi_i}\right)^{2\Xi}\right)^{\Xi}\right], \\ \left[\left(\left(1-\bigcup_{i=1}^{n}\left(1-\left(\sqrt[\Xi]{(\alpha_2\Upsilon_i^{\mathcal{IL}})^{\Xi}}\right)\right)^{\chi_i}\right)^{\Xi}\right)^{\Xi}, \left(\left(1-\bigcup_{i=1}^{n}\left(1-\left(\sqrt[\Xi]{(\alpha_2\Upsilon_i^{\mathcal{IU}})^{\Xi}}\right)\right)^{\chi_i}\right)^{\Xi}\right)^{\Xi}\right], \\ \left[\begin{array}{c}\left(1-\left(1-\sqrt[2\Xi]{\bigcup_{i=1}^{n}\left(\left(1-\left(1-\sqrt[2\Xi]{(\alpha_3\Upsilon_i^{\mathcal{FL}})}\right)^{\Xi}\right)^{2\Xi}\right)^{\chi_i}}\right)^{\Xi}\right)^{2\Xi}, \\ \left(1-\left(1-\sqrt[2\Xi]{\bigcup_{i=1}^{n}\left(\left(1-\left(1-\sqrt[2\Xi]{(\alpha_3\Upsilon_i^{\mathcal{FU}})}\right)^{\Xi}\right)^{2\Xi}\right)^{\chi_i}}\right)^{\Xi}\right)^{2\Xi}\end{array}\right]\end{array}\right].$$

Proof. It must be demonstrated that,

$$\bigcap_{i=1}^{n}\chi_i\overline{\Theta}_i^{\Xi} = \left[\begin{array}{c} \left[\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_i^{\mathcal{TL}})^{\Xi}}\right)^{\chi_i}\right)^{2\Xi}, \left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_i^{\mathcal{TU}})^{\Xi}}\right)^{\chi_i}\right)^{2\Xi}\right], \\ \left[\left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_i^{\mathcal{IL}})^{\Xi}}\right)^{\chi_i}\right)^{\Xi}, \left(1-\bigcup_{i=1}^{n}\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_i^{\mathcal{IU}})^{\Xi}}\right)^{\chi_i}\right)^{\Xi}\right], \\ \left[\bigcup_{i=1}^{n}\left(\left(1-\left(1-\sqrt[2\Xi]{(\alpha_3\Upsilon_i^{\mathcal{FL}})}\right)^{\Xi}\right)^{2\Xi}\right)^{\chi_i}, \bigcup_{i=1}^{n}\left(\left(1-\left(1-\sqrt[2\Xi]{(\alpha_3\Upsilon_i^{\mathcal{FU}})}\right)^{\Xi}\right)^{2\Xi}\right)^{\chi_i}\right]\end{array}\right]$$

. Put $n = 2$, $\chi_1\Theta_1 \oplus \chi_2\Theta_2 =$

$$\left[\begin{array}{c} \left[\begin{array}{c}\left(\begin{array}{c}\sqrt[2\Xi]{\left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_1^{\mathcal{TL}})^{\Xi}}\right)^{\chi_1}\right)^{2\Xi}} + \sqrt[2\Xi]{\left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_2^{\mathcal{TL}})^{\Xi}}\right)^{\chi_2}\right)^{2\Xi}} \\ -\sqrt[2\Xi]{\left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_1^{\mathcal{TL}})^{\Xi}}\right)^{\chi_1}\right)^{2\Xi}}\cdot\sqrt[2\Xi]{\left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_2^{\mathcal{TL}})^{\Xi}}\right)^{\chi_2}\right)^{2\Xi}}\end{array}\right)^{2\Xi}, \\ \left(\begin{array}{c}\sqrt[2\Xi]{\left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_1^{\mathcal{TU}})^{\Xi}}\right)^{\chi_1}\right)^{2\Xi}} + \sqrt[2\Xi]{\left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_2^{\mathcal{TU}})^{\Xi}}\right)^{\chi_2}\right)^{2\Xi}} \\ -\sqrt[2\Xi]{\left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_1^{\mathcal{TU}})^{\Xi}}\right)^{\chi_1}\right)^{2\Xi}}\cdot\sqrt[2\Xi]{\left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_2^{\mathcal{TU}})^{\Xi}}\right)^{\chi_2}\right)^{2\Xi}}\end{array}\right)^{2\Xi}\end{array}\right], \\ \left[\begin{array}{c}\left(\begin{array}{c}\sqrt[\Xi]{\left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_1^{\mathcal{IL}})^{\Xi}}\right)^{\chi_1}\right)^{\Xi}} + \sqrt[\Xi]{\left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_2^{\mathcal{IL}})^{\Xi}}\right)^{\chi_2}\right)^{\Xi}} \\ -\sqrt[\Xi]{\left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_1^{\mathcal{IL}})^{\Xi}}\right)^{\chi_1}\right)^{\Xi}}\cdot\sqrt[\Xi]{\left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_2^{\mathcal{IL}})^{\Xi}}\right)^{\chi_2}\right)^{\Xi}}\end{array}\right)^{\Xi}, \\ \left(\begin{array}{c}\sqrt[\Xi]{\left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_1^{\mathcal{IU}})^{\Xi}}\right)^{\chi_1}\right)^{\Xi}} + \sqrt[\Xi]{\left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_2^{\mathcal{IU}})^{\Xi}}\right)^{\chi_2}\right)^{\Xi}} \\ -\sqrt[\Xi]{\left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_1^{\mathcal{IU}})^{\Xi}}\right)^{\chi_1}\right)^{\Xi}}\cdot\sqrt[\Xi]{\left(1-\left(1-\sqrt[\Xi]{(\alpha_2\Upsilon_2^{\mathcal{IU}})^{\Xi}}\right)^{\chi_2}\right)^{\Xi}}\end{array}\right)^{\Xi}\end{array}\right], \\ \left[\begin{array}{c}\left(\left(1-\left(1-\sqrt[2\Xi]{(\alpha_3\Upsilon_1^{\mathcal{FL}})}\right)^{\Xi}\right)^{2\Xi}\right)^{\chi_1}\cdot\left(\left(1-\left(1-\sqrt[2\Xi]{(\alpha_3\Upsilon_2^{\mathcal{FL}})}\right)^{\Xi}\right)^{2\Xi}\right)^{\chi_1}, \\ \left(\left(1-\left(1-\sqrt[2\Xi]{(\alpha_3\Upsilon_1^{\mathcal{FU}})}\right)^{\Xi}\right)^{2\Xi}\right)^{\chi_1}\cdot\left(\left(1-\left(1-\sqrt[2\Xi]{(\alpha_3\Upsilon_2^{\mathcal{FU}})}\right)^{\Xi}\right)^{2\Xi}\right)^{\chi_1}\end{array}\right]\end{array}\right]$$

$$= \begin{bmatrix} \left[\left(1 - \bigcup_{i=1}^{2}\left(1 - \sqrt[2\Xi]{(\alpha_1 \Upsilon_i^{\mathcal{TL}})^{\Xi}}\right)^{\chi_i}\right)^{2\Xi}, \left(1 - \bigcup_{i=1}^{2}\left(1 - \sqrt[2\Xi]{(\alpha_1 \Upsilon_i^{\mathcal{TU}})^{\Xi}}\right)^{\chi_i}\right)^{2\Xi} \right], \\ \left[\left(1 - \bigcup_{i=1}^{2}\left(1 - \sqrt[\Xi]{(\alpha_2 \Upsilon_i^{\mathcal{IL}})^{\Xi}}\right)^{\chi_i}\right)^{\Xi}, \left(1 - \bigcup_{i=1}^{2}\left(1 - \sqrt[\Xi]{(\alpha_2 \Upsilon_i^{\mathcal{IU}})^{\Xi}}\right)^{\chi_i}\right)^{\Xi} \right], \\ \left[\bigcup_{i=1}^{2}\left(\left(1 - \left(1 - \sqrt[2\Xi]{(\alpha_3 \Upsilon_i^{\mathcal{FL}})}\right)^{\Xi}\right)^{2\Xi}\right)^{\chi_i}, \bigcup_{i=1}^{2}\left(\left(1 - \left(1 - \sqrt[2\Xi]{(\alpha_3 \Upsilon_i^{\mathcal{FU}})}\right)^{\Xi}\right)^{2\Xi}\right)^{\chi_i} \right] \end{bmatrix}.$$

In general,

$$\begin{bmatrix} \left[\left(1 - \bigcup_{i=1}^{l}\left(1 - \sqrt[2\Xi]{(\alpha_1 \Upsilon_i^{\mathcal{TL}})^{\Xi}}\right)^{\chi_i}\right)^{2\Xi}, \left(1 - \bigcup_{i=1}^{l}\left(1 - \sqrt[2\Xi]{(\alpha_1 \Upsilon_i^{\mathcal{TU}})^{\Xi}}\right)^{\chi_i}\right)^{2\Xi} \right], \\ \left[\left(1 - \bigcup_{i=1}^{l}\left(1 - \sqrt[\Xi]{(\alpha_2 \Upsilon_i^{\mathcal{IL}})^{\Xi}}\right)^{\chi_i}\right)^{\Xi}, \left(1 - \bigcup_{i=1}^{l}\left(1 - \sqrt[\Xi]{(\alpha_2 \Upsilon_i^{\mathcal{IU}})^{\Xi}}\right)^{\chi_i}\right)^{\Xi} \right], \\ \left[\bigcup_{i=1}^{l}\left(\left(1 - \left(1 - \sqrt[2\Xi]{(\alpha_3 \Upsilon_i^{\mathcal{FL}})}\right)^{\Xi}\right)^{2\Xi}\right)^{\chi_i}, \bigcup_{i=1}^{l}\left(\left(1 - \left(1 - \sqrt[2\Xi]{(\alpha_3 \Upsilon_i^{\mathcal{FU}})}\right)^{\Xi}\right)^{2\Xi}\right)^{\chi_i} \right] \end{bmatrix}.$$

If $n = l + 1$, then $\bigcap_{i=1}^{l} \chi_i \overline{\Theta}_i^{\Xi} + \chi_{l+1}\overline{\Theta}_{l+1}^{\Xi} = \bigcap_{i=1}^{l+1} \chi_i \overline{\Theta}_i^{\Xi}$.

Now, $\bigcap_{i=1}^{l} \chi_i \overline{\Theta}_i^{\Xi} + \chi_{l+1}\overline{\Theta}_{l+1}^{\Xi} = \bigcap_{i=1}^{l+1} \chi_i \overline{\Theta}_i^{\Xi} = \chi_1 \overline{\Theta}_1^{\Xi} \oplus \chi_2 \overline{\Theta}_2^{\Xi} \oplus ... \oplus \chi_l \overline{\Theta}_l^{\Xi} \oplus \chi_{l+1}\overline{\Theta}_{l+1}^{\Xi}$

$$= \begin{bmatrix} \begin{bmatrix} \begin{pmatrix} \sqrt[2\Xi]{\left(1 - \bigcup_{i=1}^{l}\left(1 - \sqrt[2\Xi]{(\alpha_1 \Upsilon_i^{\mathcal{TL}})^{\Xi}}\right)^{\chi_i}\right)^{2\Xi}} + \sqrt[2\Xi]{\left(1 - \left(1 - \sqrt[2\Xi]{(\alpha_1 \Upsilon_{l+1}^{\mathcal{TL}})^{\Xi}}\right)^{\chi_1}\right)^{2\Xi}} \\ - \sqrt[2\Xi]{\left(1 - \bigcup_{i=1}^{l}\left(1 - \sqrt[2\Xi]{(\alpha_1 \Upsilon_i^{\mathcal{TL}})^{\Xi}}\right)^{\chi_i}\right)^{2\Xi}} \cdot \sqrt[2\Xi]{\left(1 - \left(1 - \sqrt[2\Xi]{(\alpha_1 \Upsilon_{l+1}^{\mathcal{TL}})^{\Xi}}\right)^{\chi_1}\right)^{2\Xi}} \end{pmatrix}^{2\Xi}, \\ \begin{pmatrix} \sqrt[2\Xi]{\left(1 - \bigcup_{i=1}^{l}\left(1 - \sqrt[2\Xi]{(\alpha_1 \Upsilon_i^{\mathcal{TU}})^{\Xi}}\right)^{\chi_i}\right)^{2\Xi}} + \sqrt[2\Xi]{\left(1 - \left(1 - \sqrt[2\Xi]{(\alpha_1 \Upsilon_{l+1}^{\mathcal{TU}})^{\Xi}}\right)^{\chi_1}\right)^{2\Xi}} \\ - \sqrt[2\Xi]{\left(1 - \bigcup_{i=1}^{l}\left(1 - \sqrt[2\Xi]{(\alpha_1 \Upsilon_i^{\mathcal{TU}})^{\Xi}}\right)^{\chi_i}\right)^{2\Xi}} \cdot \sqrt[2\Xi]{\left(1 - \left(1 - \sqrt[2\Xi]{(\alpha_1 \Upsilon_{l+1}^{\mathcal{TU}})^{\Xi}}\right)^{\chi_1}\right)^{2\Xi}} \end{pmatrix}^{2\Xi} \end{bmatrix}, \\ \begin{bmatrix} \begin{pmatrix} \sqrt[\Xi]{\left(1 - \bigcup_{i=1}^{l}\left(1 - \sqrt[\Xi]{(\alpha_2 \Upsilon_i^{\mathcal{IL}})^{\Xi}}\right)^{\chi_i}\right)^{\Xi}} + \sqrt[\Xi]{\left(1 - \left(1 - \sqrt[\Xi]{(\alpha_2 \Upsilon_{l+1}^{\mathcal{IL}})^{\Xi}}\right)^{\chi_1}\right)^{\Xi}} \\ - \sqrt[\Xi]{\left(1 - \bigcup_{i=1}^{l}\left(1 - \sqrt[\Xi]{(\alpha_2 \Upsilon_i^{\mathcal{IL}})^{\Xi}}\right)^{\chi_i}\right)^{\Xi}} \cdot \sqrt[\Xi]{\left(1 - \left(1 - \sqrt[\Xi]{(\alpha_2 \Upsilon_{l+1}^{\mathcal{IL}})^{\Xi}}\right)^{\chi_1}\right)^{\Xi}} \end{pmatrix}^{\Xi}, \\ \begin{pmatrix} \sqrt[\Xi]{\left(1 - \bigcup_{i=1}^{l}\left(1 - \sqrt[\Xi]{(\alpha_2 \Upsilon_i^{\mathcal{IU}})^{\Xi}}\right)^{\chi_i}\right)^{\Xi}} + \sqrt[\Xi]{\left(1 - \left(1 - \sqrt[\Xi]{(\alpha_2 \Upsilon_{l+1}^{\mathcal{IU}})^{\Xi}}\right)^{\chi_1}\right)^{\Xi}} \\ - \sqrt[\Xi]{\left(1 - \bigcup_{i=1}^{l}\left(1 - \sqrt[\Xi]{(\alpha_2 \Upsilon_i^{\mathcal{IU}})^{\Xi}}\right)^{\chi_i}\right)^{\Xi}} \cdot \sqrt[\Xi]{\left(1 - \left(1 - \sqrt[\Xi]{(\alpha_2 \Upsilon_{l+1}^{\mathcal{IU}})^{\Xi}}\right)^{\chi_1}\right)^{\Xi}} \end{pmatrix}^{\Xi} \end{bmatrix}, \\ \begin{bmatrix} \bigcup_{i=1}^{l}\left(\left(1 - \left(1 - \sqrt[2\Xi]{(\alpha_3 \Upsilon_i^{\mathcal{FL}})}\right)^{\Xi}\right)^{2\Xi}\right)^{\chi_i} \cdot \left(\left(1 - \left(1 - \sqrt[2\Xi]{(\alpha_3 \Upsilon_{l+1}^{\mathcal{FL}})}\right)^{\Xi}\right)^{2\Xi}\right)^{\chi_1}, \\ \bigcup_{i=1}^{l}\left(\left(1 - \left(1 - \sqrt[2\Xi]{(\alpha_3 \Upsilon_i^{\mathcal{FU}})}\right)^{\Xi}\right)^{2\Xi}\right)^{\chi_i} \cdot \left(\left(1 - \left(1 - \sqrt[2\Xi]{(\alpha_3 \Upsilon_{l+1}^{\mathcal{FU}})}\right)^{\Xi}\right)^{2\Xi}\right)^{\chi_1} \end{bmatrix} \end{bmatrix}$$

$$\bigcap_{i=1}^{l+1} \chi_i \overline{\Theta}_i^{\Xi} = \begin{bmatrix} \left[\left(1 - \bigcup_{i=1}^{l+1}\left(1 - \sqrt[2\Xi]{(\alpha_1 \Upsilon_i^{\mathcal{TL}})^{\Xi}}\right)^{\chi_i}\right)^{2\Xi}, \left(1 - \bigcup_{i=1}^{l+1}\left(1 - \sqrt[2\Xi]{(\alpha_1 \Upsilon_i^{\mathcal{TU}})^{\Xi}}\right)^{\chi_i}\right)^{2\Xi} \right], \\ \left[\left(1 - \bigcup_{i=1}^{l+1}\left(1 - \sqrt[\Xi]{(\alpha_2 \Upsilon_i^{\mathcal{IL}})^{\Xi}}\right)^{\chi_i}\right)^{\Xi}, \left(1 - \bigcup_{i=1}^{l+1}\left(1 - \sqrt[\Xi]{(\alpha_2 \Upsilon_i^{\mathcal{IU}})^{\Xi}}\right)^{\chi_i}\right)^{\Xi} \right], \\ \left[\bigcup_{i=1}^{l+1}\left(\left(1 - \left(1 - \sqrt[2\Xi]{(\alpha_3 \Upsilon_i^{\mathcal{FL}})}\right)^{\Xi}\right)^{2\Xi}\right)^{\chi_i}, \bigcup_{i=1}^{l+1}\left(\left(1 - \left(1 - \sqrt[2\Xi]{(\alpha_3 \Upsilon_i^{\mathcal{FU}})}\right)^{\Xi}\right)^{2\Xi}\right)^{\chi_i} \right] \end{bmatrix}.$$

$$\left(\bigcup_{i=1}^{l+1} \chi_i \overline{\Theta}_i^{\Xi}\right)^{1/\Xi} =$$

https://doi.org/10.54216/IJNS.190401
Received: April 28, 2022 Accepted: November 15, 2022

$$\begin{bmatrix} \left[\left(\left(1-\bigcup_{i=1}^{l+1}\left(1-\left(\sqrt[2\Xi]{(\alpha_1\Upsilon_i^{\mathcal{TL}})^\Xi}\right)\right)^{\chi_i}\right)^{2\Xi}\right)^{\Xi}, \left(\left(1-\bigcup_{i=1}^{l+1}\left(1-\left(\sqrt[2\Xi]{(\alpha_1\Upsilon_i^{\mathcal{TU}})^\Xi}\right)\right)^{\chi_i}\right)^{2\Xi}\right)^{\Xi}\right], \\ \left[\left(\left(1-\bigcup_{i=1}^{l+1}\left(1-\left(\sqrt[\Xi]{(\alpha_2\Upsilon_i^{\mathcal{IL}})^\Xi}\right)\right)^{\chi_i}\right)^{\Xi}\right)^{\Xi}, \left(\left(1-\bigcup_{i=1}^{l+1}\left(1-\left(\sqrt[\Xi]{(\alpha_2\Upsilon_i^{\mathcal{IU}})^\Xi}\right)\right)^{\chi_i}\right)^{\Xi}\right)^{\Xi}\right], \\ \left[\begin{array}{c}\left(1-\left(1-\sqrt[2\Xi]{\bigcup_{i=1}^{l+1}\left(\left(1-\left(1-\sqrt[2\Xi]{(\alpha_3\Upsilon_i^{\mathcal{FL}})}\right)^{\Xi}\right)^{2\Xi}\right)^{\chi_i}}\right)^{\Xi}\right)^{2\Xi}, \\ \left(1-\left(1-\sqrt[2\Xi]{\bigcup_{i=1}^{l+1}\left(\left(1-\left(1-\sqrt[2\Xi]{(\alpha_3\Upsilon_i^{\mathcal{FU}})}\right)^{\Xi}\right)^{2\Xi}\right)^{\chi_i}}\right)^{\Xi}\right)^{2\Xi}\end{array}\right] \end{bmatrix}.$$

The above formula valid for any l.

The GSRDioNIVWA operator is switched to the SRDioNIVWA operator if $\Xi = 1$.

Theorem 5.11. *If all* $\overline{\Theta}_i = \Big\langle [\alpha_1\Upsilon_i^{\mathcal{TL}}, \alpha_1\Upsilon_i^{\mathcal{TU}}], [\alpha_2\Upsilon_i^{\mathcal{IL}}, \alpha_2\Upsilon_i^{\mathcal{IU}}][\alpha_3\Upsilon_i^{\mathcal{FL}}, \alpha_3\Upsilon_i^{\mathcal{FU}}]\Big\rangle (i = 1\ to\ n)$ *are equal, then GSRDioNIVWA*$(\overline{\Theta}_1, \overline{\Theta}_2, ..., \overline{\Theta}_n) = \overline{\Theta}$.

Using the GSRDioNIVWA operator, the boundedness and monotonicity properties are met.

5.4 Generalized SRDioNIVWG (GSRDioNIVWG) operator

Definition 5.12. Let $\overline{\Theta}_i = \Big\langle [\alpha_1\Upsilon_i^{\mathcal{TL}}, \alpha_1\Upsilon_i^{\mathcal{TU}}], [\alpha_2\Upsilon_i^{\mathcal{IL}}, \alpha_2\Upsilon_i^{\mathcal{IU}}], [\alpha_3\Upsilon_i^{\mathcal{FL}}, \alpha_3\Upsilon_i^{\mathcal{FU}}]\Big\rangle$ be the family of SRDioNIVNs. Then GSRDioNIVWG $(\overline{\Theta}_1, \overline{\Theta}_2, ..., \overline{\Theta}_n) = \frac{1}{\Xi}\Big(\bigcup_{i=1}^{n}(\Xi\overline{\Theta}_i)^{\chi_i}\Big)$ $(i = 1, 2, ..., n)$.

Theorem 5.13. *Let* $\overline{\Theta}_i = \Big\langle [\alpha_1\Upsilon_i^{\mathcal{TL}}, \alpha_1\Upsilon_i^{\mathcal{TU}}], [\alpha_2\Upsilon_i^{\mathcal{IL}}, \alpha_2\Upsilon_i^{\mathcal{IU}}], [\alpha_3\Upsilon_i^{\mathcal{FL}}, \alpha_3\Upsilon_i^{\mathcal{FU}}]\Big\rangle$ *be the family of SRDioNIVNs. Then GSRDioNIVWG*$(\overline{\Theta}_1, \overline{\Theta}_2, ..., \overline{\Theta}_n)$=

$$\begin{bmatrix} \left[\begin{array}{c}\left(1-\left(1-\sqrt[2\Xi]{\bigcup_{i=1}^{n}\left(\left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_i^{\mathcal{TL}})}\right)^{\Xi}\right)^{2\Xi}\right)^{\chi_i}}\right)^{\Xi}\right)^{2\Xi}, \\ \left(1-\left(1-\sqrt[2\Xi]{\bigcup_{i=1}^{n}\left(\left(1-\left(1-\sqrt[2\Xi]{(\alpha_1\Upsilon_i^{\mathcal{TU}})}\right)^{\Xi}\right)^{2\Xi}\right)^{\chi_i}}\right)^{\Xi}\right)^{2\Xi}\end{array}\right], \\ \left[\left(\left(1-\bigcup_{i=1}^{n}\left(1-\left(\sqrt[\Xi]{(\alpha_2\Upsilon_i^{\mathcal{IL}})^\Xi}\right)\right)^{\chi_i}\right)^{\Xi}\right)^{\Xi}, \left(\left(1-\bigcup_{i=1}^{n}\left(1-\left(\sqrt[\Xi]{(\alpha_2\Upsilon_i^{\mathcal{IU}})^\Xi}\right)\right)^{\chi_i}\right)^{\Xi}\right)^{\Xi}\right], \\ \left[\left(\left(1-\bigcup_{i=1}^{n}\left(1-\left(\sqrt[2\Xi]{(\alpha_3\Upsilon_i^{\mathcal{FL}})^\Xi}\right)\right)^{\chi_i}\right)^{2\Xi}\right)^{\Xi}, \left(\left(1-\bigcup_{i=1}^{n}\left(1-\left(\sqrt[2\Xi]{(\alpha_3\Upsilon_i^{\mathcal{FU}})^\Xi}\right)\right)^{\chi_i}\right)^{2\Xi}\right)^{\Xi}\right] \end{bmatrix}.$$

The GSRDioNIVWG operator becomes the SRDioNIVWG operator if $\Xi = 1$.

Using the GSRDioNIVWG operator, the boundedness and monotonicity properties are met.

Theorem 5.14. *If all* $\overline{\Theta}_i = \Big\langle [\alpha_1\Upsilon_i^{\mathcal{TL}}, \alpha_1\Upsilon_i^{\mathcal{TU}}], [\alpha_2\Upsilon_i^{\mathcal{IL}}, \alpha_2\Upsilon_i^{\mathcal{IU}}][\alpha_3\Upsilon_i^{\mathcal{FL}}, \alpha_3\Upsilon_i^{\mathcal{FU}}]\Big\rangle (i = 1\ to\ n)$ *are equal, then GSRDioNIVWG*$(\overline{\Theta}_1, \overline{\Theta}_2, ..., \overline{\Theta}_n) = \overline{\Theta}$.

6 MADM using SRDioNIV data

Let $\overline{\Theta} = \{\overline{\Theta}_1, \overline{\Theta}_2, ..., \overline{\Theta}_n\}$ be the n-alternatives, $C = \{C_1, C_2, ..., C_m\}$ be the m-attributes, $w = \{\chi_1, \chi_2, ..., \chi_m\}$ be the weights of attributes,
$\overline{\overline{\Theta}}_{ij} = \Big\langle [\alpha_1\Upsilon_{ij}^{\mathcal{TL}}, \alpha_1\Upsilon_{ij}^{\mathcal{TU}}], [\alpha_2\Upsilon_{ij}^{\mathcal{IL}}, \alpha_2\Upsilon_{ij}^{\mathcal{IU}}], [\alpha_3\Upsilon_{ij}^{\mathcal{FL}}, \alpha_3\Upsilon_{ij}^{\mathcal{FU}}]\Big\rangle$ is denoted by SRDioNIVN of $\overline{\Theta}_i$ in C_j. Here,
$\Big[\alpha_1\Upsilon_{ij}^{\mathcal{TL}}, \alpha_1\Upsilon_{ij}^{\mathcal{TU}}\Big], \Big[\alpha_2\Upsilon_{ij}^{\mathcal{IL}}, \alpha_2\Upsilon_{ij}^{\mathcal{IU}}\Big], \Big[\Upsilon_{ij}^{\mathcal{FL}}, \Upsilon_{ij}^{\mathcal{FU}}\Big] \in [0,1]$ and $0 \leq (\alpha_1\Upsilon_{ij}^{\mathcal{TU}}(\tau))^2 + \sqrt{(\alpha_2\Upsilon_{ij}^{\mathcal{IU}}(\tau))} + \sqrt{(\Upsilon_{ij}^{\mathcal{FU}}(\tau))} \leq 2$. Here, the n-alternative sets and m-attribute sets result in the $n \times m$ decision matrix, which is indicated by the equation $\mathbb{D} = (\overline{\Theta}_{ij})_{n\times m}$. In a MADM problem, one tries to select the best choice from a set of constrained options using a number of attributes with preferred weights. In this scenario, each alternative is described in connection to each attribute using the euclidean and hamming distance ideas, and SRDioNIVSs are utilized to draw a conclusion. The representation is created by adding the positive and negative ideal values of each attribute in relation to each attributes. After applying the following algorithm, a decision is made.

https://doi.org/10.54216/IJNS.190401

Received: April 28, 2022 Accepted: November 15, 2022

6.1 Algoritheorem for SRDioNIV

Step-1: SRDioNIV choice values should be entered.
Step-2: To decide on the ization decision values. The matrix of choices $\mathbb{D} = (\overline{\Theta}_{ij})_{n\times m}$ is ized into $\widehat{\mathbb{D}} = (\overline{\Theta}_{ij})_{n\times m}$; put

$$\overline{\Theta}_{ij} = \left\langle [\widehat{\alpha_1 \Upsilon_{ij}^{\mathcal{TL}}}, \widehat{\alpha_1 \Upsilon_{ij}^{\mathcal{TU}}}], [\widehat{\alpha_2 \Upsilon_{ij}^{\mathcal{IL}}}, \widehat{\alpha_2 \Upsilon_{ij}^{\mathcal{IU}}}], [\widehat{\alpha_3 \Upsilon_{ij}^{\mathcal{FL}}}, \widehat{\alpha_3 \Upsilon_{ij}^{\mathcal{FU}}}] \right\rangle$$

and

$$\widehat{\alpha_1 \Upsilon_{ij}^{\mathcal{TL}}} = \alpha_1 \Upsilon_{ij}^{\mathcal{TL}}, \widehat{\alpha_1 \Upsilon_{ij}^{\mathcal{TU}}} = \alpha_1 \Upsilon_{ij}^{\mathcal{TU}}.$$

Step-3: To determine the aggregate values. Using SRDioNIV information aggregation operators as a base, attribute C_j in $\overline{\Theta}_i$, $\overline{\Theta}_{ij} = \left\langle [\widehat{\alpha_1 \Upsilon_{ij}^{\mathcal{TL}}}, \widehat{\alpha_1 \Upsilon_{ij}^{\mathcal{TU}}}], [\widehat{\alpha_2 \Upsilon_{ij}^{\mathcal{IL}}}, \widehat{\alpha_2 \Upsilon_{ij}^{\mathcal{IU}}}], [\widehat{\alpha_3 \Upsilon_{ij}^{\mathcal{FL}}}, \widehat{\alpha_3 \Upsilon_{ij}^{\mathcal{FU}}}] \right\rangle$ is aggregated into $\overline{\Theta}_i = \left\langle [\widehat{\alpha_1 \Upsilon_{i}^{\mathcal{TL}}}, \widehat{\alpha_1 \Upsilon_{i}^{\mathcal{TU}}}], [\widehat{\alpha_2 \Upsilon_{i}^{\mathcal{IL}}}, \widehat{\alpha_2 \Upsilon_{i}^{\mathcal{IU}}}], [\widehat{\alpha_3 \Upsilon_{i}^{\mathcal{FL}}}, \widehat{\alpha_3 \Upsilon_{i}^{\mathcal{FU}}}] \right\rangle$.
Step-4: Calculate the ideal values, both positive and negative, for each alternative as follows:

$$\overline{\Theta}^P = \left\langle [1,1],[1,1],[0,0] \right\rangle,$$

$$\overline{\Theta}^N = \left\langle [0,0],[0,0],[1,1] \right\rangle.$$

Step-5: Find the ED between each option using the following two ideal values:

$$\mathbb{D}_i^P = \mathbb{D}_E\left(\overline{\Theta}_i, \overline{\Theta}^P\right);\ \mathbb{D}_i^N = \mathbb{D}_E\left(\overline{\Theta}_i, \overline{\Theta}^N\right).$$

Step-6: The values for relative closeness are calculated as follows:

$$\mathbb{D}_i^\star = \frac{\mathbb{D}_i^N}{\mathbb{D}_i^P + \mathbb{D}_i^N}.$$

Step-7: The output that produces the best value is $\sup \mathbb{D}_i^\star$. Therefore, decision is making the best option for the given problem.

6.2 Selection process based on engineering

The engineer must be fully aware of all of these things and must rtauin complete mental tranquilly. The engineer must adhere to governmental regulations and be a qualified expert. The engineer started by picking a spot close to the needed infrastructure. The site must be adjacent to the green surrounds of the fields since the engineer will select the location closest to the green surroundings. The plants will supply fresh air because the planes will be able to detect the stale air. They will consequently opt for a green atmosphere. The building plan must be created by an engineer who is educated about modern building construction techniques. Additionally, the engineer must not waste any materials because they will be used to neatly and efficiently construct the building. The structure ought to be built by the engineer in a little period of time. for the purpose of calling him an engineer. The house's design should be carefully planned by the engineer. The most critical concept to comprehend is the electrical circuit. The electrical circuit must be connected to the ground. While performing this repair, they must exercise caution and make plans for the water tap system. The main source is the water system. The engineer should authorize future building projects and provide the building owners permission to enlarge the home in the future. A building firm is aware of how crucial it is to get the best engineer for a brand-new construction project. Five engineers (alternatives) are nominated after the first round of suggestions, such as $A = \{A_1, A_2, A_3, A_4, A_5\}$. The score of the engineers evaluated by the experts is represented by $C = \{E_1$: team work, E_2: low wastage materials, E_3 : designing with planning, E_4 : future extension$\}$ and weights are $w = \{0.4, 0.3, 0.2, 0.1\}$. Based on expert analyses of the criteria,

https://doi.org/10.54216/IJNS.190401
Received: April 28, 2022 Accepted: November 15, 2022

our purpose is to select the most successful engineer from among the best number of possibilities. $E_1 = \begin{bmatrix} [0.35, 0.45], [0.2, 0.25], [0.5, 0.55], [0.25, 0.3], [0.4, 0.6], [0.1, 0.15] \\ [0.15, 0.35], [0.25, 0.3], [0.15, 0.25], [0.1, 0.15], [0.45, 0.55], [0.15, 0.2] \\ [0.45, 0.5], [0.3, 0.35], [0.4, 0.55], [0.15, 0.2], [0.3, 0.45], [0.2, 0.3] \\ [0.3, 0.35], [0.2, 0.25], [0.25, 0.35], [0.15, 0.25], [0.5, 0.55], [0.3, 0.35] \\ [0.35, 0.4], [0.35, 0.4], [0.4, 0.45], [0.3, 0.35], [0.45, 0.6], [0.1, 0.15] \end{bmatrix}$

$$E_2 = \begin{bmatrix} [0.45, 0.55], [0.15, 0.25], [0.45, 0.6], [0.2, 0.25], [0.6, 0.7], [0.1, 0.15] \\ [0.5, 0.6], [0.2, 0.3], [0.55, 0.8], [0.1, 0.15], [0.25, 0.35], [0.2, 0.25] \\ [0.15, 0.25], [0.3, 0.35], [0.6, 0.9], [0.1, 0.15], [0.35, 0.45], [0.2, 0.25] \\ [0.44, 0.45], [0.15, 0.25], [0.45, 0.7], [0.1, 0.15], [0.5, 0.65], [0.3, 0.35] \\ [0.2, 0.25], [0.3, 0.4], [0.3, 0.45], [0.25, 0.3], [0.7, 0.75], [0.1, 0.15] \end{bmatrix}$$

$$E_3 = \begin{bmatrix} [0.5, 0.55], [0.15, 0.25], [0.5, 0.7], [0.2, 0.35], [0.4, 0.45], [0.1, 0.2] \\ [0.4, 0.45], [0.2, 0.25], [0.7, 0.75], [0.15, 0.2], [0.6, 0.7], [0.15, 0.3] \\ [0.6, 0.65], [0.15, 0.2], [0.65, 0.8], [0.2, 0.25], [0.3, 0.4], [0.1, 0.2] \\ [0.4, 0.45], [0.1, 0.2], [0.55, 0.6], [0.3, 0.35], [0.7, 0.75], [0.1, 0.15] \\ [0.35, 0.6], [0.2, 0.25], [0.55, 0.7], [0.35, 0.4], [0.6, 0.65], [0.25, 0.3] \end{bmatrix}$$

$$E_4 = \begin{bmatrix} [0.45, 0.6], [0.15, 0.2], [0.55, 0.7], [0.1, 0.25], [0.45, 0.55], [0.15, 0.25] \\ [0.5, 0.65], [0.1, 0.25], [0.25, 0.4], [0.15, 0.3], [0.35, 0.5], [0.3, 0.4] \\ [0.6, 0.7], [0.15, 0.3], [0.55, 0.65], [0.2, 0.25], [0.25, 0.35], [0.3, 0.35] \\ [0.5, 0.6], [0.25, 0.3], [0.4, 0.45], [0.1, 0.15], [0.35, 0.5], [0.2, 0.25] \\ [0.6, 0.65], [0.4, 0.45], [0.65, 0.7], [0.1, 0.2], [0.45, 0.65], [0.1, 0.2] \end{bmatrix}$$

The following aggregate data for each alternative using the SRDioNIVWA operator $SRDioNIVWA\ operator$ $(\Xi = 1)$
$\overline{\Theta}_1 = \langle [0.07, 0.1256], [0.1028, 0.1783], [0.0476, 0.0983] \rangle$
$\overline{\Theta}_2 = \langle [0.0641, 0.1339], [0.0478, 0.0942], [0.0683, 0.1241] \rangle$
$\overline{\Theta}_3 = \langle [0.0921, 0.1419], [0.0795, 0.1414], [0.0559, 0.114] \rangle$
$\overline{\Theta}_4 = \langle [0.0633, 0.104], [0.0669, 0.1167], [0.1193, 0.1741] \rangle$
$\overline{\Theta}_5 = \langle [0.1023, 0.152], [0.1166, 0.1754], [0.0654, 0.1165] \rangle$
Determine the optimum values, both positive and negative, of the following alternatives:
$\overline{\Theta}^P = \langle [1, 1], [1, 1], [0, 0] \rangle$ and $\overline{\Theta}^N = \langle [0, 0], [0, 0], [1, 1] \rangle$
The following table shows the ED between each alternatives positive and negative ideal values:
$\mathbb{D}_1^P = 0.1920, \mathbb{D}_2^P = 0.1710, \mathbb{D}_3^P = 0.1843, \mathbb{D}_4^P = 0.2064, \mathbb{D}_5^P = 0.2027$
$\mathbb{D}_1^N = 0.1142, \mathbb{D}_2^N = 0.1352, \mathbb{D}_3^N = 0.1219, \mathbb{D}_4^N = 0.0998, \mathbb{D}_5^N = 0.1035.$
The values for relative closeness are calculated as follows: $\mathbb{D}_1^\star = 0.3731$, $\mathbb{D}_2^\star = 0.4414$, $\mathbb{D}_3^\star = 0.3982$, $\mathbb{D}_4^\star = 0.3260$, $\mathbb{D}_5^\star = 0.3381$.
Ranking of alternatives are as follows: $\overline{\Theta}_2 \geq \overline{\Theta}_3 \geq \overline{\Theta}_1 \geq \overline{\Theta}_5 \geq \overline{\Theta}_4$.

Similarly, we propose the SRDioNIVWG, GSRDioNIVWA, and GSRDioNIVWG approaches, which are based on ED and HD, respectively. The different distances are as follows:

$\Xi = 1$	SRDioNIVWA	SRDioNIVWG	GSRDioNIVWA	GSRDioNIVWG
TOPSIS-Euclidean distance (proposed)	$\overline{\Theta}_2 \geq \overline{\Theta}_3 \geq \overline{\Theta}_1 \geq \overline{\Theta}_5 \geq \overline{\Theta}_4$	$\overline{\Theta}_2 \geq \overline{\Theta}_3 \geq \overline{\Theta}_1 \geq \overline{\Theta}_5 \geq \overline{\Theta}_4$	$\overline{\Theta}_2 \geq \overline{\Theta}_3 \geq \overline{\Theta}_1 \geq \overline{\Theta}_5 \geq \overline{\Theta}_4$	$\overline{\Theta}_2 \geq \overline{\Theta}_3 \geq \overline{\Theta}_1 \geq \overline{\Theta}_5 \geq \overline{\Theta}_4$
TOPSIS-Hamming distance (proposed)	$\overline{\Theta}_2 \geq \overline{\Theta}_3 \geq \overline{\Theta}_1 \geq \overline{\Theta}_5 \geq \overline{\Theta}_4$	$\overline{\Theta}_2 \geq \overline{\Theta}_3 \geq \overline{\Theta}_1 \geq \overline{\Theta}_5 \geq \overline{\Theta}_4$	$\overline{\Theta}_2 \geq \overline{\Theta}_3 \geq \overline{\Theta}_1 \geq \overline{\Theta}_5 \geq \overline{\Theta}_4$	$\overline{\Theta}_2 \geq \overline{\Theta}_3 \geq \overline{\Theta}_1 \geq \overline{\Theta}_5 \geq \overline{\Theta}_4$

7 Conclusion:

We have presented the ED and HD measures in this article. These distance measures are advantageous due to their mathematical simplicity. It is established that both the ED and HD metrics are applicable. With regard to SRDioNIVWA, SRDioNIVWG, GSRDioNIVWA, and GSRDioNIVWG, we have suggested aggregation operation rules. In uncertain and inconsistent circumstances, the implementation of the SRDioNIV MADM technique can assist people in selecting the best option from a range of available options. To MADM problems depending on Ξ, we have used the SRDioNIVWA, SRDioNIVWG, GSRDioNIVWA, and GSRDioNIVWG operators. With the SRDioNIVWA, SRDioNIVWG, GSRDioNIVWA, and GSRDioNIVWG operators based for Ξ, the distinct ranking of alternatives can be discovered. The study presented above concludes by showing that the ranking of alternatives is most significantly impacted by the generalized values of Ξ. The decision-makers may choose to select the values for Ξ based on the real problem for the best. Therefore, based on the values of Ξ, the decision-maker may choose how to arrive at the result. Because this field of study is still in its early stages, the author is confident that the talks in this paper will be helpful to future academics interested in it.

Conflicts of Interest: The authors declare no conflict of interest.

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Received: April 28, 2022 Accepted: November 15, 2022