A Study of Some Neutrosophic Algebraic Games and Their Winning Strategies

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Abstract

In This work, we define two novel algebraic games over neutrosophic AH-groups. Also, we present a solution of these games in many possible cases, where some winning strategies will be discussed and illustrated by examples.

Keywords: winning strategy; algebraic game; neutrosophic group; On My Turf

Introduction

Game theory as a rich field of applied mathematical research has many applications in computer science, and economy. Algebra is a great material for game theoretic research, were we find many algebraic games defined over algebraic structures such as groups and matrices [1,2,4,7-10]. In [5], we find an application of neutrosophic groups in representing games. This side has motivated us to study two known games defined over groups by making them defined over multiplicative neutrosophic groups, where On My Turf\ Avoid the Identity games will be defined by using neutrosophic multiplicative groups and to analyze the possible winning strategies of such algebraic games.

Preliminaries

Definition:

Let (G,*) be a group. Then the neutrosophic group is generated by G and I under * denoted by N(G)={$< G \cup I >,*$}

I is called the indeterminate element (neutrosophic element) with the property $I^2 = I$

The most useful understanding of this definition has been written in [3].We consider N(G) as a union of G and GI i.e. N(G)=$\{x_1, x_2, \dots x_1 I, x_2 I, \dots . ; x_i \in G\}$ if G is a group under multiplication operation.

Definition :

Let (F,*) be a finite group then the (Avoid the identity) game [ID-Game] can be defined as :

-One and Two alternately pick unchosen members of F

-The player whose selection causes the group product of all members chosen thus far to be the group identity , loses

Example:

Let G=Z_5 then One and Two plays ID(G,+) like

Player One	Player Two	Sum mod 5

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2		2
	4	1
3		4
	0	4
1		0

Player Two wins the game.

Theorem:

If (F,*) is abelian group with odd order, then player Two has a winning strategy.

Theorem:

If (F,*) is abelian with even order, then player one has a winning strategy if and only if G has no subgroups which are isomorphic to $Z_2 \times Z_2$.

3. Main concepts

Definition :

Let G be a finite group and N(G) be the corresponding neutrosophic group, we define ON MY TURF neutrosophic game (NMT-game) over a subset F of N(G) as follows:

player ONE picks an unchosen element x_i , then player TWO picks another unchosen element y_i

The game ends when all elements have been chosen.

ONE wins if the group product $(x_i . y_i \ldots)$in this order is in F . Else TWO wins

Example:

Let G = Z_3, consider the neutrosophic group N(G) = { 0 , 1 , 2 , I , 1+I , 2+I }, and the set F ={0,1+I}. Suppose that A and B play the game as

A	B	Group product
0	I	I
2	1+I	I
2+I	1	I

The group product is I+I+I = I thus B wins.

Theorem :

Let G be a finite group with odd order and the set F does not contain I, then B has a winning strategy.

Proof:

G has no elements with order two thus for each $e \neq x \in G$ there is a different element x ≠y= x^{-1} . The winning strategy of B can be described as :

If A picked x then B should pick x^{-1}I , and if A picked xI then A should pick x^{-1}

We remark that in every step the group product of chosen elements is I , since $I^n = I$ is not in F, thus B is the winner.

Theorem :

Let G be a finite group with odd order and the set F contains I, then A has a winning strategy.

Proof :

A begins with I , if B picked an element $x \neq e$ then A should pick any x^{-1} or $x^{-1}I$, if B picked an element xI then A should pick $x^{-1}I$ or x^{-1} .

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If B picked e in any step then A should pick any element y or yI , if B picked $y^{-1} or\ y^{-1}I$ the A picks any unchosen element , but if B picked an element which is different from $y^{-1} or\ y^{-1}I$ say m, then A should pick m^{-1} or $m^{-1}I$. By the previous algorithm we will reach to the final position and the group product of elements is equal to I and A wins.

Example:

Let G = Z_5 , consider the following neutrosophic group N(G)={0 ,1 ,2 ,3 ,4 , I , 1+I ,2+I , 3+I , 4+ I}, and F={I , 2+I , 3}.

We summarize the game as :

A	B	Group product
I	1+I	1+I
4	2	1
3	0	3
2+I	4+I	1+I
1	3+I	4+I

The group product is equal to (1+I)+1+3+(1+I)+(4+I)=I and A is the winner.

Theorem:

Let G be a finite group with even order. Suppose that G has only one element with order two (a) , if F does not contain I then B has a winning strategy.

Proof :

We summarize the winning strategy of B as follows:

If A picked x $\neq a$ then B should pick $x^{-1}\ or\ x^{-1}$I , if A picked a then B should pick aI , if A picked aI then B should pick a , in the final position the group product will be equal to I and B wins.

Example:

Let G =Z_4 , with the following neutrosophic group N(G) = { 0 ,1 , 2, 3 , I , 1+I, 2+I, 3+I }, F={1,2+I}.

A	B	Group product
3	1+I	I
2	2+I	I
1	3	0
I	0	I

The group product of elements is I+I+0+I=I and B wins

Theorem:

Let G be a finite group with odd order. Suppose that G has only one element with order two (a) , if F contains I then A has a winning strategy.

Proof :

A begins with aI , if B picked a then A picks any unchosen element , if B picked an element $x\ or\ xI \neq a$ then A should pick $x^{-1}\ or\ x^{-1}I$, the group product in the final position will be equal to I and A must be the winner.

Example:

Let G =Z_4 , and N(G) = { 0 ,1 , 2, 3 , I , 1+I, 2+I, 3+I }, F={I,1,2+I}

A	B	Group product
2	1+I	3+I
3	2+I	1+I
1	0	1
I	3+I	3+I

The group product of elements is 3+I+1+I+1+3+I=I and A wins.

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Neutrosophic Identity-Game : (NID)

Definition

Let G be a finite multiplicative AH-group, and N(G) = G ∪ GI be the related neutrosophic group.Suppose that A , B are two players, we summarize the rules of NID-Game as follows :

A chooses an element a_i of N(G), then B chooses an element b_i of N(G) and so on. We denote by C_A={$a_1, \dots a_n$} to the set of chosen elements by player A and $C_B = \{b_1, \dots b_m\}$ for chosen elements by player B.

If in any step of the game, the product of elements of C_i denoted by (s) is equal to e (the identity of the group G) or I, then player i loses the game for i∈ $\{A, B\}$

If (s) is not equal to e or I, then the winner of NID-game is the player who picked up I

Example:

Let G ={1,3,5} a group under the multiplication modulo 14, then N(G) = { 1 , 3 , 5 , I , 3I , 5I }

Player A begins the game by choosing 3I, then B chooses 3, then A chooses I, then B chooses 1

We remark that now player A has two elements to choose 5 or 5I and in the two cases the element (s) is equal to I so B is the winner.

Example:

Let G =(Z_5^*, .) then N(G) = { 1 , 2 , 3 ,4 , I , 2I , 3I , 4I },. Suppose that A begins with 3I, then B takes 2I ,. We put the chosen elements as the following:

Player A	Player B
3I	2I
4I	4
2	3
I	1

We find that (s) is not equal to e or I in the both cases, so A wins because he picks up I

NIDS-Game :

Definition:

We define the NIDS-Game as NID-Game with one difference, if (s) is not equal to e or I then we compute

$s_1 = a_1 b_1 , \dots s_n = a_n b_n$, if $s_i \in C_t\, ; t \in \{A, B\}$ we give player t one point, the player with biggest number of points wins the game,. If the players have the same number of points then the winner is the player who picks up I

Example:

In the last example we found that (s) is not equal to e or I ,. By computing the elements s_i ,we found

$s_1 = (3I)(2I) = I \quad . s_2 = (4I)(4) = I \quad , s_3 = 2.3 = 1 \quad , s_4 = I.1 = I$ so the player A has 3 points and B has one point so A wins.

Example:

Let G =$Z_3^* \times Z_3^*$ with multiplication modulo 3, then N(G) = { (1,1) , (1,,2) , (2,1) , (2,2) , I , (1,2) I , (2,1)I , (2,2)I} , two players A , B choose as the following:

Player A	Player B
(2,2)I	(1,2)I
I	(1,1)
(2,1)I	(2,2)
(2,1)	(1,2)

(s) is not equal to (1,1) or I in both cases so we compute

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$$s_1 = (2,1)I\ , s_2 = I\ , s_3 = (1,2)I\ , s_4 = (2,2)$$

A , B have two points then A wins because he picks up I

Theorem:

If O(G) is odd number then B has a winning strategy.

Proof:

Suppose that O(G) is odd then there is no element with order 2 so the product of all elements in G is equal to e , and the product of all elements in GI is equal to I

The winning strategy of player B can be described as:

If A chooses x then choose xI , if A chooses xI then choose x, in the end of the game we will reach a position that A must be the loser.

Example :

Let G={11 , 9 , 1} a group under multiplication modulo 14 thus N(G)={11 , 9, 1 , I , 9I , 11I } , A begins with 3I ,.The game can be as:

A	B
9I	9
I	1

A reaches to a position which he/she has to choose between 11 , 11I , in the both cases he/she will lose.
Remark:

It is easy to see that the previous winning strategy can be applied to the NID-Game in the case of odd order group.

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