



On Neutrosophic Contra Generalized α _g-Continuous Mappings

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Abstract

This article employs the conception of neutrosophic generalized α _g-closed sets to construct a further weakly class of neutrosophic contra continuous mappings for instance neutrosophic contra generalized α _g-continuous mappings via neutrosophic topological spaces. Additionally, essential related attributes and various theorems and related features and notes have been investigated. At the same time, the closed connections between this form of mappings and the viewpoints of the other existing relevant classes of contra continuous mappings have been obtained. Further, we furnished some counterexamples where the results fail.

Keywords: Neutrosophic α _g-connected space; Neutrosophic α _g-locally indiscrete space; Neutrosophic contra- α _g-continuous mapping.

1. Introduction

At the end of the 20th century and the beginning of the 21st century, the conceptions of neutrosophy and the neutrosophic set ($\mathcal{NS}$) were grounded and built up by F. Smarandach [1,2]. Thereafter, neutrosophic sets have been successfully applied to many fields such as topology. Whereas A. A. Salama and S. A. Alblowi [3], presented the neutrosophic topological space ($\mathcal{NST}$) by utilizing the neutrosophic set.

Soon after this advent, many researchers focused on this theory, and they developed it further in different directions. Several investigations were conducted on the popularizing of the conception of contra generalized continuous mappings in many fields such as [4,5].

Research gap: Recent literature does not contain any reports of studies on neutrosophic contra generalized α _g-continuous mappings of neutrosophic generalized α _g-closed sets.

Motivation: In order to fill the research gap, we introduce the neutrosophic contra- α _g-continuous mapping.

The remainder of the paper is formatted as follows:

The definition of neutrosophic generalized α _g-closed sets, neutrosophic generalized α _g-continuous and neutrosophic contra α _g-continuous mappings that have already been established by several researchers are recalled in Section 2. In Section 3 of this article, we aspire to enunciate the thought of a neutrosophic contra α _g-continuous mappings. Also, we study the relations between this term on the one hand and the other types of neutrosophic contra generalized continuous mappings in [6-8] on the other hand with interesting counterexamples. Concluding remarks are presented in Section 4.

2. Preliminaries

The needful basic definitions of several notions in neutrosophic topological spaces are presented in this section.

Definition 2.1 [9]: An $\mathcal{NS}$ $\mathcal{E}$ in an $\mathcal{NTS}$ $(\mathcal{U}, \Gamma)$ is said to be a neutrosophic generalized α g-closed set and symbolized by $\mathcal{Ng}\alpha$ g-closed set if for any $\mathcal{N}\alpha$ g-open subset $\mathcal{K}$ of $(\mathcal{U}, \Gamma)$ such that $\mathcal{E} \subseteq \mathcal{K}$, then $\mathcal{Ncl}(\mathcal{E}) \subseteq \mathcal{K}$. Furthermore, its complement indicated by $\mathcal{Ng}\alpha$ g-open set and called a neutrosophic generalized α g-open set.

Definition 2.2 [9]: For any $\mathcal{NS}$ $\mathcal{E}$ in an $\mathcal{NTS}$ $(\mathcal{U}, \Gamma)$, the neutrosophic $\mathcal{g}\alpha$ g-interior and the neutrosophic $\mathcal{g}\alpha$ g-closure are defined as:

$$\mathcal{Ng}\alpha\text{g-int}(\mathcal{E}) = \bigcup \{ \mathcal{K} : \mathcal{K} \subseteq \mathcal{E}, \mathcal{K} \text{ is an } \mathcal{Ng}\alpha\text{g-open subset of } \mathcal{U} \}$$

$$\mathcal{Ng}\alpha\text{g-cl}(\mathcal{E}) = \bigcap \{ \mathcal{K} : \mathcal{E} \subseteq \mathcal{K}, \mathcal{K} \text{ is an } \mathcal{Ng}\alpha\text{g-closed subset of } \mathcal{U} \}$$

Definition 2.3 [9]: A mapping h from an $\mathcal{NTS}$ $(\mathcal{U}, \Gamma)$ into an $\mathcal{NTS}$ $(\mathcal{W}, T)$ is said to be neutrosophic generalized α g-continuous substantially $\mathcal{Ng}\alpha$ g-continuous if $h^{-1}(\mathcal{E})$ is an $\mathcal{Ng}\alpha$ g-closed subset of $(\mathcal{U}, \Gamma)$, for each $\mathcal{N}$ -closed subset $\mathcal{E}$ of $(\mathcal{W}, T)$.

The various varieties of neutrosophic contra-continuous mappings are explained in the following definition.

Definition 2.4: A mapping h from an $\mathcal{NTS}$ $(\mathcal{U}, \Gamma)$ into an $\mathcal{NTS}$ $(\mathcal{W}, T)$ is called:

- neutrosophic contra-continuous [7] substantially $\mathcal{N}$ -contra-continuous if $h^{-1}(\mathcal{E})$ is an $\mathcal{N}$ -closed subset of $(\mathcal{U}, \Gamma)$, for each $\mathcal{N}$ -open subset $\mathcal{E}$ of $(\mathcal{W}, T)$.
- neutrosophic contra $\mathcal{g}$ -continuous [7] substantially $\mathcal{N}$ -contra $\mathcal{g}$ -continuous if $h^{-1}(\mathcal{E})$ is an $\mathcal{Ng}$ -closed subset of $(\mathcal{U}, \Gamma)$, for each $\mathcal{N}$ -open subset $\mathcal{E}$ of $(\mathcal{W}, T)$.
- neutrosophic contra α g-continuous [6] substantially $\mathcal{N}$ -contra α g-continuous if $h^{-1}(\mathcal{E})$ is an $\mathcal{N}\alpha$ g-closed subset of $(\mathcal{U}, \Gamma)$, for each $\mathcal{N}$ -open subset $\mathcal{E}$ of $(\mathcal{W}, T)$.
- neutrosophic contra $\mathcal{g}\alpha$ -continuous [8] substantially $\mathcal{N}$ -contra $\mathcal{g}\alpha$ -continuous if $h^{-1}(\mathcal{E})$ is an $\mathcal{Ng}\alpha$ -closed subset of $(\mathcal{U}, \Gamma)$, for each $\mathcal{N}$ -open subset $\mathcal{E}$ of $(\mathcal{W}, T)$.

Definition 2.5 [9]: An $\mathcal{NTS}$ $(\mathcal{U}, \Gamma)$ is said to be a neutrosophic $T_{\frac{1}{2}}$ -space and signified by $\mathcal{N}$ - $T_{\frac{1}{2}}$ -space if for each $\mathcal{Ng}$ -closed subset of $(\mathcal{U}, \Gamma)$ is an $\mathcal{N}$ -closed set, or equivalently every $\mathcal{Ng}$ -open subset of $(\mathcal{U}, \Gamma)$ is an $\mathcal{N}$ -open set.

Definition 2.6 [9]: An $\mathcal{NTS}$ $(\mathcal{U}, \Gamma)$ is said to be a neutrosophic $T_{\mathcal{g}\alpha\mathcal{g}}$ -space and signified by $\mathcal{N}$ - $T_{\mathcal{g}\alpha\mathcal{g}}$ -space if for each $\mathcal{Ng}\alpha$ g-closed subset of $(\mathcal{U}, \Gamma)$ is an $\mathcal{N}$ -closed set, or equivalently every $\mathcal{Ng}\alpha$ g-open subset of $(\mathcal{U}, \Gamma)$ is an $\mathcal{N}$ -open set.

Definition 2.7: An $\mathcal{NTS}$ $(\mathcal{U}, \Gamma)$ is said to be a neutrosophic $\mathcal{g}\alpha$ g-locally indiscrete space if every $\mathcal{Ng}\alpha$ g-closed subset of $(\mathcal{U}, \Gamma)$ is an $\mathcal{Ng}\alpha$ g-open set, or equivalently every $\mathcal{Ng}\alpha$ g-open subset of $(\mathcal{U}, \Gamma)$ is an $\mathcal{Ng}\alpha$ g-closed set.

Definition 2.8: An $\mathcal{NTS}$ $(\mathcal{U}, \Gamma)$ is said to be:

- $\mathcal{N}$ -connected [10] if $(\mathcal{U}, \Gamma)$ can't be expressed as the union of two disjoint of non-empty $\mathcal{N}$ -open sets.
- $\mathcal{Ng}\alpha$ g-connected if $(\mathcal{U}, \Gamma)$ can't be written as the union of two disjoint of non-empty $\mathcal{Ng}\alpha$ g-open sets.

3. Neutrosophic Contra Generalized α g-Continuous Mappings

Neutrosophic contra generalized α g-continuous in neutrosophic topological space is a new class of neutrosophic contra continuous mappings described in this section. We also discuss some of its fundamental characteristics.

Definition 3.1: A mapping h from an $\mathcal{NTS}(\mathcal{U}, \Gamma)$ into an $\mathcal{NTS}(\mathcal{W}, T)$ is said to be neutrosophic contra generalized α -continuous substantially $\mathcal{N}$ -contra-g α -continuous if $h^{-1}(\mathcal{E})$ is an $\mathcal{Ng}\alpha$ -closed subset of $(\mathcal{U}, \Gamma)$, for each $\mathcal{N}$ -open subset $\mathcal{E}$ of $(\mathcal{W}, T)$.

Theorem 3.2: A mapping h from an $\mathcal{NTS}(\mathcal{U}, \Gamma)$ into an $\mathcal{NTS}(\mathcal{W}, T)$. Thereafter, h is $\mathcal{N}$ -contra-g α -continuous iff $h^{-1}(\mathcal{E})$ is an $\mathcal{Ng}\alpha$ -open subset of $(\mathcal{U}, \Gamma)$, for each $\mathcal{N}$ -closed subset of $(\mathcal{W}, T)$.

Proof: Directly from Definition 3.1.

Remark 3.3: In general, the terms $\mathcal{Ng}\alpha$ -continuous and $\mathcal{N}$ -contra-g α -continuous are not linked to each other as shown in the following examples.

Example 3.4: Consider $\Gamma = \{0_{\mathcal{N}}, \mathcal{Q}, \mathcal{S}, 1_{\mathcal{N}}\}$ and $T = \{0_{\mathcal{N}}, \mathcal{E}, 1_{\mathcal{N}}\}$ are $\mathcal{N}$ -topological spaces on $\mathcal{U} = \{m, n\}$, where $\mathcal{Q} = \langle u, (0.6, 0.7), (0.4, 0.3), (0.5, 0.2) \rangle$, $\mathcal{S} = \langle u, (0.5, 0.5), (0.5, 0.4), (0.6, 0.5) \rangle$ and $\mathcal{E} = \langle u, (0.5, 0.6), (0.5, 0.5), (0.6, 0.5) \rangle$ are the neutrosophic sets. Define a mapping $h: (\mathcal{U}, \Gamma) \rightarrow (\mathcal{U}, T)$ as $h(m) = m$ and $h(n) = n$. Then h is $\mathcal{Ng}\alpha$ -continuous, because $h^{-1}(\mathcal{E}^c)$ is an $\mathcal{Ng}\alpha$ -closed subset of $(\mathcal{U}, \Gamma)$, whenever $\mathcal{E}^c$ is an $\mathcal{N}$ -closed subset of $(\mathcal{U}, T)$. But h isn't $\mathcal{N}$ -contra-g α -continuous, because $h^{-1}(\mathcal{E})$ isn't an $\mathcal{Ng}\alpha$ -closed subset of $(\mathcal{U}, \Gamma)$, when $\mathcal{E}$ is an $\mathcal{N}$ -open subset of $(\mathcal{U}, T)$.

Example 3.5: Consider $\Gamma = \{0_{\mathcal{N}}, \mathcal{Q}, \mathcal{S}, 1_{\mathcal{N}}\}$ and $T = \{0_{\mathcal{N}}, \mathcal{E}, 1_{\mathcal{N}}\}$ are $\mathcal{N}$ -topological spaces on $\mathcal{U} = \{m, n\}$, where $\mathcal{Q} = \langle u, (0.8, 0.9), (0.6, 0.5), (0.7, 0.4) \rangle$, $\mathcal{S} = \langle u, (0.7, 0.7), (0.7, 0.6), (0.8, 0.7) \rangle$ and $\mathcal{E} = \langle u, (0.7, 0.7), (0.8, 0.6), (0.7, 0.9) \rangle$ are the neutrosophic sets. Define a mapping $h: (\mathcal{U}, \Gamma) \rightarrow (\mathcal{U}, T)$ as $h(m) = m$ and $h(n) = n$. Then h is $\mathcal{N}$ -contra-g α -continuous, because $h^{-1}(\mathcal{E})$ is an $\mathcal{Ng}\alpha$ -closed subset of $(\mathcal{U}, \Gamma)$, whenever $\mathcal{E}$ is an $\mathcal{N}$ -open subset of $(\mathcal{U}, T)$. But h isn't $\mathcal{Ng}\alpha$ -continuous, because $h^{-1}(\mathcal{E}^c)$ isn't an $\mathcal{Ng}\alpha$ -closed subset of $(\mathcal{U}, \Gamma)$, when $\mathcal{E}^c$ is an $\mathcal{N}$ -closed subset of $(\mathcal{U}, T)$.

The previous remark becomes true whenever the domain of the mapping is a neutrosophic g α -locally indiscrete space.

Theorem 3.6: A mapping h from a neutrosophic g α -locally indiscrete space $(\mathcal{U}, \Gamma)$ into any $\mathcal{NTS}(\mathcal{W}, T)$ is $\mathcal{N}$ -contra-g α -continuous iff its $\mathcal{Ng}\alpha$ -continuous.

Proof: The result is straight from the fact that every $\mathcal{Ng}\alpha$ -closed set is an $\mathcal{Ng}\alpha$ -open set and vice versa is also true in the neutrosophic g α -locally indiscrete space.

Theorem 3.7: For any mapping $h: (\mathcal{U}, \Gamma) \rightarrow (\mathcal{W}, T)$, then the subsequent clauses are valid and not vice versa:

- (a) All $\mathcal{N}$ -contra continuous mappings are $\mathcal{N}$ -contra-g α -continuous.
- (b) All $\mathcal{N}$ -contra-g α -continuous mappings are $\mathcal{N}$ -contra-g-continuous.
- (c) All $\mathcal{N}$ -contra-g α -continuous mappings are $\mathcal{N}$ -contra- α -continuous.
- (d) All $\mathcal{N}$ -contra-g α -continuous mappings are $\mathcal{N}$ -contra-g α -continuous.

Example 3.8: Consider $\Gamma = \{0_{\mathcal{N}}, \mathcal{Q}, \mathcal{S}, 1_{\mathcal{N}}\}$ and $T = \{0_{\mathcal{N}}, \mathcal{E}, 1_{\mathcal{N}}\}$ are $\mathcal{N}$ -topological spaces on $\mathcal{U} = \{m, n\}$, where $\mathcal{Q} = \langle u, (0.6, 0.7), (0.1, 0.1), (0.4, 0.2) \rangle$, $\mathcal{S} = \langle u, (0.1, 0.2), (0.1, 0.1), (0.8, 0.8) \rangle$ and $\mathcal{E} = \langle u, (0.2, 0.2), (0.1, 0.1), (0.7, 0.6) \rangle$ are the neutrosophic sets. Define a mapping $h: (\mathcal{U}, \Gamma) \rightarrow (\mathcal{U}, T)$ as $h(m) = n$ and $h(n) = m$. Then h is $\mathcal{N}$ -contra-g α -continuous, because $h^{-1}(\mathcal{E})$ is an $\mathcal{Ng}\alpha$ -closed subset of $(\mathcal{U}, \Gamma)$, whenever $\mathcal{E}$ is an $\mathcal{N}$ -open subset of $(\mathcal{U}, T)$. But h isn't $\mathcal{N}$ -contra continuous, because $h^{-1}(\mathcal{E})$ isn't an $\mathcal{N}$ -closed subset of $(\mathcal{U}, \Gamma)$, when $\mathcal{E}$ is an $\mathcal{N}$ -open subset of $(\mathcal{U}, T)$.

Example 3.9: Consider $\Gamma = \{0_{\mathcal{N}}, \mathcal{Q}, 1_{\mathcal{N}}\}$ and $T = \{0_{\mathcal{N}}, \mathcal{E}, 1_{\mathcal{N}}\}$ are $\mathcal{N}$ -topological spaces on $\mathcal{U} = \{m, n\}$, where $\mathcal{Q} = \langle u, (0.5, 0.7), (0.1, 0.3), (0.4, 0.7) \rangle$, $\mathcal{S} = \langle u, (0.5, 0.6), (0.2, 0.3), (0.3, 0.5) \rangle$ and $\mathcal{E} = \langle u, (0.6, 0.4), (0.3, 0.2), (0.7, 0.6) \rangle$ are the neutrosophic sets. Define a mapping $h: (\mathcal{U}, \Gamma) \rightarrow (\mathcal{U}, T)$ as $h(m) = n$

and $h(n) = m$. Then h is $\mathcal{N}$ -contra- g -continuous, because $h^{-1}(\mathfrak{E})$ is an $\mathcal{N}g$ -closed subset of $(\mathcal{U}, \Gamma)$, whenever $\mathfrak{E}$ is an $\mathcal{N}$ -open subset of $(\mathcal{U}, T)$. But h isn't $\mathcal{N}$ -contra- $g\alpha g$ -continuous, because $h^{-1}(\mathfrak{E})$ isn't an $\mathcal{N}g\alpha g$ -closed subset of $(\mathcal{U}, \Gamma)$, when $\mathfrak{E}$ is an $\mathcal{N}$ -open subset of $(\mathcal{U}, T)$, because $h^{-1}(\mathfrak{E}) \subseteq \mathfrak{S}$, when $\mathfrak{S}$ is an $\mathcal{N}g\alpha g$ -closed subset of $(\mathcal{U}, \Gamma)$, but $\mathcal{N}cl(h^{-1}(\mathfrak{E})) \not\subseteq \mathfrak{S}$.

Example 3.10: Consider $\Gamma = \{0_{\mathcal{N}}, \mathfrak{Q}, \mathfrak{S}, 1_{\mathcal{N}}\}$ and $T = \{0_{\mathcal{N}}, \mathfrak{E}, 1_{\mathcal{N}}\}$ are $\mathcal{N}$ -topological spaces on $\mathcal{U} = \{m, n\}$, where $\mathfrak{Q} = \langle u, (0.5, 0.5), (0.3, 0.4), (0.4, 0.2) \rangle$, $\mathfrak{S} = \langle u, (0.5, 0.3), (0.4, 0.4), (0.4, 0.6) \rangle$ and $\mathfrak{E} = \langle u, (0.4, 0.5), (0.4, 0.4), (0.5, 0.4) \rangle$ are the neutrosophic sets. Define a mapping $h: (\mathcal{U}, \Gamma) \rightarrow (\mathcal{U}, T)$ as $h(m) = m$ and $h(n) = n$. Then h is $\mathcal{N}$ -contra- $g\alpha g$ -continuous, because $h^{-1}(\mathfrak{E})$ is an $\mathcal{N}g\alpha g$ -closed subset of $(\mathcal{U}, \Gamma)$, whenever $\mathfrak{E}$ is an $\mathcal{N}$ -open subset of $(\mathcal{U}, T)$. But h isn't $\mathcal{N}$ -contra- $g\alpha g$ -continuous, because $h^{-1}(\mathfrak{E})$ isn't an $\mathcal{N}g\alpha g$ -closed subset of $(\mathcal{U}, \Gamma)$, when $\mathfrak{E}$ is an $\mathcal{N}$ -open subset of $(\mathcal{U}, T)$.

Example 3.11: In Example 3.10, we can see that h is $\mathcal{N}$ -contra- $g\alpha$ -continuous, because $h^{-1}(\mathfrak{E})$ is an $\mathcal{N}g\alpha$ -closed subset of $(\mathcal{U}, \Gamma)$, whenever $\mathfrak{E}$ is an $\mathcal{N}$ -open subset of $(\mathcal{U}, T)$. But h isn't $\mathcal{N}$ -contra- $g\alpha g$ -continuous, because $h^{-1}(\mathfrak{E})$ isn't an $\mathcal{N}g\alpha g$ -closed subset of $(\mathcal{U}, \Gamma)$, when $\mathfrak{E}$ is an $\mathcal{N}$ -open subset of $(\mathcal{U}, T)$.

Theorem 3.12: A mapping h from an $\mathcal{NT}_{g\alpha g}$ -space $(\mathcal{U}, \Gamma)$ into any $\mathcal{NTS}$ $(\mathcal{W}, T)$ is $\mathcal{N}$ -contra continuous whenever h is $\mathcal{N}$ -contra- $g\alpha g$ -continuous.

Proof: Straightly from the fact that every $\mathcal{N}g\alpha g$ -closed subset of $(\mathcal{U}, \Gamma)$ is an $\mathcal{N}$ -closed set.

Theorem 3.13: A mapping h from an $\mathcal{NT}_{\frac{1}{2}}$ -space $(\mathcal{U}, \Gamma)$ into any $\mathcal{NTS}$ $(\mathcal{W}, T)$ is $\mathcal{N}$ -contra- $g\alpha g$ -continuous whenever h is $\mathcal{N}$ -contra- g -continuous.

Proof: Directly from the fact that every $\mathcal{N}g$ -closed subset of $(\mathcal{U}, \Gamma)$ is an $\mathcal{N}$ -closed set, according to its an $\mathcal{N}g\alpha g$ -closed subset of $(\mathcal{U}, \Gamma)$.

Theorem 3.14: An injective mapping h from an $\mathcal{NTS}$ $(\mathcal{U}, \Gamma)$ onto an $\mathcal{NTS}$ $(\mathcal{W}, T)$ is $\mathcal{N}$ -contra- $g\alpha g$ -continuous if $\mathcal{N}cl(h(\mathfrak{E})) \subseteq h(\mathcal{N}g\alpha g\text{-int}(\mathfrak{E}))$, for each $\mathcal{NS}$ $\mathfrak{E}$ in $(\mathcal{U}, \Gamma)$.

Proof: Let $\mathfrak{E}$ be an $\mathcal{N}$ -closed set in $(\mathcal{W}, T)$. Then $\mathcal{N}cl(\mathfrak{E}) = \mathfrak{E}$ and $h^{-1}(\mathfrak{E})$ is an $\mathcal{NS}$ in $(\mathcal{U}, \Gamma)$. By hypothesis, $\mathcal{N}cl(h(h^{-1}(\mathfrak{E}))) \subseteq h(\mathcal{N}g\alpha g\text{-int}(h^{-1}(\mathfrak{E})))$. Since h is onto, $h(h^{-1}(\mathfrak{E})) = \mathfrak{E}$. Therefore, $\mathfrak{E} = \mathcal{N}cl(\mathfrak{E}) = \mathcal{N}cl(h(h^{-1}(\mathfrak{E}))) \subseteq h(\mathcal{N}g\alpha g\text{-int}(h^{-1}(\mathfrak{E})))$. Now, $\mathfrak{E} \subseteq h(\mathcal{N}g\alpha g\text{-int}(h^{-1}(\mathfrak{E})))$, $h^{-1}(\mathfrak{E}) \subseteq h^{-1}(h(\mathcal{N}g\alpha g\text{-int}(h^{-1}(\mathfrak{E})))) = \mathcal{N}g\alpha g\text{-int}(h^{-1}(\mathfrak{E})) \subseteq h^{-1}(\mathfrak{E})$. Hence $h^{-1}(\mathfrak{E})$ is an $\mathcal{N}g\alpha g$ -open set in $(\mathcal{U}, \Gamma)$. Thus, h is an $\mathcal{N}$ -contra- $g\alpha g$ -continuous mapping.

Theorem 3.15: If h from an $\mathcal{NTS}$ $(\mathcal{U}, \Gamma)$ into an $\mathcal{NTS}$ $(\mathcal{W}, T)$ is a mapping, then the following are equivalent statements.

- (a) h is $\mathcal{N}$ -contra- $g\alpha g$ -continuous.
- (b) for each neutrosophic point $p_{(r,s,t)}$ in $\mathcal{U}$ and for each $\mathcal{N}$ -closed subset $\mathfrak{K}$ of $(\mathcal{W}, T)$ containing $h(p_{(r,s,t)})$, there exists an $\mathcal{N}g\alpha g$ -open subset $\mathfrak{E}$ in $(\mathcal{U}, \Gamma)$ containing $p_{(r,s,t)}$ such that $\mathfrak{E} \subseteq h^{-1}(\mathfrak{K})$.
- (c) for each neutrosophic point $p_{(r,s,t)}$ in $\mathcal{U}$ and for each $\mathcal{N}$ -closed subset $\mathfrak{K}$ of $(\mathcal{W}, T)$ containing $h(p_{(r,s,t)})$, there exists an $\mathcal{N}g\alpha g$ -open subset $\mathfrak{E}$ of $(\mathcal{U}, \Gamma)$ containing $p_{(r,s,t)}$ such that $h(\mathfrak{E}) \subseteq \mathfrak{K}$.

Proof: (a) $\Rightarrow$ (b) Let h be an $\mathcal{N}$ -contra- $g\alpha g$ -continuous mapping and let $\mathfrak{K}$ be any $\mathcal{N}$ -closed subset of $(\mathcal{W}, T)$ and let $p_{(r,s,t)}$ be a neutrosophic point in $\mathcal{U}$ such that $h(p_{(r,s,t)}) \in \mathfrak{K}$, then $p_{(r,s,t)} \in h^{-1}(\mathfrak{K}) = \mathcal{N}g\alpha g\text{-int}(h^{-1}(\mathfrak{K}))$. Let $\mathfrak{E} = \mathcal{N}g\alpha g\text{-int}(h^{-1}(\mathfrak{K}))$. Then $\mathfrak{E}$ is an $\mathcal{N}g\alpha g$ -open subset of $(\mathcal{U}, \Gamma)$ and $E = \mathcal{N}g\alpha g\text{-int}(h^{-1}(\mathfrak{K})) \subseteq h^{-1}(\mathfrak{K})$.

(b) $\Rightarrow$ (c) The result follows from evident relations $h(\mathfrak{E}) \subseteq h(h^{-1}(\mathfrak{K})) \subseteq \mathfrak{K}$.

(c) $\Rightarrow$ (a) Let $\mathfrak{K}$ be any $\mathcal{N}$ -closed subset of $(\mathcal{W}, T)$ and let $p_{(r,s,t)}$ be a neutrosophic point in U such that $p_{(r,s,t)} \in h^{-1}(\mathfrak{K})$. Then $h(p_{(r,s,t)}) \in \mathfrak{K}$. According to the assumption there exists an $\mathcal{N}g\alpha g$ -open subset $\mathfrak{E}$ of $(\mathcal{U}, \Gamma)$ such that

$p_{(r,s,t)} \in \mathfrak{E}$ and $h(\mathfrak{E}) \subseteq \mathfrak{K}$. Hence $p_{(r,s,t)} \in \mathfrak{E} \subseteq h^{-1}(h(\mathfrak{E})) \subseteq h^{-1}(\mathfrak{K})$. Therefore $p_{(r,s,t)} \in \mathfrak{E} = \mathcal{N}gag\text{-}int(\mathfrak{E}) \subseteq \mathcal{N}gag\text{-}int(h^{-1}(\mathfrak{K}))$. Since $p_{(r,s,t)}$ is an arbitrary neutrosophic point and $h^{-1}(\mathfrak{K})$ is the union of all neutrosophic points in $h^{-1}(\mathfrak{K})$, we obtain that $h^{-1}(\mathfrak{K}) \subseteq \mathcal{N}gag\text{-}int(h^{-1}(\mathfrak{K}))$. Thus h is an $\mathcal{N}$ -contra- gag -continuous mapping.

Theorem 3.16: Let $h: (\mathcal{U}, \Gamma) \rightarrow (\mathfrak{W}, T)$ be a mapping along with $\mathcal{I}: \mathcal{U} \rightarrow \mathcal{U} \times \mathfrak{W}$ be the graph mapping defined by $\mathcal{I}(\kappa) = (\kappa, h(\kappa))$ being each $\kappa \in \mathcal{U}$. Whenever $\mathcal{I}$ is an $\mathcal{N}$ -contra- gag -continuous mapping, thereupon h is $\mathcal{N}$ -contra- gag -continuous.

Proof: Let $\mathfrak{E}$ be an $\mathcal{N}$ -open subset of $(\mathfrak{W}, T)$ accordingly $\mathcal{U} \times \mathfrak{E}$ is an $\mathcal{N}$ -open subset of $\mathcal{U} \times \mathfrak{W}$. In view of $\mathcal{I}$ is an $\mathcal{N}$ -contra- gag -continuous mapping, so that $h^{-1}(\mathfrak{E}) = \mathcal{U} \cap h^{-1}(\mathfrak{E}) = \mathcal{I}^{-1}(\mathcal{U} \times \mathfrak{E})$ is an $\mathcal{N}gag$ -closed subset of $(\mathcal{U}, \Gamma)$. Thus h is $\mathcal{N}$ -contra- gag -continuous.

Theorem 3.17: If $h: (\mathcal{U}, \Gamma) \rightarrow (\mathfrak{W}, T)$ is a surjective and $\mathcal{N}$ -contra- gag -continuous mapping along with $(\mathcal{U}, \Gamma)$ is an $\mathcal{N}gag$ -connected space, then $(\mathfrak{W}, T)$ is $\mathcal{N}$ -connected.

Proof: Assume that $(\mathfrak{W}, T)$ isn't an $\mathcal{N}$ -connected space. Accordingly, there exists disjoint non-empty $\mathcal{N}$ -open sets $\mathfrak{E}$ and $\mathfrak{K}$ such that $\mathfrak{W} = \mathfrak{E} \sqcup \mathfrak{K}$. Then, $\mathfrak{E}$ and $\mathfrak{K}$ are $\mathcal{N}$ -clopen subsets of $(\mathfrak{W}, T)$. As h is $\mathcal{N}$ -contra- gag -continuous, then $h^{-1}(\mathfrak{E})$ and $h^{-1}(\mathfrak{K})$ are $\mathcal{N}gag$ -open subsets of $(\mathcal{U}, \Gamma)$. In addition, $h^{-1}(\mathfrak{E})$ and $h^{-1}(\mathfrak{K})$ are disjoint non-empty $\mathcal{N}gag$ -open sets and $\mathcal{U} = h^{-1}(\mathfrak{E}) \sqcup h^{-1}(\mathfrak{K})$. It is contradiction to the fact that $(\mathcal{U}, \Gamma)$ is an $\mathcal{N}gag$ -connected space. Hence $(\mathfrak{W}, T)$ is an $\mathcal{N}$ -connected space.

Theorem 3.18: If $h: (\mathcal{U}, \Gamma) \rightarrow (\mathfrak{W}, T)$ is a mapping. Suppose one of the following properties hold:

- (a) $h^{-1}(\mathcal{N}gag\text{-}cl(\mathfrak{K})) \subseteq \mathcal{N}gag\text{-}int(\mathcal{N}gag\text{-}cl(h^{-1}(\mathfrak{K})))$, for each $\mathcal{NS}$ $\mathfrak{K}$ in $(\mathfrak{W}, T)$.
- (b) $\mathcal{N}gag\text{-}cl(\mathcal{N}gag\text{-}int(h^{-1}(\mathfrak{K}))) \subseteq h^{-1}(\mathcal{N}gag\text{-}int(\mathfrak{K}))$, for each $\mathcal{NS}$ $\mathfrak{K}$ in $(\mathfrak{W}, T)$.
- (c) $h(\mathcal{N}gag\text{-}cl(\mathcal{N}gag\text{-}int(\mathfrak{E}))) \subseteq \mathcal{N}gag\text{-}int(h(\mathfrak{E}))$, for each $\mathcal{NS}$ $\mathfrak{E}$ in $(\mathcal{U}, \Gamma)$.
- (d) $h(\mathcal{N}gag\text{-}cl(\mathfrak{E})) \subseteq \mathcal{N}gag\text{-}int(h(\mathfrak{E}))$, for each $\mathcal{NS}$ $\mathfrak{E}$ in $(\mathcal{U}, \Gamma)$.

Then h is an $\mathcal{N}$ -contra- gag -continuous mapping.

Proof: (a) $\Rightarrow$ (b) Assume that $\mathfrak{K}$ is an $\mathcal{NS}$ in $(\mathfrak{W}, T)$. Then $h^{-1}(\mathfrak{K})$ is an $\mathcal{NS}$ in $(\mathcal{U}, \Gamma)$. By hypothesis, $h^{-1}(\mathcal{N}gag\text{-}cl(\mathfrak{K})) \subseteq \mathcal{N}gag\text{-}int(\mathcal{N}gag\text{-}cl(h^{-1}(\mathfrak{K})))$. Taking complement $(h^{-1}(\mathcal{N}gag\text{-}cl(\mathfrak{K})))^c \supseteq (\mathcal{N}gag\text{-}int(\mathcal{N}gag\text{-}cl(h^{-1}(\mathfrak{K}))))^c$, $h^{-1}(\mathcal{N}gag\text{-}cl(\mathfrak{K}))^c \supseteq \mathcal{N}gag\text{-}cl(\mathcal{N}gag\text{-}cl(h^{-1}(\mathfrak{K})))^c$, $h^{-1}(\mathcal{N}gag\text{-}int(\mathfrak{K})) \supseteq \mathcal{N}gag\text{-}cl(\mathcal{N}gag\text{-}int(h^{-1}(\mathfrak{K})))^c$. Thus $\mathcal{N}gag\text{-}cl(\mathcal{N}gag\text{-}int(h^{-1}(\mathfrak{K}))) \subseteq h^{-1}(\mathcal{N}gag\text{-}int(\mathfrak{K}))$.

(b) $\Rightarrow$ (c) Let $\mathfrak{E}$ be an $\mathcal{NS}$ in $(\mathcal{U}, \Gamma)$. Put $\mathfrak{K} = h(\mathfrak{E})$, then $\mathfrak{E} \subseteq h^{-1}(\mathfrak{K})$. By hypothesis, $\mathcal{N}gag\text{-}cl(\mathcal{N}gag\text{-}int(\mathfrak{E})) \subseteq \mathcal{N}gag\text{-}cl(\mathcal{N}gag\text{-}int(h^{-1}(\mathfrak{K}))) \subseteq h^{-1}(\mathcal{N}gag\text{-}int(\mathfrak{K}))$, $\mathcal{N}gag\text{-}cl(\mathcal{N}gag\text{-}int(\mathfrak{E})) \subseteq h^{-1}(\mathcal{N}gag\text{-}int(\mathfrak{K}))$. Therefore, $h(\mathcal{N}gag\text{-}cl(\mathcal{N}gag\text{-}int(\mathfrak{E}))) \subseteq \mathcal{N}gag\text{-}int(\mathfrak{K}) = \mathcal{N}gag\text{-}int(h(\mathfrak{E}))$. This means that $h(\mathcal{N}gag\text{-}cl(\mathcal{N}gag\text{-}int(\mathfrak{E}))) \subseteq \mathcal{N}gag\text{-}int(h(\mathfrak{E}))$.

(c) $\Rightarrow$ (d) Let $\mathfrak{E}$ be any $\mathcal{N}gag$ -open subset of $(\mathcal{U}, \Gamma)$. Then $\mathcal{N}gag\text{-}int(\mathfrak{E}) = \mathfrak{E}$. By hypothesis, $h(\mathcal{N}gag\text{-}cl(\mathfrak{E})) = h(\mathcal{N}gag\text{-}cl(\mathcal{N}gag\text{-}int(\mathfrak{E}))) \subseteq \mathcal{N}gag\text{-}int(h(\mathfrak{E}))$. Thus, $h(\mathcal{N}gag\text{-}cl(\mathfrak{E})) \subseteq \mathcal{N}gag\text{-}int(h(\mathfrak{E}))$.

Suppose that (d) holds. Let $\mathfrak{K}$ be an $\mathcal{N}$ -open subset of $(\mathfrak{W}, T)$. Then $h^{-1}(\mathfrak{K}) = \mathfrak{E}$ is an $\mathcal{NS}$ in $(\mathcal{U}, \Gamma)$. By hypothesis, $h(\mathcal{N}gag\text{-}cl(\mathfrak{E})) \subseteq \mathcal{N}gag\text{-}int(h(\mathfrak{E}))$. Now $h(\mathcal{N}gag\text{-}cl(\mathfrak{E})) \subseteq \mathcal{N}gag\text{-}int(h(\mathfrak{E})) \subseteq h(\mathfrak{E})$, $h(\mathcal{N}gag\text{-}cl(\mathfrak{E})) \subseteq h(\mathfrak{E})$, $\mathcal{N}gag\text{-}cl(\mathfrak{E}) \subseteq h^{-1}(h(\mathfrak{E})) = \mathfrak{E}$. That is, $\mathcal{N}gag\text{-}cl(\mathfrak{E}) \subseteq \mathfrak{E}$. But $\mathfrak{E} \subseteq \mathcal{N}gag\text{-}cl(\mathfrak{E})$. Hence $\mathfrak{E} = \mathcal{N}gag\text{-}cl(\mathfrak{E})$. Thus $\mathfrak{E}$ is an $\mathcal{N}gag$ -closed subset of $(\mathcal{U}, \Gamma)$. Hence h is an $\mathcal{N}$ -contra- gag -continuous mapping.

4. Conclusion

To summarize, $\mathcal{N}$ -contra- gag -continuous mappings were introduced as new mathematical tools that can be realized. We also introduced some new notions that are necessary for this study. As a result, we investigated some of these mappings' characterizations and explained how they relate to other types of mappings for instance $\mathcal{N}$ -contra ag -continuous and $\mathcal{N}$ -contra ga -continuous mappings. We also showed that the opposite is not true in general except

under particular conditions by providing a series of examples that illustrate that. In the future, we anticipate that many additional studies will be able to be conducted in the future using these ideas from $\mathcal{NTS}$ s.

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