The Neutrosophic Traveling Salesman problem with Neutrosophic Edge Weight: Formulation and A Genetic Algorithm

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Abstract

The traveling salesman problem (TSP) is an important and well known classical combinatorial network optimization problem in operation research, where the TSP finds a shortest possible route through a set of n nodes such that each and every node are visited exactly one time except for the starting node. In this problem, the arc lengths are generally considered to represent the traveling time or travelling cost rather than geographical distance. It is not possible to predict the exact arc length because the traveling time or traveling cost fluctuated with payload, weather, traffic conditions and so on. neutrosophic set theory provides a new tool to handle the uncertainties in TSP. In this paper, we concentrate on TSP on a network in which neutrosophic set, Instead of real number is assigned to edge as edge weight. We propose a mathematical model for a TSP with neutrosophic arc lengths. We present the utility of neutrosophic sets as arc length for TSP. An algorithmic method based on Genetic Algorithm (GA) is proposed for solving this problem. We have designed a new heuristic crossover and heuristic mutation our proposed GA. We have used a numerical example to illustrate the effectiveness of our proposed algorithm.
Keywords: Neutrosophic Edge Weight; Formulation; Genetic Algorithm; Traveling Salesman problem

1 introduction

In 1930, the traveling salesman problem (TSP)[1–3] was first introduced as a simple mathematical problem and it becomes very popular after 1950. The TSP is one of the most fundamental and intensively studied problems in area of operation research even in recent years. In this TSP, decision maker finds a shortest possible tour for the salesman who visit each and every cities exactly one time (except the starting city) for a given list of distinct cities and come back to the starting city. It is a very popular NP-hard optimization problem. In real world scenarios, the edge cost in a path of a traffic network have the different constants which are not easy to determine precisely (i.e., traveling payment, road demand, traveling time, frequency of traffic, road capacity, etc.). However, the traveling cost, traveling time and traveling speed may vary in different weather and time or traffic condition. Therefore, decision maker cannot be estimated exact the traveling time and traveling cost between two distinct cities in such real life scenarios. The decision maker have to consider the uncertainty in traveling cost and time to solve the TSP properly. For this reason, neutrosophic set can be used to handle this type of uncertainty[4] which enables the decision maker to take decisions based on uncertain data. The

https://doi.org/10.54216/IJNS.190304

Received: June 06, 2022 Accepted: November 12, 2022

traveling cost/time between two cities mainly depend on the conveyance applied for traveling purpose. It generally changes on a day to day and hence it is good to consider the traveling costs/time of this problem as neutrosophic[?,?,5] in nature. There are several effort on neutrosophic traveling salesman problem.[?,6–8]

The TSP[3] is a NP hard problem. However, it is very easy to understand but very hard to find the solution. The computational time of this TSP increases exponentially when an increase in the number of cities. TSP is generally considered as a benchmark optimization problem to present the performance of newly introduced optimization algorithm. This problem is considered the sequence of the cities traveled and the total travel cost/distance/time. The best sequence of cities is the optimal solution that is needed to be determined. In this TSP, decision maker finds a minimum length Hamiltonian cycle of all the given cities and each and every cities (except starting city) have to be traveled exactly one time.

Many researchers have done lots of study to develop the exact algorithm, mathematical method, heuristic algorithm and metaheuristic algorithms for this TSP. Mathematical methods and exact algorithms[9] can determine the exact optimal solution of this TSP within a short time. However, if the network is very high then the exact algorithms take long time to find the solution. Because it considers all possible combination and it does not perform well. For this reason, heuristic and metaheuristic optimization algorithms are more satiable to solve the TSP. Both of the algorithms reduce the computational time to solve this problem.

There exists many different types heuristic and metaheuristic algorithms to solve TSP such as genetic algorithm, artificial bee colony algorithm, and harmony search algorithm. Genetic Algorithm is a heuristic optimization algorithm. In genetic algorithm, a chromosome/individual describes a possible solution of the given problem. A collection of different chromosomes constructs the population. In this algorithm, chromosomes are recombined to generate new chromosomes. This recombination method is performed by mainly three genetic operations, selection, mutation and crossover. In genetic algorithm, the more fitted chromosomes replace the less fitted chromosomes in the population. In this method, genetic algorithm finds the optimal solutions. Many optimization problems[10] can be solved by genetic algorithm. The genetic algorithm is also used to solve the TSP.[11–16]

In this paper, we present on neutrosophic TSP on a network in which neutrosophic set, Instead of real number is assigned to as edge weight. A mathematical model is introduced for a TSP with neutrosophic arc lengths. We present the utility of neutrosophic sets as arc length for TSP. An algorithmic method based on Genetic Algorithm (GA) is proposed for solving this problem. We have designed a new crossover and mutation our proposed GA. We have used a numerical example to illustrate the effectiveness of our proposed algorithm.

2 Preliminary

Definition 2.1. Let ξ be an universal set. The neutrosophic set[17] A on the universal set ξ categorized in to three membership functions called the true $T_A(x)$, indeterminate $I_A(x)$ and false $F_A(x)$ contained in real standard or non-standard subset of $]^-0, 1^+[$ respectively.

$$^-0 \le \sup\ T_A(x) + \sup\ I_A(x) + \sup\ F_A(x) \le 3^+ \tag{1}$$

Definition 2.2. Let ξ be a universal set. The single valued neutrosophic sets (SVNs)[18] A on the universal ξ is denoted as following

$$A = \{\langle x : T_A(x), I_A(x), F_A(x) | x \in \xi \rangle\} \tag{2}$$

The functions $T_A(x) \in [0,1]$, $I_A(x) \in [0,1]$ and $F_A(x) \in [0,1]$ are named as degree of truth, indeterminacy and falsity membership of x in A, satisfy the following condition:

$$^-0 \le \sup\ T_A(x) + \sup\ I_A(x) + \sup\ F_A(x) \le 3^+ \tag{3}$$

Definition 2.3. Let $A = (T, I, F)$ be a SVNs, a score function S,[19] based on the truth-membership degree (T), an indeterminacy membership degree (I) and a falsity membership degree (F) is defined as follow:

$$S(A) = \frac{(1 + (T - 2I - F)(2 - T - F))}{2} \tag{4}$$

https://doi.org/10.54216/IJNS.190304

Received: June 06, 2022 Accepted: November 12, 2022

3 Formulation of a neutrosophic traveling salesman problem

A salesman is needed to travel each and every distinct n cities, numbered by $c_1, c_2, ..., c_n$. The salesman starts from a "base city" numbered by c_0 and he travels all the other n cities exactly one time and returns to the "base city" (c_0). Let $\mathcal{E}_{i,j}$ be the traveling time/cost from city i to city j. It is needed to find such a path which minimizes the total traveling time of the salesman. The neutrosophic traveling salesman problem can be formulated as following mathematical model

$$\sum_{0 \leq i \neq j \leq n} \sum \mathcal{E}_{i,j} y_{i,j} \tag{5}$$

$$\sum_{i=0\ i \neq j}^{n} y_{i,j} = 1 \quad (j = 1, ...n) \tag{6}$$

$$\sum_{i=0\ j \neq i}^{n} y_{i,j} = 1 \quad (i = 1, ...n) \tag{7}$$

$$\sum_{i,j \in S} y_{i,j} \in |S| - 1 \quad (S \subset V, 2 \leq |S| \leq n-2) \tag{8}$$

$$y_{i,j} \in \{0, 1\} \quad \forall i, j \in A \tag{9}$$

4 Proposed Genetic algorithm

The TSP is a well known permutation problem in the area of operation research. The TSP finds the shortest path among N different places that the salesman considers as tour. Our proposed genetic algorithm solves the neutrosophic TSP of the neutrosophic graph $\mathcal{G}$. Our proposed algorithm for this problem is shown in Algorithm 1. [h!]

[1]

Initial Population: Create an initial population P. each chromosome(traveling path) in the P **Evaluate:** Compute the fitness value of the chromosome as shown in Section 4.4. ($generation \neq maxgen$) **Selection** Apply the roulette wheel selection method to select the chromosomes from the P to create the mating pool as presented in Section 4.5. **Crossover** Apply a crossover to generate their new children chromosome as described in Section 4.2. **Mutation** Apply a mutation to generate in the chromosome using mutation operation as presented in Section 4.3. **Update** $current\_populations$=$child\_populations$ $generations$ = $generations$ +1. **Evaluate:** Compute the fitness value of the chromosome as shown in Section 4.4. Return

(1, 1) node[circle, draw](m2)2 (1, 4) node[circle, draw](m1)1 (3.5, 2.5) node[circle, draw](m3)3 (6, 1) node[circle, draw](m5)5 (6, 4) node[circle, draw](m4)4 (9, 2) node[circle, draw](m6)6; [-] (m1) – node[left] 2 (m2) [-] (m1) – node[above] 1 (m4) [-] (m1) – node[right] 3 (m3) [-] (m2) – node[above] 9 (m3) [-] (m2) – node[below] 10 (m5) [-] (m3) – node[above] 4 (m4) [-] (m3) – node[above] 8 (m5) [-] (m4) – node[right] 7 (m5) [-] (m4) – node[above] 5 (m6) [-] (m5) – node[below] 6 (m6);

Figure 1: A neutrosophic graph with neutrosophic numbers as arc lengths for Example 1.

https://doi.org/10.54216/IJNS.190304

Received: June 06, 2022 Accepted: November 12, 2022

4.1 Encoding

We sequentially number all the cities starting from 1. The path among the cities is denoted using an array which is a chromosome. Here, a chromosome is nothing but a collection of integers and each element (integer) of the chromosome represents the different city. A chromosome describes the order in which the cities are visited to construct a route. Each and every city presents exactly one time in a chromosome.

4.2 Crossover

Crossover is the method for choosing two parent chromosomes (solutions) and generating from them a child chromosome. Crossover helps to make clones of good chromosomes (solutions). It is used in the genetic algorithm with the hope that it generates some better chromosomes. The simple crossover (single point crossover or two point crossover) may not work properly in neutrosophic traveling salesman problem because it can make some invalid chromosomes. In those chromosome, some cities may be dropped out while other are doubled.

We use the order crossover technique to avoid this problem. We describe the crossover below. [h!] [1] Two parent chromosomes P_1 and P_2 are chosen from the population. A substring is randomly chosen from a parent chromosome P_1. A child chromosome C_1 is produced by copying the selected substring into the corresponding position of it. The cities of P_2 are traversed one by one from left to right, if a city of P_2 is absent in C_1 then place it into C_1 at the first available position in child chromosome C_1.

4.3 Mutation

In our proposed algorithm, mutation technique is applied on the chromosome after crossover operation. This operation creates the diversity which prevents our algorithm to be trapped in a local maximum. The mutation technique of our algorithm is described below.

Step 1. Two positions are randomly chosen from a chromosome.

Step 2. The relative cities in those two positions are swapped.

4.4 Fitness function

In genetic algorithm, the fitness function is used to calculate fitness of a chromosome. The fitness of chromosome is the value of objective function of the problem. For finding the fitness value, the chromosome needs to be first decoded then the objective function is calculated based this. The fitness value is used to describe not only how good the chromosome is, however it also describes how near the solution(chromosome) is to the optimal solution. In this neutrosophic traveling salesman problem, the main objective is to find the shortest path to cover all the cities. The salesman describes this shortest path as a tour and the total distance traveled is minimum. In this study, the arc lengths of the neutrosophic graph are described by neutrosophic number. The path length is represented by triangular neutrosophic number. The path length is computed by summing all the neutrosophic numbers corresponding to the arc (arc lengths) present in the path. A grade mean ranking technique is applied to calculate the rank of the path. The neutrosophic traveling salesman problem is a minimization problem which determines a route having minimum grade mean rank.

4.5 Selection

In this neutrosophic traveling salesman problem, the roulette wheel selection technique is used to select the chromosomes for the crossover operation and mutation operation. In this method, chromosomes with more fitness values have higher probabilities to be picked out compared to chromosomes with less fitness value to create the mating pool.

https://doi.org/10.54216/IJNS.190304

Received: June 06, 2022 Accepted: November 12, 2022

[y=.17cm, x=.08cm,font=][scale=.50] (0,0) – coordinate (x axis mid) (80,0); (0,0) – coordinate (y axis mid) (0,35); (0,30) – (15,30); (15,30) – (16,28.4); (16,28.4) – (80,28.4);
in 0,10,...,80 (,1pt) – (,-3pt) node[anchor=north] ; [below=0.8cm] at (x axis mid) Iteration;
in 0,5,...,35 (1pt,) – (-3pt,) node[anchor=east] ;

Figure 2: Convergence curve for Example 1.

[scale=2.0] (0, 2) node[circle, draw](P1)1 (2, 2) node[circle, draw](P2)2 (2, 0) node[circle, draw](P3)3 (0, 0) node[circle, draw](P4)4; [-¿,ultra thick] (P1)– node[above] 1(P2); [-¿,ultra thick] (P3)– node[above] 6(P4); [-¿] (P1)– node[left] 7(P4); [-¿,ultra thick] (P2)– node[right] 5(P3); [-¿] (P1)– node[right] 4(P3); [-¿] (P2)– node[left] 4(P4); (P4) edge[-¿,bend left,ultra thick] node[left] 8 (P1); (P2) edge[-¿,bend right] node[above] 10 (P1); (P3) edge[-¿,bend right] node[right] 9 (P2); (P4) edge[-¿,bend right] node[below] 9 (P3); (P4) edge[-¿,bend right] node[right] 8 (P2); (P3) edge[-¿,bend left] node[left] 7 (P1);

Figure 3: A neutrosophic network for neutrosophic traveling salesman problem.

5 Results

The computational experiment of our proposed genetic algorithm is done on a two randomly generated neutrosophic graphs. Those two neutrosophic graphs are shown in Figure 1 and Figure 3.

Table 1: The arc lengths of the neutrosophic graph graph, represented as neutrosophic number.

Index	**SVNs**
1	$\langle(4.6, 5.5, 8.6)\rangle$
2	$\langle(4.7, 6.9, 8.5)\rangle$
3	$\langle(6.2, 7.6, 8.2)\rangle$
4	$\langle(6.2, 8.9, 9.1)\rangle$
5	$\langle(4.4, 5.9, 7.2)\rangle$
6	$\langle(6.6, 8.8, 10)\rangle$
7	$\langle(6.3, 7.5, 8.9), \rangle$
8	$\langle(6.2, 7.6, 8.2)\rangle$
9	$\langle(4.4, 5.9, 7.2)\rangle$
10	$\langle(4.4, 5.9, 7.2)\rangle$

Table 2: Result of the neutrosophic traveling salesman problem

Solution using Linear programming	**Solution using proposed GA**
Min Z= 26.9 $x_{14} = 1, x_{46} = 1,$ $x_{65} = 1, x_{53} = 1, x_{32} = 1$	The cost of NMST= 26.9 route =1 $\rightarrow 4 \rightarrow 6 \rightarrow 5$ $\rightarrow 3 \rightarrow 2 \rightarrow 1$

5.1 Example 1

Our proposed genetic algorithm is used to find the path of traveling salesman problem of a neutrosophic graph. The neutrosophic graph is shown in Figure 1. This neutrosophic graph has 6 nodes and 10 arcs. We have numbered arbitrarily those nodes from 1 to 6, shown in Figure 1. We have described the edge weights of the neutrosophic graph in Table 1 in the form of neutrosophic numbers. To the best of our knowledge, there exist no such neutrosophic graph in the literature whose edge weights are in the form of neutrosophic number. For this reason, we have chosen this ten possible neutrosophic number from and those neutrosophic numbers are indexed from one (1) to ten (10) as described in Table 1. The neutrosophic numbers are putted to the

Received: June 06, 2022 Accepted: November 12, 2022

edges as edge weights of the neutrosophic network. The average values of the neutrosophic path lengths are computed in different generations from 20 runs of the proposed genetic algorithm. In this experiment, the chromosome numbers, repetition number, mutation operation probability and crossover operation probability are respectively 6, 40, 0.7 and 0.4. The route of this neutrosophic traveling salesman problem is $(1 \rightarrow 4 \rightarrow 6 \rightarrow 5 \rightarrow 3 \rightarrow 2 \rightarrow 1)$. We have presented the convergence curve graph for this FTSP in Figure 2.

We have also solved the mathematical model (liner programming problem) of this neutrosophic traveling salesman problem as described in section 3. The TORA software is used to find the solution of the problem. The solution of this problem is presented in Table 2. In this linear programming problem, a decision variable $x_{a,b}$ is used to present the solution of the problem. If the variable $x_{a,b}$ =1, then the (a, b) arc presents in the route. From the Table 2, we have found that the solution (route) of our proposed genetic algorithm is exactly identical with liner programming model. However, our proposed algorithm needs less time than liner programming model. It shows the correctness and efficiency of our proposed algorithm.

5.2 Example 2

We have tested our proposed algorithm on an another neutrosophic graph, shown in Figure 3 with 4 nodes and 12 arcs. We have numbered the nodes (cities) of the neutrosophic network arbitrarily from 1 to 4. The weight of edges of the graph are presented in Table 1. They are in trapezoidal shaped neutrosophic numbers. The average values of the neutrosophic path lengths are calculated in different generations from 5 runs of the proposed genetic algorithm. We have considered the parameter (repetition number = 20, chromosome number = 5, crossover probability = 0.3 and mutation probability= 0.6) for this neutrosophic traveling salesman problem. For all the cases, the shortest route is found as$(1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1)$ for which the corresponding path length is 12.7.

6 Conclusion

In this study, we present the neutrosophic traveling salesman problem, where arc lengths are denoted by triangular neutrosophic number. We describe the utility of neutrosophic number in traveling salesman problem. We describe a mathematical model for neutrosophic traveling salesman problem. The TORA software is used to solve this mathematical model. A genetic algorithm is also introduced to solve the neutrosophic traveling salesman problem. In our genetic algorithm, we describe the encoding technique, fitness/objective function, selection operator, crossover technique, and mutation technique. One simple numerical example is shown to describe the performance and efficiency of our proposed genetic algorithm.

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Received: June 06, 2022 Accepted: November 12, 2022

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https://doi.org/10.54216/IJNS.190304
Received: June 06, 2022 Accepted: November 12, 2022