# Different Types of Operations on Neutrosophic Graphs

**Arindam Dey [1,*], Ranjan Kumar[2], Said Broumi[3,4], Pritam Bhowmik[5]**
[1] School of Computer Science and Engineering (SCOPE), VIT-AP University, Inavolu, Beside AP Secretariat, Amaravati AP, India
[2]Department of Mathematics, VIT-AP University, Inavolu, Beside AP Secretariat, Amaravati AP, India
[3] Faculty of Science Ben M'Sik, University Hassan II, Casablanca, Morocco
[4] Regional Center for the Professions of Education and Training (CRMEF),Casablanca-Settat, Morocco
[5] School of Electrical Engineering, Vellore Institute of Technology, Chennai, India
Emails: arindam84nit@gmail.com; ranjank.nit52@gmail.com; broumisaid78@gmail.com; pritambhowmikjan@gmail.com

**Abstract**

The fuzzy graph theory uses a substantial and important role in modelling and structuring many optimization problems. DIfferent type of uncertainties exist in most of the optimization problems in real lIfe scenarios due to indeterminate and incomplete information and it is a challenging task for the expert to design those optimization problems applying fuzzy graph. To design the incomplete, uncertainty and vagueness in graphical optimization problems, several extensions of graph theoretical ideas are proposed. The idea of neutrosophic graph plays an important role to manage the uncertainty, linked with the indeterminate and incomplete data/information of any optimization problem. In this manuscript, we present the idea of regular neutrosophic graph, strong neutrosophic graph, bipartite neutrosophic graph, regular neutrosophic graph, and regular strong neutrosophic graph. We also introduce six different operations on neutrosophic graph, viz., cartesian product, composition, join, direct product, lexicographic and strong product.

**Keywords:** Neutrosophic; uncertainty; neutrosophic graph; bipartite neutrosophic graph.

## 1 Introduction

Graph theory has many real-life applications to problems in computer applications, systems analysis, computer networking, transportation, operations research, and economics. A graph is essentially a relational model and is used to represent a real-world problem consisting of a relationship between objects. The vertices and edges of the graph are used to represent the objects and relationships between the object of the problem. In many optimization problems, the available information is usually imprecise or imprecise for various reasons such as information loss, lack of evidence, imperfect statistical data, and insufficient information. In general, uncertainty in real-life problems can appear in the problem information that defines the problem.

Classical graph theory uses the concept of classical set theory introduced by Cantor. In a classical graph, there are two possibilities for any vertex or edge: either it is in the graph or it is not a graph. Thus, a classical graph cannot model an uncertain optimization problem. Because real-life problems are often uncertain and it is very difficult to model uncertain real-life problems using a classical graph. A fuzzy set[1] is an extended version of a classical set, where objects have different degrees of membership. A fuzzy set gives its objects different degrees of membership between 0 and 1. Degree of membership is not the same as probability; rather, it describes membership in vaguely defined sets. Atanassov used a new component that finds the degree of

https://doi.org/10.54216/IJNS.190208
Received: March 19, 2022 Accepted: September 05, 2022

non-membership in a fuzzy set. Fuzzy set uses degree of membership while intuitionistic fuzzy set uses both degree of membership and degree of non-membership.

Smarandache[2] presented the concept of neutrosophic set in 1995, by altering the concept of fuzzy set and its extension. The idea of neutrosophic set can manage with indeterminate, uncertain, vague, and inconsistent data of any uncertain optimization real world problem. It is mainly modified idea of classical (crisp) set, type 1 and intuitionistic fuzzy set. It is presented by of three membership degree: true, indeterminate and false membership value. All the 3 different membership degrees are independent to each other and the values lie between the non-standard unit interval] 0, 1[. Neutrosophic graph[3] can handle inconsistent information about several types of real life problem. Recently, several researchers have more actively researched on neutrosophic graph theory, for instance, Yang et al.,[4] Ye,[5] Broumi,[6] Naz et al.,[7] and Arkam.[8–11]

## 2 Preliminaries

In this portion, we will discuss about single valued neutrosophic graph , adjacent node, path, isolated node, strength of a path, strong neutrosophic graph, complement neutrosophic graph and complete neutrosophic graph, which is efficient for the present work.

**Definition 2.1.** Let $G = (P, R)$ be a neutrosophic graph[12,13] where $P$ and $R$ are represented two neutrosophic sets on $V$ and $E$ respectively which satisfy the following.

$$\mathcal{T}_P(w, x) \leq \min\left(\mathcal{T}_R(w), \mathcal{T}_R(x)\right)$$
$$\mathcal{I}_P(w, x) \geq \max\left(\mathcal{I}_R(w), \mathcal{I}_R(x)\right)$$
$$\mathcal{F}_P(w, x) \geq \max\left(\mathcal{F}_R(w), \mathcal{F}_R(x)\right)$$

Here, $w$ and $x$ are two vertices of $G$ and $(w, x) \in E$ is an edge of $G$.

**Definition 2.2.** The two nodes $w$ and $x$ of a neutrosophic graph $G = (P, R)$ are defined as adjacent node in $G$ If and only If $(\mathcal{T}_P(w, x), \mathcal{I}_P(w, x), \mathcal{F}_P(w, x)) > 0$. The two nodes $w$ and $x$ are called neighbor node and $(w, x)$ is incident at $w$ and $x$.

**Definition 2.3.** Let $G = (P, R)$ be a neutrosophic graph and $P$ be a path of $G$. $P$ is a collection of dIfferent nodes, $w_0, w_1, w_2, ..., w_n$ such that $\left(\mathcal{T}_P(w_{i-1}, w_i), \mathcal{I}_P(w_{i-1}, w_i), \mathcal{F}_P(w_{i-1}, w_i)\right) > 0$ for $0 \leq i \leq n$. Here, $n$ represents the neutrosophic length of the path $P$. A single neutrosophic vertex, i.e., $w_i$ in $G$ is also assumed as a path. The path length of a single node $w_i$ is $(0, 0, 0)$. We define the order pair $(w_{i-1}, w_i)$ as the edge of the path. $P$ is said to be a neutrosophic cycle If $w_0 = w_n$ and $n \geq 3$.

**Definition 2.4.** A node $w_i \in V$ of a neutrosophic graph $G$ is called the isolated node If there exists no incident arc to the vertex $w_i$.

**Definition 2.5.** Let $G = (P, R)$ be a neutrosophic graph. $G$ having a path $P$ of path length[12] $k$ from node $w$ to node $x$ in $G$ such as $P = \{a = w_1, (w_1, w_2), w_2, ..., w_{k-1}(w_{k-1}, w_k), w_k = b)\}$ then $T_M^k(w, x)$ , $I_M^k(w, x)$ and $F_M^k(w, x)$ are described as follows.

$$T_M^k(w, x) = \sup\left(\mathcal{T}_P(a, w_1) \wedge \mathcal{T}_P(w_1, w_2) \wedge ... \wedge T_R(w_{k-1}, w_k)\right)$$
$$I_M^k(w, x) = \inf\left(\mathcal{I}_P(a, w_1) \vee \mathcal{I}_P(w_1, w_2) \vee ... \vee \mathcal{I}_P(w_{k-1}, w_k)\right)$$
$$F_M^k(w, x) = \inf\left(\mathcal{F}_P(a, w_1) \vee \mathcal{F}_P(w_1, w_2) \vee ... \vee \mathcal{F}_P(w_{k-1}, w_k)\right)$$

**Definition 2.6.** Let $G = (P, R)$ is a neutrosophic graph. The strength of connection of a path $P$ between two nodes $w$ and $x$ is defined by $(\mathcal{T}_M^\infty(w, x), \mathcal{I}_M^\infty(w, x), \mathcal{F}_M^\infty(w, x))$ where

$$\mathcal{T}_M^\infty(w, x) = \sup\{T_M^k(w, x) \,|k = 1, 2, ...\}$$


https://doi.org/10.54216/IJNS.190208
Received: March 19, 2022 Accepted: September 05, 2022

$$\mathcal{I}_M^{\infty}(w,x) = \inf\{I_M^k(w,x)\,|k=1,2,...\}$$
$$\mathcal{F}_M^{\infty}(w,x) = \inf\{F_M^k(w,x)\,|k=1,2,...\}$$

**Definition 2.7.** Let $G = (P, R)$ is a neutrosophic graph. $G$ is said to be connected neutrosophic graph[12] If there exists no isolated node is in $G$.

**Definition 2.8.** Let $G = (P, R)$ be a neutrosophic graph and $a \in V$ is node of $G$. The degree of node $w$ is the sum of the truth membership values, sum of the indeterminacy membership values and sum of the membership values of falsity of all the arcs which are adjacent to the node $w$. The degree of node[7,11,14] $w$ is denoted by $d(w) = (d_{\mathcal{T}}(w), d_{\mathcal{I}}(w), d_{\mathcal{F}}(w))$ where

$$d_{\mathcal{T}}(w) = \sum_{\substack{w\in V\\ w\neq x}} \mathcal{T}_P(w,x)\,, d_{\mathcal{I}}(w) = \sum_{\substack{w\in V\\ w\neq x}} \mathcal{I}_P(w,x)\,, d_{\mathcal{F}}(w) = \sum_{\substack{w\in V\\ w\neq x}} \mathcal{F}_P(w,x)$$

Here, $d_{\mathcal{T}}(w)$, $d_{\mathcal{I}}(w)$ and $d_{\mathcal{F}}(w)$ are the degree of truth membership value, degree of indeterminacy membership value and degree falsity membership value respectively of the vertex $w$.

**Definition 2.9.** Let $G = (P, R)$ be a neutrosophic graph. Then the order of $G$ is denoted by $O(G) = (O_{\mathcal{T}}(G), O_{\mathcal{I}}(G), O_{\mathcal{F}}(G))$ where

$$O_{\mathcal{T}}(G) = \sum_{a\in V} \mathcal{T}_R(w)\,, O_{\mathcal{I}}(G) = \sum_{a\in V} \mathcal{I}_R(w)\,, O_{\mathcal{F}}(G) = \sum_{a\in V} \mathcal{F}_R(w)$$

Here, $O_{\mathcal{T}}(G)$, $O_{\mathcal{I}}(G)$ and $O_{\mathcal{F}}(G)$ are the order of the membership degree of truth value, indeterminacy value and falsity value of $G$ respectively.

**Definition 2.10.** Let $G = (P, R)$ be a neutrosophic graph and the size graph of $G$ is described as $S(G) = (S_{\mathcal{T}}(G), S_{\mathcal{I}}(G), S_{\mathcal{F}}(G))$ where

$$S_{\mathcal{T}}(G) = \sum_{\substack{w,x\in V\\ a\neq b}} \mathcal{T}_P(w,x)\,, S_{\mathcal{I}}(G) = \sum_{\substack{w,x\in V\\ a\neq b}} \mathcal{I}_P(w,x)\,, S_{\mathcal{F}}(G) = \sum_{\substack{w,x\in V\\ a\neq b}} \mathcal{F}_P(w,x)$$

Here, $S_{\mathcal{T}}(G)$, $S_{\mathcal{I}}(G)$ and $S_{\mathcal{F}}(G)$ are respectively the order of the membership degree of truth, indeterminacy and falsity of $G$.

**Definition 2.11.** Let $G = (P, R)$ is a neutrosophic graph. $G$ is said to be a strong neutrosophic graph[7,11,14] If

$$\mathcal{T}_P(w,x) = \min(\mathcal{T}_R(w), \mathcal{T}_R(x))$$
$$\mathcal{I}_P(w,x) = \max(\mathcal{I}_R(w), \mathcal{I}_R(x))$$
$$\mathcal{F}_P(w,x) = \max(\mathcal{F}_R(w), \mathcal{F}_R(x)), \forall (w,x) \in E$$

**Definition 2.12.** Let $G = (P, R)$ be a neutrosophic graph and $G$ is said to be a complete neutrosophic graph[7,11,14] If

$$\mathcal{T}_P(w,x) = \min(\mathcal{T}_R(w), \mathcal{T}_R(x))$$
$$\mathcal{I}_P(w,x) = \max(\mathcal{I}_R(w), \mathcal{I}_R(x))$$
$$\mathcal{F}_P(w,x) = \max(\mathcal{F}_R(w), \mathcal{F}_R(x)), \forall w,x \in V$$

**Definition 2.13.** Let $G = (P, R)$ be a neutrosophic graph. The $G^c = (P^c, R^c)$ is the complement of a neutrosophic graph[7,11,14] If $P^c = P$ and $R^c$ is computed as below.

$$\mathcal{T}_P^c(w,x) = \min(\mathcal{T}_R(w), \mathcal{T}_R(x)) - \mathcal{T}_P(w,x)$$
$$\mathcal{I}_P^c(w,x) = \max(\mathcal{I}_R(w), \mathcal{I}_R(x)) - \mathcal{I}_P(w,x)$$
$$\mathcal{F}_P^c(w,x) = \max(\mathcal{F}_R(w), \mathcal{F}_R(x)) - \mathcal{F}_P(w,x), \forall w,x \in V$$

Here, $\mathcal{T}_P^c(w,x)$, $\mathcal{I}_P^c(w,x)$ and $\mathcal{F}_P^c(w,x)$ are denoted the true, intermediate and false membership degree for edge $(w,x)$ of $G^c$.

https://doi.org/10.54216/IJNS.190208
Received: March 19, 2022 Accepted: September 05, 2022

## 3 Operations on neutrosophic graph

In this section, we introduce six operations on neutrosophic graph, viz., cartesian product, composition, join, direct product, lexicographic and strong product.

**Definition 3.1.** Let $G_1 = (P_1, R_1)$ and $G_2 = (P_2, R_2)$ are two neutrosophic graph of $G_1^* = (\mathcal{V}_1, \mathcal{E}_1)$ and $G_2^* = (\mathcal{V}_2, \mathcal{E}_2)$ respectively. The cartesian product $G_1 \times G_2$ of neutrosophic graph $G_1$ and $G_2$ is defined by $(A, B)$, where $A = (\mathcal{T}_P, \mathcal{I}_P, \mathcal{F}_P)$ and $B = (\mathcal{T}_R, \mathcal{I}_R, \mathcal{F}_R)$ are two neutrosophic sets on $V$=$(V_1 \times V_2)$, and $E = \{(x, w_2), (x, x_2) | x \in \mathcal{V}_1, w_2x_2 \in \mathcal{E}_2\} \cup \{(w_1, z), (x_1, z) | z \in \mathcal{V}_2, w_1x_1 \in \mathcal{E}_1\}$ respectively which satisfies the followings:

$(i)$ $\forall (w_1, w_2) \in V_1 \times V_2$,

(a) $\mathcal{T}_P((w_1, w_2)) = \mathcal{T}_{P_1}(w_1) \wedge \mathcal{T}_{P_2}(w_2)$

(b) $\mathcal{I}_P((w_1, w_2)) = \mathcal{I}_{P_1}(w_1) \wedge \mathcal{I}_{P_2}(w_2)$

(c) $\mathcal{F}_P((w_1, w_2)) = \mathcal{F}_{P_1}(w_1) \vee \mathcal{F}_{P_2}(w_2)$

$(ii)$ $\forall x \in V_1$ and $\forall (w_2, x_2) \in E_2$,

(a) $\mathcal{T}_R((x, w_2)(x, x_2)) = \mathcal{T}_{P_1}(w_1) \wedge \mathcal{T}_{R_2}(w_2x_2)$

(b) $\mathcal{I}_R((x, w_2)(x, x_2)) = \mathcal{I}_{P_1}(w_1) \vee \mathcal{I}_{R_2}(w_2x_2)$

(c) $\mathcal{F}_R((x, w_2)(x, x_2)) = \mathcal{F}_{P_1}(w_1) \vee \mathcal{F}_{R_2}(w_2x_2)$

$(iii)$ $\forall z \in V_2$ and $\forall (w_1, w_2) \in E_1$,

(a) $\mathcal{T}_R((w_1, z)(x_1, z)) = \mathcal{T}_{R_1}(w_1x_1) \wedge \mathcal{T}_{P_2}(z)$

(b) $\mathcal{I}_R((w_1, z)(x_1, z) = \mathcal{I}_{R_1}(w_1x_1) \vee \mathcal{I}_{P_2}(z)$

(c) $\mathcal{F}_R((w_1, z)(x_1, z)) = \mathcal{F}_{R_1}(w_1x_1) \vee \mathcal{F}_{P_2}(z)$

**Definition 3.2.** The composition $G_1G_2$ of two neutrosophic graph $G_1 = (P_1, R_1)$ and $G_2 = (P_2, R_2)$ defined as a pair $(A, B)$, where $A = (\mathcal{T}_P, \mathcal{I}_P, \mathcal{F}_P)$ and $B = (\mathcal{T}_R, \mathcal{I}_R, \mathcal{F}_R)$ are two neutrosophic sets on $V$=$(V_1 \times V_2)$, and $E = \{(x, w_2), (x, x_2) | x \in \mathcal{V}_1, w_2x_2 \in \mathcal{E}_2\} \cup \{(w_1, z), (x_1, z) | z \in \mathcal{V}_2, w_1x_1 \in \mathcal{E}_1\} \cup \{(w_1, w_2), (x_1, x_2) | w_2x_2 \in \mathcal{V}_2 \neq x_2, w_1x_1 \in \mathcal{E}_1\}$ respectively, which satisfies the followings:

$(i)$ $\forall (w_1, w_2) \in V_1 \times V_2$,

(a) $\mathcal{T}_P((w_1, w_2)) = \mathcal{T}_{P_1}(w_1) \wedge \mathcal{T}_{P_2}(w_2)$

(b) $\mathcal{I}_P((w_1, w_2)) = \mathcal{I}_{P_1}(w_1) \vee \mathcal{I}_{P_2}(w_2)$

(c) $\mathcal{F}_P((w_1, w_2)) = \mathcal{F}_{P_1}(w_1) \vee \mathcal{F}_{P_2}(w_2)$

$(ii)$ $\forall x \in V_1$ and $\forall (w_2, x_2) \in E_2$,

(a) $\mathcal{T}_R((x, w_2)(x, x_2)) = \mathcal{T}_{P_1}(w_1) \wedge \mathcal{T}_{R_2}(w_2x_2)$

(b) $\mathcal{I}_R((x, w_2)(x, x_2)) = \mathcal{I}_{P_1}(w_1) \vee \mathcal{I}_{R_2}(w_2x_2)$

(c) $\mathcal{F}_R((x, w_2)(x, x_2)) = \mathcal{F}_{P_1}(w_1) \vee \mathcal{F}_{R_2}(w_2x_2)$

https://doi.org/10.54216/IJNS.190208
Received: March 19, 2022 Accepted: September 05, 2022

$(iii)$ $\forall z \in V_2$ and $\forall (w_1, w_2) \in E_1$,

(a) $\mathcal{T}_R((w_1, z)(x_1, z)) = \mathcal{T}_{R_1}(w_1 x_1) \wedge \mathcal{T}_{P_2}(z)$

(b) $\mathcal{I}_R((w_1, z)(x_1, z) = \mathcal{I}_{R_1}(w_1 x_1) \vee \mathcal{I}_{P_2}(z)$

(c) $\mathcal{F}_R((w_1, z)(x_1, z)) = \mathcal{F}_{R_1}(w_1 x_1) \vee \mathcal{F}_{P_2}(z)$

$(iv)$ $\forall w_2 x_2 \in V_2$, $w_2 \neq x_2$ and $\forall (w_1 x_1) \in E_1$,

(a) $\mathcal{T}_R((w_1, w_2)(x_1, x_2)) = \mathcal{T}_{P_2}(w_2) \wedge \mathcal{T}_{P_2}(x_2) \wedge \mathcal{T}_{R_1}(w_1 x_1)$

(b) $\mathcal{I}_R((w_1, w_2)(x_1, x_2)) = \mathcal{I}_{P_2}(w_2) \wedge \mathcal{I}_{P_2}(x_2) \wedge \mathcal{I}_{R_1}(w_1 x_1)$

(c) $\mathcal{F}_R((w_1, w_2)(x_1, x_2)) = \mathcal{F}_{P_2}(w_2) \wedge \mathcal{F}_{P_2}(x_2) \wedge \mathcal{F}_{R_1}(w_1 x_1)$

**Definition 3.3.** The union $G_1 \cup G_2$ of two neutrosophic graph $G_1 = (P_1, R_1)$ and $G_2 = (P_2, R_2)$ is defined as $(A, B)$, where $A = (\mathcal{T}_P, \mathcal{I}_P, \mathcal{F}_P)$ is a neutrosophic set on $V = \mathcal{V}_1 \cup \mathcal{V}_2$ and $B = (\mathcal{T}_R, \mathcal{I}_R, \mathcal{F}_R)$ is an another neutrosophic set on $E = \mathcal{E}_1 \cup \mathcal{E}_2$, which satisfies the following:

$(i)$ (a) $\mathcal{T}_P(x) = \mathcal{T}_{P_1}(x)$ If $x \in \mathcal{V}_1$ and $x \notin \mathcal{V}_2$

(b) $\mathcal{T}_P(x) = \mathcal{T}_{P_2}(x)$ If $x \in \mathcal{V}_2$ and $x \notin \mathcal{V}_1$

(c) $\mathcal{T}_P(x) = \mathcal{T}_{P_1}(x) \wedge \mathcal{T}_{P_2}(x)$ If $x \in \mathcal{V}_1 \cap \mathcal{V}_2$

$(ii)$ (a) $\mathcal{I}_P(x) = \mathcal{I}_{P_1}(x)$ If $x \in \mathcal{V}_1$ and $x \notin \mathcal{V}_2$

(b) $\mathcal{I}_P(x) = \mathcal{I}_{P_2}(x)$ If $x \in \mathcal{V}_2$ and $x \notin \mathcal{V}_1$

(c) $\mathcal{I}_P(x) = \mathcal{I}_{P_1}(x) \wedge \mathcal{I}_{P_2}(x)$ If $x \in \mathcal{V}_1 \cap \mathcal{V}_2$

$(iii)$ (a) $\mathcal{F}_P(x) = \mathcal{F}_{P_1}(x)$ If $x \in \mathcal{V}_1$ and $x \notin \mathcal{V}_2$

(b) $\mathcal{F}_P(x) = \mathcal{F}_{P_2}(x)$ If $x \in \mathcal{V}_2$ and $x \notin \mathcal{V}_1$

(c) $\mathcal{F}_P(x) = \mathcal{F}_{P_1}(x) \wedge \mathcal{I}_{P_2}(x)$ If $x \in \mathcal{V}_1 \cap \mathcal{V}_2$

$(iv)$ (a) $\mathcal{T}_R(xy) = \mathcal{T}_{R_1}(xy)$ If $xy \in \mathcal{E}_1$ and $xy \notin \mathcal{E}_2$

(b) $\mathcal{T}_R(xy) = \mathcal{T}_{R_2}(xy)$ If $xy \in \mathcal{E}_2$ and $xy \notin \mathcal{E}_1$

(c) $\mathcal{T}_R(xy) = \mathcal{T}_{R_1}(xy) \wedge \mathcal{T}_{R_2}(xy)$ If $xy \in \mathcal{E}_1 \cap \mathcal{E}_2$

$(v)$ (a) $\mathcal{I}_R(xy) = \mathcal{I}_{R_1}(xy)$ If $xy \in \mathcal{E}_1$ and $xy \notin \mathcal{E}_2$

(b) $\mathcal{I}_R(xy) = \mathcal{I}_{R_2}(xy)$ If $xy \in \mathcal{E}_2$ and $xy \notin \mathcal{E}_1$

(c) $\mathcal{I}_R(xy) = \mathcal{I}_{R_1}(xy) \wedge \mathcal{I}_{R_2}(xy)$ If $xy \in \mathcal{E}_1 \cap \mathcal{E}_2$

https://doi.org/10.54216/IJNS.190208
Received: March 19, 2022 Accepted: September 05, 2022

$(vi)$ (a) $\mathcal{F}_R(xy) = \mathcal{F}_{R_1}(xy)$ If $xy \in \mathcal{E}_1$ and $xy \notin \mathcal{E}_2$

(b) $\mathcal{F}_R(xy) = \mathcal{F}_{R_2}(xy)$ If $xy \in \mathcal{E}_2$ and $xy \notin \mathcal{E}_1$

(c) $\mathcal{F}_R(xy) = \mathcal{F}_{R_1}(xy) \wedge \mathcal{F}_{R_2}(xy)$ If $xy \in \mathcal{E}_1 \cap \mathcal{E}_2$

**Definition 3.4.** The join $G_1 + G_2$ of two neutrosophic graph $G_1 = (P_1, R_1)$ and $G_2 = (P_2, R_2)$ is defined as $(A, B)$, where $A = (\mathcal{T}_P, \mathcal{I}_P, \mathcal{F}_P)$ is a neutrosophic set on $V = \mathcal{V}_1 \cup \mathcal{V}_2$ and $B = (\mathcal{T}_R, \mathcal{I}_R, \mathcal{F}_R)$ is an another neutrosophic set on $E = \mathcal{E}_1 \cup \mathcal{E}_2 \cup \mathcal{E}^{'}$(($\mathcal{E}^{'}$ represents all edges joining the vertex of $\mathcal{V}_1$ and $\mathcal{V}_2$ ), which satisfies the following:

$(i)$ (a) $\mathcal{T}_P(x) = \mathcal{T}_{P_1}(x)$ If $x \in \mathcal{V}_1$ and $x \notin \mathcal{V}_2$

(b) $\mathcal{T}_P(x) = \mathcal{T}_{P_2}(x)$ If $x \in \mathcal{V}_2$ and $x \notin \mathcal{V}_1$

(c) $\mathcal{T}_P(x) = \mathcal{T}_{P_1}(x) \wedge \mathcal{T}_{P_2}(x)$ If $x \in \mathcal{V}_1 \cap \mathcal{V}_2$

$(ii)$ (a) $\mathcal{I}_P(x) = \mathcal{I}_{P_1}(x)$ If $x \in \mathcal{V}_1$ and $x \notin \mathcal{V}_2$

(b) $\mathcal{I}_P(x) = \mathcal{I}_{P_2}(x)$ If $x \in \mathcal{V}_2$ and $x \notin \mathcal{V}_1$

(c) $\mathcal{I}_P(x) = \mathcal{I}_{P_1}(x) \wedge \mathcal{I}_{P_2}(x)$ If $x \in \mathcal{V}_1 \cap \mathcal{V}_2$

$(iii)$ (a) $\mathcal{F}_P(x) = \mathcal{F}_{P_1}(x)$ If $x \in \mathcal{V}_1$ and $x \notin \mathcal{V}_2$

(b) $\mathcal{F}_P(x) = \mathcal{F}_{P_2}(x)$ If $x \in \mathcal{V}_2$ and $x \notin \mathcal{V}_1$

(c) $\mathcal{F}_P(x) = \mathcal{F}_{P_1}(x) \wedge \mathcal{I}_{P_2}(x)$ If $x \in \mathcal{V}_1 \cap \mathcal{V}_2$

$(iv)$ (a) $\mathcal{T}_R(xy) = \mathcal{T}_{R_1}(xy)$ If $xy \in \mathcal{E}_1$ and $xy \notin \mathcal{E}_2$

(b) $\mathcal{T}_R(xy) = \mathcal{T}_{R_2}(xy)$ If $xy \in \mathcal{E}_2$ and $xy \notin \mathcal{E}_1$

(c) $\mathcal{T}_R(xy) = \mathcal{T}_{R_1}(xy) \wedge \mathcal{T}_{R_2}(xy)$ If $xy \in \mathcal{E}_1 \cap \mathcal{E}_2$

$(v)$ (a) $\mathcal{I}_R(xy) = \mathcal{I}_{R_1}(xy)$ If $xy \in \mathcal{E}_1$ and $xy \notin \mathcal{E}_2$

(b) $\mathcal{I}_R(xy) = \mathcal{I}_{R_2}(xy)$ If $xy \in \mathcal{E}_2$ and $xy \notin \mathcal{E}_1$

(c) $\mathcal{I}_R(xy) = \mathcal{I}_{R_1}(xy) \wedge \mathcal{I}_{R_2}(xy)$ If $xy \in \mathcal{E}_1 \cap \mathcal{E}_2$

$(vi)$ (a) $\mathcal{F}_R(xy) = \mathcal{F}_{R_1}(xy)$ If $xy \in \mathcal{E}_1$ and $xy \notin \mathcal{E}_2$

(b) $\mathcal{F}_R(xy) = \mathcal{F}_{R_2}(xy)$ If $xy \in \mathcal{E}_2$ and $xy \notin \mathcal{E}_1$

(c) $\mathcal{F}_R(xy) = \mathcal{F}_{R_1}(xy) \wedge \mathcal{F}_{R_2}(xy)$ If $xy \in \mathcal{E}_1 \cap \mathcal{E}_2$

$(vii)$ (a) $\mathcal{T}_R(xy) = \mathcal{T}_{R_1}(x) \vee \mathcal{T}_{R_2}(y)$ If $xy \in \mathcal{E}^{'}$


Received: March 19, 2022 Accepted: September 05, 2022

(b) $\mathcal{I}_R(xy) = \mathcal{I}_{R_1}(x) \wedge \mathcal{I}_{R_2}(y)$ If $xy \in \mathcal{E}^{'}$

(c) $\mathcal{F}_R(xy) = \mathcal{F}_{R_1}(x) \wedge \mathcal{F}_{R_2}(y)$ If $xy \in \mathcal{E}^{'}$

**Definition 3.5.** The direct product $G_1 * G_2$ of two neutrosophic graph $G_1$ and $G_2$ is defined as a pair $(A, B)$, where $A = (\mathcal{T}_P, \mathcal{I}_P, \mathcal{F}_P)$ is a neutrosophic set on $V = \mathcal{V}_1 \times \mathcal{V}_2$ and $B = (\mathcal{T}_R, \mathcal{I}_R, \mathcal{F}_R)$ is an another neutrosophic set on $E = \{(w_1, w_2)(x_1, x_2) | w_1 x_1 \in \mathcal{E}_1, w_2 x_2 \in \mathcal{E}_2\}$, which satisfies the followings:

$(i)$ $\forall (w_1, w_2) \in \mathcal{V}_1 \times$

(a) $\mathcal{T}_P(w_1, w_2) = \mathcal{T}_{P_1}(w_1) \vee \mathcal{T}_{P_2}(w_2)$
(b) $\mathcal{I}_P(w_1, w_2) = \mathcal{I}_{P_1}(w_1) \wedge \mathcal{I}_{P_2}(w_2)$
(c) $\mathcal{F}_P(w_1, w_2) = \mathcal{F}_{P_1}(w_1) \wedge \mathcal{F}_{P_2}(w_2)$

$(ii)$ $\forall (w_1 x_1) \in \mathcal{E}_1, \forall (w_2 x_2) \in \mathcal{E}_2$

(a) $\mathcal{T}_R(w_1, w_2)(x_1, x_2) = \mathcal{T}_{R_1}(w_1 x_1) \vee \mathcal{T}_{R_2}(w_2 x_2)$
(b) $\mathcal{I}_R(w_1, w_2)(x_1, x_2) = \mathcal{I}_{R_1}(w_1 x_1) \wedge \mathcal{I}_{R_2}(w_2 x_2)$
(c) $\mathcal{F}_R(w_1, w_2)(x_1, x_2) = \mathcal{F}_{R_1}(w_1 x_1) \wedge \mathcal{T}_{R_2}(w_2 x_2)$

## 4 Conclusion

The idea of graph/network theory has applied to several types to the problems in the area of computer network, operations research, economics, wireless networking, computer systems analysis, transportation and traffic planning. Uncertainty exists in almost every graph theory problem in real life scenarios. Neutrosophic set is a popular and useful method to manage the uncertainty. To model the complex real life problems, a number of extension of neutrosophic graph have been studied. The main objective of this manuscript is to present the idea of different type of neutrosophic graph and its different operation. We also present some different types of neutrosophic graph such as regular picture fuzzy graph, strong picture fuzzy graph, complete picture fuzzy graph, and complement picture fuzzy graph. Five different operations on neutrosophic graph, viz. cartesian product, composition, join, direct product, lexicographic and strong product are described in this paper. .

## References

[1] L. A. Zadeh, "Fuzzy sets," *Information and control*, vol. 8, no. 3, pp. 338–353, 1965.

[2] F. Smarandache, *A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability: Neutrsophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability*. Infinite Study, 2005.

[3] G. Ghorai and M. Pal, "Certain types of product bipolar fuzzy graphs," *International Journal of Applied and Computational Mathematics*, vol. 3, no. 2, pp. 605–619, 2017.

[4] H.-L. Yang, Z.-L. Guo, Y. She, and X. Liao, "On single valued neutrosophic relations," *Journal of Intelligent & Fuzzy Systems*, vol. 30, no. 2, pp. 1045–1056, 2016.

[5] J. Ye, "Single-valued neutrosophic minimum spanning tree and its clustering method," *Journal of intelligent Systems*, vol. 23, no. 3, pp. 311–324, 2014.

[6] R. A. Borzooei, H. Rashmanlou, S. Samanta, and M. Pal, "Regularity of vague graphs," *Journal of Intelligent & Fuzzy Systems*, vol. 30, no. 6, pp. 3681–3689, 2016.

[7] S. Naz, H. Rashmanlou, and M. A. Malik, "Operations on single valued neutrosophic graphs with application," *Journal of Intelligent & Fuzzy Systems*, vol. 32, no. 3, pp. 2137–2151, 2017.

*https://doi.org/10.54216/IJNS.190208*

*Received: March 19, 2022 Accepted: September 05, 2022*

[8] M. Akram, N. Waseem, and W. A. Dudek, "Certain types of edge m-polar fuzzy graphs," *Iranian Journal of Fuzzy Systems*, vol. 14, no. 4, pp. 27–50, 2017.

[9] M. Akram, "Certain bipolar neutrosophic competition graphs," *Journal of the Indonesian Mathematical Society*, vol. 24, no. 1, pp. 1–25, 2017.

[10] M. Akram and S. Siddique, "Neutrosophic competition graphs with applications," *Journal of Intelligent & Fuzzy Systems*, vol. 33, no. 2, pp. 921–935, 2017.

[11] M. Akrama, *Single-valued neutrosophic planar graphs*. Infinite Study, 2016.

[12] S. Broumi, M. Talea, A. Bakali, and F. Smarandache, "On bipolar single valued neutrosophic graphs," *Journal of New Theory*, vol. 11, pp. 84–102, 2016.

[13] F. Karaaslan and B. Davvaz, "Properties of single-valued neutrosophic graphs," *Journal of Intelligent & Fuzzy Systems*, vol. 34, no. 1, pp. 57–79, 2018.

[14] S. Broumi, F. Smarandache, M. Talea, and A. Bakali, *Operations on interval valued neutrosophic graphs*. Infinite Study, 2016.


https://doi.org/10.54216/IJNS.190208

Received: March 19, 2022 Accepted: September 05, 2022