Generalized of A-Closed Set and Ƈ- Closed Set in Fuzzy Neutrosophic Topological Spaces

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Abstract

In this research paper, a new two classes of sets called fuzzy neutrosophic generalized A-closed sets and fuzzy neutrosophic generalized Ƈ-Closed sets in fuzzy neutrosophic topology are introduced and some of their properties have been investigated. We give some theorems, propositions and some necessary examples related to presented definitions. Then, we discuss the relations among the new defined sets.

Keywords: Fuzzy Neutrosophic Sets; Fuzzy Neutrosophic Generalized A-Closed Set; Generalized Fuzzy Neutrosophic Ƈ-Closed Set; Fuzzy Neutrosophic Topology.

1.Introduction

In 1965, L. Zadeh [1] was introducing the significant theory on fuzzy sets "FSs" where the membership of any object having by the a unit interval [0,1]. Then, in 1986 K. Atanassov [2-4] generalization of the term of "FSs" and introduced the concept of intuitionstic fuzzy sets "IFSs" whose the elements had the non-membership values in addition to the membership ones with the same unit interval [0,1]. Later, in 1995 F. Smarsndache [5,6] introduced neutrosophic logic , regarded as new logic differs from the last two logics (fuzzy logic and intuitionistic fuzzy logic) and called it, neutrosophic theory "NSs", then in 2012, A.Salama et.al. [7] studied the term of neutrosophic topological spae"NTS". As generalization of "NTS" and exactly before five years ago in 2017, Y.Veereswari [8] gave an introduction of fuzzy neutrosophic topological spaces "FNTSs".
In this studied paper, some new classes of sets called, generalized A-closed and generalized Ƈ-closed sets via fuzzy neutrosophic topological spaces were introduce and study as generalization of F. Mohammed [9-12], and T.M. Nour et al. [13-16]. Finally, we discussed some relationships between them and we comparative them with the other sets.

2. Preliminaries

Definition 2.1[8] "Let Ҳ be any non-empty fixed set. The fuzzy neutrosophic set (FNS, for short), λ_{FN} is an object define as the form λ_{FN} ={<x, $\mu_{\lambda FN}$ (x), $\sigma_{\lambda FN}$ (x), $\nu_{\lambda FN}$ (x) >:x∈Ҳ} where the functions $\mu_{\lambda FN}$, $\sigma_{\lambda FN}$, $\nu_{\lambda FN}$: Ҳ → [0, 1] denote the degree of membership function (namely $\mu_{\lambda FN}$(x)), also the degree of indeterminacy function (namely $\sigma_{\lambda FN}$ (x)) and the degree of non-membership (namely $\nu_{\lambda FN}$ (x)) respectively of each element x∈ Ҳ to the set λ_{FN} and 0 ≤ $\mu_{\lambda FN}$ (x) + $\sigma_{\lambda FN}$ (x) + $\nu_{\lambda FN}$ (x) ≤ 3, for each x∈Ҳ."

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Remark 2.2 [8] FNS λ_{FN} = {< x, $\mu_{\lambda FN}$ (x), $\sigma_{\lambda FN}$ (x), $\nu_{\lambda FN}$ (x)>: x ∈ Ҳ} can be identified to an ordered triple <x, $\mu_{\lambda FN}$, $\sigma_{\lambda FN}$, $\nu_{\lambda FN}$ > in [0, 1] on Ҳ.

Definition 2.3 [8] "Let Ҳ be a non-empty set and the FNSs λ_{FN} and β_{FN} be in the form:

$$\lambda_{FN} = \{<x, \mu_{\lambda FN}(x), \sigma_{\lambda FN}(x), \nu_{\lambda FN}(x)>: x \in Ҳ\} \text{ and}$$

$$\beta_{FN} = \{<x, \mu_{\beta FN}(x), \sigma_{\beta FN}(x), \nu_{\beta FN}(x)>: x \in Ҳ\} \text{ on } Ҳ.$$

Then,

- **i.** $\lambda_{FN} \subseteq \beta_{FN}$ iff $\mu_{\lambda FN}(x) \leq \mu_{\beta FN}(x)$, $\sigma_{\lambda FN}(x) \leq \sigma_{\beta FN}(x)$ and $\nu_{\lambda FN}(x) \geq \nu_{\beta FN}(x)$ for all x ∈ Ҳ,
- **ii.** $\lambda_{FN} = \beta_{FN}$ iff $\lambda_{FN} \subseteq \beta_{FN}$ and $\beta_{FN} \subseteq \lambda_{FN}$,
- **iii.** $(\lambda_{FN})^c = 1_{FN}-\lambda_{FN}$ = {<x, $\nu_{\lambda FN}$ (x), 1- $\sigma_{\lambda FN}$ (x), $\mu_{\lambda FN}$ (x)>: x ∈ Ҳ},
- **iv.** $\lambda_{FN} \cup \beta_{FN}$ = {<x, Max($\mu_{\lambda FN}$ (x), $\mu_{\beta FN}$ (x)), Max($\sigma_{\lambda FN}$ (x), $\sigma_{\beta FN}$ (x)),Min ($\nu_{\lambda FN}$ (x), $\nu_{\beta FN}$ (x)) >: x ∈ Ҳ},
- **v.** $\lambda_{FN} \cap \beta_{FN}$ = {<x, Min($\mu_{\lambda FN}$ (x), $\mu_{\beta FN}$ (x)), Min($\sigma_{\lambda FN}$ (x), $\sigma_{\beta FN}$ (x)),Max($\nu_{\lambda FN}$ (x), $\nu_{\beta FN}$ (x)) >: x ∈ Ҳ},
- **vi.** 0_{FN} = <x, 0, 0, 1> and 1_{FN} = <x, 1, 1, 0>. "

Proposition 2.4 [13] "Let λ_{FN} and β_{FN} are FNSs in Ҳ. Then,

- **i.** $1_{FN}-(\lambda_{FN} \cup \beta_{FN}) = (1_{FN}-\lambda_{FN}) \cap (1_{FN}-\beta_{FN})$,
- **ii.** $1_{FN}-(\lambda_{FN} \cap \beta_{FN}) = (1_{FN}-\lambda_{FN}) \cup (1_{FN}-\beta_{FN})$,
- **iii.** $\lambda_{FN} \subseteq \beta_{FN} \Leftrightarrow 1_{FN}-\beta_{FN} \subseteq (1_{FN}-\lambda_{FN})$,
- **iv.** $1_{FN}-(1_{FN}-\lambda_{FN}) = \lambda_{FN}$,
- **v.** $(1_{FN}-1_{FN}) = 0_{FN}$ and $(1_{FN}-0_{FN}) = 1_{FN}$."

Definition 2.5 [8]"Fuzzy neutrosophic topology (FNT, for short) on a non-empty set Ҳ is a family $\mathcal{T}_{FN}$ of fuzzy neutrosophic subsets in Ҳ satisfying the following axioms:

- **i.** $0_{FN}, 1_{FN} \in \mathcal{T}_{FN}$,
- **ii.** $\lambda_{FN1} \cap \lambda_{FN2} \in \mathcal{T}_{FN}$ for any $\lambda_{FN1}, \lambda_{FN2} \in \mathcal{T}_{FN}$,
- **iii.** $\cup \lambda_{FNj} \in \mathcal{T}_{FN}$, ∀ { λ_{FNj}: j ∈ J} ⊆ $\mathcal{T}_{FN}$.

In this case, the pair (Ҳ, $\mathcal{T}_{FN}$) is called fuzzy neutrosophic topological space (FNTS, for short). The elements of $\mathcal{T}_{FN}$ are called fuzzy neutrosophic-open sets (FN-OS, for short).The complement of FN-open set in the FNTS (Ҳ, $\mathcal{T}_{FN}$) is called fuzzy neutrosophic-closed set (FN-CS, for short)."

Definition 2.6 [8]" Let (Ҳ , $\mathcal{T}_{FN}$) is FNTS and λ_{FN}= <x, $\mu_{\lambda FN}$, $\sigma_{\lambda FN}$, $\nu_{\lambda FN}$> is FNS in Ҳ.Then, the fuzzy neutrosophic-closure (FNcl, for short) and the fuzzy neutrosophic-interior (FNint, for short) of λ_{FN} are defined by:

$$\text{FNcl}(\lambda_{FN}) = \cap \{\beta_{FN}: \beta_{FN} \text{ is FN-CS in Ҳ and } \lambda_{FN} \subseteq \beta_{FN}\},$$

$$\text{FNint}(\lambda_{FN}) = \cup\{\beta_{FN}: \beta_{FN} \text{ is FN-OS in Ҳ and } \beta_{FN} \subseteq \lambda_{FN}\}.$$

Now, the FNcl (λ_{FN}) is FN-CS and FNint (λ_{FN}) is FN-OS in Ҳ.

Further,

- **i.** λ_{FN} is FN-CS in Ҳ iff FNcl (λ_{FN}) = λ_{FN}
- **ii.** λ_{FN} is FN-OS in Ҳ iff FNint(λ_{FN}) = λ_{FN} ."

Proposition 2.7 [9,10] "For any FNS λ_{FN} in FNTS (Ҳ, $\mathcal{T}_{FN}$) we have,

i. FNcl ($1_{FN}-\lambda_{FN}$) = 1_{FN}-(FNint (λ_{FN})),

ii. FNint ($1_{FN}-\lambda_{FN}$) =1_{FN} - (FNcl (λ_{FN}))".

Proposition 2.8 [8] "Let (Ҳ,$\mathcal{T}_{FN}$) is FNTS and λ_{FN}, β_{FN} are FNSs in Ҳ. Then the following properties hold:

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i. FNint (λ_{FN}) ⊆ λ_{FN} and λ_{FN} ⊆ FNcl (λ_{FN}),
ii. $\lambda_{FN} \subseteq \beta_{FN} \Longrightarrow$ FNint (λ_{FN}) ⊆FNint (β_{FN}) and $\lambda_{FN} \subseteq \beta_{FN} \Longrightarrow$ FNcl(λ_{FN}) ⊆ FNcl(β_{FN}),
iii. FNint(FNint(λ_{FN})) = FNint(λ_{FN}) and FNcl(FNcl(λ_{FN})) = FNcl(λ_{FN}),
iv. FNint($\lambda_{FN} \cap \beta_{FN}$) = FNint(λ_{FN}) ∩ FNint(β_{FN}) and FNcl ($\lambda_{FN} \cup \beta_{FN}$) = FNcl ($\lambda_{FN}$) ∪ FNcl ($\beta_{FN}$),
v. FNint (1_{FN}) = 1_{FN} and FNcl (0_{FN}) =0_{FN}".

Definition 2.10 [9,10] " FNS λ_{FN} in FNTS (X,$\mathcal{T}_{FN}$) is called:

i. Fuzzy neutrosophic regular-open set (FNR-open set, for short) if λ_{FN}= FNint (FNcl (λ_{FN})).
ii. Fuzzy neutrosophic regular-closed set (FNR-closed set, for short) if λ_{FN}= FNcl (FNint (λ_{FN})).
iii. Fuzzy neutrosophic pre-open set (FNP-open set, for short) if λ_{FN}⊆ FNint (FNcl (λ_{FN})).
iv. Fuzzy neutrosophic pre-closed set (FNP-closed set, for short) if FNcl (FNint (λ_{FN})) ⊆ λ_{FN}."

3. Generalized of A-Closed Set and C̃- Closed Set in Fuzzy Neutrosophic Topological Spaces

In this part of our work has been presented new classes of sets in fuzzy neutrosophic topological spaces. As well as, some of its relationships with defined sets and the other sets have been proposed in the same space.

Definition 3.1: A fuzzy neutrosophic set λ_{FN} in FNTS (X, $\mathcal{T}_{FN}$) called:

i- Fuzzy neutrosophic A-closed set (briefly, FNA-CS) if $\lambda_{FN} = \alpha_{FN} \wedge \beta_{FN}$ where α_{FN} is FN-OS and β_{FN} is FNR-closed set in X. The complement of FNA-CS is called fuzzy neutrosophic A-open set (briefly, FNA-OS).
ii- Fuzzy neutrosophic C̃-closed set (briefly, FNC̃-CS) if $\lambda_{FN} = \alpha_{FN} \wedge \beta_{FN}$ where α_{FN} is FN-OS and β_{FN} is FNP-closed set in X. The complement of FN C̃-CS is called fuzzy neutrosophic C̃-open set (briefly, FNC̃-OS).

Definition 3.2: Let (X, $\mathcal{T}_{FN}$) be FNTS and λ_{FN}= <x, $\mu_{\lambda FN}$, $\sigma_{\lambda FN}$, $\nu_{\lambda FN}$> be a FNA-set (FNC̃-set) in X.Then, the fuzzy neutrosophic A-closure (FNA-cl, for short) (fuzzy neutrosophic C̃-closure (FNC̃-cl, for short)) of (λ_{FN}) is defined by:

1- FNA-cl (λ_{FN}) = ∩ {β_{FN}: β_{FN} is FNA-CS in X and $\lambda_{FN} \subseteq \beta_{FN}$},
2- FNC̃-cl (λ_{FN}) = ∩ {β_{FN}: β_{FN} is FNC̃-CS in X and $\lambda_{FN} \subseteq \beta_{FN}$}

Now, the FNA-cl (λ_{FN}) and FNC̃-cl (λ_{FN}) are FN-closed set in X.

Further,

i- λ_{FN} is FNA-CS in X iff FNA-cl (λ_{FN}) = λ_{FN},
ii- λ_{FN} is FNC̃-CS in X iff FNC̃-cl (λ_{FN}) = λ_{FN}.

Definition 3.3: A fuzzy neutrosophic set λ_{FN} in FNTS (X, $\mathcal{T}_{FN}$) called:

i. Fuzzy neutrosophic generalized closed set (FN-GCS) if FN-cl (λ_{FN}) ⊆ W_{FN} where $\lambda_{FN} \subseteq W_{FN}$ and W_{FN} is FN-OS in X.
ii. Fuzzy neutrosophic generalized A-closed set (FN-gA-CS) if FNA-cl (λ_{FN}) ⊆ W_{FN} where $\lambda_{FN} \subseteq W_{FN}$ and W_{FN} is FN-OS in X.
iii. Fuzzy neutrosophic generalized C̃-closed set (FN-gC̃-CS) if FNC̃-cl (λ_{FN}) ⊆ W_{FN} where $\lambda_{FN} \subseteq W_{FN}$ and W_{FN} is FN-OS in X.

Example 3.4:

i- Let X ={a,b} define FNSs η_{FN} and ω_{FN} in X as follows:

$\eta_{FN} = < x, \left(\frac{a}{0.3}, \frac{b}{0.5}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}\right), \left(\frac{a}{0.2}, \frac{b}{0.5}\right) >$

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$\omega_{FN} = < x, (\frac{a}{0.2}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.3}, \frac{b}{0.5})>$

The family $\mathcal{T}_{FN} = \{0_{FN}, 1_{FN}, \eta_{FN}, \omega_{FN}\}$

Now, $\mathcal{T}^{c}_{FN} = \{0_{FN}, 1_{FN}, \eta^{c}_{FN}, \omega^{c}_{FN}\}$

Where, $\eta^{c}_{FN} = < x, (\frac{a}{0.2}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.3}, \frac{b}{0.5})>$

And, $\omega^{c}_{FN} =< x, (\frac{a}{0.3}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.2}, \frac{b}{0.5})>$

Now, FNcl (FNint (ω^{c}_{FN}) = FNcl (η^{c}_{FN}) = ω^{c}_{FN} that is $\omega^{c}_{FN} = \omega^{c}_{FN}$.

Then, ω^{c}_{FN} FNR-closed set.

Now, let $\alpha_{FN} = \eta_{FN}$ and $\beta_{FN} = \omega^{c}_{FN}$ so,

$\lambda_{FN} = \eta_{FN} \cap \omega^{c}_{FN} =< x, (\frac{a}{0.3}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.2}, \frac{b}{0.5})> \cap < x, (\frac{a}{0.3}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.2}, \frac{b}{0.5})>$

$=< x, (\frac{a}{0.3}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.2}, \frac{b}{0.5})>$. Therefore, ω^{c}_{FN} is FNA-CS.

ii- Let Ӽ = {a, b} define FNS Ψ_{FN} in Ӽ as follows:

$\Psi_{FN} = < x, (\frac{a}{0.5}, \frac{b}{0.2}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.7}) >$

Let the family $\mathcal{T}_{FN} = \{0_{FN}, 1_{FN}, \Psi_{FN}\}$ be FNT.

Now, $\omega_{FN} = < x, (\frac{a}{0.9}, \frac{b}{0.7}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.1}, \frac{b}{0.1}) >$ and let $\Psi_{FN} = < x, (\frac{a}{0.5}, \frac{b}{0.7}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.2}) >$ so,

FNcl (FNint (ω_{FN})) $\subseteq$ FNcl (Ψ_{FN}) $\subseteq$ (Ψ^{c}_{FN}) $\subseteq \omega_{FN}$. Then, ω_{FN} is FNP-closed set.

Now, $\lambda_{FN} = \Psi_{FN} \cap \beta_{FN}$ that is,

$\lambda_{FN} = < x, (\frac{a}{0.5}, \frac{b}{0.2}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.7}) > \cap < x, (\frac{a}{0.9}, \frac{b}{0.7}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.1}, \frac{b}{0.1}) >$ so we have,

$\lambda_{FN} = < (\frac{a}{0.5}, \frac{b}{0.2}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.7}) >$ is FNC*-CS.

iii. Take $\lambda_{FN} =< x, (\frac{a}{0.3}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.2}, \frac{b}{0.5})>$ in i which is FNA-CS.

And put $W_{FN} = \eta_{FN} = < x, (\frac{a}{0.3}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.2}, \frac{b}{0.5})>$ where W_{FN} is FN-open set so,

$\omega^{c}_{FN} \subseteq W_{FN}$ such that $< x, (\frac{a}{0.3}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.2}, \frac{b}{0.5}) > \subseteq < x, (\frac{a}{0.3}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.2}, \frac{b}{0.5})>$

So, FNA-cl (λ_{FN}) = ω^{c}_{FN}. Then, FNA-cl (λ_{FN}) $\subseteq W_{FN}$.

Since, $< x, (\frac{a}{0.3}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.2}, \frac{b}{0.5}) > \subseteq < x, (\frac{a}{0.3}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.2}, \frac{b}{0.5}) >$. That is λ_{FN} FN-gA-CS.

iv. Take **ii.** So, we know $\lambda_{FN} =< x, (\frac{a}{0.5}, \frac{b}{0.2}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.7}) >$ is FNC*-set.

Now, put $W_{FN} = 1_{FN}$ where W_{FN} is FN-open set such that $1_{FN} =< x, 1, 1, 0 >$. Then

$\lambda_{FN} \subseteq 1_{FN}$ and we have FNC*-cl (λ_{FN}) = Ψ^{c}_{FN}

So, FNC*-cl (λ_{FN}) $\subseteq W_{FN}$

Since, $< x, (\frac{a}{0.5}, \frac{b}{0.2}), (\frac{a}{0.5}, \frac{b}{0.5}), (\frac{a}{0.5}, \frac{b}{0.7}) > \subseteq 1_{FN}$

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Hence, λ_{FN} is FN-gC*-CS.

Theorem 3.5: If ($\mathfrak{X}$, $\mathcal{T}_{FN}$) is FNTS with λ_{FN} and $\beta_{FN} \in \mathfrak{X}$. The FNA-cl (FNC*-cl) satisfy the following statements:

i-

1) FNA-cl (0_{FN}) = 0_{FN} and FNA-cl (1_{FN}) = 1_{FN}.
2) $\lambda_{FN} \subseteq$ FNA-cl (λ_{FN}).
3) If $\lambda_{FN} \subseteq \beta_{FN}$, then FNA-cl ($\lambda_{FN}$) $\subseteq$ FNA-cl (β_{FN}).
4) λ_{FN} is FNA-CS iff FNA-cl (λ_{FN}) = λ_{FN}.
5) FNA-cl (FNA-cl (λ_{FN})) = FNA-cl (λ_{FN}).
6) FNA-cl (λ_{FN}) ∪ FNA-cl (β_{FN}) = FNA-cl ($\lambda_{FN} \cup \beta_{FN}$)

ii-

1) FNC*-cl (0_{FN}) = 0_{FN} and FNC*-cl (1_{FN}) =1_{FN}.

2) $\lambda_{FN} \subseteq$ FNC*-cl (λ_{FN})

3) If $\lambda_{FN} \subseteq \beta_{FN}$. Then, FNC*-cl ($\lambda_{FN}$) $\subseteq$ FNC*-cl (β_{FN})

4) λ_{FN} is FNC*-CS iff FNC*-cl (λ_{FN}) = λ_{FN}.

5) FNC*-cl (FNC*-cl (λ_{FN})) = FNC*-cl (λ_{FN}).

6) FNC*-cl (λ_{FN}) ∪ FNC*-cl (β_{FN}) = FNC*-cl ($\lambda_{FN} \cup \beta_{FN}$).

Proof: By def.3.2 we have,

1) FNA-cl (0_{FN}) = ∩ {β_{FN}: β_{FN} is FNA-CS in $\mathfrak{X}$ and $0_{FN} \subseteq \beta_{FN}$} = 0_{FN}.

FNA-cl (1_{FN}) = ∩ {β_{FN} : β_{FN} is FNA-CS in $\mathfrak{X}$ and $1_{FN} \subseteq \beta_{FN}$} = $1_{FN.}$

2) $\lambda_{FN} \subseteq$ ∩ {β_{FN} : β_{FN} is FNA-CS in $\mathfrak{X}$ and $\lambda_{FN} \subseteq \beta_{FN}$}= FNA-cl($\lambda_{FN}$).

3) we know FNA-cl (λ_{FN}) = ∩ {β_{FN}: β_{FN} is FNA-CS in $\mathfrak{X}$ and $\lambda_{FN} \subseteq \beta_{FN}$}

Since, $\lambda_{FN} \subseteq \beta_{FN}$ then, ∩ {β_{FN} : β_{FN} is FNA-CS in $\mathfrak{X}$ and $\lambda_{FN} \subseteq \beta_{FN}$}

$\subseteq$ ∩ {υ_{FN} : υ_{FN} is FNA-closed set in $\mathfrak{X}$ and $\beta_{FN} \subseteq \upsilon_{FN}$}

$\Rightarrow$ FNA-cl (λ_{FN}) $\subseteq$ FNA-cl (β_{FN}).

4) If λ_{FN} is FNA-CS so, ∩ {β_{FN} : β_{FN} is FNA-CS in $\mathfrak{X}$ and $\lambda_{FN} \subseteq \beta_{FN}$} and

$\lambda_{FN} \subseteq$ FNA-cl (λ_{FN}), but λ_{FN} necessarily to be the smallest set.

Thus, λ_{FN} = ∩ {β_{FN} : β_{FN} is FNA-CS in $\mathfrak{X}$ and $\lambda_{FN} \subseteq \beta_{FN}$}, So, λ_{FN} = FNA-cl (λ_{FN}).

Conversely, let β_{FN} be any subset in $\mathfrak{X}$. Then,

FNA-cl (λ_{FN}) = ∩ {β_{FN} : β_{FN} is FNA-CS in $\mathfrak{X}$ and $\lambda_{FN} \subseteq \beta_{FN}$}.

But, $\lambda_{FN} \subseteq$ ∩{β_{FN} : β_{FN} is FNA-CS in $\mathfrak{X}$ and $\lambda_{FN} \subseteq \beta_{FN}$}.

Since, the intersection of all FNA-CS is FNA-CS. It follows that FNA-cl (λ_{FN}) be FN-CS in $\mathfrak{X}$.

5) Suppose that, λ_{FN} = FNA-cl (λ_{FN}) and we have,

FNA-cl (λ_{FN}) is FN-CS $\Rightarrow$ λ_{FN} is FN-CS.

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Since, λ_{FN} = FNA-cl (λ_{FN}) we take FNA-cl by two sides we get,

FNA-cl (λ_{FN}) = FNA-cl (FNA-cl (λ_{FN})).

6) Let λ_{FN} and β_{FN} are FNA-CS, then

FNA-cl (λ_{FN}) = $\cap$ {υ_{FN} : υ_{FN} is FNA-CS in X and $\lambda_{FN} \subseteq \upsilon_{FN}$}… (1)

FNA-cl (β_{FN}) = $\cap$ {υ_{FN} : υ_{FN} is FNA-CS in X and $\beta_{FN} \subseteq \upsilon_{FN}$}… (2)

Then, by (1) and (2) we get,

FNA-cl (λ_{FN}) $\cup$ FNA-cl (β_{FN}) =

[$\cap$ {υ_{FN} : υ_{FN} is FNA-CS in X and $\lambda_{FN} \subseteq \upsilon_{FN}$}] $\cup$ [$\cap$ {υ_{FN} : υ_{FN} is FNA-CS in X and $\beta_{FN} \subseteq \upsilon_{FN}$}]

= FNA-cl ($\lambda_{FN} \cup \beta_{FN}$)… (3)

ii- By the same way in i, we can proof ii.

Theorem 3.6: Let λ_{FN} be FNS in FNTS (X, $\mathcal{T}_{FN}$), then:

i- λ_{FN} is FN-gA-CS iff λ_{FN} FNA-CS.
ii- λ_{FN} is FN-gC̃-CS iff λ_{FN} FNC̃-CS.

Proof:

i- $\Rightarrow$ Let λ_{FN} be FNA-CS and let β_{FN} be any FN-OS in X such that $\lambda_{FN} \subseteq \beta_{FN}$.

Then, from **Definition 3.2 (1)** we have FNA-cl (λ_{FN}) = $\lambda_{FN.}$

But, $\lambda_{FN} \subseteq \beta_{FN}$. So, FNA-cl ($\lambda_{FN}$) $\subseteq \beta_{FN}$.

Therefore, λ_{FN} is FN-gA-CS.

$\Leftarrow$ Let λ_{FN} be FN-gA-CS, then FNA-cl (λ_{FN}) $\subseteq \beta_{FN}$ for any FN-open set β_{FN} in X such that $\lambda_{FN} \subseteq \beta_{FN.}$

But, $\lambda_{FN} \subseteq$ FNA-cl (λ_{FN}) $\subseteq \beta_{FN}$.

That is λ_{FN} is FNA-CS.

ii. By the same way in i, we can proof ii.

Corollary 3.7:

i- The FNS λ_{FN} is called FNA-clopen (briefly, FN-gA-clopen) iff λ_{FN} is FNA-OS and FNA-CS (briefly, FN-gA-OS) and FN-gA-CS).
ii- The FNS λ_{FN} is called FNC̃-clopen (briefly, FN-gC̃-clopen) iff λ_{FN} FNC̃-OS and FNC̃-CS (briefly, FN-gC̃-OS and FN-gC̃-CS).

Theorem 3.8:

i. Let (X, $\mathcal{T}_{FN}$) be FNTS. If λ_{FN} is FN-gA-clopen set, then λ_{FN} be FNA-CS.
ii. Let (X, $\mathcal{T}_{FN}$) be FNTS. If λ_{FN} is FN-gC̃-clopen set, then λ_{FN} be FNC̃-CS.

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Proof:

i) Suppose λ_{FN} and β_{FN} FNS in X. β_{FN} is FN-OS such that $\lambda_{FN} \subseteq \beta_{FN}$. By corollary 3.7 (i) of complement, we get λ_{FN} is FN-gA-CS, so

$$\text{FNA-cl}\,(\lambda_{FN}) = \lambda_{FN} \subseteq \beta_{FN}$$

However, λ_{FN} is FN-gA-clopen set by hypothesis that is FNA-CS.

ii) We can proof that by the similar way in (i).

Theorem 3.9: Let ($\mathfrak{X}$, $\mathcal{T}_{FN}$) be FNTS. Then:

i- If λ_{FN} and β_{FN} are FN-gA-CS then, $\lambda_{FN} \cup \beta_{FN}$ is FN-gA-CS.
ii- If λ_{FN} and β_{FN} are FN-g$\mathcal{C}$-CS then, $\lambda_{FN} \cup \beta_{FN}$ is FN-g$\mathcal{C}$-CS.

Proof:

i- Suppose that λ_{FN} and β_{FN} are FN-gA-CS and υ_{FN} be any FN-OS such that $\lambda_{FN} \cup \beta_{FN} \subseteq \upsilon_{FN}$.

Thus, either $\lambda_{FN} \subseteq \upsilon_{FN}$

Or $\beta_{FN} \subseteq \upsilon_{FN}$

But, λ_{FN} is FN-gA-CS and υ_{FN} is FN-OS so,

$$\text{FNA-cl}\,(\lambda_{FN}) \subseteq \upsilon_{FN} \ldots (1)$$

And, β_{FN} is FN-gA-CS and υ_{FN} be FN-OS so,

$$\text{FNA-cl}\,(\beta_{FN}) \subseteq \upsilon_{FN} \ldots (2).$$

Take the union of (1) and (2) we get,

FNA-cl (λ_{FN}) ∪ FNA-cl (β_{FN}) = FNA-cl ($\lambda_{FN} \cup \beta_{FN}$) $\subseteq \upsilon_{FN}$

That is ($\lambda_{FN} \cup \beta_{FN}$) is FN-gA-CS

ii. We can proof that by the same way in (i).

Theorem 3.10: Let ($\mathfrak{X}$, $\mathcal{T}_{FN}$) be FNTS and λ_{FN} with β_{FN} are FN-gA-CS (FN-g$\mathcal{C}$-CS). Then:

i. The intersection of two FN-gA-CS is FN-gA-CS.
ii. The intersection of two FN-g$\mathcal{C}$-CS is FN-gA-CS.

Proof:

i) Let λ_{FN} and β_{FN} be FN-gA-CS and υ_{FN} be any FN-OS such that

$$\text{FNA-cl}\,(\lambda_{FN}) \subseteq \upsilon_{FN} \ldots (1)$$

$$\text{FNA-cl}\,(\beta_{FN}) \subseteq \upsilon_{FN} \ldots (2)$$

Since, FNA-cl (λ_{FN}) $\subseteq \upsilon_{FN}$ and FNA-cl (β_{FN}) $\subseteq \upsilon_{FN}$.

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Take the intersection of (1) and (2).

So we get, FNA-cl (λ_{FN}) ∩ FNA-cl (β_{FN}) ⊆ υ_{FN}

By theorem 3.5 (6), we have,

FNA-cl (λ_{FN}) ∩ FNA-cl (β_{FN}) = FNA-cl ($\lambda_{FN} \cap \beta_{FN}$) ⊆ υ_{FN}.

Then, the intersection of two FN-gA-CS is FN-gA-CS.

ii) We can proof that by the same way in (i).

Theorem 3.11:

i- The union of FN-gA-CS and FN-gC^*-CS will be FN-gC^*-CS.
ii- The intersection of FN-gA-CS and FN-gC^*-CS will be FN-gA-CS.

Proof:

i- Let λ_{FN} be FN-gA-CS and β_{FN} be FN-gC^*-CS and υ_{FN} be FN-OS, such that $\lambda_{FN} \cup \beta_{FN} \subseteq \upsilon_{FN}$

Now, FNA-cl (λ_{FN}) ⊆ υ_{FN} … (1)

And, FNC^*-cl (β_{FN}) ⊆ υ_{FN} … (2)

Now, from (1) and (2) so we get,

FNA-cl (λ_{FN}) ⊆ υ_{FN} or FNC^*-cl (β_{FN}) ⊆ υ_{FN}

So, FNA-cl (λ_{FN}) ∪ FNC^*-cl (β_{FN}) ⊆ υ_{FN}

Since, β_{FN} is the largest set and $\lambda_{FN} \subseteq \beta_{FN}$ then,

FNA-cl (λ_{FN}) ∪ FNC^*-cl (β_{FN}) ⊆ FNC^*-cl (β_{FN})

So, FNC^*-cl (β_{FN}) ⊆ υ_{FN}

Then, the union be FN-gC^*-CS.

ii- Let λ_{FN} be FN-gA-CS and β_{FN} be FN-gC^*-CS with υ_{FN} be FN-OS such that $\lambda_{FN} \cap \beta_{FN} \subseteq \upsilon_{FN}$.

Now, FNA-cl (λ_{FN}) ⊆ υ_{FN} … (1)

And, FNC^*-cl (β_{FN}) ⊆ υ_{FN} … (2)

Now, by (1) and (2) we have,

FNA-cl (λ_{FN}) ⊆ υ_{FN} and FNC^*-cl (β_{FN}) ⊆ υ_{FN}

Then we get, FNA-cl (λ_{FN}) ∩ FNC^*-cl (β_{FN}) ⊆ υ_{FN}

Since, λ_{FN} is smallest and $\lambda_{FN} \cap \beta_{FN} \subseteq \lambda_{FN}$

Then, FNA-cl (λ_{FN}) ∩ FNC^*-cl (β_{FN}) ⊆ FNA-cl (λ_{FN})

So, FNA-cl (λ_{FN}) ⊆ υ_{FN}

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Then, the intersection be FN-gA-CS.

Remark 3.12:

i- The union of FN-gA-CS and FN-gC*-CS be FN-gA-CS. We can show that by the next example.
ii- The intersection of FN-gA-CS and FN-gC*-CS be FN-gA-CS and FN-gC*-CS. We can show that by the next example.

Example 3.13: Take Example 3.4 we have $D^c_{FN} = < x, \left(\frac{a}{0.6}, \frac{b}{0.6}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}\right) >$ is FNA-CS such that

FNcl (FNint (D^c_{FN}) = FNcl (D_{FN}) = D^c_{FN}. So, D^c_{FN} is FNR-closed set.

$\lambda_{FN} = 1_{FN} \cap D^c_{FN} = D^c_{FN}$

Then, $\lambda_{FN} = D^c_{FN}$ is FNA-CS

Put $W_{FN} = 1_{FN}$ where W_{FN} is FN-OS such that $D^c_{FN} \subseteq 1_{FN}$.

Now, FNA-cl (D^c_{FN}) $\subseteq D^c_{FN} \subseteq W_{FN}$

Then, D^c_{FN} is FN-gA-CS.

In addition, we have B^c_{FN} is FNP-closed set. If we take $\alpha_{FN} = 1_{FN}$ where α_{FN} is FN-open set. Then,

$\lambda_{FN} = 1_{FN} \cap B^c_{FN} = B^c_{FN}$. So, $\lambda_{FN} = B^c_{FN}$ is FNC*-CS.

Let $W_{FN} = 1_{FN}$ where W_{FN} is FN-open set such that $B^c_{FN} \subseteq 1_{FN}$.

Now, FNC*-cl (B^c_{FN}) $\subseteq W_{FN}$

$B^c_{FN} \subseteq W_{FN}$

Then, B^c_{FN} is FN-gC*-CS.

i- $D^c_{FN} \cup B^c_{FN} = B^c_{FN}$. Since B^c_{FN} is FN-gC*-CS then, $D^c_{FN} \cup B^c_{FN}$ is FN-gC*-CS but not FN-gA-CS because B^c_{FN} not FNA-CS.
ii- $D^c_{FN} \cap B^c_{FN} = D^c_{FN}$. Since D^c_{FN} is FN-gA-CS then $D^c_{FN} \cap B^c_{FN}$ also FN-gA-CS and FN-gC*-CS.

Theorem 3.14

i- Every FN-gA-CS is FNG-CS.
ii- Every FN-gC*-CS is FNG-CS.

Proof:

i- If λ_{FN} is FN-gA-CS. Let λ_{FN} be FN-clopen set and υ_{FN} be FN-OS such that $\lambda_{FN} \subseteq \upsilon_{FN}$

Since λ_{FN} is FN-gA-CS so, we have FNA-cl (λ_{FN}) $\subseteq \upsilon_{FN.}$

But, FNA-cl (λ_{FN}) $\subseteq$ FNcl (λ_{FN}) $\subseteq \lambda_{FN}$.

Therefore, λ_{FN} is FNG-CS.

ii- We can prove that by similar way in (i).

Remark 3.15: The convers of Theorem 3.14 is not true. We can show that by the next example.

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Example 3.16:

i- Let X = {a, b} define FNS λ_{FN} in X as follows:

λ_{FN} = <x, $(\frac{a}{0.5}, \frac{b}{0.5})$, $(\frac{a}{0.5}, \frac{b}{0.5})$, $(\frac{a}{0.4}, \frac{b}{0.5})$ >. And, the family, $\mathcal{T}_{FN} = \{0_{FN}, 1_{FN}, \lambda_{FN}\}$ be FNT.

Now if, ω_{FN} = <x, $(\frac{a}{0.5}, \frac{b}{0.4})$, $(\frac{a}{0.5}, \frac{b}{0.5})$, $(\frac{a}{0.6}, \frac{b}{0.5})$ > and, $W_{FN} = 1_{FN}$ where 1_{FN} = < x, 1, 1, 0 > is FN-OS such that, $\omega_{FN} \subseteq 1_{FN}$.

Hence, ω_{FN} is FNG-CS where FNcl (ω_{FN}) = 1 $\nsubseteq W_{FN}$

So, ω_{FN} is not FN-gA-CS, not FNA-CS such that,

FNcl (FNint (ω_{FN})) = FNcl (0) = 0 $\neq \omega_{FN}$

Therefore, ω_{FN} is not FNP-closed set and not FN-gA-CS set.

ii- Let Ẋ = {a, b} define FNS λ_{FN} in Ẋ as follows:

λ_{FN} = <x, $(\frac{a}{0.5}, \frac{b}{0.2})$, $(\frac{a}{0.5}, \frac{b}{0.5})$, $(\frac{a}{0.5}, \frac{b}{0.7})$ >.

The family, $\mathcal{T}_{FN} = \{0_{FN}, 1_{FN}, \lambda_{FN}\}$ is FNT.

Now, if ω_{FN}= < x, $(\frac{a}{0.9}, \frac{b}{0.3})$, $(\frac{a}{0.5}, \frac{b}{0.5})$, $(\frac{a}{0.1}, \frac{b}{0.6})$ >.

And $W_{FN} = 1_{FN}$, where 1_{FN} =< 1, 1, 0 > is FN-OS such that $\omega_{FN} \subseteq W_{FN}$.

Then, FNcl (ω_{FN}) =1_{FN}. So, FNcl (ω_{FN}) $\subseteq W_{FN}$. Since, $1_{FN} \subseteq 1_{FN}$.

Hence, ω_{FN} is FNG-CS. But, not FN-gC̊-CS and is not FNC̊-CS such that,

FNcl (FNint (ω_{FN})) = FNcl (λ_{FN}) $\subseteq \lambda^{c}_{FN} \subseteq \omega_{FN}$.

Therefore, ω_{FN} is not FNP-closed set and not FNC̊-CS also is not FN-gC̊-CS.

4.Conclusions

In this paper, we introduced the concept of new classess of closed sets which is including fuzzy neutrosophic generalized A-closed set and fuzzy neutrosophic generalized C̊-closed set and some of their properties were discussed via fuzzy neutrosophic topological spaces. This study can be extended and generalized to several functions such as continuous and irresolute functions.

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