



A Study of Neutrosophic Bernoulli's and Recati Differential Equations

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Abstract

In this paper, we study the neutrosophic Bernoulli and Ricatti differential equations by using one dimensional AH-isometry. Also, we illustrate many examples to clarify the validity of our work.

Keywords:One-Dimensional Geometric AH-Isometry; Neutrosophi Ricatti Differential Equation.

1.Introduction

Neutrosophic logic. Neutrosophy, Neutrosophic set, Neutrosophic probability, are recently creations of Smarandache, being characterized by having the indeterminacy as component of their framework, and a notable feature of neutrosophic logic is that can be considered a generalization of fuzzy logics, encompassing the classical logic as well [1]. Also, F. Smarandache, has defined the concept of continuation of a neutrosophic function in year 2015 in [1], and neutrosophic mereo-limit [1], mereo-continuity. Moreover, in 2014, he has defined the concept of a neutro-oscillator differential in [3], and mereo-derivative. Finally in 2013 he introduced neutrosophic integration in [2], and mereo-integral, besides the classical definitions of limit, continuity, derivative, and integral respectively. Among the recent applications there are: neutrosophic crisp set theory in image processing, neutrosophic sets medical field [6-10], in information geographic systems and possible applications to database. Also, neutrosophic triplet group application to physics. Moreover Several researches have made multiple contributions to neutrosophic topology and algebra [14-20, 34-50], Also More researches have made multiple contributions to neutrosophic analysis [21 – 33]. Finally the neutrosophic integration may have application in calculus the areas between two neutrosophic functions.

2. Preliminaries

Definition: Neutrosophic Real Number

Suppose that w is a neutrosophic number, then it takes the following standard form: $w = a + bI$ where a, b are real coefficients, and I represents the indeterminacy, where $0.I = 0$ and $I^n = I$ for all positive integers n .

Doi: <https://doi.org/10.54216/GJMSA.010205>

Received: February 26, 2022 Accepted: May 15, 2022

For example:

$$w = 1 + 2I, w = 3 = 3 + 0I.$$

Definition: Division of neutrosophic real numbers

Suppose that w_1, w_2 are two neutrosophic number, where

$$w_1 = a_1 + b_1I, w_2 = a_2 + b_2I$$

Then:

$$\frac{w_1}{w_2} = \frac{a_1 + b_1I}{a_2 + b_2I} = \frac{a_1}{a_2} + \frac{a_2b_1 - a_1b_2}{a_2(a_2 + b_2)}I$$

Definition :

Let $R(I) = \{a + bI ; a, b \in R\}$ where $I^2 = I$ be the neutrosophic field of reals. The one-dimensional isometry (AH-Isometry) is defined as follows

$$T: R(I) \rightarrow R \times R$$

$$T(a + bI) = (a, a + b)$$

Remark :

T is an algebraic isomorphism between two rings, it has the following properties:

- 1) T is bijective.
- 2) T preserves addition and multiplication, i.e.:

$$T[(a + bI) + (c + dI)] = T(a + bI) + T(c + dI)$$

And

$$T[(a + bI) \cdot (c + dI)] = T(a + bI) \cdot T(c + dI)$$

- 3) Since T is bijective, then it is invertible by:

$$T^{-1}: R \times R \rightarrow R(I)$$

$$T^{-1}(a, b) = a + (b - a)I$$

- 4) T preserves distances, i.e.:

The distance on $R(I)$ can be defined as follows:

$$\text{Let } A = a + bI, B = c + dI \text{ be two neutrosophic real numbers, then } L = \|\overline{AB}\| = d[(a + bI, c + dI)] = |a + bI - (c + dI)| = |(a - c) + I(b - d)| = |a - c| + I[|a + b - c - d| - |a - c|].$$

On the other hand, we have:

$$T(\|\overline{AB}\|) = (|a - c|, |(a + b) - (c + d)|) = (d(a, c), d(a + b, c + d)) = d[(a, a + b), (c, c + d)] = d(T(a + bI), T(c + dI))$$

$$= \|T(\overline{AB})\|.$$

This implies that the distance is preserved up to isometry. i.e. $\|T(AB)\| = T(\|AB\|)$

NeutrosophicBernoulli's equation.

In this section is defined a Neutrosophic Bernoulli's equation by Using the One-Dimensional Geometric AH-Isometry and solutions are found for this equation.

Definition

Let $Y = y_1 + y_2I$, $X = x_1 + x_2I$ We define the Neutrosophic Bernoulli's equation by Using the One-Dimensional Geometric AH-Isometry as form:

$$\dot{Y} + f(X)Y = g(X)Y^n$$

This equation can be written as follow:

$$\begin{aligned} (y_1' + I[(y_1 + y_2)' - y_1'])(f(x_1) + I[f(x_1 + x_2) - f(x_1)])(y_1 + y_2I) \\ = (g(x_1) + I[g(x_1 + x_2) - g(x_1)])(y_1 + y_2I)^n \\ \Rightarrow y_1' + f(x_1)y_1 + I[(y_1 + y_2)' - f(x_1 + x_2)(y_1 + y_2) - (y_1' + f(x_1)y_1)] \\ = (g(x_1) + I[g(x_1 + x_2) - g(x_1)])((y_1)^n + I[(y_1 + y_2)^n - (y_1)^n]) \end{aligned}$$

Method of solution.

1. Take AH-Isometry for the differential equation, we have.

$$\begin{aligned} T(y_1' + f(x_1)y_1 + I[(y_1 + y_2)' - f(x_1 + x_2)(y_1 + y_2) - (y_1' + f(x_1)y_1)]) \\ = (g(x_1) + I[g(x_1 + x_2) - g(x_1)])((y_1)^n + I[(y_1 + y_2)^n - (y_1)^n]) \end{aligned}$$

$$\begin{aligned} T(y_1' + f(x_1)y_1 + I[(y_1 + y_2)' - f(x_1 + x_2)(y_1 + y_2) - (y_1' + f(x_1)y_1)]) \\ = T(g(x_1) + I[g(x_1 + x_2) - g(x_1)]).T((y_1)^n + I[(y_1 + y_2)^n - (y_1)^n]) \end{aligned}$$

$$[y_1' + f(x_1)y_1, (y_1 + y_2)' - f(x_1 + x_2)(y_1 + y_2)] = [g(x_1), g(x_1 + x_2)]. [(y_1)^n, (y_1 + y_2)^n]$$

Then.

$$\begin{cases} y_1' + f(x_1)y_1 = g(x_1)(y_1)^n \dots \dots (1) \\ (y_1 + y_2)' - f(x_1 + x_2)(y_1 + y_2) = g(x_1 + x_2)(y_1 + y_2)^n \dots \dots (2) \end{cases}$$

The equations (1) and (2) are two Bernoulli's differential equation classical.

2. We find the solution to the equations classical (1) and (2), we have.

y_1 the solution to the equation (1).

$(y_1 + y_2)$ the solution to the equation (2).

3. We Take invertible AH-Isometry, then, we have the solution of a Neutrosophic identical linear differential equation.

$$Y = y_1 + y_2I = T^{-1}(y_1, y_1 + y_2) = y_1 + ((y_1 + y_2) - y_1)I$$

Example Find a solution to the equation:

$$\dot{Y} + \frac{1}{X}Y = XY^3$$

Solution.

Let $Y = y_1 + y_2I, X = x_1 + x_2I$. Then.

$$y_1' + \frac{1}{x_1}y_1 + I \left[(y_1 + y_2)' - \frac{1}{(x_1 + x_2)}(y_1 + y_2) - \left(y_1' + \frac{1}{x_1}y_1 \right) \right] \\ = (x_1 + I[(x_1 + x_2) - (x_1)])(y_1)^3 + I[(y_1 + y_2)^3 - (y_1)^3]$$

Now, Take AH-Isometry for the differential equation, we have.

$$\begin{cases} y_1' + \frac{1}{x_1}y_1 = x_1(y_1)^3 \dots \dots (3) \\ (y_1 + y_2)' - \frac{1}{x_1 + x_2}(y_1 + y_2) = (x_1 + x_2)(y_1 + y_2)^3 \dots \dots (4) \end{cases}$$

The solution of equation (3) written as follow:

$$y_1 = \left\{ \frac{1}{\mu(x_1)} \left(a + \int \mu(x_1)g(x_1)dx_1 \right) \right\}^{\frac{1}{-n+1}}$$

$$y_1 = \left\{ \frac{1}{(x_1)^2} \left(a + \int -2(x_1)^3 d(x_1) \right) \right\}^{\frac{-1}{2}}$$

$$y_1 = \left\{ \frac{1}{(x_1)^2} \left(a - \frac{(x_1)^4}{2} \right) \right\}^{\frac{-1}{2}} = \left\{ \frac{a}{(x_1)^2} - \frac{(x_1)^2}{2} \right\}^{\frac{-1}{2}}, \text{ where } a \in R.$$

The solution of equation (4) written as follow:

$$(y_1 + y_2) = \left\{ \frac{1}{(x_1+x_2)^2} \left(b - \frac{(x_1+x_2)^4}{2} \right) \right\}^{\frac{-1}{2}} = \left\{ \frac{b}{(x_1+x_2)^2} - \frac{(x_1+x_2)^2}{2} \right\}^{\frac{-1}{2}}, \text{ where } b \in R$$

Now, Take invertibleAH-Isometry, then, we have the solution of aNeutrosophic identical linear differential equation

$$Y = y_1 + y_2I = T^{-1} \left(\left\{ \frac{a}{(x_1)^2} - \frac{(x_1)^2}{2} \right\}^{\frac{-1}{2}}, \left\{ \frac{b}{(x_1 + x_2)^2} - \frac{(x_1 + x_2)^2}{2} \right\}^{\frac{-1}{2}} \right)$$

$$Y = y_1 + y_2I = \left\{ \frac{a}{(x_1)^2} - \frac{(x_1)^2}{2} \right\}^{\frac{-1}{2}} + \left(\left\{ \frac{b}{(x_1 + x_2)^2} - \frac{(x_1 + x_2)^2}{2} \right\}^{\frac{-1}{2}} - \left\{ \frac{a}{(x_1)^2} - \frac{(x_1)^2}{2} \right\}^{\frac{-1}{2}} \right) I$$

By Definition 2.7, we have.

$$Y = y_1 + y_2I = \left\{ \frac{a}{(x_1)^2} - \frac{(x_1)^2}{2} + \left(\frac{b}{(x_1 + x_2)^2} - \frac{(x_1 + x_2)^2}{2} \right) I \right\}^{\frac{-1}{2}}$$

$$Y = y_1 + y_2 I = \left\{ \frac{a}{(x_1)^2} + \left(\frac{b}{(x_1 + x_2)^2} \right) I - \frac{(x_1)^2}{2} - \left(\frac{(x_1 + x_2)^2}{2} \right) I \right\}^{-\frac{1}{2}}$$

$$Y = y_1 + y_2 I = \left\{ \frac{a}{(x_1)^2} + \left(\frac{b}{(x_1 + x_2)^2} \right) I - \left[\frac{(x_1)^2}{2} + \left(\frac{(x_1 + x_2)^2}{2} \right) I \right] \right\}^{-\frac{1}{2}}$$

By Definition 2.5, we have.

$$\frac{a}{(x_1)^2} + \left(\frac{b}{(x_1 + x_2)^2} \right) I = \frac{a}{(x_1)^2} + \frac{b}{(x_1 + x_2)^2} I = \frac{a + bI}{(x_1 + x_2 I)^2}$$

Then.

$$Y = y_1 + y_2 I = \left\{ \frac{a + bI}{(x_1 + x_2 I)^2} + \frac{-(x_1 + x_2 I)^2}{2} \right\}^{-\frac{1}{2}} = \left\{ \frac{a + bI}{X^2} + \frac{-X^2}{2} \right\}^{-\frac{1}{2}}$$

So that,

$$Y = y_1 + y_2 I = \left\{ \frac{a + bI}{X^2} + \frac{-X^2}{2} \right\}^{-\frac{1}{2}}$$

where $a + bI \in R(I)$.

Example Find a solution to the equation:

$$\dot{Y} + \tan(X)Y = \sin(X)Y^2$$

Solution.

Let $Y = y_1 + y_2 I, X = x_1 + x_2 I$. Then.

$$y_1' + \tan(x_1)y_1 + I[(y_1 + y_2)' + \tan(x_1 + x_2)(y_1 + y_2) - (y_1' + \tan(x_1)y_1)] = (\sin(x_1) + I[\sin(x_1 + x_2) - \sin(x_1)])(y_1)^2 + I[(y_1 + y_2)^2 - (y_1)^2]$$

Now, Take AH-Isometry for the differential equation, we have.

$$\left\{ \begin{aligned} y_1' + \tan(x_1)y_1 &= \sin(x_1)(y_1)^2 \dots \dots (5) \end{aligned} \right.$$

$$\left\{ \begin{aligned} (y_1 + y_2)' + \tan(x_1 + x_2)(y_1 + y_2) &= \sin(x_1 + x_2)(y_1 + y_2)^2 \dots \dots (6) \end{aligned} \right.$$

The solution of equation (5s) written as follow:

$$y_1 = \left\{ \frac{1}{\mu(x_1)} \left(a + \int \mu(x_1)g(x_1)dx_1 \right) \right\}^{-\frac{1}{n+1}}$$

$$y_1 = \left\{ \frac{1}{\cos(x_1)} \left(a + \int \cos(x_1) \cdot \sin(x_1)d(x_1) \right) \right\}^{-1}$$

$$y_1 = \left\{ \frac{1}{\cos(x_1)} \left(a + \frac{1}{4} \cos 2(x_1) \right) \right\}^{-1} = \left\{ \frac{a}{\cos(x_1)} + \frac{1}{4} \frac{\cos 2(x_1)}{\cos(x_1)} \right\}^{-1}, \text{ where } a \in R.$$

The solution of equation (6) written as follow:

$$(y_1 + y_2) = \left\{ \frac{1}{\cos(x_1+x_2)} \left(b + \frac{1}{4} \cos 2(x_1 + x_2) \right) \right\}^{-1} = \left\{ \frac{b}{\cos(x_1+x_2)} + \frac{1}{4} \frac{\cos 2(x_1+x_2)}{\cos(x_1)} \right\}^{-1}, \text{ where } b \in R$$

Now, Take invertible AH-Isometry, then, we have the solution of a Neutrosophic identical linear differential equation

$$Y = y_1 + y_2 I = T^{-1} \left(\left\{ \frac{a}{\cos(x_1)} + \frac{1}{4} \frac{\cos 2(x_1)}{\cos(x_1)} \right\}^{-1}, \left\{ \frac{b}{\cos(x_1+x_2)} + \frac{1}{4} \frac{\cos 2(x_1+x_2)}{\cos(x_1)} \right\}^{-1} \right)$$

$$Y = y_1 + y_2 I = \left\{ \frac{a}{\cos(x_1)} + \frac{1}{4} \frac{\cos 2(x_1)}{\cos(x_1)} \right\}^{-1} + \left(\left\{ \frac{b}{\cos(x_1+x_2)} + \frac{1}{4} \frac{\cos 2(x_1+x_2)}{\cos(x_1)} \right\}^{-1} - \left\{ \frac{a}{\cos(x_1)} + \frac{1}{4} \frac{\cos 2(x_1)}{\cos(x_1)} \right\}^{-1} \right) I$$

we have.

$$Y = y_1 + y_2 I = \left\{ \frac{a}{\cos(x_1)} + \frac{1}{4} \frac{\cos 2(x_1)}{\cos(x_1)} + \left(\frac{b}{\cos(x_1+x_2)} + \frac{1}{4} \frac{\cos 2(x_1+x_2)}{\cos(x_1)} \right) I \right\}^{-1}$$

$$Y = y_1 + y_2 I = \left\{ \frac{a}{\cos(x_1)} + \left(\frac{b}{\cos(x_1+x_2)} \right) I + \frac{1}{4} \frac{\cos 2(x_1)}{\cos(x_1)} + \left(\frac{1}{4} \frac{\cos 2(x_1+x_2)}{\cos(x_1)} \right) I \right\}^{-1}$$

we have.

$$\frac{a}{\cos(x_1)} + \left(\frac{b}{\cos(x_1+x_2)} \right) I = \frac{a}{\cos(x_1)} + \frac{b}{\cos(x_1+x_2)} I = \frac{a + bI}{\cos(x_1+x_2)I}$$

Then.

$$Y = y_1 + y_2 I = \left\{ \frac{a + bI}{\cos(x_1+x_2)I} + \frac{1}{4} \frac{\cos 2(x_1+x_2)I}{\cos(x_1+x_2)I} \right\}^{-1} = \left\{ \frac{a + bI}{\cos(X)} + \frac{1}{4} \frac{\cos 2X}{\cos X} \right\}^{-1}$$

So that,

$$Y = y_1 + y_2 I = \left\{ \frac{a + bI}{\cos(X)} + \frac{1}{4} \frac{\cos 2X}{\cos X} \right\}^{-1}$$

where $a + bI \in R(I)$.

Neutrosophic Recati equation.

In this section is defined a Neutrosophic Recati equation by Using the One-Dimensional Geometric AH-Isometry and solutions are found for this equation.

Definition .

Let $Y = y_1 + y_2I$, $X = x_1 + x_2I$ We define the NeutrosophicRecati equation by Using the One-Dimensional Geometric AH-Isometry as form:

$$\dot{Y} + f(X)Y^2 + g(X)Y + h(X) = 0$$

And takes a particular solution:

$$Z = z_1 + z_2I = r(X) = r(x_1 + x_2I)$$

This equation can be written as follow:

$$(y_1' + I[(y_1 + y_2)' - y_1'])(f(x_1) + I[f(x_1 + x_2) - f(x_1)])(y_1 + y_2I)^2 + (g(x_1) + I[g(x_1 + x_2) - g(x_1)])(y_1 + y_2I) + h(x_1) + I[h(x_1 + x_2) - h(x_1)] = 0$$

$$y_1' + f(x_1)(y_1)^2 + I[(y_1 + y_2)' - f(x_1 + x_2)(y_1 + y_2) - (y_1' + f(x_1)(y_1)^2)] + (g(x_1) + I[g(x_1 + x_2) - g(x_1)])(y_1 + I[(y_1 + y_2) - (y_1)]) + h(x_1) + I[h(x_1 + x_2) - h(x_1)] = 0$$

And:

$$Z = z_1 + z_2I = r(x_1 + x_2I) = r(x_1) + I[r(x_1 + x_2) - r(x_1)]$$

Method of solution.

1. Take AH-Isometry for the differential equation, and Take AH-Isometry for a particular solution, we have.

$$T(y_1' + f(x_1)(y_1)^2 + I[(y_1 + y_2)' - f(x_1 + x_2)(y_1 + y_2) - (y_1' + f(x_1)(y_1)^2)] + (g(x_1) + I[g(x_1 + x_2) - g(x_1)])(y_1 + I[(y_1 + y_2) - (y_1)]) + h(x_1) + I[h(x_1 + x_2) - h(x_1)]) = T(0)$$

$$[y_1' + f(x_1)(y_1)^2 + g(x_1)y_1 + h(x_1), (y_1 + y_2)' + f(x_1 + x_2)(y_1 + y_2)^2 + g(x_1 + x_2)(y_1 + y_2) + h(x_1 + x_2)] = [0, 0]$$

Then.

$$\begin{cases} y_1' + f(x_1)(y_1)^2 + g(x_1)y_1 + h(x_1) = 0 \dots \dots (7) \\ (y_1 + y_2)' + f(x_1 + x_2)(y_1 + y_2)^2 + g(x_1 + x_2)(y_1 + y_2) + h(x_1 + x_2) = 0 \dots \dots (8) \end{cases}$$

And:

$$T(z_1 + z_2I) = T(r(x_1) + I[r(x_1 + x_2) - r(x_1)])$$

$$[z_1, (z_1 + z_2)] = [r(x_1), r(x_1 + x_2)]$$

$$\begin{cases} z_1 = r(x_1) \dots \dots (9) \\ (z_1 + z_2) = r(x_1 + x_2) \dots \dots (10) \end{cases}$$

The equations (7) and (8) are two Recati differential equation classical with two particular solution (9) and (10).

2. We find the solution to the equations classical (7) and (8), we have.

y_1 the solution to the equation (7).

$(y_1 + y_2)$ the solution to the equation (8).

3. We Take invertible AH-Isometry, then, we have the solution of a Neutrosophic identical linear differential equation.

$$Y = y_1 + y_2 I = T^{-1}(y_1, y_1 + y_2) = y_1 + ((y_1 + y_2) - y_1)I$$

Example Find the general solution for the following neutrosophic ricati equation:

$$\dot{Y} + \left(\frac{\cos X}{1 - \sin X \cos X}\right) Y^2 + \left(\frac{-1}{1 - \sin X \cos X}\right) Y + \frac{\sin X}{1 - \sin X \cos X} = 0$$

If a particular solution is:

$$Z = z_1 + z_2 I = \cos X$$

Solution.

Let $Y = y_1 + y_2 I, X = x_1 + x_2 I$. Then.

$$\begin{aligned} & y_1' + \frac{\cos x_1}{1 - \sin x_1 \cdot \cos x_1} (y_1)^2 \\ & + I \left[(y_1 + y_2)' + \frac{\cos(x_1 + x_2)}{1 - \sin(x_1 + x_2) \cdot \cos(x_1 + x_2)} (y_1 + y_2) - \left(y_1' + \frac{\cos x_1}{1 - \sin x_1 \cdot \cos x_1} (y_1)^2 \right) \right] \\ & + \left(\frac{-1}{1 - \sin x_1 \cdot \cos x_1} + I \left[\frac{-1}{1 - \sin(x_1 + x_2) \cdot \cos(x_1 + x_2)} - \left(\frac{-1}{1 - \sin x_1 \cdot \cos x_1} \right) \right] \right) (y_1) \\ & + I[(y_1 + y_2) - (y_1)] + \frac{\sin x_1}{1 - \sin x_1 \cdot \cos x_1} \\ & + I \left[\frac{\sin(x_1 + x_2)}{1 - \sin(x_1 + x_2) \cdot \cos(x_1 + x_2)} - \frac{\sin x_1}{1 - \sin x_1 \cdot \cos x_1} \right] = 0 \end{aligned}$$

And:

$$Z = z_1 + z_2 I = \cos X = \cos x_1 + I[\cos(x_1 + x_2) - \cos x_1]$$

Now, Take AH-Isometry for the differential equation, and Take AH-Isometry for a particular solution, we have..

$$\left\{ \begin{aligned} & y_1' + \frac{\cos x_1}{1 - \sin x_1 \cdot \cos x_1} (y_1)^2 - \frac{1}{1 - \sin x_1 \cdot \cos x_1} y_1 + \frac{\sin x_1}{1 - \sin x_1 \cdot \cos x_1} = 0 \dots \dots (11) \\ & (y_1 + y_2)' + \frac{\cos(x_1 + x_2)}{1 - \sin(x_1 + x_2) \cdot \cos(x_1 + x_2)} (y_1 + y_2)^2 - \frac{1}{1 - \sin(x_1 + x_2) \cdot \cos(x_1 + x_2)} (y_1 + y_2) \\ & \quad + \frac{\sin(x_1 + x_2)}{1 - \sin(x_1 + x_2) \cdot \cos(x_1 + x_2)} = 0 \dots \dots (12) \end{aligned} \right.$$

And:

$$\begin{cases} z_1 = \cos x_1 \\ (z_1 + z_2) = \cos(x_1 + x_2) \end{cases}$$

The solution of equation (11) written as follow:

$$y_1 = \cos x_1 + \left\{ \frac{1}{1 - \sin x_1 \cdot \cos x_1} (a + \sin x_1) \right\}^{-1}, \text{ where } a \in R.$$

By the method same, The solution of equation (12) written as follow:

$$(y_1 + y_2) = \cos(x_1 + x_2) + \left\{ \frac{1}{1 - \sin(x_1 + x_2) \cdot \cos(x_1 + x_2)} (b + \sin(x_1 + x_2)) \right\}^{-1}, \text{ where } b \in R$$

Now, Take invertible AH-Isometry, then, we have the solution of a Neutrosophic identical linear differential equation

$$Y = y_1 + y_2 I = T^{-1} \left(\cos x_1 + \left\{ \frac{1}{1 - \sin x_1 \cdot \cos x_1} (a + \sin x_1) \right\}^{-1}, \cos(x_1 + x_2) + \left\{ \frac{1}{1 - \sin(x_1 + x_2) \cdot \cos(x_1 + x_2)} (b + \sin(x_1 + x_2)) \right\}^{-1} \right)$$

$$Y = y_1 + y_2 I = \cos x_1 + \left\{ \frac{1}{1 - \sin x_1 \cdot \cos x_1} (a + \sin x_1) \right\}^{-1} + I \left[\cos(x_1 + x_2) + \left\{ \frac{1}{1 - \sin(x_1 + x_2) \cdot \cos(x_1 + x_2)} (b + \sin(x_1 + x_2)) \right\}^{-1} - \left(\cos x_1 + \left\{ \frac{1}{1 - \sin x_1 \cdot \cos x_1} (a + \sin x_1) \right\}^{-1} \right) \right]$$

$$Y = y_1 + y_2 I = \cos x_1 + I(\cos(x_1 + x_2) - \cos x_1) + \left\{ \frac{1}{1 - \sin x_1 \cdot \cos x_1} (a + \sin x_1) \right\}^{-1} + I \left[\left\{ \frac{1}{1 - \sin(x_1 + x_2) \cdot \cos(x_1 + x_2)} (b + \sin(x_1 + x_2)) \right\}^{-1} - \left(\left\{ \frac{1}{1 - \sin x_1 \cdot \cos x_1} (a + \sin x_1) \right\}^{-1} \right) \right]$$

$$Y = y_1 + y_2 I = \cos(x_1 + x_2) I + \left\{ \frac{a}{1 - \sin x_1 \cdot \cos x_1} + \frac{\sin x_1}{1 - \sin x_1 \cdot \cos x_1} \right\}^{-1} + I \left[\left\{ \frac{b}{1 - \sin(x_1 + x_2) \cdot \cos(x_1 + x_2)} + \frac{\sin(x_1 + x_2)}{1 - \sin(x_1 + x_2) \cdot \cos(x_1 + x_2)} \right\}^{-1} - \left\{ \frac{a}{1 - \sin x_1 \cdot \cos x_1} + \frac{\sin x_1}{1 - \sin x_1 \cdot \cos x_1} \right\}^{-1} \right]$$

$$Y = y_1 + y_2I = \cos(X) + \left\{ \frac{a}{1 - \sin x_1 \cdot \cos x_1} + \frac{\sin x_1}{1 - \sin x_1 \cdot \cos x_1} \right\}^{-1} + I \left[\left\{ \frac{b}{1 - \sin(x_1 + x_2) \cdot \cos(x_1 + x_2)} + \frac{\sin(x_1 + x_2)}{1 - \sin(x_1 + x_2) \cdot \cos(x_1 + x_2)} \right\}^{-1} - \left\{ \frac{a}{1 - \sin x_1 \cdot \cos x_1} + \frac{\sin x_1}{1 - \sin x_1 \cdot \cos x_1} \right\}^{-1} \right]$$

we have.

$$\begin{aligned} & \left\{ \frac{a}{1 - \sin x_1 \cdot \cos x_1} + \frac{\sin x_1}{1 - \sin x_1 \cdot \cos x_1} \right\}^{-1} \\ & + I \left[\left\{ \frac{b}{1 - \sin(x_1 + x_2) \cdot \cos(x_1 + x_2)} + \frac{\sin(x_1 + x_2)}{1 - \sin(x_1 + x_2) \cdot \cos(x_1 + x_2)} \right\}^{-1} - \left\{ \frac{a}{1 - \sin x_1 \cdot \cos x_1} + \frac{\sin x_1}{1 - \sin x_1 \cdot \cos x_1} \right\}^{-1} \right] \\ & = \left\{ \frac{a}{1 - \sin x_1 \cdot \cos x_1} + \frac{\sin x_1}{1 - \sin x_1 \cdot \cos x_1} + \left(\frac{b}{1 - \sin(x_1 + x_2) \cdot \cos(x_1 + x_2)} + \frac{\sin(x_1 + x_2)}{1 - \sin(x_1 + x_2) \cdot \cos(x_1 + x_2)} \right) I \right\}^{-1} \\ & = \left\{ \frac{a}{1 - \sin x_1 \cdot \cos x_1} + \frac{\sin x_1}{1 - \sin x_1 \cdot \cos x_1} + \frac{b}{1 - \sin(x_1 + x_2) \cdot \cos(x_1 + x_2)} I + \frac{\sin(x_1 + x_2)}{1 - \sin(x_1 + x_2) \cdot \cos(x_1 + x_2)} I \right\}^{-1} \\ & = \left\{ \frac{a}{1 - \sin x_1 \cdot \cos x_1} + \frac{b}{1 - \sin(x_1 + x_2) \cdot \cos(x_1 + x_2)} I + \frac{\sin x_1}{1 - \sin x_1 \cdot \cos x_1} + \frac{\sin(x_1 + x_2)}{1 - \sin(x_1 + x_2) \cdot \cos(x_1 + x_2)} I \right\}^{-1} \end{aligned}$$

By Definition 2.5, we have.

$$\frac{a}{1 - \sin x_1 \cdot \cos x_1} + \frac{b}{1 - \sin(x_1 + x_2) \cdot \cos(x_1 + x_2)} I = \frac{a + bI}{1 - \sin(x_1 + x_2)I \cdot \cos(x_1 + x_2)I} = \frac{a + bI}{1 - \sin(X) \cdot \cos(X)}$$

And.

$$\frac{\sin x_1}{1 - \sin x_1 \cdot \cos x_1} + \frac{\sin(x_1 + x_2)}{1 - \sin(x_1 + x_2) \cdot \cos(x_1 + x_2)} I = \frac{\sin(x_1 + x_2)I}{1 - \sin(x_1 + x_2)I \cdot \cos(x_1 + x_2)I} = \frac{\sin(X)}{1 - \sin(X) \cdot \cos(X)}$$

So that,

$$Y = y_1 + y_2I = \cos(X) + \left\{ \frac{a + bI}{1 - \sin(X) \cdot \cos(X)} + \frac{\sin(X)}{1 - \sin(X) \cdot \cos(X)} \right\}^{-1}$$

where $a + bI \in R(I)$.

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