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A Novel Approach on Neutrosophic Binary αgs Neighborhood Points and its Operators

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Abstract

The main idea of this paper is to introduce neutrosophic binary αgs-neighborhood points and neutrosophic binary αgs interior and closure operators. Furthermore, some of its properties are contemplated.
Keywords: Neutrosophic binary αgs-neighborhood points; neutrosophic binary αgs interior and neutrosophic binary αgs closure.

1 Introduction

Topology between two sets is said to be the binary topology as it is the binary structure from X to Y which is defined to be the ordered pairs (A, B) where $A \subseteq X$ and $B \subseteq Y$. This binary topology was initiated by S.N.Jothi and P.Thangavelu[6] in 2011. The concept of neutrosophic sets was presented by Smarandache[8] which is a generalisation of intuitionistic fuzzy sets. Using this sets, A.A.Salama[10] in 2012 introduced the neutrosophic topological spaces. The neutrosophic set contains the degree of membership, the degree of indeterminancy and the degree of non-membership of each components of X. In continuation, by combining these topologies, S.S.Surekha, J.Elekiah and G.Sindhu[11] in 2022 formulated neutrosophic binary topological spaces. Again, in 2022, S.S.Surekha and G.Sindhu[12] defined N_b-αgs closed sets in neutrosophic binary topological spaces. In this paper, we have introduced a new concept called neutrosophic binary αgs-neighborhood points and discussed some of its properties. Also, we discussed about the neutrosophic binary αgs interior and closure operators and proved some theorems which is analyzed using the examples.

2 Preliminaries

Definition 2.1. [11] A Neutrosophic binary topology from X to Y is a binary structure $\mathcal{M}_N \subseteq P(X) \times P(Y)$ that satisfies the following conditions:

1. $(0_X, 0_Y) \in \mathcal{M}_N$ and $(1_X, 1_Y) \in \mathcal{M}_N$.
2. $(A_1 \cap A_2, B_1 \cap B_2) \in \mathcal{M}_N$ whenever $(A_1, B_1) \in \mathcal{M}_N$ and $(A_2, B_2) \in \mathcal{M}_N$.
3. If $(A_\alpha, B_\alpha)_{\alpha \in A}$ is a family of members of $\mathcal{M}_N$, then $(\cup_{\alpha \in A} A_\alpha, \cup_{\alpha \in A} B_\alpha) \in \mathcal{M}_N$.

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The triplet $(X, Y, \mathcal{M}_N)$ is called Neutrosophic Binary Topological space. The members of $\mathcal{M}_N$ are called the neutrosophic binary open sets and the complement of neutrosophic binary open sets are called the neutrosophic binary closed sets in the binary topological space $(X, Y, \mathcal{M}_N)$.

Definition 2.2. [11] $(0_X, 0_Y)$ can be defined as
(0_1) $0_X = \{< x, 0, 0, 1 >: x \in X\}, 0_Y = \{< y, 0, 0, 1 >: y \in Y\}$
(0_2) $0_X = \{< x, 0, 1, 1 >: x \in X\}, 0_Y = \{< y, 0, 1, 1 >: y \in Y\}$
(0_3) $0_X = \{< x, 0, 1, 0 >: x \in X\}, 0_Y = \{< y, 0, 1, 0 >: y \in Y\}$
(0_4) $0_X = \{< x, 0, 0, 1 >: x \in X\}, 0_Y = \{< y, 0, 0, 0 >: y \in Y\}$

$(1_X, 1_Y)$ can be defined as
(1_1) $1_X = \{< x, 1, 0, 0 >: x \in X\}, 1_Y = \{< y, 1, 0, 0 >: y \in Y\}$
(1_2) $1_X = \{< x, 1, 0, 1 >: x \in X\}, 1_Y = \{< y, 1, 0, 1 >: y \in Y\}$
(1_3) $1_X = \{< x, 1, 1, 0 >: x \in X\}, 1_Y = \{< y, 1, 1, 0 >: y \in Y\}$
(1_4) $1_X = \{< x, 1, 1, 1 >: x \in X\}, 1_Y = \{< y, 1, 1, 1 >: y \in Y\}$

Definition 2.3. [11] Let $(A, B) = \{< \mu_A, \sigma_A, \gamma_A >, < \mu_B, \sigma_B, \gamma_B >\}$ be a neutrosophic binary set on $(X, Y, \mathcal{M}_N)$, then the complement of the set $C(A, B)$ may be defined as

$$\begin{aligned}
(C_1)\quad C(A,B) =& \{x, < 1-\mu_A(x), \sigma_A(x), 1-\gamma_A(x) >: x \in X, \\
& < y, 1-\mu_B(y), \sigma_B(y), 1-\gamma_B(y) >: y \in Y\} \\
(C_2)\quad C(A,B) =& \{x, < \gamma_A(x), \sigma_A(x), \mu_A(x) >: x \in X, \\
& < y, \gamma_B(y), \sigma_B(y), \mu_B(y) >: y \in Y\} \\
(C_3)\quad C(A,B) =& \{x, < \gamma_A(x), 1-\sigma_A(x), \mu_A(x) >: x \in X, \\
& < y, \gamma_B(y), 1-\sigma_B(y), \mu_B(y) >: y \in Y\}
\end{aligned}$$

Definition 2.4. [11] Let (A, B) and (C, D) be two neutrosophic binary sets which is in the form
$(A, B) = \{< \mu_A, \sigma_A, \gamma_A >, < \mu_B, \sigma_B, \gamma_B >\}$ and
$(C, D) = \{< \mu_C, \sigma_C, \gamma_C >, < \mu_D, \sigma_D, \gamma_D >\}$.
Then $(A, B) \subseteq (C, D)$ can be defined as

1. $(A, B) \subseteq (C, D) \iff \mu_A(x) \leq \mu_C(x), \sigma_A(x) \leq \sigma_C(x), \gamma_A(x) \geq \gamma_C(x) \forall x \in X$
   $\mu_B(y) \leq \mu_D(y), \sigma_B(y) \leq \sigma_D(y), \gamma_B(y) \geq \gamma_D(y) \forall y \in Y$
2. $(A, B) \subseteq (C, D) \iff \mu_A(x) \leq \mu_C(x), \sigma_A(x) \geq \sigma_C(x), \gamma_A(x) \geq \gamma_C(x) \forall x \in X$
   $\mu_B(y) \leq \mu_D(y), \sigma_B(y) \geq \sigma_D(y), \gamma_B(y) \geq \gamma_D(y) \forall y \in Y$

Definition 2.5. [11] Let (A, B) and (C, D) be two neutrosophic binary sets which is in the form
$(A, B) = \{< \mu_A, \sigma_A, \gamma_A >, < \mu_B, \sigma_B, \gamma_B >\}$ and
$(C, D) = \{< \mu_C, \sigma_C, \gamma_C >, < \mu_D, \sigma_D, \gamma_D >\}$.
(1) $(A, B) \cap (C, D)$ can be defined as

$$\begin{aligned}
(A,B) \cap (C,D) = \{ & < x, \mu_A(x) \wedge \mu_C(x), \sigma_A(x) \wedge \sigma_C(x), \gamma_A(x) \vee \gamma_C(x) > \\
& < y, \mu_B(y) \wedge \mu_D(y), \sigma_B(y) \wedge \sigma_D(y), \gamma_B(y) \vee \gamma_D(y) >\}
\end{aligned}$$

$$\begin{aligned}
(A,B) \cap (C,D) = \{ & < x, \mu_A(x) \wedge \mu_C(x), \sigma_A(x) \vee \sigma_C(x), \gamma_A(x) \vee \gamma_C(x) > \\
& < y, \mu_B(y) \wedge \mu_D(y), \sigma_B(y) \vee \sigma_D(y), \gamma_B(y) \vee \gamma_D(y) >\}
\end{aligned}$$

(2) $(A, B) \cup (C, D)$ can be defined as

$$\begin{aligned}
(A,B) \cup (C,D) = \{ & < x, \mu_A(x) \vee \mu_C(x), \sigma_A(x) \vee \sigma_C(x), \gamma_A(x) \wedge \gamma_C(x) > \\
& < y, \mu_B(y) \vee \mu_D(y), \sigma_B(y) \vee \sigma_D(y), \gamma_B(y) \wedge \gamma_D(y) >\}
\end{aligned}$$

$$\begin{aligned}
(A,B) \cap (C,D) = \{ & < x, \mu_A(x) \vee \mu_C(x), \sigma_A(x) \wedge \sigma_C(x), \gamma_A(x) \wedge \gamma_C(x) > \\
& < y, \mu_B(y) \vee \mu_D(y), \sigma_B(y) \wedge \sigma_D(y), \gamma_B(y) \wedge \gamma_D(y) >\}
\end{aligned}$$

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Definition 2.6. [11] Let $(X, Y, \mathcal{M}_N)$ be a Neutrosophic Binary Topological Space. Then,
$(A, B)^{1^\star_N} = \cap\{A_\alpha : (A_\alpha, B_\alpha)$ is neutrosophic binary closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$
$(A, B)^{2^\star_N} = \cap\{B_\alpha : (A_\alpha, B_\alpha)$ is neutrosophic binary closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$.
The ordered pair $((A, B)^{1^\star_N}, (A, B)^{2^\star_N})$ is called the neutrosophic binary closure of (A, B).

Definition 2.7. [11] Let $(X, Y, \mathcal{M}_N)$ be a Neutrosophic Binary Topological Space. Then,
$(A, B)^{1^0_N} = \cup\{A_\alpha : (A_\alpha, B_\alpha)$ is neutrosophic binary open and $(A_\alpha, B_\alpha) \subseteq (A, B)\}$
$(A, B)^{2^0_N} = \cup\{B_\alpha : (A_\alpha, B_\alpha)$ is neutrosophic binary open and $(A_\alpha, B_\alpha) \subseteq (A, B)\}$.
The ordered pair $((A, B)^{1^0_N}, (A, B)^{2^0_N})$ is called the neutrosophic binary interior of (A, B).

Definition 2.8. [12] Let $(X, Y, \mathcal{M}_N)$ be a Neutrosophic Binary Topological Space. Then (A, B) is called

1. Neutrosophic binary α open if $(A, B) \subseteq N_bint(N_bcl(N_bint(A, B)))$.
2. Neutrosophic binary semiopen if $(A, B) \subseteq N_bcl(N_bint(A, B))$.

Definition 2.9. [12] Let $(X, Y, \mathcal{M}_N)$ be a Neutrosophic Binary Topological Space. Then,
$(A, B)^{1^\star_N} = \cap\{A_\alpha : (A_\alpha, B_\alpha)$ is neutrosophic binary α closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$
$(A, B)^{2^\star_N} = \cap\{B_\alpha : (A_\alpha, B_\alpha)$ is neutrosophic binary α closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$.
The ordered pair $((A, B)^{1^\star_N}, (A, B)^{2^\star_N})$ is called the neutrosophic binary α closure of (A, B) and is denoted by $N_b\alpha cl(A, B)$.

Definition 2.10. [12] Let $(X, Y, \mathcal{M}_N)$ be a Neutrosophic Binary Topological Space. Let $(A, B) \subseteq (X, Y)$. Then (A, B) is called a Neutrosophic Binary α generalised semiclosed set (shortly $N_b\alpha gs$-closed set) if $N_b\alpha cl(A, B) \subseteq (U, V)$ whenever (U, V) is Neutrosophic Binary Semiopen.

Definition 2.11. [4] Let x be a point of a topological space (X, τ). A set $U \subseteq X$ is called an α-neighborhood of x in X if there exists an α-open set A containing x and $A \subseteq U$.

3 Neutrosophic Binary αgs neighbourhoods

Definition 3.1. Let $(X, Y, \mathcal{M}_N)$ be a neutrosophic binary topological space. Let $\mathcal{M}_N(X)$ be the set of all neutrosophic binary sets over X and $\mathcal{M}_N(Y)$ be the set of all neutrosophic binary sets over Y. A neutrosophic binary set

$$(S, T) = \{< x, T(x), I(x), F(x) >: x \in X, < y, T(y), I(y), F(y) >: y \in Y\}$$

is called the binary point if and only if for any element $(P, Q) \in (X, Y)$,

$$\begin{cases} T(P) = \alpha_1; T(Q) = \alpha_2 \\ I(P) = \beta_1; I(Q) = \beta_2; \quad for(P, Q) = (X, Y) \\ F(P) = \gamma_1; F(Q) = \gamma_2 \end{cases} \qquad \begin{cases} T(P) = 0; T(Q) = 0 \\ I(P) = 1; I(Q) = 1; \quad for(P, Q) \neq (X, Y) \\ F(P) = 1; F(Q) = 1 \end{cases}$$

The neutrosophic binary point

$$(S, T) = \{< x, T(x), I(x), F(x) >: x \in X, < y, T(y), I(y), F(y) >: y \in Y\}$$

will be denoted by $(S^x_{\alpha_1,\beta_1,\gamma_1}, T^y_{\alpha_2,\beta_2,\gamma_2})$ or $(S < x, \alpha_1, \beta_1, \gamma_1 >, T < y, \alpha_2, \beta_2, \gamma_2 >)$ or simply $(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2})$. For the neutrosophic binary point $(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2})$, (x, y) is said to be the binary support. The complement of the neutrosophic binary point is denoted as $(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2})^c$.

Definition 3.2. Let $(X, Y, \mathcal{M}_N)$ be a neutrosophic binary topological space. Let (A, B) be the neutrosophic binary subset of the Neutrosophic Binary Topological sapce. The subset (A, B) is called as neutrosophic binary αgs- neighborhood of a neutrosophic binary point $(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2})$ if and only if there exists an neutrosophic binary αgs open set (C, D) such that $(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2}) \in (C, D) \subseteq (A, B)$.

Definition 3.3. The neutrosophic binary neighborhood (A, B) of the neutrosophic binary point $(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2})$ is said to be neutrosophic binary αgs- neighborhood of $(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2})$ if (A, B) is a neutrosophic binary αgs open set.

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Definition 3.4. The family consisting of all the neutrosophic binary αgs-neighborhoods of neutrosophic binary point $(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2})$ is called the system of neutrosophic binary αgs-neigborhoods. The family is denoted by $N_b(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2})$.

Proposition 3.5. *A neutrosophic binary set (A,B) in a neutrosophic binary topological space $(X,Y,\mathcal{M}_N)$ is said to be neutrosophic binary αgs-open if and only if it is a neutrosophic binary αgs-neighborhood of each of its neutrosophic binary points.*

Proof. Let $(X,Y,\mathcal{M}_N)$ be a neutrosophic binary topological space. Let (A,B) be a subset of Neutrosophic Binary Topological Space. Suppose (A,B) is neutrosophic binary αgs-open set. Then for every neutrosophic binary point $(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2}) \in (A,B) \subseteq (A,B)$. This implies that, (A,B) is a neutrosophic binary αgs-neighborhood of $(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2})$. Thus, (A,B) is a neutrosophic binary αgs-neighborhood of each of its neutrosophic binary points.
Conversely, suppose that (A,B) is a neutrosophic binary αgs- neighborhood of each of its neutrosophic binary points.
If $(A,B) = (0_X, 0_Y)$. Then, obviously (A,B) is a neutrosophic binary αgs open set.
If $(A,B) \neq (0_X, 0_Y)$. Then for each $(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2}) \in (A,B)$, there exists an neutrosophic binary αgs-open set (C,D) such that $(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2}) \in (C,D) \subseteq (A,B)$.
Obviously, $(A,B) = \cup(C(x_{\alpha_1,\beta_1,\gamma_1}), D(y_{\alpha_2,\beta_2,\gamma_2}))$.
Hence, (A,B) is a neutrosophic binary αgs-open set. □

Proposition 3.6. *Two neutrosophic binary topologies on the same set are identical if and only if they admit the same neutrosophic bianry αgs-neighborhoods.*

Proof. The neccesary part is obvious.
Sufficient part: Let $(X,Y,\mathcal{M}_{N_1})$ and $(X,Y,\mathcal{M}_{N_2})$ be two neutrosophic binary topological spaces having the same neutrosophic binary αgs-neighborhoods of the neutrosophic binary points over (X,Y). Now,
let (A,B) be the neutrosophic binary αgs-open set of $(X,Y,\mathcal{M}_{N_1})$.
$\iff$ (A,B) is a neutrosophic binary αgs-neighborhood of its neutrosophic binary points in $(X,Y,\mathcal{M}_{N_1})$.
$\iff$ (A,B) is a neutrosophic binary αgs-neighborhood of its neutrosophic binary points in $(X,Y,\mathcal{M}_{N_2})$.
$\iff$ (A,B) is a neutrosophic binary αgs-open set in $(X,Y,\mathcal{M}_{N_2})$.
Therefore, $\mathcal{M}_{N_1} = \mathcal{M}_{N_2}$. □

3.1 Properties of a neutrosophic binary αgs-neighborhoods

Let $(X,Y,\mathcal{M}_N)$ be a neutrosophic binary topological space and let $(x,y) \in (X,Y)$. If $\mathcal{M}_N(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2})$ be the collection of all neutrosophic binary αgs-neighborhoods of the neutrosophic binary point $(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2})$, then

1. $\mathcal{M}_N(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2}) \neq 0$ for every neutrosophic binary point $(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2}) \in \mathcal{M}_N(X,Y)$.
2. $(A,B) \in \mathcal{M}_N(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2}) \implies (x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2}) \in (A,B)$.
3. $(A,B) \in \mathcal{M}_N(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2})$ and $(A,B) \subseteq (C,D) \implies (C,D) \in \mathcal{M}_N(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2})$.
4. $(A,B) \in \mathcal{M}_N(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2})$,then there exists $(C,D) \in \mathcal{M}_N(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2})$ such that $(C,D) \subseteq (A,B)$ and $(C,D) \in \mathcal{M}_N(x'_{\alpha_{1'},\beta_{1'},\gamma_{1'}}, y_{\alpha_{2'},\beta_{2'},\gamma_{2'}})$ for all $(x'_{\alpha_{1'},\beta_{1'},\gamma_{1'}}, y_{\alpha_{2'},\beta_{2'},\gamma_{2'}}) \in (C,D)$

Proof. 1. Since (X,Y) is an neutrosophic binary αgs-open set, it is a neutrosophic binary αgs neighborhood of every neutrosophic binary point $(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2})$. Thus there exists atleast one neutrosophic binary αgs-neighborhood for every neutrosophic binary point $(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2})$. Therefore, $\mathcal{M}_N(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2}) \neq 0$ for every neutrosophic binary point $(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2}) \in \mathcal{M}_N(X,Y)$.

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2. Let $(A,B) \in \mathcal{M}_N(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2})$.
$\implies$ (A,B) is a neutrosophic binary αgs-neighborhood of $(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2})$.
$\implies$ $(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2}) \in (A,B)$.

3. Let $(A,B) \in \mathcal{M}_N(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2})$.
$\implies$ (A,B) is a neutrosophic binary αgs-neighborhood of $(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2})$.
$\implies$ there exists an neutrosophic binary αgs-open set (A_1,B_1) such that $(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2}) \in (A_1,B_1) \subseteq (A,B)$.
Since, $(A,B) \subseteq (C,D)$,
$\implies$ there exists an neutrosophic binary αgs-open set (A_1,B_1) such that $(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2}) \in (A_1,B_1) \subseteq (C,D)$.
$\implies$ (C,D) is a neutrosophic binary αgs-neighborhood of $(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2})$.
$\implies$ $(C,D) \in \mathcal{M}_N(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2})$.

4. Let $(A,B) \in \mathcal{M}_N(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2})$. Then, there exists an neutrosophic binary αgs-open (C,D) such that $(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2}) \in (C,D) \subseteq (A,B)$. Since, (C,D) is a neutrosophic binary αgs-open set and since $(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2}) \in (C,D) \subseteq (C,D)$, we have $(C,D) \in \mathcal{M}_N(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2})$ and also $(A,B) \subseteq (C,D)$. Further, since (C,D) is a neutrosophic binary αgs-open set, (C,D) is a neutrosophic binary αgs-neighborhood of each of its neutrosophic binary points. Therefore, $(C,D) \in \mathcal{M}_N(x'_{\alpha_1',\beta_1',\gamma_1'}, y_{\alpha_2',\beta_2',\gamma_2'})$ for all $(x'_{\alpha_1',\beta_1',\gamma_1'}, y_{\alpha_2',\beta_2',\gamma_2'}) \in (C,D)$.

□

4 Neutrosophic binary αgs interior of a set

Definition 4.1. A neutrosophic binary point $(x_{\alpha_1,\beta_1,\gamma_1}, y_{\alpha_2,\beta_2,\gamma_2})$ in a neutrosophic binary topological space $(X,Y,\mathcal{M}_N)$ is said to be neutrosophic binary αgs-interior point of (A,B) if and only if there exists an neutrosophic binary αgs-open set (U,V) in (X,Y) such that $(U,V) \subseteq (A,B)$.

Definition 4.2. Let $(X,Y,\mathcal{M}_N)$ be a Neutrosophic Binary Topological Space. Then,
$(A,B)^{1^0_N}_{\alpha gs} = \cup\{A_\alpha : (A_\alpha, B_\alpha)$ is neutrosophic binary αgs-open and $(A_\alpha,B_\alpha) \subseteq (A,B)\}$

$(A,B)^{2^0_N}_{\alpha gs} = \cup\{B_\alpha : (A_\alpha, B_\alpha)$ is neutrosophic binary αgs-open and $(A_\alpha,B_\alpha) \subseteq (A,B)\}$.

The ordered pair $((A,B)^{1^0_N}, (A,B)^{2^0_N})$ is called the neutrosophic binary αgs interior of (A,B).

Remark 4.3. The set of all neutrosophic binary αgs-interior points of $(A,B) \subseteq (X,Y)$ is said to be the neutrosophic binary αgs-interior of (A,B). Equivalently, the union of all neutrosophic binary αgs-open sets contained in (A,B) is called the neutrosophic binary αgs-interior of (A,B) and is denoted by $N_b\alpha gs$-$int(A,B)$.
The union of all neutrosophic binary αgs-open set in a Neutrosopic Binary Topological Space $(X,Y,\mathcal{M}_N)$ is neutrosophic binary αgs-open. This implies that the $N_b\alpha gs$-$int(A,B)$ is the neutrosophic binary αgs-open set.

Remark 4.4. Since by theorem 5.3[12], every neutrosophic binary open set is neutrosophic binary αgs-open set, it follows that every interior point of $(A,B) \subseteq (X,Y)$ is a neutrosophic binary αgs-interior point of (A,B). Therefore, N_b-$int(A,B) \subseteq N_b\alpha gs$-$int(A,B)$. But the converse is not true as shown by the following example.

Example 4.5. Let $X = \{a,b\}$ and $Y = \{c,d\}$. The neutrosophic binary topological space is given by $\mathcal{M}_N = \{(0_X,0_Y),(1_X,1_Y),(V_1,W_1),(V_2,W_2)\}$ where,
$(V_1,W_1) = \{< x,(0.4,0.5,0.5),(0.3,0.5,0.6) >,< y,(0.3,0.5,0.5),(0.4,0.5,0.7) >\}$ and
$(V_2,W_2) = \{< x,(0.3,0.5,0.6),(0.2,0.5,0.7) >,< y,(0.2,0.5,0.6),(0.3,0.5,0.7) >\}$.
Let $(A,B) = \{< x,(0.8,0.5,0.1),(0.8,0.5,0.1) >,< y,(0.7,0.5,0.2),(0.6,0.5,0.1) >\}$.
Here N_b-$int(A,B) = \{< x,(0.4,0.5,0.5),(0.3,0.5,0.6) >,< y,(0.3,0.5,0.5),(0.4,0.5,0.7) >\}$ and $N_b\alpha gs$-$int(A,B) = \{< x,(0.2,0.5,0.9),(0.2,0.5,0.9) >,< y,(0.3,0.5,0.8),(0.4,0.5,0.9) >\}$, which implies that $N_b\alpha gs$-$int(A,B) \nsubseteq N_b$-$int(A,B)$.

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Remark 4.6. Let $(X, Y, \mathcal{M}_N)$ be a neutrosophic binary topological space. The subset (A, B) is said to be neutrosophic bianry αgs-open if and only if $(A, B) = N_b\alpha gs\text{-}int(A, B)$.

Proposition 4.7. *Suppose $(A, B) \subseteq (C, D) \subseteq (X, Y)$ and $(X, Y, \mathcal{M}_N)$ is a neutrosophic binary topological space. Then,*

(i) $N_b\alpha gs\text{-}int(0_X, 0_Y) = (0_X, 0_Y)$
$N_b\alpha gs\text{-}int(1_X, 1_Y) = (1_X, 1_Y)$

(ii) $N_b\alpha gs\text{-}int(A, B) \subseteq (A, B)$

(iii) $(A, B)^{1^0_N}_{\alpha gs} \subseteq (C, D)^{1^0_N}_{\alpha gs}$

(iv) $(A, B)^{2^0_N}_{\alpha gs} \subseteq (C, D)^{2^0_N}_{\alpha gs}$

(v) $N_b\alpha gs\text{-}int(A, B) \subseteq N_b\alpha gs\text{-}int(C, D)$

(vi) $N_b\alpha gs\text{-}int(N_b\alpha gs\text{-}int(A, B)) = N_b\alpha gs\text{-}int(A, B)$

Proof. (i) and (ii) are obvious.

$$\begin{aligned}(iii)\ (A, B)^{1^0_N}_{\alpha gs} &= \cup\{A_\alpha : (A_\alpha, B_\alpha) \text{ is neutrosophic binary} \alpha gs\text{-open and } (A_\alpha, B_\alpha) \subseteq (A, B)\}\\ &\subseteq \cup\{A_\alpha : (A_\alpha, B_\alpha) \text{ is neutrosophic binary} \alpha gs\text{-open and } (A_\alpha, B_\alpha) \subseteq (C, D)\}\\ &= (C, D)^{1^0_N}_{\alpha gs}.\end{aligned}$$

Similarly, (iv) also holds.

$$\begin{aligned}(v)\ N_b\alpha gs - int(A, B) &= ((A, B)^{1^0_N}_{\alpha gs}, (A, B)^{2^0_N}_{\alpha gs})\\ &\subseteq ((C, D)^{1^0_N}_{\alpha gs}, (C, D)^{2^0_N}_{\alpha gs})\\ &= N_b\alpha gs - int(C, D).\end{aligned}$$

(vi) It follows from the remark 4.6. □

5 Neutrosophic binary αgs closure of a set

Definition 5.1. Let $(X, Y, \mathcal{M}_N)$ be a Neutrosophic Binary Topological Space. Let $(A, B) \subseteq (X, Y)$. The intersection of all the neutrosophic binary αgs-closed sets in (X, Y) which contains (A, B) is called the neutrosophic binary αgs-closure of (A, B).

Definition 5.2. Let $(X, Y, \mathcal{M}_N)$ be a Neutrosophic Binary Topological Space. Then,
$(A, B)^{1^\star_N}_{\alpha gs} = \cap\{A_\alpha : (A_\alpha, B_\alpha)$ is neutrosophic binary αgs-closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$

$(A, B)^{2^\star_N}_{\alpha gs} = \cap\{B_\alpha : (A_\alpha, B_\alpha)$ is neutrosophic binary αgs-closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$

The ordered pair $((A, B)^{1^\star_N}, (A, B)^{2^\star_N})$ is called the neutrosophic binary αgs-closure of (A, B) and it is denoted by $N_b\alpha gs\text{-}cl(A, B)$.

Remark 5.3. Obviously, $N_b\alpha gs\text{-}cl(A, B)$ is the neutrosophic binary αgs-closed set and every neutrosophic binary closed set is neutrosophic binary αgs-closed set. It is clear that $N_b\alpha gs\text{-}cl(A, B) \subseteq N_b\text{-}cl(A, B)$. The converse is not true.

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Example 5.4. Let $E_1 = \{a_1, a_2, a_3\}$ and $E_2 = \{b_1, b_2, b_3\}$ be the universe of the neutrosophic binary topological space $(X, Y, \mathcal{M}_N) = \{(0_X, 0_Y), (1_X, 1_Y), (A_1, A_2), (B_1, B_2), (C_1, C_2), (D_1, D_2)\}$. Here

$$\begin{aligned}
(A_1, A_2) = \{ &< E_1, (0.4, 0.5, 0.2), (0.3, 0.5, 0.1), (0.9, 0.6, 0.8) >,\\
&< E_2, (0.2, 0.5, 0.5), (0.1, 0.5, 0.2), (0.6, 0.5, 0.8) >\}\\
(B_1, B_2) = \{ &< E_1, (0.5, 0.6, 0.2), (0.4, 0.5, 0.1), (0.7, 0.6, 0.7) >,\\
&< E_2, (0.3, 0.5, 0.4), (0.3, 0.5, 0.1), (0.7, 0.5, 0.6) >\}\\
(C_1, C_2) = \{ &< E_1, (0.5, 0.5, 0.2), (0.4, 0.5, 0.1), (0.9, 0.6, 0.7) >,\\
&< E_2, (0.3, 0.5, 0.4), (0.3, 0.5, 0.1), (0.7, 0.5, 0.6) >\}\\
(D_1, D_2) = \{ &< E_1, (0.4, 0.6, 0.2), (0.3, 0.5, 0.1), (0.7, 0.6, 0.8) >,\\
&< E_2, (0.2, 0.5, 0.5), (0.1, 0.5, 0.2), (0.6, 0.5, 0.8) >\}
\end{aligned}$$

$$\begin{aligned}
(A, B) = \{ &< E_1, (0.4, 0.5, 0.2), (0.3, 0.5, 0.1), (0.9, 0.4, 0.8) >,\\
&< E_2, (0.2, 0.5, 0.5), (0.1, 0.5, 0.2), (0.6, 0.5, 0.8) >\}
\end{aligned}$$

be the $N_b\alpha gs$-closed sets in $(X, Y, \mathcal{M}_N)$. Here,

$$\begin{aligned}
N_b\alpha gs - cl(A, B) = \{ &< E_1, (0.4, 0.5, 0.2), (0.3, 0.5, 0.1), (0.9, 0.4, 0.8) >,\\
&< E_2, (0.2, 0.5, 0.5), (0.1, 0.5, 0.2), (0.6, 0.5, 0.8) >\}
\end{aligned}$$

and

$$\begin{aligned}
N_bcl(A, B) = \{ &< E_1, (0.5, 0.5, 0.2), (0.4, 0.5, 0.1), (0.7, 0.4, 0.7) >,\\
&< (0.3, 0.5, 0.4), (0.3, 0.5, 0.1), (0.7, 0.5, 0.6) >\}
\end{aligned}$$

Clearly, $N_b\text{-}cl(A, B) \nsubseteq N_b\alpha gs\text{-}cl(A, B)$.

Remark 5.5. Let $(X, Y, \mathcal{M}_N)$ be a neutrosophic binary topological space. The subset (A, B) is said to be neutrosophic binary αgs-closed if and only if $(A, B) = N_b\alpha gs\text{-}cl(A, B)$.

Proposition 5.6. *Suppose $(A, B) \subseteq (C, D) \subseteq (X, Y)$ and $(X, Y, \mathcal{M}_N)$ is a neutrosophic binary topological space. Then,*

(i) $N_b\alpha gs\text{-}cl(0_X, 0_Y) = (0_X, 0_Y)$
$N_b\alpha gs\text{-}cl(1_X, 1_Y) = (1_X, 1_Y)$

(ii) $(A, B) \subseteq N_b\alpha gs\text{-}cl(A, B)$

(iii) $(A, B)^{1_N^\star}_{\alpha gs} \subseteq (C, D)^{1_N^\star}_{\alpha gs}$

(iv) $(A, B)^{2_N^\star}_{\alpha gs} \subseteq (C, D)^{2_N^\star}_{\alpha gs}$

(v) $N_b\alpha gs\text{-}cl(A, B) \subseteq N_b\alpha gs\text{-}cl(C, D)$

(vi) $N_b\alpha gs\text{-}cl(N_b\alpha gs\text{-}cl(A, B)) = N_b\alpha gs\text{-}cl(A, B)$

Proof. (i) and (ii) are obvious.

$$\begin{aligned}
(iii)\ (A, B)^{1_N^\star}_{\alpha gs} &= \cap\{A_\alpha : (A_\alpha, B_\alpha) \text{ is neutrosophic binary} \alpha gs\text{-closed and } (A, B) \subseteq (A_\alpha, B_\alpha)\}\\
&\subseteq \cap\{B_\alpha : (A_\alpha, B_\alpha) \text{ is neutrosophic binary} \alpha gs\text{-closed and } (C, D) \subseteq (A_\alpha, B_\alpha)\}\\
&= (C, D)^{1_N^\star}_{\alpha gs}.
\end{aligned}$$

Similarly, (iv) also holds.

$$\begin{aligned}
(v)\ N_b\alpha gs - cl(A, B) &= ((A, B)^{1_N^\star}_{\alpha gs}, (A, B)^{2_N^\star}_{\alpha gs})\\
&\subseteq ((C, D)^{1_N^\star}_{\alpha gs}, (C, D)^{2_N^\star}_{\alpha gs})\\
&= N_b\alpha gs - cl(C, D).
\end{aligned}$$

(vi) It follows from the remark 5.5. □

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6 Conclusion

Neutrosophic Binary αgs neighborhood points and neutrosphic binary αgs interior and closure operators were introduced in this paper. The properties are thoroughly analyzed and verified using the examples. Thus the concept of neutrosophic αgs neighborhood points and neutrosphic binary αgs interior and closure operators has been formulated. In future, more operators like neutrosphic binary αgs border, neutrosphic binary αgs frontier and neutrosphic binary αgs exterior will be defined.

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