On The Neutrosophic Formula of Some Matrix Equations Derived from Data Mining Theory and Control Systems

S. Obaid Jameel[1], A. Mahdi Salih[2], R. Adnan Jaleel[3], Musaddak M. A. Zahra[4]

[1]University of Wasit, College of Education for Pure Sciences, Mathematics department, Wasit, Iraq
[2]Wasit University, College of Administration and Economics, Statistics Department, Wasit, Iraq
[3]Al-Nahrain University, Department of Information and Communication Engineering, Baghdad, Iraq,
[4]Al-Mustaqbal University College, Department of Computer Technology Engineering, Babylon, Iraq
[4]Unvirsity of Babylon, Department of Electrical Engineering, Babylon, Iraq

Emails: sjameel@uowasit.edu.iq; amahdi@uowasit.edu.iq; *iraq_it_2010@yahoo.com;* musaddaqmahir@mustaqbal-college.edu.iq

Abstract

This paper is dedicated to studying the neutrosophic formula of some famous matrix equations used in theoretical data mining algorithms and control systems by using neutrosophic matrices and refined neutrosophic matrices over neutrosophic real fields. On the other hand, we concentrate on the neutrosophic formula of the Sylvester equation, and Lyapunov equation, where we study their formulas and properties in terms of theorems in the neutrosophic real number field and refined real number field. Also, we illustrate many different examples to clarify the validity of our work.

Keywords: Data mining; Control System; Matrix equation; Neutrosophic matrix; Refined neutrosophic matrix.

Introduction

In the theory of data mining algorithms and control systems, we find many essential matrix equations that describe the state of a system in real-life action[1,2]. For example, three famous matrix equations are used in these two research fields Ricatti, Sylvester, and Lyapunov equations [11,12,13].

In the literature, neutrosophic matrices and refined neutrosophic matrices were defined as generalizations of classical and fuzzy matrices [3]. Where these matrices were built by using the Smarandache idea about indeterminacy I. Many results about neutrosophic matrices and their related spaces, representations, and computations can be found in [4-10].

The motivation of this work is to find a novel application of neutrosophic algebraic structures in applied mathematics fields such as control systems theory and data mining algorithms. We will present a discussion of the neutrosophic formula and refined neutrosophic formula of some special and famous matrix equations.

Doi: https://doi.org/10.54216/IJNS.190122
Received: March 04, 2022 Accepted: September 16, 2022

Main Discussion

Definition (Sylvester Matrix Equation):

It is defined as follows:

$AX + XB = W;\ A \in R^{n\times n}, B \in R^{m\times m}, W \in R^{n\times m}$.

Definition (Lyapunov Matrix equation):

$AX + XA^T = W;\ A \in R^{n\times n}, W = W^T \in R^{n\times n}$.

Definition (Sylvester neutrosophic matrix equation):

We define it as follows:

$(A_0 + A_1I)(X_0 + X_1I) + (X_0 + X_1I)(B_0 + B_1I) = W_0 + W_1I;\ A_i \in R^{n\times n},\ B_i \in R^{m\times m}, W_i \in R^{n\times m}$, and X=$X_0 + X_1I$.

Example:

Let $A_0 = \begin{pmatrix} 1 & 6 \\ 0 & -2 \end{pmatrix}, A_1 = \begin{pmatrix} 3 & 3 \\ 11 & 4 \end{pmatrix}, B_0 = \begin{pmatrix} 1 & 2 & -5 \\ 7 & 7 & 0 \\ 67 & 1 & 0 \end{pmatrix}, B_1 = \begin{pmatrix} 2 & 2 & 1 \\ 13 & 2 & 13 \\ 16 & -4 & -6 \end{pmatrix}, W_0 = \begin{pmatrix} 2 & 1 & 1 \\ 9 & 7 & 5 \end{pmatrix}, W_1 = \begin{pmatrix} -2 & 0 & 8 \\ 1 & 1 & 3 \end{pmatrix}$. We get:

$$\begin{pmatrix} 1+3I & 6+3I \\ 11I & -2+4I \end{pmatrix}X + X\begin{pmatrix} 1+2I & 2+2I & -5+I \\ 7+13I & 7+2I & 13I \\ 67+16I & 1-4I & -6I \end{pmatrix} = \begin{pmatrix} 2-2I & 1 & 1+8I \\ 9+I & 7+I & 5+3I \end{pmatrix}.$$

Definition (Sylvester refined neutrosophic matrix equation):

We define it as follows:

$(A_0 + A_1I_1 + A_2I_2)(X_0 + X_1I_1 + X_2I_2) + (X_0 + X_1I_1 + X_2I_2)(B_0 + B_1I_1 + B_2I_2) = W_0 + W_1I_1 + W_2I_2; A_i \in R^{n\times n}, B_i \in R^{m\times m}, W_i \in R^{n\times m}$.

Example:

Let $A_0 = \begin{pmatrix} 1 & 6 \\ 0 & -2 \end{pmatrix}, A_1 = \begin{pmatrix} 3 & 3 \\ 11 & 4 \end{pmatrix}, A_2 = \begin{pmatrix} 0 & 1 \\ 22 & -3 \end{pmatrix}, B_0 = \begin{pmatrix} 1 & 2 & -5 \\ 7 & 7 & 0 \\ 67 & 1 & 0 \end{pmatrix}, B_1 = \begin{pmatrix} 2 & 2 & 1 \\ 13 & 2 & 13 \\ 16 & -4 & -6 \end{pmatrix}, B_2 = \begin{pmatrix} 1 & 0 & 2 \\ -4 & -1 & 1 \\ 5 & 0 & 0 \end{pmatrix}, W_0 = \begin{pmatrix} 2 & 1 & 1 \\ 9 & 7 & 5 \end{pmatrix}, W_1 = \begin{pmatrix} -2 & 0 & 8 \\ 1 & 1 & 3 \end{pmatrix}, W_2 = \begin{pmatrix} 1 & 0 & 1 \\ -5 & 3 & -8 \end{pmatrix}$. We get:

$$\begin{pmatrix} 1+3I_1 & 6+3I_1+I_2 \\ 11I_1+22I_2 & -2+4I_1-3I_2 \end{pmatrix}X + X\begin{pmatrix} 1+2I_1+I_2 & 2+2I_1 & -5+I_1+2I_2 \\ 7+13I_1-4I_2 & 7+2I_1-I_2 & 13I_1+I_2 \\ 67+16I_1+5I_2 & 1-4I_1 & -6I_1 \end{pmatrix} = \begin{pmatrix} 2-2I_1+I_2 & 1 & 1+8I_1+I_2 \\ 9+I_1-5I_2 & 7+I_1+3I_2 & 5+3I_1-8I_2 \end{pmatrix}.$$

Theorem:

Let $(A_0 + A_1I)(X_0 + X_1I) + (X_0 + X_1I)(B_0 + B_1I) = W_0 + W_1I$ be the neutrosophic Sylvester matrix equation, then it is equivalent to the following two classical Sylvester equations:

(1)$A_0X_0 + X_0B_0 = W_0$.

(2)$(A_0 + A_1)(X_0 + X_1) + (X_0 + X_1)(B_0 + B_1) = W_0 + W_1$.

Proof:

By computing the value of the multiplication, we get:

$A_0X_0 + X_0B_0 + I(A_0X_1 + A_1X_0 + A_1X_1 + X_0B_1 + X_1B_0 + X_1B_1) = W_0 + W_1I$, thus

Doi: https://doi.org/10.54216/IJNS.190122
Received: March 04, 2022 Accepted: September 16, 2022

$A_0X_0 + X_0B_0 = W_0, and \left((A_0 + A_1)(X_0 + X_1)\right)\left((X_0 + X_1)(B_0 + B_1)\right) - (A_0X_0 + X_0B_0) = W_1$, so that $(A_0 + A_1)(X_0 + X_1) + (X_0 + X_1)(B_0 + B_1) = W_0 + W_1$.

Theorem:

Let $(A_0 + A_1I_1 + A_2I_2)(X_0 + X_1I_1 + X_2I_2) + (X_0 + X_1I_1 + X_2I_2)(B_0 + B_1I_1 + B_2I_2) = W_0 + W_1I_1 + W_2I_2;\ A_i \in R^{n\times n},\ B_i \in R^{m\times m}, W_i \in R^{n\times m}$ be the refined neutrosophic Sylvester matrix equation, then it is equivalent to the following classical system of three Sylvester equations:

(1)$A_0X_0 + X_0B_0 = W_0$

(2)$(A_0 + A_1 + A_2)(X_0 + X_1 + X_2) + (X_0 + X_1 + X_2)(B_0 + B_1 + B_2) = W_0 + W_1 + W_2$.

(3)$(A_0 + A_2)(X_0 + X_2) + (X_0 + X_2)(B_0 + B_2) = W_0 + W_2$.

Proof:

By computing the value of the multiplication, we get:

$A_0X_0 + X_0B_0 + I_1[A_0X_1 + A_1X_0 + A_1X_1 + A_1X_2 + X_0B_1 + X_1B_0 + X_1B_1 + X_1B_2 + A_2X_1 + X_2B_1] + I_2[A_0X_2 + A_2X_0 + A_2X_2 + X_0B_2 + X_2B_0 + X_2B_2] = W_0 + W_1I_1 + W_2I_2$.

The previous formula can be written as follows:

$A_0X_0 + X_0B_0 = W_0, (A_0 + A_1 + A_2)(X_0 + X_1 + X_2) + (X_0 + X_1 + X_2)(B_0 + B_1 + B_2) = W_0 + W_1 + W_2, (A_0 + A_2)(X_0 + X_2) + (X_0 + X_2)(B_0 + B_2) = W_0 + W_2$.

Example:

Consider the following neutrosophic Sylvester equation:

$$\begin{pmatrix} 1+3I_1 & 6+3I_1+I_2 \\ 11I_1+22I_2 & -2+4I_1-3I_2 \end{pmatrix}X + X\begin{pmatrix} 1+2I_1+I_2 & 2+2I_1 & -5+I_1+2I_2 \\ 7+13I_1-4I_2 & 7+2I_1-I_2 & 13I_1+I_2 \\ 67+16I_1+5I_2 & 1-4I_1 & -6I_1 \end{pmatrix} = \begin{pmatrix} 2-2I_1+I_2 & 1 & 1+8I_1+I_2 \\ 9+I_1-5I_2 & 7+I_1+3I_2 & 5+3I_1-8I_2 \end{pmatrix}$$

It can be written by classical description as follows:

$$\left(\begin{pmatrix} 1 & 6 \\ 0 & -2 \end{pmatrix}\right)(X_0) + (X_0)\begin{pmatrix} 1 & 2 & -5 \\ 7 & 7 & 0 \\ 67 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 \\ 9 & 7 & 5 \end{pmatrix}.$$

$$\begin{pmatrix} 4 & 9 \\ 11 & 2 \end{pmatrix}(X_0 + X_1) + (X_0 + X_1)\begin{pmatrix} 3 & 4 & 4 \\ 20 & 9 & 13 \\ 87 & -3 & -6 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 9 \\ 10 & 8 & 8 \end{pmatrix}.$$

Definition (LyapunovNeutrosophic matrix equation):

We define the neutrosophic version of the Lyapunov matrix equation as follows:

$(A_0 + A_1I)(X_0 + X_1I) + (X_0 + X_1I)({A_0}^T + {A_1}^T I) = W_0 + W_1I;\ A_i \in R^{n\times n}, W_i = W_i^T \in R^{n\times n}$.

Example :

$$A_0 = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 0 & 3 \\ 6 & -2 & 5 \end{pmatrix}, A_1 = \begin{pmatrix} 4 & -2 & -4 \\ 0 & 3 & 1 \\ 5 & 9 & 9 \end{pmatrix}, W_0 = \begin{pmatrix} 2 & 3 & 5 \\ 3 & 1 & 7 \\ 5 & 7 & 6 \end{pmatrix}, W_1 = \begin{pmatrix} 6 & -3 & 4 \\ -3 & 8 & 2 \\ 4 & 2 & 1 \end{pmatrix}.$$

Thus, the neutrosophic Lyapunov equation is:

$$\begin{pmatrix} 2+4I & 1-2I & 1-4I \\ 0 & 3I & 3+I \\ 6+5I & -2+9I & 5+9I \end{pmatrix}(X_0 + X_1I) + (X_0 + X_1I)\begin{pmatrix} 2+4I & 0 & 6+5I \\ 1-2I & 3I & -2+9I \\ 1-4I & 3+I & 5+9I \end{pmatrix} = \begin{pmatrix} 2+6I & 3-3I & 5+4I \\ 3-3I & 1+8I & 7+2I \\ 5+4I & 7+2I & 6+I \end{pmatrix}.$$

Doi: https://doi.org/10.54216/IJNS.190122
Received: March 04, 2022 Accepted: September 16, 2022

Definition (Lyapunov refined neutrosophic equation):

We define the refined neutrosophic version of the Lyapunov matrix equation as follows:

$(A_0 + A_1I_1 + A_2I_2)(X_0 + X_1I_1 + X_2I_2) + (X_0 + X_1I_1 + X_2I_2)(A_0^T + A_1^TI_1 + A_1^TI_2) = W_0 + W_1I_1 + W_2I_2; A_i \in R^{n\times n}, W_i = W_i^{\,T} \in R^{n\times n}$.

Example:

$A_0 = \begin{pmatrix} 2 & 1 & 4 \\ 3 & 3 & 2 \\ 0 & 6 & -1 \end{pmatrix}, A_1 = \begin{pmatrix} 1 & 0 & 7 \\ 7 & 2 & 5 \\ 8 & 8 & 2 \end{pmatrix}, A_2 = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 6 & 1 \\ 7 & 3 & -1 \end{pmatrix}, W_0 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & 3 & 2 \end{pmatrix}, W_1 = \begin{pmatrix} 2 & 5 & 2 \\ 5 & 1 & 4 \\ 2 & 4 & 1 \end{pmatrix}, W_2 = \begin{pmatrix} -4 & 3 & -4 \\ 3 & 1 & 1 \\ -4 & 1 & 0 \end{pmatrix}$. So, the corresponding refined neutrosophic Lyapunov equation is:

$$\begin{pmatrix} 2+I_1 & 1 & 4+7I_1 \\ 3+7I_1+I_2 & 3+2I_1+6I_2 & 2+5I_1+I_2 \\ 8I_1+7I_2 & 6+8I_1+2I_2 & -1+2I_1-I_2 \end{pmatrix}(X_0 + X_1I_1 + X_2I_2) + (X_0 + X_1I_1 + X_2I_2)\begin{pmatrix} 2+I_1 & 3+7I_1+I_2 & 8I_1+7I_2 \\ 1 & 3+2I_1+6I_2 & 6+8I_1+2I_2 \\ 4+7I_1 & 2+5I_1+I_2 & -1+2I_1-I_2 \end{pmatrix} = \begin{pmatrix} 1+2I_1-4I_2 & 1+5I_1+3I_2 & 1+2I_1-4I_2 \\ 1+5I_1+3I_2 & I_1+I_2 & 3+4I_1+I_2 \\ 1+2I_1-4I_2 & 3+4I_1+I_2 & 2+I_1 \end{pmatrix}.$$

Theorem:

Let $(A_0 + A_1I)(X_0 + X_1I) + (X_0 + X_1I)(A_0^{\,T} + A_1^{\,T}I) = W_0 + W_1I;\ A_i \in R^{n\times n}, W_i = W_i^{\,T} \in R^{n\times n}$ be the neutrosophic Lyapunov equation, hence it is equivalent to the following classical system of matrix equations:

(1) $A_0X_0 + X_0A_0^{\,T} = W_0$.

(2) $(A_0 + A_1)(X_0 + X_1) + (X_0 + X_1)\left(A_0^{\,T} + A_1^{\,T}\right) = W_0 + W_1$.

Theorem:

Let $(A_0 + A_1I_1 + A_2I_2)(X_0 + X_1I_1 + X_2I_2) + (X_0 + X_1I_1 + X_2I_2)(A_0^T + A_1^TI_1 + A_1^TI_2) = W_0 + W_1I_1 + W_2I_2;\ A_i \in R^{n\times n}, W_i = W_i^{\,T} \in R^{n\times n}$ be the refined neutrosophic Lyapunov equation, hence it is equivalent to:

(1)$A_0X_0 + X_0A_0^{\,T} = W_0$

(2)$(A_0 + A_1 + A_2)(X_0 + X_1 + X_2) + (X_0 + X_1 + X_2)\left(A_0^{\,T} + A_1^{\,T} + A_2^{\,T}\right) = W_0 + W_1 + W_2$.

(3)$(A_0 + A_2)(X_0 + X_2) + (X_0 + X_2)\left(A_0^{\,T} + A_2^{\,T}\right) = W_0 + W_2$

Example:

The neutrosophic Lyapunov equation

$$\begin{pmatrix} 2+4I & 1-2I & 1-4I \\ 0 & 3I & 3+I \\ 6+5I & -2+9I & 5+9I \end{pmatrix}(X_0 + X_1I) + (X_0 + X_1I)\begin{pmatrix} 2+4I & 0 & 6+5I \\ 1-2I & 3I & -2+9I \\ 1-4I & 3+I & 5+9I \end{pmatrix} = \begin{pmatrix} 2+6I & 3-3I & 5+4I \\ 3-3I & 1+8I & 7+2I \\ 5+4I & 7+2I & 6+I \end{pmatrix}$$ is equivalent to:

(1) $\begin{pmatrix} 2 & 1 & 1 \\ 0 & 0 & 3 \\ 6 & -2 & 5 \end{pmatrix}(X_0) + (X_0)\begin{pmatrix} 2 & 0 & 6 \\ 1 & 0 & -2 \\ 1 & 3 & 5 \end{pmatrix} = \begin{pmatrix} 2 & 3 & 5 \\ 3 & 1 & 7 \\ 5 & 7 & 6 \end{pmatrix}$.

(2) $\begin{pmatrix} 6 & -1 & -3 \\ 0 & 3 & 4 \\ 11 & 7 & 14 \end{pmatrix}(X_0 + X_1) + (X_0 + X_1)\begin{pmatrix} 6 & 0 & 11 \\ -1 & 3 & 7 \\ -3 & 4 & 14 \end{pmatrix} = \begin{pmatrix} 8 & 0 & 9 \\ 0 & 9 & 9 \\ 9 & 9 & 7 \end{pmatrix}$.

Doi: https://doi.org/10.54216/IJNS.190122
Received: March 04, 2022 Accepted: September 16, 2022

Conclusion

In this paper, we have presented the neutrosophic formula of some matrix equations derived from the theory of control systems, where we have discussed the Lyapunov, and Sylvester matrix equation in the neutrosophic real field and refined neutrosophic real field. On the other hand, we have illustrated many examples to clarify the validity of this work.

References

[1] R. Bartels and G. W. Stewart. Solution of the matrix equation AX + XB = C. Comm A.C.M., 15(9):820–826, 1972.

[2] L. Grubiši´c and D. Kressner. On the eigenvalue decay of solutions to operator Lyapunov equations. Syst. Control Lett., 73:42–47, 2014.

[3] Abobala, M., Bal, M., and Hatip, A., "A Review On Recent Advantages In Algebraic Theory Of Neutrosophic Matrices", International Journal of Neutrosophic Science, Vol. 17, 2021.

[4] Olgun, N., Hatip, A., Bal, M., and Abobala, M., " A Novel Approach To Necessary and Sufficient Conditions For The Diagonalization of Refined Neutrosophic Matrices", International Journal of Neutrosophic Science, Vol. 16, pp. 72-79, 2021.

[5] Abobala, M., Hatip, A., Olgun, N., Broumi, S., Salama, A, A., and Khaled, E, H., The algebraic creativity In The Neutrosophic Square Matrices, Neutrosophic Sets and Systems, Vol. 40, pp. 1-11, 2021.

[6] Abobala, M., On The Representation of Neutrosophic Matrices by Neutrosophic Linear Transformations, Journal of Mathematics, Hindawi, 2021.

[7] Prem Kumar Singh, A Note on Basic Proof of Some Famous Mathematical Theorem and Its Illustration, Journal of Neutrosophic and Fuzzy Systems, Vol. 3 , No. 1 , (2022) : 39-53

[8] Abobala, M., "On Some Algebraic Properties of n-Refined Neutrosophic Elements and n-Refined Neutrosophic Linear Equations", Mathematical Problems in Engineering, Hindawi, 2021

[9] E.O. Adeleke , A.A.A. Agboola , F. Smarandache, Refined Neutrosophic Rings II, International Journal of Neutrosophic Science, Vol. 2 , No. 2 , (2020) : 89-94.

[10] Mikail Bal , Katy D. Ahmad , Rozina Ali, A Review On Recent Developments In Neutrosophic Linear Diophantine Equations, Journal of Neutrosophic and Fuzzy Systems, Vol. 2 , No. 1 , (2022) : 61-75

[11] V. Simoncini. A new iterative method for solving large-scale Lyapunov matrix equations. SIAM J. Sci. Comput., 29(3):1268–1288, 2007.

[12] T. Stykel. Stability and inertia theorems for generalized Lyapunov equations. Linear Algebra Appl., 355:297–314, 2002.

[13] Smarandache, F., " *n*-Valued Refined Neutrosophic Logic and Its Applications in Physics", Progress in Physics, 143-146, Vol. 4, 2013.

Doi: https://doi.org/10.54216/IJNS.190122

Received: March 04, 2022 Accepted: September 16, 2022