



Time Q-Neutrosophic Soft Expert Set

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Abstract

This paper is a combination of time neutrosophic soft sets and Q- neutrosophic soft expert sets. We define the general concept of time Q-neutrosophic soft expert sets and study its properties. Some operations such as complement, union, and intersection are explored. Then a hypothetical application of this concept in the decision-making problem is given.

Keywords: Soft Set; Neutrosophic Set; Neutrosophic Soft Set; Neutrosophic Soft Expert Set; Time-Neutrosophic Soft Expert Set; Q-Neutrosophic Soft Expert Set; Time Q-Neutrosophic Soft Expert Set.

1. Introduction

Fuzzy sets were developed by Zadeh [1] to solve problems that contain uncertain information. Atanassov [2] extended the fuzzy set to an intuitionistic fuzzy set. Different problems in economics, engineering, and medical science are solved by the theory of soft set, which was introduced by Molodtsov [3]. Problems involving imprecise information were solved by the theory of neutrosophic set introduced by Smaradache [4].

Maji et al. [5] as a combination of fuzzy set and soft set have also introduced the concept of fuzzy soft set, a more general concept, which is and studied its properties. Where Zou and Xian [6] introduced respectively the soft set and fuzzy soft set into the incomplete environment. Soft multiset as a generalization of soft set introduced by Alkhazaleh et al. [7]. They also defined the concepts of fuzzy parameterized interval-valued fuzzy soft sets [8] and the possibility of fuzzy soft sets [9] and studied their applications in DM and MD problems. The concept of the Q-fuzzy soft set was developed by Adam and Hassan [10], they introduced the basic properties of this concept with illustrative examples.

The theory of the Q-fuzzy soft (Q-FSS) set was extended to the Q-neutrosophic soft set (Q-NSS) [11]. They studied relations between Q-NSSs. Abu Qamar and Hassan studied measures of distance, similarity, and entropy in [12].

Alkhazaleh and Salleh [13] designed a new model where the user can know the opinion of all experts in one model without any operations by introducing the concept of a soft expert set (SES). Even after any operation, the user can know the opinion of all experts.

The concept of a soft set was generalized by Alkazaleh and Salleh [14] to a fuzzy soft expert set (FSES) which was a combination of a fuzzy set and soft expert set. They studied the basic operations and gave an application of this concept in decision-making problems.

Maji [15] introduced a neutrosophic soft set (NSS), which was a combination between a neutrosophic set and a soft set. He also, introduced some definitions, operations, and properties of this concept.

Neutrosophic soft expert set (NSES) was defined by Şahin et.al.[16]. The concept of NSES developed to generalized neutrosophic soft expert set (GNSES) by Vakkas et. al. [17]. They discussed the basic operations of complement, union, intersection, AND, and OR of GNSES. They also applied this concept to decision-making problems.

Hassan et. al. [18] combined NSES and Q-fuzzy soft sets and introduced a new concept called Q-neutrosophic soft expert set (QNSES). This concept extended the single dimension of the soft expert set to two dimensions. The concept of generalized Q-neutrosophic soft expert set (GQNSES) was the development of QNSES by Abu Qamar et. al. [19]. They extended the novel concept to two dimensions. Also, they construct an algorithm based on this concept.

In 2013, Hazaymeh [20] considered a collection of time (periods) in his PhD thesis by generalizing the concept of fuzzy soft set into the time-fuzzy soft set (TFSS) and studied some of its properties, and explained this concept in the decision-making problem. Alkhazaleh introduced the concept of time neutrosophic soft set (TNSS) and studied its operations namely: complement, union, and intersection. He gave an application to solve decision-making problems in [21]. Ulucay et.al. introduced a time-neutrosophic soft expert set (TNSES) with basic operation and suitable examples. Also, they gave an application in a decision-making problem of this concept in [22]. In 2018, Smarandache [23] extended the soft set to the hypersoft set (HSS) by replacing a single attribute-valued function with a multi-attribute-valued function. In 2020, Saeed et al. [24,25] generalized this concept and studied the fundamentals of the hypersoft set.

In 2021, Debnath [26] introduced the concept of fuzzy hypersoft set (FHSS), which was more flexible. FHSS was a combination between fuzzy set and hypersoft set. Yolcu et al. [27] discussed some properties such as complement, union, intersection, etc. of FHSS. Also, they gave an application of FHSS in object recognition problems. The concept of FSES was generalized to the Fuzzy hypersoft expert set (FHSES) by Ihsan et al. [28]. They discussed some properties, fundamental results, and set-theoretic operations. Also, they gave an algorithm to solve decision-making problems and applied it to the best product. In 2022, Ihsan et al. [29] defined the concept of a neutrosophic hypersoft expert set (NHSES) and its application in decision-making problems. This model solved the problems with different experts which of different parametric valued sets parallel to different characteristics.

In this paper, the concept of QNSES extends to the time Q-neutrosophic soft expert set (T-QNSES) by redefining a time on QNSES. We also discuss the properties of a new concept. An application of this concept applies to decision-making problems

2. Preliminary

Necessary definitions are presented in this section such as:

neutrosophic set, soft set, soft neutrosophic set, soft expert set, fuzzy soft expert set, neutrosophic soft expert set, time neutrosophic soft set, Q-neutrosophic soft expert set theory, and time Q-neutrosophic soft expert set.

Definition 2.1. [3] Let U be a universe and E be a set of parameters. Let $P(U)$ denote the power set of U and $A \subseteq E$. A pair (F, A) is called a soft set over U , where F is a mapping $F: A \rightarrow P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U . For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε -approximate elements of the soft set (F, A) .

Definition 2.2. [15] A neutrosophic set A on the universe of discourse X is defined as $A = \{ \langle x; T_A(x), I_A(x), F_A(x) \rangle; x \in X \}$ where $T; I; F: X \rightarrow [-0, 1^+]$ and $-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

Definition 2.3. [3] Let U be an initial universe set and E be a set of parameters. Consider $A \subseteq E$. Let $N(U)$ denotes the set of all neutrosophic sets of U . The collection (F, A) is termed to be the neutrosophic soft set over U , where F is a mapping given by $F: A \rightarrow P(U)$.

Definition 2.4. [19] Let U be an initial universal set and let E be a set of parameters. Let I^U denote the power set of all fuzzy subsets of U , and let $A \subseteq E$ and T be a set of times where $T = \{t_1, t_2, \dots, t_n\}$. A collection of pairs $(F, E)t, \forall t \in T$ is called a time-fuzzy soft set (TFSS) over U where F is a mapping given by $F_t: A \rightarrow I^U$.

Alkhazaleh et al. defined the soft expert set in the following way. Let U be a universe, E a set of parameters, X a set of experts (agents), and $O = \{1 = agree, 0 = disagree\}$ a set of opinions. Let $Z = E \times X \times O$ and $A \subseteq Z$.

Definition 2.5. [13] A pair (F, A) is called a soft expert set over U , where F is a mapping given by $F: A \rightarrow P(U)$ where $P(U)$ denotes the set of U .

Definition 2.6. [14] A pair (F, A) is called a fuzzy soft expert set over U , where F is a mapping given by $F: A \rightarrow I^U$ where I^U denotes the set of all fuzzy subsets of U

Definition 2.7. [15] A pair (F, A) is called a neutrosophic soft expert set over U , where F is a mapping given by $F: A \rightarrow N(U)$ where $N(U)$ denotes the set of all neutrosophic subsets of U . Alkhazaleh and Salleh (2014) gave the following definition on power set followed by Sahin etl al. (2015) on the power of the neutrosophic set.

Definition 2.8. [17] Let U be an initial universal set and let E be a set of parameters, Q be a nonempty set, X is the set of experts, $O = \{agree = 1, disagree = 0\}$ a set of opinions. Let $N(U \times Q)$ denote the power set of all neutrosophic subsets of U , and let $A \subseteq Z = E \times X \times O$. A collection of pairs (F_Q, A) is called Q neutrosophic soft Expert set over $U \times Q$ where F_Q is a mapping given by $F_Q: A \rightarrow N(U \times Q)$ such that QNSES is the set of all QNSES over $U \times Q$.

In the following, we define the basic operations of QNSES introduced in [17].

Definition 2.9. [17] For two QNSES (F_Q, A) and (G_Q, B) over $U \times Q$, (G_Q, B) is called a Q-a neutrosophic soft expert subset of (F_Q, A) if

1. $B \subseteq A$
2. $G_Q(\varepsilon)$ is Q-neutrosophic soft subset $F_Q(\varepsilon)$ for all $\varepsilon \in B$.

Definition 2.10. [17] Two QNSES (F_Q, A) and (G_Q, B) over $U \times Q$ are equal if (F_Q, A) is a QNSES subset of (G_Q, B) , and (G_Q, B) is a QNSES subset of (F_Q, A) .

Definition 2.11. [17] Agree on QNSES, $(F_Q, A)_1$ over U is a subset of (F_Q, A) , defined as $(F_Q, A)_1 = \{F_{Q1}(\alpha) : \alpha \in E \times X \times \{1\}\}$.

Definition 2.12. [17] A disagree QNSES, $(F_Q, A)_0$ over U is a subset of (F_Q, A) , defined as $(F_Q, A)_0 = \{F_{Q0}(\alpha) : \alpha \in E \times X \times \{0\}\}$.

Definition 2.13. [17] The complement of a QNSES (F_Q, A) is defined as $(F_Q, A)^c = (F_Q^c, A)$, where $F_Q^c: A \rightarrow P(X \times Q)$ and $F_Q^c(x) = \{T_{F_Q^c(x)} = F_{F_Q(x)}, I_{F_Q^c(x)} = I_{F_Q(x)}, F_{F_Q^c(x)} = T_{F_Q(x)}; \forall x \in E\}$.

Definition 2.14. [17] The union of two QNSESs (F_Q, A) and (G_Q, B) over the common universe U . denoted by $(F_Q, A) \tilde{\cup} (G_Q, B)$ is the QNSES and is defined by $(F_Q, A) \tilde{\cup} (G_Q, B) = (K_Q, C)$, where $C = A \cup B$ and the truth membership, indeterminacy-membership and falsity-membership of (K_Q, C) are as follows:

$$T_{K_Q(e)}(m) = \{T_{F_Q(e)}(m) \quad \text{if } e \in$$

$$\begin{aligned}
 &A - B \quad T_{G_Q(e)}(m) && \text{if } e \in B - A \max(T_{F_Q(e)}(m), T_{G_Q(e)}(m)) && \text{if } e \in \\
 &A \cap B && && \\
 &I_{K_Q(e)}(m) = \begin{cases} I_{F_Q(e)}(m) & \text{if } e \\ \in A - B I_{G_Q(e)}(m) & \text{if } e \\ \in B - A \min(I_{F_Q(e)}(m), I_{G_Q(e)}(m)) & \text{if } e \in A \cap B \end{cases} \\
 &F_{K_Q(e)}(m) = \begin{cases} F_{F_Q(e)}(m) & \text{if } e \\ \in A - B F_{G_Q(e)}(m) & \text{if } e \\ \in B - A \min(T_{F_Q(e)}(m), T_{G_Q(e)}(m)) & \text{if } e \in A \cap B \end{cases}
 \end{aligned}$$

Definition 2.15. [17] The intersection of two QNSEs (F_Q, A) and (G_Q, B) over the common universe U . denoted by $(F_Q, A) \tilde{\cap} (G_Q, B)$ is the QNSES and is defined by $(F_Q, A) \tilde{\cap} (G_Q, B) = (K_Q, C)$, where $C = A \cap B$ and the truth membership, indeterminacy-membership and falsity-membership of (K_Q, C) are as follows:

$$\begin{aligned}
 &T_{K_Q(e)}(m) = \begin{cases} T_{F_Q(e)}(m) & \text{if } e \in \\ A - B T_{G_Q(e)}(m) & \text{if } e \in B - A \min(T_{F_Q(e)}(m), T_{G_Q(e)}(m)) \\ A \cap B & \end{cases} && \text{if } e \in \\
 &I_{K_Q(e)}(m) = \begin{cases} I_{F_Q(e)}(m) & \text{if } e \\ \in A - B I_{G_Q(e)}(m) & \text{if } e \\ \in B - A \min(I_{F_Q(e)}(m), I_{G_Q(e)}(m)) & \text{if } e \in A \cap B \end{cases} \\
 &F_{K_Q(e)}(m) = \begin{cases} F_{F_Q(e)}(m) & \text{if } e \in \\ A - B F_{G_Q(e)}(m) & \text{if } e \in B - A \max(T_{F_Q(e)}(m), T_{G_Q(e)}(m)) \\ A \cap B & \end{cases} && \text{if } e \in
 \end{aligned}$$

Definition 2.16. [21] A pair $(F, A)_t$ is called a time- neutrosophic soft expert set over U , where F is a mapping and $N(U)$ is the set of all neutrosophic soft expert subsets of U . Let $A \subseteq Z$ and T be a set of time where $T = \{t_1, t_2, t_3, \dots, t_n\}$. Then F_t^μ is called a time-neutrosophic soft expert set and defined as follows:

$$F_t^\mu = \{F(e), \mu(e) : e \in A, F(e) \in N(U), \mu(e) \in [0,1] \}$$

where F is a mapping given by $F: A \rightarrow N(U)$ and μ is a fuzzy set such that $\mu: A \rightarrow I = [0,1]$. Here F_t^μ is a mapping defined by $F_t^\mu: A \rightarrow N(U)$. For any parameter $e \in A$, $F(e)$ is referred to as the neutrosophic value set of parameter e , i.e,

$$F_t^\mu = \left\{ \left(\frac{u_1^t}{F(e)(u_1)}, \frac{u_2^t}{F(e)(u_2)}, \frac{u_3^t}{F(e)(u_3)}, \dots, \frac{u_n^t}{F(e)(u_n)} \right), \mu(e) \right\},$$

where $T, I, F: U \rightarrow [0,1]$ are the membership function of truth, indeterminacy, and falsity of the element $u \in U$ respectively. For any $u \in U$ and $e \in A$

$$0 \leq T_A(u) + I_A(u) + F_A(u) \leq 3$$

In fact, F_t^μ is a parameterized family of neutrosophic soft expert sets on U , which has the degree of the possibility of the approximate value set which is represented by $\mu(e)$ for each parameter e .

Note: This is a definition of generalized time neutrosophic soft expert set not a definition of time neutrosophic soft expert set. So, in this paper we will use our definition which defined as follows:

$$F_t = \left\{ \left(\frac{u_1^t}{F(e)(u_1)}, \frac{u_2^t}{F(e)(u_2)}, \frac{u_3^t}{F(e)(u_3)}, \dots, \frac{u_n^t}{F(e)(u_n)} \right) \right\}$$

3. Time-Q-Neutrosophic Soft Expert Set (T-QNSEs)

Definition 3.1. Let U be an initial universal set and let E be a set of parameters, Q be a nonempty set, X is the set of experts, $O = \{agree = 1, disagree = 0\}$ a set of opinions. Let $N(U \times Q)$ denote the power set of all neutrosophic subsets of $U \times Q$, let $A \subseteq Z = E \times X \times O$ and T be a set of time where $T = \{t_1, t_2, \dots, t_n\}$. A collection of pairs $(F_Q, E)_t \forall t \in T$ is called a time-Q neutrosophic soft expert set (T-QNSEs) over $U \times Q$ where F_Q^t is a mapping given by $F_Q^t: A \rightarrow N(U \times Q)$. subsets of $U \times Q$.

Example 3.1. Let $U = \{u_1, u_2, u_3\}$ be a set of universe, $Q = \{s_1, s_2\}$ be a set of supply, $E = \{e_1, e_2, e_3\}$ a set of parameters and $T = \{t_1, t_2, t_3\}$ be a set of time, $X = \{p, q, r\}$ is the set of expert, $O = \{agree = 1, disagree = 0\}$ a set of opinion. Define a function

$F_Q^t: A \rightarrow N(U \times Q)$ as follows:

$$(F_Q, A)_t =$$

$$\langle (e_1, p, 1), \left(\frac{(u_1, s_1)^{t_1}}{(0.1, 0.1, 0.3)}, \frac{(u_2, s_1)^{t_1}}{(0.9, 0.7, 0.2)}, \frac{(u_3, s_1)^{t_1}}{(0.1, 0.2, 0.9)}, \frac{(u_1, s_2)^{t_1}}{(0.0, 0.1, 0.4)}, \frac{(u_2, s_2)^{t_1}}{(0.0, 0.7, 0.8)}, \frac{(u_3, s_2)^{t_1}}{(0.4, 0.2, 0.8)} \right) \rangle,$$

$$\langle (e_1, r, 1), \left(\frac{(u_1, s_1)^{t_1}}{(0.4, 0.0, 0.1)}, \frac{(u_2, s_1)^{t_1}}{(0.2, 0.4, 0.6)}, \frac{(u_3, s_1)^{t_1}}{(0.9, 0.2, 0.4)}, \frac{(u_1, s_2)^{t_1}}{(0.6, 0.4, 0.5)}, \frac{(u_2, s_2)^{t_1}}{(0.2, 0.4, 0.2)}, \frac{(u_3, s_2)^{t_1}}{(0.9, 0.0, 0.5)} \right) \rangle,$$

$$\langle (e_1, q, 1), \left(\frac{(u_1, s_1)^{t_1}}{(0.8, 0.7, 0.5)}, \frac{(u_2, s_1)^{t_1}}{(0.7, 0.6, 0.7)}, \frac{(u_3, s_1)^{t_1}}{(0.0, 0.2, 0.9)}, \frac{(u_1, s_2)^{t_1}}{(0.3, 0.6, 0.4)}, \frac{(u_2, s_2)^{t_1}}{(0.5, 0.1, 0.9)}, \frac{(u_3, s_2)^{t_1}}{(0.3, 0.5, 0.0)} \right) \rangle,$$

$$\langle (e_2, p, 1), \left(\frac{(u_1, s_1)^{t_1}}{(0.5, 0.5, 0.3)}, \frac{(u_2, s_1)^{t_1}}{(0.1, 0.9, 0.7)}, \frac{(u_3, s_1)^{t_1}}{(0.7, 0.5, 0.6)}, \frac{(u_1, s_2)^{t_1}}{(0.5, 0.1, 0.9)}, \frac{(u_2, s_2)^{t_1}}{(0.1, 0.3, 0.8)}, \frac{(u_3, s_2)^{t_1}}{(0.4, 0.8, 0.9)} \right) \rangle,$$

$$\langle (e_2, r, 1), \left(\frac{(u_1, s_1)^{t_1}}{(0.2, 0.4, 0.5)}, \frac{(u_2, s_1)^{t_1}}{(0.6, 0.1, 0.0)}, \frac{(u_3, s_1)^{t_1}}{(0.1, 0.0, 0.3)}, \frac{(u_1, s_2)^{t_1}}{(0.0, 0.6, 0.4)}, \frac{(u_2, s_2)^{t_1}}{(0.7, 0.5, 0.0)}, \frac{(u_3, s_2)^{t_1}}{(0.9, 0.4, 0.0)} \right) \rangle,$$

$$\langle (e_2, q, 1), \left(\frac{(u_1, s_1)^{t_1}}{(0.8, 0.9, 0.4)}, \frac{(u_2, s_1)^{t_1}}{(0.8, 0.0, 0.0)}, \frac{(u_3, s_1)^{t_1}}{(0.1, 0.2, 0.6)}, \frac{(u_1, s_2)^{t_1}}{(0.2, 0.2, 0.3)}, \frac{(u_2, s_2)^{t_1}}{(0.8, 0.9, 0.2)}, \frac{(u_3, s_2)^{t_1}}{(0.5, 0.9, 0.4)} \right) \rangle,$$

$$\langle (e_3, p, 1), \left(\frac{(u_1, s_1)^{t_1}}{(0.5, 0.2, 0.2)}, \frac{(u_2, s_1)^{t_1}}{(0.3, 0.8, 0.7)}, \frac{(u_3, s_1)^{t_1}}{(0.5, 0.7, 0.4)}, \frac{(u_1, s_2)^{t_1}}{(0.8, 0.6, 0.8)}, \frac{(u_2, s_2)^{t_1}}{(0.7, 0.0, 0.0)}, \frac{(u_3, s_2)^{t_1}}{(0.5, 0.2, 0.3)} \right) \rangle,$$

$$\langle (e_3, r, 1), \left(\frac{(u_1, s_1)^{t_1}}{(0.6, 0.5, 0.2)}, \frac{(u_2, s_1)^{t_1}}{(0.0, 0.6, 0.7)}, \frac{(u_3, s_1)^{t_1}}{(0.9, 0.6, 0.0)}, \frac{(u_1, s_2)^{t_1}}{(0.0, 0.5, 0.8)}, \frac{(u_2, s_2)^{t_1}}{(0.1, 0.2, 0.5)}, \frac{(u_3, s_2)^{t_1}}{(0.3, 0.4, 0.2)} \right) \rangle,$$

$$\langle (e_3, q, 1), \left(\frac{(u_1, s_1)^{t_1}}{(0.0, 0.6, 0.0)}, \frac{(u_2, s_1)^{t_1}}{(0.5, 0.0, 0.1)}, \frac{(u_3, s_1)^{t_1}}{(0.4, 0.0, 0.7)}, \frac{(u_1, s_2)^{t_1}}{(0.8, 0.1, 0.5)}, \frac{(u_2, s_2)^{t_1}}{(0.9, 0.3, 0.8)}, \frac{(u_3, s_2)^{t_1}}{(0.0, 0.0, 0.3)} \right) \rangle,$$

$$\langle (e_1, p, 0), \left(\frac{(u_1, s_1)^{t_1}}{(0.2, 0.5, 0.3)}, \frac{(u_2, s_1)^{t_1}}{(0.8, 0.8, 0.2)}, \frac{(u_3, s_1)^{t_1}}{(0.8, 0.2, 0.4)}, \frac{(u_1, s_2)^{t_1}}{(0.1, 0.9, 0.9)}, \frac{(u_2, s_2)^{t_1}}{(0.2, 0.4, 0.3)}, \frac{(u_3, s_2)^{t_1}}{(0.6, 0.7, 0.0)} \right) \rangle,$$

$$\langle (e_1, r, 0), \left(\frac{(u_1, s_1)^{t_1}}{(0.5, 0.9, 0.1)}, \frac{(u_2, s_1)^{t_1}}{(0.3, 0.4, 0.1)}, \frac{(u_3, s_1)^{t_1}}{(0.1, 0.7, 0.2)}, \frac{(u_1, s_2)^{t_1}}{(0.1, 0.0, 0.9)}, \frac{(u_2, s_2)^{t_1}}{(0.9, 0.6, 0.1)}, \frac{(u_3, s_2)^{t_1}}{(0.1, 0.7, 0.4)} \right) \rangle,$$

$$\langle (e_1, q, 0), \left(\frac{(u_1, s_1)^{t_1}}{(0.4, 0.7, 0.5)}, \frac{(u_2, s_1)^{t_1}}{(0.7, 0.5, 0.5)}, \frac{(u_3, s_1)^{t_1}}{(0.1, 0.5, 0.8)}, \frac{(u_1, s_2)^{t_1}}{(0.8, 0.3, 0.1)}, \frac{(u_2, s_2)^{t_1}}{(0.4, 0.3, 0.8)}, \frac{(u_3, s_2)^{t_1}}{(0.6, 0.4, 0.0)} \right) \rangle,$$

$$\langle (e_2, p, 0), \left(\frac{(u_1, s_1)^{t_1}}{(0.6, 0.2, 0.4)}, \frac{(u_2, s_1)^{t_1}}{(0.4, 0.1, 0.3)}, \frac{(u_3, s_1)^{t_1}}{(0.4, 0.8, 0.7)}, \frac{(u_1, s_2)^{t_1}}{(0.2, 0.8, 0.9)}, \frac{(u_2, s_2)^{t_1}}{(0.9, 0.2, 0.9)}, \frac{(u_3, s_2)^{t_1}}{(0.0, 0.8, 0.5)} \right) \rangle,$$

$$\begin{aligned}
& \langle (e_2, r, 0), \left(\frac{(u_1, s_1)^{t_1}}{(0.3, 0.9, 0.4)}, \frac{(u_2, s_1)^{t_1}}{(0.4, 0.8, 0.5)}, \frac{(u_3, s_1)^{t_1}}{(0.7, 0.2, 0.7)}, \frac{(u_1, s_2)^{t_1}}{(0.8, 0.9, 0.9)}, \frac{(u_2, s_2)^{t_1}}{(0.4, 0.9, 0.8)}, \frac{(u_3, s_2)^{t_1}}{(0.3, 0.2, 0.1)} \right) \rangle, \\
& \langle (e_2, q, 0), \left(\frac{(u_1, s_1)^{t_1}}{(0.3, 0.6, 0.1)}, \frac{(u_2, s_1)^{t_1}}{(0.1, 0.3, 0.3)}, \frac{(u_3, s_1)^{t_1}}{(0.3, 0.0, 0.0)}, \frac{(u_1, s_2)^{t_1}}{(0.6, 0.6, 0.6)}, \frac{(u_2, s_2)^{t_1}}{(0.8, 0.8, 0.9)}, \frac{(u_3, s_2)^{t_1}}{(0.4, 0.0, 0.8)} \right) \rangle \\
& \langle (e_3, p, 0), \left(\frac{(u_1, s_1)^{t_1}}{(0.5, 0.3, 0.1)}, \frac{(u_2, s_1)^{t_1}}{(0.0, 0.3, 0.1)}, \frac{(u_3, s_1)^{t_1}}{(0.3, 0.9, 0.9)}, \frac{(u_1, s_2)^{t_1}}{(0.4, 0.3, 0.7)}, \frac{(u_2, s_2)^{t_1}}{(0.0, 0.2, 0.9)}, \frac{(u_3, s_2)^{t_1}}{(0.7, 0.7, 0.0)} \right) \rangle, \\
& \langle (e_3, r, 0), \left(\frac{(u_1, s_1)^{t_1}}{(0.7, 0.8, 0.0)}, \frac{(u_2, s_1)^{t_1}}{(0.8, 0.3, 0.2)}, \frac{(u_3, s_1)^{t_1}}{(0.1, 0.5, 0.3)}, \frac{(u_1, s_2)^{t_1}}{(0.3, 0.5, 0.3)}, \frac{(u_2, s_2)^{t_1}}{(0.1, 0.7, 0.0)}, \frac{(u_3, s_2)^{t_1}}{(0.2, 0.2, 0.7)} \right) \rangle, \\
& \langle (e_3, q, 0), \left(\frac{(u_1, s_1)^{t_1}}{(0.5, 0.0, 0.4)}, \frac{(u_2, s_1)^{t_1}}{(0.9, 0.5, 0.6)}, \frac{(u_3, s_1)^{t_1}}{(0.7, 0.2, 0.1)}, \frac{(u_1, s_2)^{t_1}}{(0.0, 0.6, 0.9)}, \frac{(u_2, s_2)^{t_1}}{(0.9, 0.8, 0.8)}, \frac{(u_3, s_2)^{t_1}}{(0.4, 0.3, 0.6)} \right) \rangle, \\
& \langle (e_1, p, 1), \left(\frac{(u_1, s_1)^{t_2}}{(0.8, 0.7, 0.5)}, \frac{(u_2, s_1)^{t_2}}{(0.7, 0.1, 0.7)}, \frac{(u_3, s_1)^{t_2}}{(0.3, 0.3, 0.1)}, \frac{(u_1, s_2)^{t_2}}{(0.2, 0.4, 0.0)}, \frac{(u_2, s_2)^{t_2}}{(0.7, 0.1, 0.2)}, \frac{(u_3, s_2)^{t_2}}{(0.8, 0.6, 0.1)} \right) \rangle, \\
& \langle (e_1, r, 1), \left(\frac{(u_1, s_1)^{t_2}}{(0.1, 0.6, 0.0)}, \frac{(u_2, s_1)^{t_2}}{(0.4, 0.5, 0.8)}, \frac{(u_3, s_1)^{t_2}}{(0.2, 0.5, 0.9)}, \frac{(u_1, s_2)^{t_2}}{(0.7, 0.5, 0.2)}, \frac{(u_2, s_2)^{t_2}}{(0.8, 0.5, 0.8)}, \frac{(u_3, s_2)^{t_2}}{(0.6, 0.6, 0.5)} \right) \rangle, \\
& \langle (e_1, q, 1), \left(\frac{(u_1, s_1)^{t_2}}{(0.2, 0.3, 0.6)}, \frac{(u_2, s_1)^{t_2}}{(0.4, 0.5, 0.2)}, \frac{(u_3, s_1)^{t_2}}{(0.5, 0.3, 0.7)}, \frac{(u_1, s_2)^{t_2}}{(0.6, 0.0, 0.6)}, \frac{(u_2, s_2)^{t_2}}{(0.4, 0.2, 0.2)}, \frac{(u_3, s_2)^{t_2}}{(0.7, 0.5, 0.9)} \right) \rangle, \\
& \langle (e_2, p, 1), \left(\frac{(u_1, s_1)^{t_2}}{(0.9, 0.3, 0.7)}, \frac{(u_2, s_1)^{t_2}}{(0.0, 0.2, 0.7)}, \frac{(u_3, s_1)^{t_2}}{(0.9, 0.8, 0.5)}, \frac{(u_1, s_2)^{t_2}}{(0.7, 0.5, 0.7)}, \frac{(u_2, s_2)^{t_2}}{(0.9, 0.6, 0.5)}, \frac{(u_3, s_2)^{t_2}}{(0.2, 0.3, 0.3)} \right) \rangle, \\
& \langle (e_2, r, 1), \left(\frac{(u_1, s_1)^{t_1}}{(0.7, 0.7, 0.8)}, \frac{(u_2, s_1)^{t_1}}{(0.5, 0.4, 0.4)}, \frac{(u_3, s_1)^{t_1}}{(0.4, 0.9, 0.6)}, \frac{(u_1, s_2)^{t_1}}{(0.8, 0.9, 0.3)}, \frac{(u_2, s_2)^{t_1}}{(0.1, 0.1, 0.8)}, \frac{(u_3, s_2)^{t_1}}{(0.0, 0.7, 0.0)} \right) \rangle, \\
& \langle (e_2, q, 1), \left(\frac{(u_1, s_1)^{t_1}}{(0.2, 0.2, 0.8)}, \frac{(u_2, s_1)^{t_1}}{(0.7, 0.8, 0.9)}, \frac{(u_3, s_1)^{t_1}}{(0.3, 0.9, 0.7)}, \frac{(u_1, s_2)^{t_1}}{(0.6, 0.5, 0.4)}, \frac{(u_2, s_2)^{t_1}}{(0.9, 0.1, 0.6)}, \frac{(u_3, s_2)^{t_1}}{(0.8, 0.6, 0.2)} \right) \rangle, \\
& \langle (e_3, p, 1), \left(\frac{(u_1, s_1)^{t_2}}{(0.5, 0.0, 0.3)}, \frac{(u_2, s_1)^{t_2}}{(0.9, 0.4, 0.8)}, \frac{(u_3, s_1)^{t_2}}{(0.4, 0.6, 0.3)}, \frac{(u_1, s_2)^{t_2}}{(0.4, 0.6, 0.2)}, \frac{(u_2, s_2)^{t_2}}{(0.7, 0.2, 0.0)}, \frac{(u_3, s_2)^{t_2}}{(0.4, 0.3, 0.2)} \right) \rangle, \\
& \langle (e_3, r, 1), \left(\frac{(u_1, s_1)^{t_2}}{(0.8, 0.0, 0.2)}, \frac{(u_2, s_1)^{t_2}}{(0.6, 0.3, 0.9)}, \frac{(u_3, s_1)^{t_2}}{(0.4, 0.6, 0.1)}, \frac{(u_1, s_2)^{t_2}}{(0.1, 0.6, 0.7)}, \frac{(u_2, s_2)^{t_2}}{(0.2, 0.3, 0.3)}, \frac{(u_3, s_2)^{t_2}}{(0.1, 0.6, 0.8)} \right) \rangle, \\
& \langle (e_3, q, 1), \left(\frac{(u_1, s_1)^{t_2}}{(0.0, 0.5, 0.3)}, \frac{(u_2, s_1)^{t_2}}{(0.7, 0.7, 0.8)}, \frac{(u_3, s_1)^{t_2}}{(0.1, 0.9, 0.8)}, \frac{(u_1, s_2)^{t_2}}{(0.2, 0.7, 0.5)}, \frac{(u_2, s_2)^{t_2}}{(0.4, 0.7, 0.1)}, \frac{(u_3, s_2)^{t_2}}{(0.0, 0.0, 0.6)} \right) \rangle, \\
& \langle (e_1, p, 0), \left(\frac{(u_1, s_1)^{t_2}}{(0.7, 0.0, 0.3)}, \frac{(u_2, s_1)^{t_2}}{(0.0, 0.7, 0.0)}, \frac{(u_3, s_1)^{t_2}}{(0.2, 0.7, 0.1)}, \frac{(u_1, s_2)^{t_2}}{(0.1, 0.6, 0.4)}, \frac{(u_2, s_2)^{t_2}}{(0.7, 0.6, 0.4)}, \frac{(u_3, s_2)^{t_2}}{(0.3, 0.3, 0.0)} \right) \rangle, \\
& \langle (e_1, r, 0), \left(\frac{(u_1, s_1)^{t_2}}{(0.7, 0.1, 0.6)}, \frac{(u_2, s_1)^{t_2}}{(0.0, 0.2, 0.0)}, \frac{(u_3, s_1)^{t_2}}{(0.3, 0.2, 0.2)}, \frac{(u_1, s_2)^{t_2}}{(0.3, 0.1, 0.4)}, \frac{(u_2, s_2)^{t_2}}{(0.3, 0.7, 0.9)}, \frac{(u_3, s_2)^{t_2}}{(0.0, 0.7, 0.7)} \right) \rangle, \\
& \langle (e_1, q, 0), \left(\frac{(u_1, s_1)^{t_2}}{(0.3, 0.6, 0.7)}, \frac{(u_2, s_1)^{t_2}}{(0.9, 0.0, 0.1)}, \frac{(u_3, s_1)^{t_2}}{(0.3, 0.1, 0.6)}, \frac{(u_1, s_2)^{t_2}}{(0.4, 0.3, 0.6)}, \frac{(u_2, s_2)^{t_2}}{(0.1, 0.0, 0.8)}, \frac{(u_3, s_2)^{t_2}}{(0.1, 0.8, 0.3)} \right) \rangle, \\
& \langle (e_2, p, 0), \left(\frac{(u_1, s_1)^{t_2}}{(0.7, 0.3, 0.5)}, \frac{(u_2, s_1)^{t_2}}{(0.1, 0.2, 0.8)}, \frac{(u_3, s_1)^{t_2}}{(0.3, 0.7, 0.0)}, \frac{(u_1, s_2)^{t_2}}{(0.6, 0.3, 0.8)}, \frac{(u_2, s_2)^{t_2}}{(0.3, 0.9, 0.1)}, \frac{(u_3, s_2)^{t_2}}{(0.3, 0.5, 0.7)} \right) \rangle,
\end{aligned}$$

$$\begin{aligned}
& \langle (e_2, r, 0), \left(\frac{(u_1, s_1)^{t_2}}{(0.6, 0.3, 0.0)}, \frac{(u_2, s_1)^{t_2}}{(0.0, 0.3, 0.0)}, \frac{(u_3, s_1)^{t_2}}{(0.1, 0.1, 0.0)}, \frac{(u_1, s_2)^{t_2}}{(0.4, 0.8, 0.0)}, \frac{(u_2, s_2)^{t_2}}{(0.8, 0.8, 0.3)}, \frac{(u_3, s_2)^{t_2}}{(0.5, 0.5, 0.4)} \right) \rangle, \\
& \langle (e_2, q, 0), \left(\frac{(u_1, s_1)^{t_2}}{(0.7, 0.3, 0.0)}, \frac{(u_2, s_1)^{t_2}}{(0.2, 0.1, 0.6)}, \frac{(u_3, s_1)^{t_2}}{(0.1, 0.1, 0.2)}, \frac{(u_1, s_2)^{t_2}}{(0.4, 0.1, 0.2)}, \frac{(u_2, s_2)^{t_2}}{(0.5, 0.1, 0.2)}, \frac{(u_3, s_2)^{t_2}}{(0.1, 0.4, 0.2)} \right) \rangle, \\
& \langle (e_3, p, 0), \left(\frac{(u_1, s_1)^{t_2}}{(0.4, 0.5, 0.2)}, \frac{(u_2, s_1)^{t_2}}{(0.6, 0.0, 0.8)}, \frac{(u_3, s_1)^{t_2}}{(0.5, 0.6, 0.7)}, \frac{(u_1, s_2)^{t_2}}{(0.7, 0.2, 0.5)}, \frac{(u_2, s_2)^{t_2}}{(0.9, 0.6, 0.2)}, \frac{(u_3, s_2)^{t_2}}{(0.2, 0.5, 0.9)} \right) \rangle, \\
& \langle (e_3, r, 0), \left(\frac{(u_1, s_1)^{t_2}}{(0.1, 0.6, 0.6)}, \frac{(u_2, s_1)^{t_2}}{(0.4, 0.2, 0.8)}, \frac{(u_3, s_1)^{t_2}}{(0.3, 0.9, 0.8)}, \frac{(u_1, s_2)^{t_2}}{(0.5, 0.7, 0.7)}, \frac{(u_2, s_2)^{t_2}}{(0.5, 0.5, 0.4)}, \frac{(u_3, s_2)^{t_2}}{(0.2, 0.1, 0.5)} \right) \rangle, \\
& \langle (e_3, q, 0), \left(\frac{(u_1, s_1)^{t_2}}{(0.8, 0.1, 0.8)}, \frac{(u_2, s_1)^{t_2}}{(0.5, 0.0, 0.2)}, \frac{(u_3, s_1)^{t_2}}{(0.7, 0.6, 0.8)}, \frac{(u_1, s_2)^{t_2}}{(0.4, 0.4, 0.8)}, \frac{(u_2, s_2)^{t_2}}{(0.7, 0.1, 0.4)}, \frac{(u_3, s_2)^{t_2}}{(0.2, 0.2, 0.2)} \right) \rangle, \\
& \langle (e_1, p, 1), \left(\frac{(u_1, s_1)^{t_3}}{(0.9, 0.2, 0.6)}, \frac{(u_2, s_1)^{t_3}}{(0.5, 0.9, 0.9)}, \frac{(u_3, s_1)^{t_3}}{(0.0, 0.6, 0.4)}, \frac{(u_1, s_2)^{t_3}}{(0.8, 0.9, 0.7)}, \frac{(u_2, s_2)^{t_3}}{(0.7, 0.2, 0.2)}, \frac{(u_3, s_2)^{t_3}}{(0.6, 0.8, 0.7)} \right) \rangle, \\
& \langle (e_1, r, 1), \left(\frac{(u_1, s_1)^{t_3}}{(0.4, 0.1, 0.3)}, \frac{(u_2, s_1)^{t_3}}{(0.3, 0.7, 0.0)}, \frac{(u_3, s_1)^{t_3}}{(0.7, 0.2, 0.2)}, \frac{(u_1, s_2)^{t_3}}{(0.6, 0.9, 0.8)}, \frac{(u_2, s_2)^{t_3}}{(0.7, 0.7, 0.4)}, \frac{(u_3, s_2)^{t_3}}{(0.1, 0.3, 0.6)} \right) \rangle, \\
& \langle (e_1, q, 1), \left(\frac{(u_1, s_1)^{t_3}}{(0.9, 0.7, 0.8)}, \frac{(u_2, s_1)^{t_3}}{(0.3, 0.6, 0.2)}, \frac{(u_3, s_1)^{t_3}}{(0.0, 0.0, 0.0)}, \frac{(u_1, s_2)^{t_3}}{(0.8, 0.0, 0.4)}, \frac{(u_2, s_2)^{t_3}}{(0.9, 0.3, 0.7)}, \frac{(u_3, s_2)^{t_3}}{(0.0, 0.4, 0.1)} \right) \rangle, \\
& \langle (e_2, p, 1), \left(\frac{(u_1, s_1)^{t_3}}{(0.1, 0.9, 0.7)}, \frac{(u_2, s_1)^{t_3}}{(0.8, 0.0, 0.5)}, \frac{(u_3, s_1)^{t_3}}{(0.9, 0.7, 0.2)}, \frac{(u_1, s_2)^{t_3}}{(0.5, 0.3, 0.8)}, \frac{(u_2, s_2)^{t_3}}{(0.3, 0.5, 0.5)}, \frac{(u_3, s_2)^{t_3}}{(0.4, 0.9, 0.1)} \right) \rangle, \\
& \langle (e_2, r, 1), \left(\frac{(u_1, s_1)^{t_3}}{(0.7, 0.7, 0.9)}, \frac{(u_2, s_1)^{t_3}}{(0.7, 0.5, 0.6)}, \frac{(u_3, s_1)^{t_3}}{(0.5, 0.3, 0.2)}, \frac{(u_1, s_2)^{t_3}}{(0.7, 0.6, 0.1)}, \frac{(u_2, s_2)^{t_3}}{(0.6, 0.5, 0.9)}, \frac{(u_3, s_2)^{t_3}}{(0.4, 0.3, 0.9)} \right) \rangle, \\
& \langle (e_2, q, 1), \left(\frac{(u_1, s_1)^{t_3}}{(0.8, 0.9, 0.6)}, \frac{(u_2, s_1)^{t_3}}{(0.1, 0.3, 0.4)}, \frac{(u_3, s_1)^{t_3}}{(0.3, 0.3, 0.2)}, \frac{(u_1, s_2)^{t_3}}{(0.6, 0.1, 0.3)}, \frac{(u_2, s_2)^{t_3}}{(0.9, 0.5, 0.3)}, \frac{(u_3, s_2)^{t_3}}{(0.8, 0.5, 0.8)} \right) \rangle, \\
& \langle (e_3, p, 1), \left(\frac{(u_1, s_1)^{t_3}}{(0.5, 0.0, 0.5)}, \frac{(u_2, s_1)^{t_3}}{(0.9, 0.0, 0.5)}, \frac{(u_3, s_1)^{t_3}}{(0.9, 0.8, 0.2)}, \frac{(u_1, s_2)^{t_3}}{(0.3, 0.5, 0.2)}, \frac{(u_2, s_2)^{t_3}}{(0.6, 0.0, 0.8)}, \frac{(u_3, s_2)^{t_3}}{(0.5, 0.1, 0.8)} \right) \rangle, \\
& \langle (e_3, r, 1), \left(\frac{(u_1, s_1)^{t_3}}{(0.3, 0.0, 0.7)}, \frac{(u_2, s_1)^{t_3}}{(0.1, 0.7, 0.2)}, \frac{(u_3, s_1)^{t_3}}{(0.9, 0.3, 0.7)}, \frac{(u_1, s_2)^{t_3}}{(0.4, 0.6, 0.6)}, \frac{(u_2, s_2)^{t_3}}{(0.7, 0.5, 0.7)}, \frac{(u_3, s_2)^{t_3}}{(0.8, 0.0, 0.2)} \right) \rangle, \\
& \langle (e_3, q, 1), \left(\frac{(u_1, s_1)^{t_3}}{(0.6, 0.3, 0.9)}, \frac{(u_2, s_1)^{t_3}}{(0.3, 0.4, 0.3)}, \frac{(u_3, s_1)^{t_3}}{(0.1, 0.7, 0.1)}, \frac{(u_1, s_2)^{t_3}}{(0.0, 0.0, 0.4)}, \frac{(u_2, s_2)^{t_3}}{(0.0, 0.6, 0.7)}, \frac{(u_3, s_2)^{t_3}}{(0.3, 0.5, 0.2)} \right) \rangle, \\
& \langle (e_1, p, 0), \left(\frac{(u_1, s_1)^{t_3}}{(0.2, 0.1, 0.6)}, \frac{(u_2, s_1)^{t_3}}{(0.0, 0.2, 0.1)}, \frac{(u_3, s_1)^{t_3}}{(0.2, 0.6, 0.3)}, \frac{(u_1, s_2)^{t_3}}{(0.1, 0.7, 0.9)}, \frac{(u_2, s_2)^{t_3}}{(0.7, 0.9, 0.5)}, \frac{(u_3, s_2)^{t_3}}{(0.3, 0.8, 0.8)} \right) \rangle, \\
& \langle (e_1, r, 0), \left(\frac{(u_1, s_1)^{t_3}}{(0.4, 0.6, 0.3)}, \frac{(u_2, s_1)^{t_3}}{(0.7, 0.1, 0.1)}, \frac{(u_3, s_1)^{t_3}}{(0.3, 0.1, 0.3)}, \frac{(u_1, s_2)^{t_3}}{(0.6, 0.5, 0.3)}, \frac{(u_2, s_2)^{t_3}}{(0.2, 0.0, 0.3)}, \frac{(u_3, s_2)^{t_3}}{(0.6, 0.8, 0.6)} \right) \rangle, \\
& \langle (e_1, q, 0), \left(\frac{(u_1, s_1)^{t_3}}{(0.0, 0.5, 0.2)}, \frac{(u_2, s_1)^{t_3}}{(0.1, 0.4, 0.0)}, \frac{(u_3, s_1)^{t_3}}{(0.6, 0.6, 0.6)}, \frac{(u_1, s_2)^{t_3}}{(0.8, 0.8, 0.8)}, \frac{(u_2, s_2)^{t_3}}{(0.6, 0.4, 0.7)}, \frac{(u_3, s_2)^{t_3}}{(0.7, 0.7, 0.0)} \right) \rangle, \\
& \langle (e_2, p, 0), \left(\frac{(u_1, s_1)^{t_3}}{(0.4, 0.4, 0.0)}, \frac{(u_2, s_1)^{t_3}}{(0.3, 0.6, 0.3)}, \frac{(u_3, s_1)^{t_3}}{(0.6, 0.7, 0.5)}, \frac{(u_1, s_2)^{t_3}}{(0.4, 0.3, 0.3)}, \frac{(u_2, s_2)^{t_3}}{(0.2, 0.6, 0.1)}, \frac{(u_3, s_2)^{t_3}}{(0.7, 0.5, 0.8)} \right) \rangle,
\end{aligned}$$

$$\begin{aligned} & \langle (e_2, r, 0), \left(\frac{(u_1, s_1)^{t_3}}{(0.9, 0.1, 0.2)}, \frac{(u_2, s_1)^{t_3}}{(0.8, 0.8, 0.8)}, \frac{(u_3, s_1)^{t_3}}{(0.9, 0.2, 0.2)}, \frac{(u_1, s_2)^{t_3}}{(0.7, 0.8, 0.3)}, \frac{(u_2, s_2)^{t_3}}{(0.0, 0.9, 0.7)}, \frac{(u_3, s_2)^{t_3}}{(0.3, 0.8, 0.7)} \right) \rangle, \\ & \langle (e_2, q, 0), \left(\frac{(u_1, s_1)^{t_3}}{(0.4, 0.1, 0.5)}, \frac{(u_2, s_1)^{t_3}}{(0.7, 0.8, 0.5)}, \frac{(u_3, s_1)^{t_3}}{(0.6, 0.2, 0.9)}, \frac{(u_1, s_2)^{t_3}}{(0.5, 0.9, 0.5)}, \frac{(u_2, s_2)^{t_3}}{(0.1, 0.1, 0.5)}, \frac{(u_3, s_2)^{t_3}}{(0.6, 0.8, 0.2)} \right) \rangle, \\ & \langle (e_3, p, 0), \left(\frac{(u_1, s_1)^{t_3}}{(0.8, 0.7, 0.0)}, \frac{(u_2, s_1)^{t_3}}{(0.2, 0.1, 0.2)}, \frac{(u_3, s_1)^{t_3}}{(0.7, 0.4, 0.5)}, \frac{(u_1, s_2)^{t_3}}{(0.1, 0.6, 0.6)}, \frac{(u_2, s_2)^{t_3}}{(0.9, 0.8, 0.3)}, \frac{(u_3, s_2)^{t_3}}{(0.8, 0.8, 0.7)} \right) \rangle, \\ & \langle (e_3, r, 0), \left(\frac{(u_1, s_1)^{t_3}}{(0.5, 0.1, 0.3)}, \frac{(u_2, s_1)^{t_3}}{(0.5, 0.8, 0.2)}, \frac{(u_3, s_1)^{t_3}}{(0.7, 0.5, 0.0)}, \frac{(u_1, s_2)^{t_3}}{(0.0, 0.7, 0.3)}, \frac{(u_2, s_2)^{t_3}}{(0.7, 0.4, 0.8)}, \frac{(u_3, s_2)^{t_3}}{(0.5, 0.2, 0.5)} \right) \rangle, \\ & \langle (e_3, q, 0), \left(\frac{(u_1, s_1)^{t_3}}{(0.4, 0.3, 0.8)}, \frac{(u_2, s_1)^{t_3}}{(0.0, 0.7, 0.7)}, \frac{(u_3, s_1)^{t_3}}{(0.5, 0.5, 0.6)}, \frac{(u_1, s_2)^{t_3}}{(0.7, 0.7, 0.3)}, \frac{(u_2, s_2)^{t_3}}{(0.3, 0.7, 0.1)}, \frac{(u_3, s_2)^{t_3}}{(0.6, 0.3, 0.8)} \right) \rangle \} \end{aligned}$$

Definition 3.2. Let $(F_Q, A)_t$ and $(G_Q, B)_t$ be two T-QNSEs over $U \times Q$. Then $(F_Q, A)_t$ is called a T-QNSE subset of $(G_Q, B)_t$ if

1. $A \subseteq B$
2. $\forall t \in T, c \in A, F_Q(c)$ is a Q-neutrosophic soft expert subset of $G_Q(c)$.

Definition 3.3. Two T-QNSSs $(F_Q, A)_t$ and $(G_Q, B)_t$ over $U \times Q$, are said to be equal if $(F_Q, A)_t$ is a T-QNSE subset of $(G_Q, B)_t$ and $(G_Q, B)_t$ is a T-QNSE subset of $(F_Q, A)_t$.

Example 3.2. Consider Example 3.1 and suppose that the

$$\begin{aligned} (F_Q, A)_t &= \{ \langle (e_1, p, 1), \left(\frac{(u_1, s_1)^{t_1}}{(0.1, 0.1, 0.3)}, \frac{(u_2, s_1)^{t_1}}{(0.9, 0.7, 0.2)}, \frac{(u_3, s_1)^{t_1}}{(0.1, 0.2, 0.8)}, \frac{(u_1, s_2)^{t_1}}{(0.0, 0.1, 0.4)}, \frac{(u_2, s_2)^{t_1}}{(0.0, 0.7, 0.8)}, \frac{(u_3, s_2)^{t_1}}{(0.4, 0.2, 0.8)} \right) \rangle, \right. \\ & \langle (e_2, p, 0), \left(\frac{(u_1, s_1)^{t_1}}{(0.6, 0.2, 0.4)}, \frac{(u_2, s_1)^{t_1}}{(0.4, 0.1, 0.3)}, \frac{(u_3, s_1)^{t_1}}{(0.4, 0.8, 0.7)}, \frac{(u_1, s_2)^{t_1}}{(0.2, 0.8, 0.9)}, \frac{(u_2, s_2)^{t_1}}{(0.9, 0.2, 0.9)}, \frac{(u_3, s_2)^{t_1}}{(0.0, 0.8, 0.5)} \right) \rangle, \\ & \langle (e_2, q, 1), \left(\frac{(u_1, s_1)^{t_2}}{(0.2, 0.2, 0.8)}, \frac{(u_2, s_1)^{t_2}}{(0.7, 0.8, 0.9)}, \frac{(u_3, s_1)^{t_2}}{(0.3, 0.9, 0.7)}, \frac{(u_1, s_2)^{t_2}}{(0.6, 0.5, 0.4)}, \frac{(u_2, s_2)^{t_2}}{(0.9, 0.1, 0.6)}, \frac{(u_3, s_2)^{t_2}}{(0.8, 0.6, 0.2)} \right) \rangle, \\ & \langle (e_2, q, 0), \left(\frac{(u_1, s_1)^{t_2}}{(0.7, 0.3, 0.0)}, \frac{(u_2, s_1)^{t_2}}{(0.2, 0.1, 0.6)}, \frac{(u_3, s_1)^{t_2}}{(0.1, 0.1, 0.2)}, \frac{(u_1, s_2)^{t_2}}{(0.4, 0.1, 0.2)}, \frac{(u_2, s_2)^{t_2}}{(0.5, 0.1, 0.2)}, \frac{(u_3, s_2)^{t_2}}{(0.1, 0.4, 0.2)} \right) \rangle, \\ & \left. \langle (e_3, r, 1), \left(\frac{(u_1, s_1)^{t_3}}{(0.3, 0.0, 0.7)}, \frac{(u_2, s_1)^{t_3}}{(0.1, 0.7, 0.2)}, \frac{(u_3, s_1)^{t_3}}{(0.9, 0.3, 0.7)}, \frac{(u_1, s_2)^{t_3}}{(0.4, 0.6, 0.6)}, \frac{(u_2, s_2)^{t_3}}{(0.7, 0.5, 0.7)}, \frac{(u_3, s_2)^{t_3}}{(0.8, 0.0, 0.2)} \right) \rangle \} \right. \\ (G_Q, B)_t &= \{ \langle (e_1, p, 1), \left(\frac{(u_1, s_1)^{t_1}}{(0.0, 0.1, 0.5)}, \frac{(u_2, s_1)^{t_1}}{(0.6, 0.5, 0.4)}, \frac{(u_3, s_1)^{t_1}}{(0.0, 0.1, 0.9)}, \frac{(u_1, s_2)^{t_1}}{(0.0, 0.1, 0.6)}, \frac{(u_2, s_2)^{t_1}}{(0.0, 0.5, 0.8)}, \frac{(u_3, s_2)^{t_1}}{(0.2, 0.2, 0.9)} \right) \rangle, \right. \\ & \langle (e_2, p, 0), \left(\frac{(u_1, s_1)^{t_1}}{(0.3, 0.1, 0.8)}, \frac{(u_2, s_1)^{t_1}}{(0.2, 0.0, 0.7)}, \frac{(u_3, s_1)^{t_1}}{(0.2, 0.5, 0.9)}, \frac{(u_1, s_2)^{t_1}}{(0.1, 0.6, 0.9)}, \frac{(u_2, s_2)^{t_1}}{(0.7, 0.1, 0.9)}, \frac{(u_3, s_2)^{t_1}}{(0.0, 0.3, 0.5)} \right) \rangle, \\ & \left. \langle (e_2, q, 1), \left(\frac{(u_1, s_1)^{t_2}}{(0.0, 0.2, 0.9)}, \frac{(u_2, s_1)^{t_2}}{(0.5, 0.5, 0.9)}, \frac{(u_3, s_1)^{t_2}}{(0.1, 0.5, 0.9)}, \frac{(u_1, s_2)^{t_2}}{(0.4, 0.3, 0.7)}, \frac{(u_2, s_2)^{t_2}}{(0.5, 0.0, 0.9)}, \frac{(u_3, s_2)^{t_2}}{(0.6, 0.2, 0.6)} \right) \rangle \} \right. \end{aligned}$$

Therefore, $(G_Q, B)_t$ is the Q-neutrosophic soft expert subset of $(F_Q, A)_t$.

It's clear that $B \subset A$, and $\forall t \in T, G_Q$ is a QNSE subset of F_Q . Then $(G_Q, B)_t$ is a T-QNSE subset of $(F_Q, A)_t$.

Definition 3.4. Agree on T-QNSEs $(F_Q, A)_t$ over $U \times Q$ is a T-QNSE subset of $(F_Q, A)_t$ defined as $(F_Q, A)_t^1 = \{(F_Q^t(\alpha))^1: \alpha \in E \times X \times \{1\}\}$.

Example 3.3 Recall example 3.1. The agreed time Q-neutrosophic soft expert sets $(F_{Q_1}, A)_t$ over $U \times Q$ is

$$\begin{aligned}
 &(F_{Q_1}, A)_t \\
 &= \{ \langle (e_1, p, 1), \left(\frac{(u_1, s_1)^{t_1}}{(0.1, 0.1, 0.3)}, \frac{(u_2, s_1)^{t_1}}{(0.9, 0.7, 0.2)}, \frac{(u_3, s_1)^{t_1}}{(0.1, 0.2, 0.9)}, \frac{(u_1, s_2)^{t_1}}{(0.0, 0.1, 0.4)}, \frac{(u_2, s_2)^{t_1}}{(0.0, 0.7, 0.8)}, \frac{(u_3, s_2)^{t_1}}{(0.4, 0.2, 0.8)} \right) \rangle, \right. \\
 &\langle (e_1, r, 1), \left(\frac{(u_1, s_1)^{t_1}}{(0.4, 0.0, 0.1)}, \frac{(u_2, s_1)^{t_1}}{(0.2, 0.4, 0.6)}, \frac{(u_3, s_1)^{t_1}}{(0.9, 0.2, 0.4)}, \frac{(u_1, s_2)^{t_1}}{(0.6, 0.4, 0.5)}, \frac{(u_2, s_2)^{t_1}}{(0.2, 0.4, 0.2)}, \frac{(u_3, s_2)^{t_1}}{(0.9, 0.0, 0.5)} \right) \rangle, \\
 &\langle (e_1, q, 1), \left(\frac{(u_1, s_1)^{t_1}}{(0.8, 0.7, 0.5)}, \frac{(u_2, s_1)^{t_1}}{(0.7, 0.6, 0.7)}, \frac{(u_3, s_1)^{t_1}}{(0.0, 0.2, 0.9)}, \frac{(u_1, s_2)^{t_1}}{(0.3, 0.6, 0.4)}, \frac{(u_2, s_2)^{t_1}}{(0.5, 0.1, 0.9)}, \frac{(u_3, s_2)^{t_1}}{(0.3, 0.5, 0.0)} \right) \rangle, \\
 &\langle (e_2, p, 1), \left(\frac{(u_1, s_1)^{t_1}}{(0.5, 0.5, 0.3)}, \frac{(u_2, s_1)^{t_1}}{(0.1, 0.9, 0.7)}, \frac{(u_3, s_1)^{t_1}}{(0.7, 0.5, 0.6)}, \frac{(u_1, s_2)^{t_1}}{(0.5, 0.1, 0.9)}, \frac{(u_2, s_2)^{t_1}}{(0.1, 0.3, 0.8)}, \frac{(u_3, s_2)^{t_1}}{(0.4, 0.8, 0.9)} \right) \rangle, \\
 &\langle (e_2, r, 1), \left(\frac{(u_1, s_1)^{t_1}}{(0.2, 0.4, 0.5)}, \frac{(u_2, s_1)^{t_1}}{(0.6, 0.1, 0.0)}, \frac{(u_3, s_1)^{t_1}}{(0.1, 0.0, 0.3)}, \frac{(u_1, s_2)^{t_1}}{(0.0, 0.6, 0.4)}, \frac{(u_2, s_2)^{t_1}}{(0.7, 0.5, 0.0)}, \frac{(u_3, s_2)^{t_1}}{(0.9, 0.4, 0.0)} \right) \rangle, \\
 &\langle (e_2, q, 1), \left(\frac{(u_1, s_1)^{t_1}}{(0.8, 0.9, 0.4)}, \frac{(u_2, s_1)^{t_1}}{(0.8, 0.0, 0.0)}, \frac{(u_3, s_1)^{t_1}}{(0.1, 0.2, 0.6)}, \frac{(u_1, s_2)^{t_1}}{(0.2, 0.2, 0.3)}, \frac{(u_2, s_2)^{t_1}}{(0.8, 0.9, 0.2)}, \frac{(u_3, s_2)^{t_1}}{(0.5, 0.9, 0.4)} \right) \rangle, \\
 &\langle (e_3, p, 1), \left(\frac{(u_1, s_1)^{t_1}}{(0.5, 0.2, 0.2)}, \frac{(u_2, s_1)^{t_1}}{(0.3, 0.8, 0.7)}, \frac{(u_3, s_1)^{t_1}}{(0.5, 0.7, 0.4)}, \frac{(u_1, s_2)^{t_1}}{(0.8, 0.6, 0.8)}, \frac{(u_2, s_2)^{t_1}}{(0.7, 0.0, 0.0)}, \frac{(u_3, s_2)^{t_1}}{(0.5, 0.2, 0.3)} \right) \rangle, \\
 &\langle (e_3, r, 1), \left(\frac{(u_1, s_1)^{t_1}}{(0.6, 0.5, 0.2)}, \frac{(u_2, s_1)^{t_1}}{(0.0, 0.6, 0.7)}, \frac{(u_3, s_1)^{t_1}}{(0.9, 0.6, 0.0)}, \frac{(u_1, s_2)^{t_1}}{(0.0, 0.5, 0.8)}, \frac{(u_2, s_2)^{t_1}}{(0.1, 0.2, 0.5)}, \frac{(u_3, s_2)^{t_1}}{(0.3, 0.4, 0.2)} \right) \rangle, \\
 &\langle (e_3, q, 1), \left(\frac{(u_1, s_1)^{t_1}}{(0.0, 0.6, 0.0)}, \frac{(u_2, s_1)^{t_1}}{(0.5, 0.0, 0.1)}, \frac{(u_3, s_1)^{t_1}}{(0.4, 0.0, 0.7)}, \frac{(u_1, s_2)^{t_1}}{(0.8, 0.1, 0.5)}, \frac{(u_2, s_2)^{t_1}}{(0.9, 0.3, 0.8)}, \frac{(u_3, s_2)^{t_1}}{(0.0, 0.0, 0.3)} \right) \rangle, \\
 &\langle (e_1, p, 1), \left(\frac{(u_1, s_1)^{t_2}}{(0.8, 0.7, 0.5)}, \frac{(u_2, s_1)^{t_2}}{(0.7, 0.1, 0.7)}, \frac{(u_3, s_1)^{t_2}}{(0.3, 0.3, 0.1)}, \frac{(u_1, s_2)^{t_2}}{(0.2, 0.4, 0.0)}, \frac{(u_2, s_2)^{t_2}}{(0.7, 0.1, 0.2)}, \frac{(u_3, s_2)^{t_2}}{(0.8, 0.6, 0.1)} \right) \rangle, \\
 &\langle (e_1, r, 1), \left(\frac{(u_1, s_1)^{t_2}}{(0.1, 0.6, 0.0)}, \frac{(u_2, s_1)^{t_2}}{(0.4, 0.5, 0.8)}, \frac{(u_3, s_1)^{t_2}}{(0.2, 0.5, 0.9)}, \frac{(u_1, s_2)^{t_2}}{(0.7, 0.5, 0.2)}, \frac{(u_2, s_2)^{t_2}}{(0.8, 0.5, 0.8)}, \frac{(u_3, s_2)^{t_2}}{(0.6, 0.6, 0.5)} \right) \rangle, \\
 &\langle (e_1, q, 1), \left(\frac{(u_1, s_1)^{t_2}}{(0.2, 0.3, 0.6)}, \frac{(u_2, s_1)^{t_2}}{(0.4, 0.5, 0.2)}, \frac{(u_3, s_1)^{t_2}}{(0.5, 0.3, 0.7)}, \frac{(u_1, s_2)^{t_2}}{(0.6, 0.0, 0.6)}, \frac{(u_2, s_2)^{t_2}}{(0.4, 0.2, 0.2)}, \frac{(u_3, s_2)^{t_2}}{(0.7, 0.5, 0.9)} \right) \rangle, \\
 &\langle (e_2, p, 1), \left(\frac{(u_1, s_1)^{t_2}}{(0.9, 0.3, 0.7)}, \frac{(u_2, s_1)^{t_2}}{(0.0, 0.2, 0.7)}, \frac{(u_3, s_1)^{t_2}}{(0.9, 0.8, 0.5)}, \frac{(u_1, s_2)^{t_2}}{(0.7, 0.5, 0.7)}, \frac{(u_2, s_2)^{t_2}}{(0.9, 0.6, 0.5)}, \frac{(u_3, s_2)^{t_2}}{(0.2, 0.3, 0.3)} \right) \rangle, \\
 &\langle (e_2, r, 1), \left(\frac{(u_1, s_1)^{t_1}}{(0.7, 0.7, 0.8)}, \frac{(u_2, s_1)^{t_1}}{(0.5, 0.4, 0.4)}, \frac{(u_3, s_1)^{t_1}}{(0.4, 0.9, 0.6)}, \frac{(u_1, s_2)^{t_1}}{(0.8, 0.9, 0.3)}, \frac{(u_2, s_2)^{t_1}}{(0.1, 0.1, 0.8)}, \frac{(u_3, s_2)^{t_1}}{(0.0, 0.7, 0.0)} \right) \rangle, \\
 &\langle (e_2, q, 1), \left(\frac{(u_1, s_1)^{t_1}}{(0.2, 0.2, 0.8)}, \frac{(u_2, s_1)^{t_1}}{(0.7, 0.8, 0.9)}, \frac{(u_3, s_1)^{t_1}}{(0.3, 0.9, 0.7)}, \frac{(u_1, s_2)^{t_1}}{(0.6, 0.5, 0.4)}, \frac{(u_2, s_2)^{t_1}}{(0.9, 0.1, 0.6)}, \frac{(u_3, s_2)^{t_1}}{(0.8, 0.6, 0.2)} \right) \rangle, \\
 &\}
 \end{aligned}$$

$$\begin{aligned} &\langle (e_3, p, 1), \left(\frac{(u_1, s_1)^{t_2}}{(0.5, 0.0, 0.3)}, \frac{(u_2, s_1)^{t_2}}{(0.9, 0.4, 0.8)}, \frac{(u_3, s_1)^{t_2}}{(0.4, 0.6, 0.3)}, \frac{(u_1, s_2)^{t_2}}{(0.4, 0.6, 0.2)}, \frac{(u_2, s_2)^{t_2}}{(0.7, 0.2, 0.0)}, \frac{(u_3, s_2)^{t_2}}{(0.4, 0.3, 0.2)} \right) \rangle, \\ &\langle (e_3, r, 1), \left(\frac{(u_1, s_1)^{t_2}}{(0.8, 0.0, 0.2)}, \frac{(u_2, s_1)^{t_2}}{(0.6, 0.3, 0.9)}, \frac{(u_3, s_1)^{t_2}}{(0.4, 0.6, 0.1)}, \frac{(u_1, s_2)^{t_2}}{(0.1, 0.6, 0.7)}, \frac{(u_2, s_2)^{t_2}}{(0.2, 0.3, 0.3)}, \frac{(u_3, s_2)^{t_2}}{(0.1, 0.6, 0.8)} \right) \rangle, \\ &\langle (e_3, q, 1), \left(\frac{(u_1, s_1)^{t_2}}{(0.0, 0.5, 0.3)}, \frac{(u_2, s_1)^{t_2}}{(0.7, 0.7, 0.8)}, \frac{(u_3, s_1)^{t_2}}{(0.1, 0.9, 0.8)}, \frac{(u_1, s_2)^{t_2}}{(0.2, 0.7, 0.5)}, \frac{(u_2, s_2)^{t_2}}{(0.4, 0.7, 0.1)}, \frac{(u_3, s_2)^{t_2}}{(0.0, 0.0, 0.6)} \right) \rangle, \\ &\langle (e_1, p, 1), \left(\frac{(u_1, s_1)^{t_3}}{(0.9, 0.2, 0.6)}, \frac{(u_2, s_1)^{t_3}}{(0.5, 0.9, 0.9)}, \frac{(u_3, s_1)^{t_3}}{(0.0, 0.6, 0.4)}, \frac{(u_1, s_2)^{t_3}}{(0.8, 0.9, 0.7)}, \frac{(u_2, s_2)^{t_3}}{(0.7, 0.2, 0.2)}, \frac{(u_3, s_2)^{t_3}}{(0.6, 0.8, 0.7)} \right) \rangle, \\ &\langle (e_1, r, 1), \left(\frac{(u_1, s_1)^{t_3}}{(0.4, 0.1, 0.3)}, \frac{(u_2, s_1)^{t_3}}{(0.3, 0.7, 0.0)}, \frac{(u_3, s_1)^{t_3}}{(0.7, 0.2, 0.2)}, \frac{(u_1, s_2)^{t_3}}{(0.6, 0.9, 0.8)}, \frac{(u_2, s_2)^{t_3}}{(0.7, 0.7, 0.4)}, \frac{(u_3, s_2)^{t_3}}{(0.1, 0.3, 0.6)} \right) \rangle, \\ &\langle (e_1, q, 1), \left(\frac{(u_1, s_1)^{t_3}}{(0.9, 0.7, 0.8)}, \frac{(u_2, s_1)^{t_3}}{(0.3, 0.6, 0.2)}, \frac{(u_3, s_1)^{t_3}}{(0.0, 0.0, 0.0)}, \frac{(u_1, s_2)^{t_3}}{(0.8, 0.0, 0.4)}, \frac{(u_2, s_2)^{t_3}}{(0.9, 0.3, 0.7)}, \frac{(u_3, s_2)^{t_3}}{(0.0, 0.4, 0.1)} \right) \rangle, \\ &\langle (e_2, p, 1), \left(\frac{(u_1, s_1)^{t_3}}{(0.1, 0.9, 0.7)}, \frac{(u_2, s_1)^{t_3}}{(0.8, 0.0, 0.5)}, \frac{(u_3, s_1)^{t_3}}{(0.9, 0.7, 0.2)}, \frac{(u_1, s_2)^{t_3}}{(0.5, 0.3, 0.8)}, \frac{(u_2, s_2)^{t_3}}{(0.3, 0.5, 0.5)}, \frac{(u_3, s_2)^{t_3}}{(0.4, 0.9, 0.1)} \right) \rangle, \\ &\langle (e_2, r, 1), \left(\frac{(u_1, s_1)^{t_3}}{(0.7, 0.7, 0.9)}, \frac{(u_2, s_1)^{t_3}}{(0.7, 0.5, 0.6)}, \frac{(u_3, s_1)^{t_3}}{(0.5, 0.3, 0.2)}, \frac{(u_1, s_2)^{t_3}}{(0.7, 0.6, 0.1)}, \frac{(u_2, s_2)^{t_3}}{(0.6, 0.5, 0.9)}, \frac{(u_3, s_2)^{t_3}}{(0.4, 0.3, 0.9)} \right) \rangle, \\ &\langle (e_2, q, 1), \left(\frac{(u_1, s_1)^{t_3}}{(0.8, 0.9, 0.6)}, \frac{(u_2, s_1)^{t_3}}{(0.1, 0.3, 0.4)}, \frac{(u_3, s_1)^{t_3}}{(0.3, 0.3, 0.2)}, \frac{(u_1, s_2)^{t_3}}{(0.6, 0.1, 0.3)}, \frac{(u_2, s_2)^{t_3}}{(0.9, 0.5, 0.3)}, \frac{(u_3, s_2)^{t_3}}{(0.8, 0.5, 0.8)} \right) \rangle, \\ &\langle (e_3, p, 1), \left(\frac{(u_1, s_1)^{t_3}}{(0.5, 0.0, 0.5)}, \frac{(u_2, s_1)^{t_3}}{(0.9, 0.0, 0.5)}, \frac{(u_3, s_1)^{t_3}}{(0.9, 0.8, 0.2)}, \frac{(u_1, s_2)^{t_3}}{(0.3, 0.5, 0.2)}, \frac{(u_2, s_2)^{t_3}}{(0.6, 0.0, 0.8)}, \frac{(u_3, s_2)^{t_3}}{(0.5, 0.1, 0.8)} \right) \rangle, \\ &\langle (e_3, r, 1), \left(\frac{(u_1, s_1)^{t_3}}{(0.3, 0.0, 0.7)}, \frac{(u_2, s_1)^{t_3}}{(0.1, 0.7, 0.2)}, \frac{(u_3, s_1)^{t_3}}{(0.9, 0.3, 0.7)}, \frac{(u_1, s_2)^{t_3}}{(0.4, 0.6, 0.6)}, \frac{(u_2, s_2)^{t_3}}{(0.7, 0.5, 0.7)}, \frac{(u_3, s_2)^{t_3}}{(0.8, 0.0, 0.2)} \right) \rangle, \\ &\langle (e_3, q, 1), \left(\frac{(u_1, s_1)^{t_3}}{(0.6, 0.3, 0.9)}, \frac{(u_2, s_1)^{t_3}}{(0.3, 0.4, 0.3)}, \frac{(u_3, s_1)^{t_3}}{(0.1, 0.7, 0.1)}, \frac{(u_1, s_2)^{t_3}}{(0.0, 0.0, 0.4)}, \frac{(u_2, s_2)^{t_3}}{(0.0, 0.6, 0.7)}, \frac{(u_3, s_2)^{t_3}}{(0.3, 0.5, 0.2)} \right) \rangle \} \end{aligned}$$

Definition 3.5. A disagree T-QNSES $(F_Q, A)_t$ over $U \times Q$ is a T-QNSE subset of $(F_Q, A)_t$ defined as $(F_Q, A)_t^0 = \{(F_Q^t(\alpha))^0 : \alpha \in E \times X \times \{0\}\}$.

Example 3.4 Recall example 3.1. The agreed time Q-neutrosophic soft expert sets $(F_{Q_0}, A)_t$ over $U \times Q$ is $(F_{Q_0}, Z)_t =$

$$\begin{aligned} &\{ \langle (e_1, p, 0), \left(\frac{(u_1, s_1)^{t_1}}{(0.2, 0.5, 0.3)}, \frac{(u_2, s_1)^{t_1}}{(0.8, 0.8, 0.2)}, \frac{(u_3, s_1)^{t_1}}{(0.8, 0.2, 0.4)}, \frac{(u_1, s_2)^{t_1}}{(0.1, 0.9, 0.9)}, \frac{(u_2, s_2)^{t_1}}{(0.2, 0.4, 0.3)}, \frac{(u_3, s_2)^{t_1}}{(0.6, 0.7, 0.0)} \right) \rangle, \\ &\langle (e_1, r, 0), \left(\frac{(u_1, s_1)^{t_1}}{(0.5, 0.9, 0.1)}, \frac{(u_2, s_1)^{t_1}}{(0.3, 0.4, 0.1)}, \frac{(u_3, s_1)^{t_1}}{(0.1, 0.7, 0.2)}, \frac{(u_1, s_2)^{t_1}}{(0.1, 0.0, 0.9)}, \frac{(u_2, s_2)^{t_1}}{(0.9, 0.6, 0.1)}, \frac{(u_3, s_2)^{t_1}}{(0.1, 0.7, 0.4)} \right) \rangle, \\ &\langle (e_1, q, 0), \left(\frac{(u_1, s_1)^{t_1}}{(0.4, 0.7, 0.5)}, \frac{(u_2, s_1)^{t_1}}{(0.7, 0.5, 0.5)}, \frac{(u_3, s_1)^{t_1}}{(0.1, 0.5, 0.8)}, \frac{(u_1, s_2)^{t_1}}{(0.8, 0.3, 0.1)}, \frac{(u_2, s_2)^{t_1}}{(0.4, 0.3, 0.8)}, \frac{(u_3, s_2)^{t_1}}{(0.6, 0.4, 0.0)} \right) \rangle, \end{aligned}$$

$$\begin{aligned}
& \langle (e_2, p, 0), \left(\frac{(u_1, s_1)^{t_1}}{(0.6, 0.2, 0.4)}, \frac{(u_2, s_1)^{t_1}}{(0.4, 0.1, 0.3)}, \frac{(u_3, s_1)^{t_1}}{(0.4, 0.8, 0.7)}, \frac{(u_1, s_2)^{t_1}}{(0.2, 0.8, 0.9)}, \frac{(u_2, s_2)^{t_1}}{(0.9, 0.2, 0.9)}, \frac{(u_3, s_2)^{t_1}}{(0.0, 0.8, 0.5)} \right) \rangle, \\
& \langle (e_2, r, 0), \left(\frac{(u_1, s_1)^{t_1}}{(0.3, 0.9, 0.4)}, \frac{(u_2, s_1)^{t_1}}{(0.4, 0.8, 0.5)}, \frac{(u_3, s_1)^{t_1}}{(0.7, 0.2, 0.7)}, \frac{(u_1, s_2)^{t_1}}{(0.8, 0.9, 0.9)}, \frac{(u_2, s_2)^{t_1}}{(0.4, 0.9, 0.8)}, \frac{(u_3, s_2)^{t_1}}{(0.3, 0.2, 0.1)} \right) \rangle, \\
& \langle (e_2, q, 0), \left(\frac{(u_1, s_1)^{t_1}}{(0.3, 0.6, 0.1)}, \frac{(u_2, s_1)^{t_1}}{(0.1, 0.3, 0.3)}, \frac{(u_3, s_1)^{t_1}}{(0.3, 0.0, 0.0)}, \frac{(u_1, s_2)^{t_1}}{(0.6, 0.6, 0.6)}, \frac{(u_2, s_2)^{t_1}}{(0.8, 0.8, 0.9)}, \frac{(u_3, s_2)^{t_1}}{(0.4, 0.0, 0.8)} \right) \rangle, \\
& \langle (e_3, p, 0), \left(\frac{(u_1, s_1)^{t_1}}{(0.5, 0.3, 0.1)}, \frac{(u_2, s_1)^{t_1}}{(0.0, 0.3, 0.1)}, \frac{(u_3, s_1)^{t_1}}{(0.3, 0.9, 0.9)}, \frac{(u_1, s_2)^{t_1}}{(0.4, 0.3, 0.7)}, \frac{(u_2, s_2)^{t_1}}{(0.0, 0.2, 0.9)}, \frac{(u_3, s_2)^{t_1}}{(0.7, 0.7, 0.0)} \right) \rangle, \\
& \langle (e_3, r, 0), \left(\frac{(u_1, s_1)^{t_1}}{(0.7, 0.8, 0.0)}, \frac{(u_2, s_1)^{t_1}}{(0.8, 0.3, 0.2)}, \frac{(u_3, s_1)^{t_1}}{(0.1, 0.5, 0.3)}, \frac{(u_1, s_2)^{t_1}}{(0.3, 0.5, 0.3)}, \frac{(u_2, s_2)^{t_1}}{(0.1, 0.7, 0.0)}, \frac{(u_3, s_2)^{t_1}}{(0.2, 0.2, 0.7)} \right) \rangle, \\
& \langle (e_3, q, 0), \left(\frac{(u_1, s_1)^{t_1}}{(0.5, 0.0, 0.4)}, \frac{(u_2, s_1)^{t_1}}{(0.9, 0.5, 0.6)}, \frac{(u_3, s_1)^{t_1}}{(0.7, 0.2, 0.1)}, \frac{(u_1, s_2)^{t_1}}{(0.0, 0.6, 0.9)}, \frac{(u_2, s_2)^{t_1}}{(0.9, 0.8, 0.8)}, \frac{(u_3, s_2)^{t_1}}{(0.4, 0.3, 0.6)} \right) \rangle, \\
& \langle (e_1, p, 0), \left(\frac{(u_1, s_1)^{t_2}}{(0.7, 0.0, 0.3)}, \frac{(u_2, s_1)^{t_2}}{(0.0, 0.7, 0.0)}, \frac{(u_3, s_1)^{t_2}}{(0.2, 0.7, 0.1)}, \frac{(u_1, s_2)^{t_2}}{(0.1, 0.6, 0.4)}, \frac{(u_2, s_2)^{t_2}}{(0.7, 0.6, 0.4)}, \frac{(u_3, s_2)^{t_2}}{(0.3, 0.3, 0.0)} \right) \rangle, \\
& \langle (e_1, r, 0), \left(\frac{(u_1, s_1)^{t_2}}{(0.7, 0.1, 0.6)}, \frac{(u_2, s_1)^{t_2}}{(0.0, 0.2, 0.0)}, \frac{(u_3, s_1)^{t_2}}{(0.3, 0.2, 0.2)}, \frac{(u_1, s_2)^{t_2}}{(0.3, 0.1, 0.4)}, \frac{(u_2, s_2)^{t_2}}{(0.3, 0.7, 0.9)}, \frac{(u_3, s_2)^{t_2}}{(0.0, 0.7, 0.7)} \right) \rangle, \\
& \langle (e_1, q, 0), \left(\frac{(u_1, s_1)^{t_2}}{(0.3, 0.6, 0.7)}, \frac{(u_2, s_1)^{t_2}}{(0.9, 0.0, 0.1)}, \frac{(u_3, s_1)^{t_2}}{(0.3, 0.1, 0.6)}, \frac{(u_1, s_2)^{t_2}}{(0.4, 0.3, 0.6)}, \frac{(u_2, s_2)^{t_2}}{(0.1, 0.0, 0.8)}, \frac{(u_3, s_2)^{t_2}}{(0.1, 0.8, 0.3)} \right) \rangle, \\
& \langle (e_2, p, 0), \left(\frac{(u_1, s_1)^{t_2}}{(0.7, 0.3, 0.5)}, \frac{(u_2, s_1)^{t_2}}{(0.1, 0.2, 0.8)}, \frac{(u_3, s_1)^{t_2}}{(0.3, 0.7, 0.0)}, \frac{(u_1, s_2)^{t_2}}{(0.6, 0.3, 0.8)}, \frac{(u_2, s_2)^{t_2}}{(0.3, 0.9, 0.1)}, \frac{(u_3, s_2)^{t_2}}{(0.3, 0.5, 0.7)} \right) \rangle, \\
& \langle (e_2, r, 0), \left(\frac{(u_1, s_1)^{t_2}}{(0.6, 0.3, 0.0)}, \frac{(u_2, s_1)^{t_2}}{(0.0, 0.3, 0.0)}, \frac{(u_3, s_1)^{t_2}}{(0.1, 0.1, 0.0)}, \frac{(u_1, s_2)^{t_2}}{(0.4, 0.8, 0.0)}, \frac{(u_2, s_2)^{t_2}}{(0.8, 0.8, 0.3)}, \frac{(u_3, s_2)^{t_2}}{(0.5, 0.5, 0.4)} \right) \rangle, \\
& \langle (e_2, q, 0), \left(\frac{(u_1, s_1)^{t_2}}{(0.7, 0.3, 0.0)}, \frac{(u_2, s_1)^{t_2}}{(0.2, 0.1, 0.6)}, \frac{(u_3, s_1)^{t_2}}{(0.1, 0.1, 0.2)}, \frac{(u_1, s_2)^{t_2}}{(0.4, 0.1, 0.2)}, \frac{(u_2, s_2)^{t_2}}{(0.5, 0.1, 0.2)}, \frac{(u_3, s_2)^{t_2}}{(0.1, 0.4, 0.2)} \right) \rangle, \\
& \langle (e_3, p, 0), \left(\frac{(u_1, s_1)^{t_2}}{(0.4, 0.5, 0.2)}, \frac{(u_2, s_1)^{t_2}}{(0.6, 0.0, 0.8)}, \frac{(u_3, s_1)^{t_2}}{(0.5, 0.6, 0.7)}, \frac{(u_1, s_2)^{t_2}}{(0.7, 0.2, 0.5)}, \frac{(u_2, s_2)^{t_2}}{(0.9, 0.6, 0.2)}, \frac{(u_3, s_2)^{t_2}}{(0.2, 0.5, 0.9)} \right) \rangle, \\
& \langle (e_3, r, 0), \left(\frac{(u_1, s_1)^{t_2}}{(0.1, 0.6, 0.6)}, \frac{(u_2, s_1)^{t_2}}{(0.4, 0.2, 0.8)}, \frac{(u_3, s_1)^{t_2}}{(0.3, 0.9, 0.8)}, \frac{(u_1, s_2)^{t_2}}{(0.5, 0.7, 0.7)}, \frac{(u_2, s_2)^{t_2}}{(0.5, 0.5, 0.4)}, \frac{(u_3, s_2)^{t_2}}{(0.2, 0.1, 0.5)} \right) \rangle, \\
& \langle (e_3, q, 0), \left(\frac{(u_1, s_1)^{t_2}}{(0.8, 0.1, 0.8)}, \frac{(u_2, s_1)^{t_2}}{(0.5, 0.0, 0.2)}, \frac{(u_3, s_1)^{t_2}}{(0.7, 0.6, 0.8)}, \frac{(u_1, s_2)^{t_2}}{(0.4, 0.4, 0.8)}, \frac{(u_2, s_2)^{t_2}}{(0.7, 0.1, 0.4)}, \frac{(u_3, s_2)^{t_2}}{(0.2, 0.2, 0.2)} \right) \rangle, \\
& \langle (e_1, p, 0), \left(\frac{(u_1, s_1)^{t_3}}{(0.2, 0.1, 0.6)}, \frac{(u_2, s_1)^{t_3}}{(0.0, 0.2, 0.1)}, \frac{(u_3, s_1)^{t_3}}{(0.2, 0.6, 0.3)}, \frac{(u_1, s_2)^{t_3}}{(0.1, 0.7, 0.9)}, \frac{(u_2, s_2)^{t_3}}{(0.7, 0.9, 0.5)}, \frac{(u_3, s_2)^{t_3}}{(0.3, 0.8, 0.8)} \right) \rangle, \\
& \langle (e_1, r, 0), \left(\frac{(u_1, s_1)^{t_3}}{(0.4, 0.6, 0.3)}, \frac{(u_2, s_1)^{t_3}}{(0.7, 0.1, 0.1)}, \frac{(u_3, s_1)^{t_3}}{(0.3, 0.1, 0.3)}, \frac{(u_1, s_2)^{t_3}}{(0.6, 0.5, 0.3)}, \frac{(u_2, s_2)^{t_3}}{(0.2, 0.0, 0.3)}, \frac{(u_3, s_2)^{t_3}}{(0.6, 0.8, 0.6)} \right) \rangle, \\
& \langle (e_1, q, 0), \left(\frac{(u_1, s_1)^{t_3}}{(0.0, 0.5, 0.2)}, \frac{(u_2, s_1)^{t_3}}{(0.1, 0.4, 0.0)}, \frac{(u_3, s_1)^{t_3}}{(0.6, 0.6, 0.6)}, \frac{(u_1, s_2)^{t_3}}{(0.8, 0.8, 0.8)}, \frac{(u_2, s_2)^{t_3}}{(0.6, 0.4, 0.7)}, \frac{(u_3, s_2)^{t_3}}{(0.7, 0.7, 0.0)} \right) \rangle,
\end{aligned}$$

$$\begin{aligned} & \langle (e_2, p, 0), \left(\frac{(u_1, s_1)^{t_3}}{(0.4, 0.4, 0.0)}, \frac{(u_2, s_1)^{t_3}}{(0.3, 0.6, 0.3)}, \frac{(u_3, s_1)^{t_3}}{(0.6, 0.7, 0.5)}, \frac{(u_1, s_2)^{t_3}}{(0.4, 0.3, 0.3)}, \frac{(u_2, s_2)^{t_3}}{(0.2, 0.6, 0.1)}, \frac{(u_3, s_2)^{t_3}}{(0.7, 0.5, 0.8)} \right) \rangle, \\ & \langle (e_2, r, 0), \left(\frac{(u_1, s_1)^{t_3}}{(0.9, 0.1, 0.2)}, \frac{(u_2, s_1)^{t_3}}{(0.8, 0.8, 0.8)}, \frac{(u_3, s_1)^{t_3}}{(0.9, 0.2, 0.2)}, \frac{(u_1, s_2)^{t_3}}{(0.7, 0.8, 0.3)}, \frac{(u_2, s_2)^{t_3}}{(0.0, 0.9, 0.7)}, \frac{(u_3, s_2)^{t_3}}{(0.3, 0.8, 0.7)} \right) \rangle, \\ & \langle (e_2, q, 0), \left(\frac{(u_1, s_1)^{t_3}}{(0.4, 0.1, 0.5)}, \frac{(u_2, s_1)^{t_3}}{(0.7, 0.8, 0.5)}, \frac{(u_3, s_1)^{t_3}}{(0.6, 0.2, 0.9)}, \frac{(u_1, s_2)^{t_3}}{(0.5, 0.9, 0.5)}, \frac{(u_2, s_2)^{t_3}}{(0.1, 0.1, 0.5)}, \frac{(u_3, s_2)^{t_3}}{(0.6, 0.8, 0.2)} \right) \rangle, \\ & \langle (e_3, p, 0), \left(\frac{(u_1, s_1)^{t_3}}{(0.8, 0.7, 0.0)}, \frac{(u_2, s_1)^{t_3}}{(0.2, 0.1, 0.2)}, \frac{(u_3, s_1)^{t_3}}{(0.7, 0.4, 0.5)}, \frac{(u_1, s_2)^{t_3}}{(0.1, 0.6, 0.6)}, \frac{(u_2, s_2)^{t_3}}{(0.9, 0.8, 0.3)}, \frac{(u_3, s_2)^{t_3}}{(0.8, 0.8, 0.7)} \right) \rangle, \\ & \langle (e_3, r, 0), \left(\frac{(u_1, s_1)^{t_3}}{(0.5, 0.1, 0.3)}, \frac{(u_2, s_1)^{t_3}}{(0.5, 0.8, 0.2)}, \frac{(u_3, s_1)^{t_3}}{(0.7, 0.5, 0.0)}, \frac{(u_1, s_2)^{t_3}}{(0.0, 0.7, 0.3)}, \frac{(u_2, s_2)^{t_3}}{(0.7, 0.4, 0.8)}, \frac{(u_3, s_2)^{t_3}}{(0.5, 0.2, 0.5)} \right) \rangle, \\ & \langle (e_3, q, 0), \left(\frac{(u_1, s_1)^{t_3}}{(0.4, 0.3, 0.8)}, \frac{(u_2, s_1)^{t_3}}{(0.0, 0.7, 0.7)}, \frac{(u_3, s_1)^{t_3}}{(0.5, 0.5, 0.6)}, \frac{(u_1, s_2)^{t_3}}{(0.7, 0.7, 0.3)}, \frac{(u_2, s_2)^{t_3}}{(0.3, 0.7, 0.1)}, \frac{(u_3, s_2)^{t_3}}{(0.6, 0.3, 0.8)} \right) \rangle \} \end{aligned}$$

4. Operations on TQNSSES.

Definitions of complement, union, and the intersection of the new concept T-QNSSES are introduced in this section with some properties and illustrative examples.

Definition 4.1. The complement of a time Q-neutrosophic soft expert set $(F_Q, A)_t$ is denoted by $(F_Q, A)_t^c$ is defined by $(F_Q, A)_t^c = (F_Q^c, A)_t, \forall t \in T$ such that $c(F_Q^t): A \rightarrow N(U \times Q)$

$$(F_Q, A)_t^c = \{T_{F^c(\alpha)} = F_{F(\alpha)}, I_{F^c(\alpha)} = 1 - I_{F(\alpha)}, F_{F^c(\alpha)} = T_{F(\alpha)}: \alpha \in A, \text{ and } t \in T\}$$

Example 4.1 Consider example 3.2.

$$\begin{aligned} (F_Q, A)_t &= \{ \langle (e_1, p, 1), \left(\frac{(u_1, s_1)^{t_1}}{(0.1, 0.1, 0.3)}, \frac{(u_2, s_1)^{t_1}}{(0.9, 0.7, 0.2)}, \frac{(u_3, s_1)^{t_1}}{(0.1, 0.2, 0.8)}, \frac{(u_1, s_2)^{t_1}}{(0.0, 0.1, 0.4)}, \frac{(u_2, s_2)^{t_1}}{(0.0, 0.7, 0.8)}, \frac{(u_3, s_2)^{t_1}}{(0.4, 0.2, 0.8)} \right) \rangle, \\ & \langle (e_2, p, 0), \left(\frac{(u_1, s_1)^{t_1}}{(0.6, 0.2, 0.4)}, \frac{(u_2, s_1)^{t_1}}{(0.4, 0.1, 0.3)}, \frac{(u_3, s_1)^{t_1}}{(0.4, 0.8, 0.7)}, \frac{(u_1, s_2)^{t_1}}{(0.2, 0.8, 0.9)}, \frac{(u_2, s_2)^{t_1}}{(0.9, 0.2, 0.9)}, \frac{(u_3, s_2)^{t_1}}{(0.0, 0.8, 0.5)} \right) \rangle, \\ & \langle (e_2, q, 1), \left(\frac{(u_1, s_1)^{t_2}}{(0.2, 0.2, 0.8)}, \frac{(u_2, s_1)^{t_2}}{(0.7, 0.8, 0.9)}, \frac{(u_3, s_1)^{t_2}}{(0.3, 0.9, 0.7)}, \frac{(u_1, s_2)^{t_2}}{(0.6, 0.5, 0.4)}, \frac{(u_2, s_2)^{t_2}}{(0.9, 0.1, 0.6)}, \frac{(u_3, s_2)^{t_2}}{(0.8, 0.6, 0.2)} \right) \rangle, \\ & \langle (e_2, q, 0), \left(\frac{(u_1, s_1)^{t_2}}{(0.7, 0.3, 0.0)}, \frac{(u_2, s_1)^{t_2}}{(0.2, 0.1, 0.6)}, \frac{(u_3, s_1)^{t_2}}{(0.1, 0.1, 0.2)}, \frac{(u_1, s_2)^{t_2}}{(0.4, 0.1, 0.2)}, \frac{(u_2, s_2)^{t_2}}{(0.5, 0.1, 0.2)}, \frac{(u_3, s_2)^{t_2}}{(0.1, 0.4, 0.2)} \right) \rangle, \\ & \langle (e_3, r, 1), \left(\frac{(u_1, s_1)^{t_3}}{(0.3, 0.0, 0.7)}, \frac{(u_2, s_1)^{t_3}}{(0.1, 0.7, 0.2)}, \frac{(u_3, s_1)^{t_3}}{(0.9, 0.3, 0.7)}, \frac{(u_1, s_2)^{t_3}}{(0.4, 0.6, 0.6)}, \frac{(u_2, s_2)^{t_3}}{(0.7, 0.5, 0.7)}, \frac{(u_3, s_2)^{t_3}}{(0.8, 0.0, 0.2)} \right) \rangle \} \end{aligned}$$

Then, $(F_Q, A)_t^c =$

$$\begin{aligned} & \{ \langle (e_1, p, 1), \left(\frac{(u_1, s_1)^{t_1}}{(0.3, 0.9, 0.1)}, \frac{(u_2, s_1)^{t_1}}{(0.2, 0.3, 0.9)}, \frac{(u_3, s_1)^{t_1}}{(0.8, 0.8, 0.1)}, \frac{(u_1, s_2)^{t_1}}{(0.4, 0.9, 0.0)}, \frac{(u_2, s_2)^{t_1}}{(0.8, 0.3, 0.0)}, \frac{(u_3, s_2)^{t_1}}{(0.8, 0.8, 0.4)} \right) \rangle, \right. \\ & \langle (e_2, p, 0), \left(\frac{(u_1, s_1)^{t_1}}{(0.4, 0.8, 0.6)}, \frac{(u_2, s_1)^{t_1}}{(0.3, 0.9, 0.4)}, \frac{(u_3, s_1)^{t_1}}{(0.7, 0.2, 0.4)}, \frac{(u_1, s_2)^{t_1}}{(0.9, 0.2, 0.2)}, \frac{(u_2, s_2)^{t_1}}{(0.9, 0.8, 0.9)}, \frac{(u_3, s_2)^{t_1}}{(0.5, 0.2, 0.0)} \right) \rangle, \\ & \langle (e_2, q, 1), \left(\frac{(u_1, s_1)^{t_2}}{(0.8, 0.8, 0.2)}, \frac{(u_2, s_1)^{t_2}}{(0.9, 0.2, 0.7)}, \frac{(u_3, s_1)^{t_2}}{(0.7, 0.1, 0.3)}, \frac{(u_1, s_2)^{t_2}}{(0.4, 0.5, 0.6)}, \frac{(u_2, s_2)^{t_2}}{(0.6, 0.9, 0.9)}, \frac{(u_3, s_2)^{t_2}}{(0.2, 0.4, 0.8)} \right) \rangle, \\ & \langle (e_2, q, 0), \left(\frac{(u_1, s_1)^{t_2}}{(0.0, 0.7, 0.7)}, \frac{(u_2, s_1)^{t_2}}{(0.6, 0.9, 0.2)}, \frac{(u_3, s_1)^{t_2}}{(0.2, 0.9, 0.1)}, \frac{(u_1, s_2)^{t_2}}{(0.2, 0.9, 0.4)}, \frac{(u_2, s_2)^{t_2}}{(0.2, 0.9, 0.5)}, \frac{(u_3, s_2)^{t_2}}{(0.2, 0.6, 0.1)} \right) \rangle, \\ & \left. \langle (e_3, r, 1), \left(\frac{(u_1, s_1)^{t_3}}{(0.7, 0.1, 0.3)}, \frac{(u_2, s_1)^{t_3}}{(0.2, 0.3, 0.1)}, \frac{(u_3, s_1)^{t_3}}{(0.7, 0.7, 0.9)}, \frac{(u_1, s_2)^{t_3}}{(0.6, 0.4, 0.4)}, \frac{(u_2, s_2)^{t_3}}{(0.7, 0.5, 0.7)}, \frac{(u_3, s_2)^{t_3}}{(0.2, 1, 0.8)} \right) \rangle \right\} \end{aligned}$$

Proposition 4.1. If $(F_Q, A)_t$ is a time Q-neutrosophic soft expert sets over $U \times Q$ then $((F_Q, A)_t^c)^c = (F_Q, A)_t$

Proof. $\forall t \in T$, from definition (complement 4.1) we have $(F_Q, A)_t^c = (F_Q^c, A)_t$

Let t_0 be fixed time $(F_Q, A)_{t_0}^c$ is a Q-NSES at this time. Then:

$$(F_Q^{t_0}(\alpha))^c = \{ T_{F^c(\alpha)} = F_{F(\alpha)}, I_{F^c(\alpha)} = 1 - I_{F(\alpha)}, F_{F^c(\alpha)} = T_{F(\alpha)}, \forall \alpha \in A \}.$$

Now, $((F_Q, A)_t^c)^c = ((F_Q^c, A)_t)^c$, where $(F_Q^{t_0}(\alpha))^c = [T_{F^c(\alpha)} = F_{F(\alpha)}, I_{F^c(\alpha)} = 1 - I_{F(\alpha)}, F_{F^c(\alpha)} = T_{F(\alpha)}]^c$

$$\begin{aligned} &= [T_{F(\alpha)} = F_{F^c(\alpha)}, I_{F(\alpha)} = 1 - I_{F^c(\alpha)}, F_{F(\alpha)} = T_{F^c(\alpha)}] \\ &= [T_{F(\alpha)} = F_{F^c(\alpha)}, I_{F(\alpha)} = 1 - (1 - I_{F(\alpha)}), F_{F(\alpha)} = T_{F^c(\alpha)}] \\ &= [T_{F(\alpha)} = F_{F^c(\alpha)}, I_{F(\alpha)} = I_{F(\alpha)}, F_{F(\alpha)} = T_{F^c(\alpha)}] = (F_Q, A)_t \quad \forall \alpha \in A \end{aligned}$$

Since t_0 is arbitrary, then $((F_Q, A)_t^c)^c = (F_Q, A)_t$ is true $\forall t \in T$.

Definition 4.2. Let $(F_Q, A)_t$ and $(G_Q, B)_t$ be two T-QNSEs over $U \times Q$. The union $(H_Q, C)_t = (F_Q, A)_t \tilde{\cup} (G_Q, B)_t$ is the T-QNSES where $C = A \cup B$ and is defined as follows: $\forall t \in T$, Let $c \in C$

$$H_Q^t(c) = \begin{cases} (F_Q, A)_t & c \in A - B \\ (G_Q, B)_t & c \in B - A \\ (F_Q, A)_t \cup^Q (G_Q, B)_t & c \in A \cup B \\ (F_Q, A)_t \cap^Q (G_Q, B)_t & c \in A \cap B \end{cases}$$

where \cup^Q is a QNSES union.

Example 4.2. Suppose that $(F_Q, A)_t$ and $(G_Q, B)_t$ are two T-QNSES over $U \times Q$ such that

$$\begin{aligned} (F_Q, A)_t &= \{ \langle (e_1, p, 1), \left(\frac{(u_1, s_1)^{t_1}}{(0.5, 0.1, 0.7)}, \frac{(u_2, s_1)^{t_1}}{(0.4, 0.5, 0.1)}, \frac{(u_3, s_1)^{t_1}}{(0.7, 0.1, 0.4)}, \frac{(u_1, s_2)^{t_1}}{(0.2, 0.1, 0.7)}, \frac{(u_2, s_2)^{t_1}}{(0.4, 0.3, 0.1)}, \frac{(u_3, s_2)^{t_1}}{(0.3, 0.1, 0.2)} \right) \rangle, \right. \\ & \langle (e_2, p, 0), \left(\frac{(u_1, s_1)^{t_1}}{(0.5, 0.7, 0.3)}, \frac{(u_2, s_1)^{t_1}}{(0.7, 0.5, 0.8)}, \frac{(u_3, s_1)^{t_1}}{(0.4, 0.1, 0.9)}, \frac{(u_1, s_2)^{t_1}}{(0.4, 0.5, 0.6)}, \frac{(u_2, s_2)^{t_1}}{(0.5, 0.6, 0.7)}, \frac{(u_3, s_2)^{t_1}}{(0.4, 0.3, 0.2)} \right) \rangle, \\ & \left. \langle (e_2, q, 1), \left(\frac{(u_1, s_1)^{t_2}}{(0.2, 0.2, 0.8)}, \frac{(u_2, s_1)^{t_2}}{(0.7, 0.8, 0.9)}, \frac{(u_3, s_1)^{t_2}}{(0.3, 0.9, 0.7)}, \frac{(u_1, s_2)^{t_2}}{(0.6, 0.5, 0.4)}, \frac{(u_2, s_2)^{t_2}}{(0.9, 0.1, 0.6)}, \frac{(u_3, s_2)^{t_2}}{(0.8, 0.6, 0.2)} \right) \rangle \right\}, \end{aligned}$$

$$\begin{aligned} & \langle (e_2, q, 0), \left(\frac{(u_1, s_1)^{t_2}}{(0.7, 0.3, 0.0)}, \frac{(u_2, s_1)^{t_2}}{(0.2, 0.1, 0.6)}, \frac{(u_3, s_1)^{t_2}}{(0.1, 0.1, 0.2)}, \frac{(u_1, s_2)^{t_2}}{(0.4, 0.1, 0.2)}, \frac{(u_2, s_2)^{t_2}}{(0.5, 0.1, 0.2)}, \frac{(u_3, s_2)^{t_2}}{(0.1, 0.4, 0.2)} \right) \rangle, \\ & \langle (e_3, r, 1), \left(\frac{(u_1, s_1)^{t_3}}{(0.3, 0.0, 0.7)}, \frac{(u_2, s_1)^{t_3}}{(0.1, 0.7, 0.2)}, \frac{(u_3, s_1)^{t_3}}{(0.9, 0.3, 0.7)}, \frac{(u_1, s_2)^{t_3}}{(0.4, 0.6, 0.6)}, \frac{(u_2, s_2)^{t_3}}{(0.7, 0.5, 0.7)}, \frac{(u_3, s_2)^{t_3}}{(0.8, 0.0, 0.2)} \right) \rangle \} \\ (G_Q, B)_t = & \{ \langle (e_1, p, 1), \left(\frac{(u_1, s_1)^{t_1}}{(0.6, 0.4, 0.5)}, \frac{(u_2, s_1)^{t_1}}{(0.5, 0.3, 0.6)}, \frac{(u_3, s_1)^{t_1}}{(0.3, 0.5, 0.2)}, \frac{(u_1, s_2)^{t_1}}{(0.2, 0.4, 0.6)}, \frac{(u_2, s_2)^{t_1}}{(0.4, 0.1, 0.7)}, \frac{(u_3, s_2)^{t_1}}{(0.3, 0.4, 0.8)} \right) \rangle, \right. \\ & \langle (e_2, p, 0), \left(\frac{(u_1, s_1)^{t_1}}{(0.6, 0.7, 0.1)}, \frac{(u_2, s_1)^{t_1}}{(0.8, 0.4, 0.7)}, \frac{(u_3, s_1)^{t_1}}{(0.5, 0.1, 0.7)}, \frac{(u_1, s_2)^{t_1}}{(0.5, 0.3, 0.1)}, \frac{(u_2, s_2)^{t_1}}{(0.6, 0.4, 0.5)}, \frac{(u_3, s_2)^{t_1}}{(0.5, 0.3, 0.2)} \right) \rangle, \\ & \left. \langle (e_3, r, 0), \left(\frac{(u_1, s_1)^{t_3}}{(0.5, 0.1, 0.3)}, \frac{(u_2, s_1)^{t_3}}{(0.5, 0.8, 0.2)}, \frac{(u_3, s_1)^{t_3}}{(0.7, 0.5, 0.0)}, \frac{(u_1, s_2)^{t_3}}{(0.0, 0.7, 0.3)}, \frac{(u_2, s_2)^{t_3}}{(0.7, 0.4, 0.8)}, \frac{(u_3, s_2)^{t_3}}{(0.5, 0.2, 0.5)} \right) \rangle \right\} \end{aligned}$$

Then $(F_Q, A)_t \tilde{\cup} (G_Q, B)_t = (H_Q, C)_t$ where

$$\begin{aligned} (H_Q, C)_t = & \{ \langle (e_1, p, 1), \left(\frac{(u_1, s_1)^{t_1}}{(0.6, 0.1, 0.5)}, \frac{(u_2, s_1)^{t_1}}{(0.5, 0.3, 0.1)}, \frac{(u_3, s_1)^{t_1}}{(0.7, 0.1, 0.2)}, \frac{(u_1, s_2)^{t_1}}{(0.2, 0.1, 0.6)}, \frac{(u_2, s_2)^{t_1}}{(0.4, 0.1, 0.1)}, \frac{(u_3, s_2)^{t_1}}{(0.3, 0.1, 0.2)} \right) \rangle, \right. \\ & \langle (e_2, p, 0), \left(\frac{(u_1, s_1)^{t_1}}{(0.6, 0.7, 0.1)}, \frac{(u_2, s_1)^{t_1}}{(0.8, 0.4, 0.7)}, \frac{(u_3, s_1)^{t_1}}{(0.5, 0.1, 0.7)}, \frac{(u_1, s_2)^{t_1}}{(0.5, 0.3, 0.1)}, \frac{(u_2, s_2)^{t_1}}{(0.6, 0.4, 0.5)}, \frac{(u_3, s_2)^{t_1}}{(0.5, 0.3, 0.2)} \right) \rangle \} \\ & \langle (e_2, q, 1), \left(\frac{(u_1, s_1)^{t_2}}{(0.2, 0.2, 0.8)}, \frac{(u_2, s_1)^{t_2}}{(0.7, 0.8, 0.9)}, \frac{(u_3, s_1)^{t_2}}{(0.3, 0.9, 0.7)}, \frac{(u_1, s_2)^{t_2}}{(0.6, 0.5, 0.4)}, \frac{(u_2, s_2)^{t_2}}{(0.9, 0.1, 0.6)}, \frac{(u_3, s_2)^{t_2}}{(0.8, 0.6, 0.2)} \right) \rangle, \\ & \langle (e_2, q, 0), \left(\frac{(u_1, s_1)^{t_2}}{(0.7, 0.3, 0.0)}, \frac{(u_2, s_1)^{t_2}}{(0.2, 0.1, 0.6)}, \frac{(u_3, s_1)^{t_2}}{(0.1, 0.1, 0.2)}, \frac{(u_1, s_2)^{t_2}}{(0.4, 0.1, 0.2)}, \frac{(u_2, s_2)^{t_2}}{(0.5, 0.1, 0.2)}, \frac{(u_3, s_2)^{t_2}}{(0.1, 0.4, 0.2)} \right) \rangle, \\ & \langle (e_3, r, 1), \left(\frac{(u_1, s_1)^{t_3}}{(0.3, 0.0, 0.7)}, \frac{(u_2, s_1)^{t_3}}{(0.1, 0.7, 0.2)}, \frac{(u_3, s_1)^{t_3}}{(0.9, 0.3, 0.7)}, \frac{(u_1, s_2)^{t_3}}{(0.4, 0.6, 0.6)}, \frac{(u_2, s_2)^{t_3}}{(0.7, 0.5, 0.7)}, \frac{(u_3, s_2)^{t_3}}{(0.8, 0.0, 0.2)} \right) \rangle, \\ & \left. \langle (e_3, r, 0), \left(\frac{(u_1, s_1)^{t_3}}{(0.5, 0.1, 0.3)}, \frac{(u_2, s_1)^{t_3}}{(0.5, 0.8, 0.2)}, \frac{(u_3, s_1)^{t_3}}{(0.7, 0.5, 0.0)}, \frac{(u_1, s_2)^{t_3}}{(0.0, 0.7, 0.3)}, \frac{(u_2, s_2)^{t_3}}{(0.7, 0.4, 0.8)}, \frac{(u_3, s_2)^{t_3}}{(0.5, 0.2, 0.5)} \right) \rangle \right\} \end{aligned}$$

Proposition 4.2. If $(F_Q, A)_t, (G_Q, B)_t$, and $(H_Q, C)_t$ are three T-QNSEs over $U \times Q$, then

- (i) $(F_Q, A)_t \tilde{\cup} ((G_Q, B)_t \tilde{\cup} (H_Q, C)_t) = ((F_Q, A)_t \tilde{\cup} (G_Q, B)_t) \tilde{\cup} (H_Q, C)_t$
- (ii) $(F_Q, A)_t \tilde{\cup} (G_Q, B)_t \cong (F_Q, A)_t$

Proof: The proof is straightforward.

Definition 4.3. The intersection of two T-QNSEs $(F_Q, A)_t$ and $(G_Q, B)_t$ over $U \times Q$ denoted by $(F_Q, A)_t \tilde{\cap} (G_Q, B)_t$ is the T-QNSEs $(H_Q, C)_t$ where $C = A \cap B$ and is defined as follows:

$$\forall t \in T, \text{ Let } c \in C$$

$$H_Q^t(c) = ((F_Q, A)_t \cap^Q (G_Q, B)_t) \quad c \in A \cap B$$

where \cap^Q is a QNSEs intersection.

Example 4.3 Consider example 3.2. Then the intersection $(F_Q, A)_t \tilde{\cap} (G_Q, B)_t = (K_Q, C)_t$ give as follows:

$$(K_Q, C)_t = \left\{ \left((e_1, p, 1), \left(\frac{(u_1, s_1)^{t_1}}{(0.5, 0.1, 0.7)}, \frac{(u_2, s_1)^{t_1}}{(0.4, 0.3, 0.6)}, \frac{(u_3, s_1)^{t_1}}{(0.3, 0.1, 0.4)}, \frac{(u_1, s_2)^{t_1}}{(0.2, 0.1, 0.7)}, \frac{(u_2, s_2)^{t_1}}{(0.4, 0.1, 0.7)}, \frac{(u_3, s_2)^{t_1}}{(0.3, 0.1, 0.8)} \right) \right), \right. \\ \left. \left((e_2, p, 0), \left(\frac{(u_1, s_1)^{t_1}}{(0.5, 0.7, 0.3)}, \frac{(u_2, s_1)^{t_1}}{(0.7, 0.4, 0.8)}, \frac{(u_3, s_1)^{t_1}}{(0.4, 0.1, 0.9)}, \frac{(u_1, s_2)^{t_1}}{(0.4, 0.3, 0.6)}, \frac{(u_2, s_2)^{t_1}}{(0.5, 0.4, 0.7)}, \frac{(u_3, s_2)^{t_1}}{(0.4, 0.3, 0.2)} \right) \right) \right\}$$

Proposition 4.3. If $(F_Q, A)_t$, $(G_Q, B)_t$, and $(H_Q, C)_t$ are three T-QNSEs over $U \times Q$, then

- (i) $(F_Q, A)_t \tilde{\cap} ((G_Q, B)_t \tilde{\cap} (H_Q, C)_t) = ((F_Q, A)_t \tilde{\cap} (G_Q, B)_t) \tilde{\cap} (H_Q, C)_t$
- (ii) $(F_Q, A)_t \tilde{\cap} (G_Q, B)_t \subseteq (F_Q, A)_t$

Proof: The proof is straightforward.

Proposition 4.4. If $(F_Q, A)_t$, $(G_Q, B)_t$ and $(K_Q, C)_t$ are three T-QNSEs over $U \times Q$ then $\forall t \in T$

- (i) $(F_Q, A)_t \tilde{\cup} ((G_Q, B)_t \tilde{\cap} (K_Q, C)_t) = ((F_Q, A)_t \tilde{\cup} (G_Q, B)_t) \tilde{\cap} ((F_Q, A)_t \tilde{\cup} (K_Q, C)_t)$
- (ii) $(F_Q, A)_t \tilde{\cap} ((G_Q, B)_t \tilde{\cup} (K_Q, C)_t) = ((F_Q, A)_t \tilde{\cap} (G_Q, B)_t) \tilde{\cup} ((F_Q, A)_t \tilde{\cap} (K_Q, C)_t)$

where $\tilde{\cap}$ $\tilde{\cup}$ are the T-QNSE union and intersection respectively?

Proof: The proof can be obtained from the relative definition.

Proposition 4.5. If $(F_Q, A)_t$ and $(G_Q, B)_t$ are two T-QNSEs over $U \times Q$ then $\forall t \in T$

- (i) $((F_Q, A)_t \tilde{\cap} ((G_Q, B)_t)^c) = ((F_Q, A)_t)^c \tilde{\cup} ((G_Q, B)_t)^c$
- (ii) $((F_Q, A)_t \tilde{\cup} ((G_Q, B)_t)^c) = ((F_Q, A)_t)^c \tilde{\cap} ((G_Q, B)_t)^c$

Proof: (i) By using definition (compliment) and considering the case of $\alpha \in A \cap B$ since the other cases are trivial. Let $t_0 \in T$ be fixed time.

$$\begin{aligned} & \text{We have } ((F_Q, A)_{t_0})^c \tilde{\cap} ((G_Q, B)_{t_0})^c \\ &= \{T_{F^c(\alpha)} = F_{F(\alpha)}, I_{F^c(\alpha)} = 1 - I_{F(\alpha)}, F_{F^c(\alpha)} = T_{F(\alpha)}\} \tilde{\cap} \{T_{G^c(\alpha)} = F_{G(\alpha)}, I_{G^c(\alpha)} = 1 - I_{G(\alpha)}, F_{G^c(\alpha)} = T_{G(\alpha)}\} \\ &= \{(u_i, s_i)^{t_0} / \min(T_{F^c(\alpha)}, T_{G^c(\alpha)}), \min(I_{F^c(\alpha)}, I_{G^c(\alpha)}), \max(F_{F^c(\alpha)}, F_{G^c(\alpha)})\}, \text{ For each } (u_i, s_i)^{t_0} \ i = 1, \dots, n \\ &= \{(u_i, s_i)^{t_0} / \min(F_{F(\alpha)}, F_{G(\alpha)}), \min(1 - I_{F(\alpha)}, 1 - I_{G(\alpha)}), \max(T_{F(\alpha)}, T_{G(\alpha)})\} \\ &= \{(u_i, s_i)^{t_0} / \max(T_{F(\alpha)}, T_{G(\alpha)}), (1 - I_{F(\alpha)}, 1 - I_{G(\alpha)}), (F_{F(\alpha)}, F_{G(\alpha)})\}^c \\ &= ((F_Q, A)_{t_0} \tilde{\cup} ((G_Q, B)_{t_0})^c \end{aligned}$$

Since t_0 is arbitrary, then $((F_Q, A)_t)^c \tilde{\cap} ((G_Q, B)_t)^c = ((F_Q, A)_t \tilde{\cup} ((G_Q, B)_t)^c)^c$ is true $\forall t \in T$.

(ii) proof is similar to (i)

5. An Application of Time Q-Neutrosophic Soft Expert Set in Decision Making.

In this section, an application of T-QNSEs in decision-making problems. Hassan et al. introduced the following method:

Definition 5.1. [9] Hassan et. al. Algorithm.

- Input the QNSEs (F_Q, Z) .
- Compute the QNSEs $(u, s) = |T - I - F|$.

- Find the agree- QNSEs and disagree-QNSEs.
- Calculate $c_j = \sum_i (u, s)_{ij}$ for agree- QNSEs.
- Calculate $k_j = \sum_i (u, s)_{ij}$ for disagree- QNSEs
- Determine $s_j = c_j - k_j$.
- Determine r for which $s_r = \max s_j$, If there is more than one value of r , then the college can have alternative choices.

Definition 5.2. [14] Comparison Matrix is a matrix whose rows are labeled by the object names h_1, h_2, \dots, h_n and the columns are labeled by the parameters e_1, e_2, \dots, e_m . The entries c_{ij} are calculated by $c_{ij} = a + b - c$, where 'a' is the integer calculated as 'how many times $T_{h_i}(e_j)$ exceeds or equal to $T_{h_k}(e_j)$, for $h_i \neq h_k$, $\forall h_k \in U$, 'b' is the integer calculated as 'how many times $I_{h_i}(e_j)$ exceeds or equal to $I_{h_k}(e_j)$, for $h_i \neq h_k, \forall h_k \in U$, and 'c' is the integer 'how many times $F_{h_i}(e_j)$ exceeds or equal to $F_{h_k}(e_j)$, for $h_i \neq h_k, \forall h_k \in U$.

Definition 5.3. [14] The score of an object h_i is S_i and is calculated as $S_i = \sum_j c_{ij}$.

Algorithm

Transforming T-QNSEs to QNSEs is our aim and then use Hassan et al. algorithm to find the optimum resolution. The following algorithm applies to transform T-QNSEs to QNSEs as follows:

1. Input the T-QNSEs of (F_Q, E) .
2. Compute the QNSEs of $F_Q(E)$, where $F_Q(E)$ is defined as follows:

$$F_Q(e) = \left\{ \frac{(u,s)}{(T(e),I(e),F(e))} : (u, s) \in U \times Q, e \in E \right\} \dots\dots (1)$$

such that

$$T(e) = \frac{\sum_{i=1}^n \alpha_{t_i} T_{ti}(e)}{n \sum_{i=1}^n \alpha_{t_i}(e)}, I(e) = \frac{\sum_{i=1}^n \alpha_{t_i} I_{ti}(e)}{n \sum_{i=1}^n \alpha_{t_i}(e)}, F(e) = \frac{\sum_{i=1}^n \alpha_{t_i} F_{ti}(e)}{n \sum_{i=1}^n \alpha_{t_i}(e)}$$

where $n = |T|$ and α_{t_i} the weight of it

3. Use Hassan et. al. Algorithm.

Example 5.1. Suppose that the Ministry of Agriculture wants to evaluate agricultural lands for the establishment of a certain agricultural project in one of these lands through specific parameters for three previous time periods and these parameters are mentioned below. Let $U = \{u_1, u_2, u_3\}$, be a set of lands, there may be three parameters. Let $E = \{e_1, e_2, e_3\}$ be a set of decision parameters to evaluate lands. For $i = 1, 2, 3$ the parameters e_i ($i = 1, 2, 3$) stand for "Humidity rate", "Rainfall", "Temperature" and let $T = \{t_1, t_2, t_3\}$. From the findings of this study, it will be clear to identify the best land which satisfies the above-mentioned parameters. Let $Q = \{s_1, s_2\}$ be a set of supply, $T = \{t_1, t_2, t_3\}$ be a set of time, $X = \{p, q, r\}$ is the set of expert and $O = \{agree = 1, disagree = 0\}$ a set of opinion. Define a function $F_Q^t: A \rightarrow N(U \times Q)$ as in example 3.1

Table 1: Representation of Agree T-QNSEs $F_Q^t(E)$.

$U \times Q$	(u_1, s_1)	(u_2, s_1)	(u_3, s_1)	(u_1, s_2)	(u_2, s_2)	(u_3, s_2)
(e_1, p, t_1)	(0.1,0.1,0.3)	(0.9,0.7,0.2)	(0.1,0.2,0.9)	(0.0,0.1,0.4)	(0.0,0.7,0.8)	(0.4,0.2,0.8)
(e_1, r, t_1)	(0.4,0.0,0.1)	(0.2,0.4,0.6)	(0.9,0.2,0.4)	(0.6,0.4,0.5)	(0.2,0.4,0.2)	(0.9,0.0,0.5)
(e_1, q, t_1)	(0.8,0.7,0.5)	(0.7,0.6,0.7)	(0.0,0.2,0.9)	(0.3,0.6,0.4)	(0.5,0.1,0.9)	0.3,0.5,0.0)
(e_2, p, t_1)	(0.5,0.5,0.3)	(0.1,0.9,0.7)	(0.7,0.5,0.6)	(0.5,0.1,0.9)	(0.1,0.3,0.8)	(0.4,0.8,0.9)
(e_2, r, t_1)	(0.2,0.4,0.5)	(0.6,0.1,0.0)	(0.1,0.0,0.3)	(0.0,0.6,0.4)	(0.7,0.5,0.0)	(0.9,0.4,0.0)
(e_2, q, t_1)	(0.8,0.9,0.4)	(0.8,0.0,0.0)	(0.1,0.2,0.6)	(0.2,0.2,0.3)	(0.8,0.9,0.2)	(0.5,0.9,0.4)
(e_3, p, t_1)	(0.5,0.2,0.2)	(0.3,0.8,0.7)	(0.5,0.7,0.4)	(0.8,0.6,0.8)	(0.7,0.0,0.0)	(0.5,0.2,0.3)
(e_3, r, t_1)	(0.6,0.5,0.2)	(0.0,0.6,0.7)	(0.9,0.6,0.0)	(0.0,0.5,0.8)	(0.1,0.2,0.5)	(0.3,0.4,0.2)
(e_3, q, t_1)	(0.0,0.6,0.0)	(0.5,0.0,0.1)	(0.4,0.0,0.7)	(0.8,0.1,0.5)	(0.9,0.3,0.8)	(0.0,0.0,0.3)
(e_1, p, t_2)	(0.8,0.7,0.5)	(0.7,0.1,0.7)	(0.3,0.3,0.1)	(0.2,0.4,0.0)	(0.7,0.1,0.2)	(0.8,0.6,0.1)
(e_1, r, t_2)	(0.1,0.6,0.0)	(0.4,0.5,0.8)	(0.2,0.5,0.9)	(0.7,0.5,0.2)	(0.8,0.5,0.8)	(0.6,0.6,0.5)

(e_1, q, t_2)	(0.2,0.3,0.6)	(0.4,0.5,0.2)	(0.5,0.3,0.7)	(0.6,0.0,0.6)	(0.4,0.2,0.2)	(0.7,0.5,0.9)
(e_2, p, t_2)	(0.9,0.3,0.7)	(0.0,0.2,0.7)	(0.9,0.8,0.5)	(0.7,0.5,0.7)	(0.9,0.6,0.5)	(0.2,0.3,0.3)
(e_2, r, t_2)	(0.7,0.7,0.8)	(0.5,0.4,0.4)	(0.4,0.9,0.6)	(0.8,0.9,0.3)	(0.1,0.1,0.8)	(0.0,0.7,0.0)
(e_2, q, t_2)	(0.2,0.2,0.8)	(0.7,0.8,0.9)	(0.3,0.9,0.7)	(0.6,0.5,0.4)	(0.9,0.1,0.6)	(0.8,0.6,0.2)
(e_3, p, t_2)	(0.5,0.0,0.3)	(0.9,0.4,0.8)	(0.4,0.6,0.3)	(0.4,0.6,0.2)	(0.7,0.2,0.0)	(0.4,0.3,0.2)
(e_3, r, t_2)	(0.8,0.0,0.2)	(0.6,0.3,0.9)	(0.4,0.6,0.1)	(0.1,0.6,0.7)	(0.2,0.3,0.3)	(0.1,0.6,0.8)
(e_3, q, t_2)	(0.0,0.5,0.3)	(0.7,0.7,0.8)	(0.1,0.9,0.8)	(0.2,0.7,0.5)	(0.4,0.7,0.1)	(0.0,0.0,0.6)
(e_1, p, t_3)	(0.9,0.2,0.6)	(0.5,0.9,0.9)	(0.0,0.6,0.4)	(0.8,0.9,0.7)	(0.7,0.2,0.2)	(0.6,0.8,0.7)
(e_1, r, t_3)	(0.4,0.1,0.3)	(0.3,0.7,0.0)	(0.7,0.2,0.2)	(0.6,0.9,0.8)	(0.7,0.7,0.4)	(0.1,0.3,0.6)
(e_1, q, t_3)	(0.9,0.7,0.8)	(0.3,0.6,0.2)	(0.0,0.0,0.0)	(0.8,0.0,0.4)	(0.9,0.3,0.7)	(0.0,0.4,0.1)
(e_2, p, t_3)	(0.1,0.9,0.7)	(0.8,0.0,0.5)	(0.9,0.7,0.2)	(0.5,0.3,0.8)	(0.3,0.5,0.5)	(0.4,0.9,0.1)
(e_2, r, t_3)	(0.7,0.7,0.9)	(0.7,0.5,0.6)	(0.5,0.3,0.2)	(0.7,0.6,0.1)	(0.6,0.5,0.9)	(0.4,0.3,0.9)
(e_2, q, t_3)	(0.8,0.9,0.6)	(0.1,0.3,0.4)	(0.3,0.3,0.2)	(0.6,0.1,0.3)	(0.9,0.5,0.3)	(0.8,0.5,0.8)
(e_3, p, t_3)	(0.5,0.0,0.5)	(0.9,0.0,0.5)	(0.9,0.8,0.2)	(0.3,0.5,0.2)	(0.6,0.0,0.8)	(0.5,0.1,0.8)
(e_3, r, t_3)	(0.3,0.0,0.7)	(0.1,0.7,0.2)	(0.9,0.3,0.7)	(0.4,0.6,0.6)	(0.7,0.5,0.7)	(0.8,0.0,0.2)
(e_3, q, t_3)	(0.6,0.3,0.9)	(0.3,0.4,0.3)	(0.1,0.7,0.1)	(0.0,0.0,0.4)	(0.0,0.6,0.7)	(0.3,0.5,0.2)

Table 2: Representation of Disagree T-QNSEs $F_Q^t(E)$.

$U \times Q$	(u_1, s_1)	(u_2, s_1)	(u_3, s_1)	(u_1, s_2)	(u_2, s_2)	(u_3, s_2)
(e_1, p, t_1)	(0.2,0.5,0.3)	(0.8,0.8,0.2)	(0.8,0.2,0.4)	(0.1,0.9,0.9)	(0.2,0.4,0.3)	(0.6,0.7,0.0)
(e_1, r, t_1)	(0.5,0.9,0.1)	(0.3,0.4,0.1)	(0.1,0.7,0.2)	(0.1,0.0,0.9)	(0.9,0.6,0.1)	(0.1,0.7,0.4)
(e_1, q, t_1)	(0.4,0.7,0.4)	(0.7,0.5,0.5)	(0.1,0.5,0.8)	(0.8,0.3,0.1)	(0.4,0.3,0.8)	(0.6,0.4,0.0)
(e_2, p, t_1)	(0.6,0.2,0.4)	(0.4,0.1,0.3)	(0.4,0.8,0.7)	(0.2,0.8,0.9)	(0.9,0.2,0.9)	(0.0,0.8,0.5)
(e_2, r, t_1)	(0.3,0.9,0.4)	(0.4,0.8,0.5)	(0.7,0.2,0.7)	(0.8,0.9,0.9)	(0.4,0.9,0.8)	(0.3,0.2,0.1)
(e_2, q, t_1)	(0.3,0.6,0.1)	(0.1,0.3,0.3)	(0.3,0.0,0.0)	(0.6,0.6,0.6)	(0.8,0.8,0.9)	(0.4,0.0,0.8)
(e_3, p, t_1)	(0.5,0.3,0.1)	(0.0,0.3,0.1)	(0.3,0.9,0.9)	(0.4,0.3,0.7)	(0.0,0.2,0.9)	(0.7,0.7,0.0)
(e_3, r, t_1)	(0.7,0.8,0.0)	(0.8,0.3,0.2)	(0.1,0.5,0.3)	(0.3,0.5,0.3)	(0.1,0.7,0.0)	(0.2,0.2,0.7)
(e_3, q, t_1)	(0.5,0.0,0.4)	(0.9,0.5,0.6)	(0.7,0.2,0.1)	(0.0,0.6,0.9)	(0.9,0.8,0.8)	(0.4,0.3,0.6)
(e_1, p, t_2)	(0.7,0.0,0.3)	(0.0,0.7,0.0)	(0.2,0.7,0.1)	(0.1,0.6,0.4)	(0.7,0.6,0.4)	(0.3,0.3,0.0)
(e_1, r, t_2)	(0.7,0.1,0.6)	(0.0,0.2,0.0)	(0.3,0.2,0.2)	(0.3,0.1,0.4)	(0.3,0.7,0.9)	(0.0,0.7,0.7)
(e_1, q, t_2)	(0.3,0.6,0.7)	(0.9,0.1,0.1)	(0.3,0.1,0.6)	(0.4,0.3,0.6)	(0.1,0.0,0.8)	(0.1,0.8,0.3)
(e_2, p, t_2)	(0.7,0.3,0.5)	(0.1,0.2,0.8)	(0.7,0.3,0.0)	(0.6,0.3,0.8)	(0.3,0.9,0.1)	(0.3,0.5,0.7)
(e_2, r, t_2)	(0.6,0.3,0.0)	(0.0,0.3,0.0)	(0.1,0.1,0.0)	(0.4,0.8,0.0)	(0.8,0.8,0.3)	(0.5,0.5,0.4)
(e_2, q, t_2)	(0.7,0.3,0.0)	(0.2,0.1,0.6)	(0.1,0.1,0.2)	(0.4,0.1,0.2)	(0.5,0.1,0.2)	(0.1,0.4,0.2)
(e_3, p, t_2)	(0.4,0.5,0.2)	(0.6,0.0,0.8)	(0.5,0.6,0.7)	(0.7,0.2,0.5)	(0.9,0.6,0.2)	(0.2,0.5,0.9)
(e_3, r, t_2)	(0.1,0.6,0.6)	(0.4,0.2,0.8)	(0.3,0.9,0.8)	(0.5,0.7,0.7)	(0.5,0.5,0.4)	(0.2,0.1,0.5)
(e_3, q, t_2)	(0.8,0.1,0.8)	(0.5,0.0,0.2)	(0.7,0.6,0.8)	(0.4,0.4,0.8)	(0.7,0.1,0.4)	(0.2,0.2,0.2)
(e_1, p, t_3)	(0.2,0.1,0.6)	(0.0,0.2,0.1)	(0.2,0.6,0.3)	(0.1,0.7,0.9)	(0.7,0.9,0.5)	(0.3,0.8,0.8)
(e_1, r, t_3)	(0.4,0.6,0.3)	(0.7,0.1,0.1)	(0.3,0.1,0.3)	(0.6,0.5,0.3)	(0.2,0.0,0.3)	(0.6,0.8,0.6)
(e_1, q, t_3)	(0.0,0.5,0.2)	(0.1,0.4,0.0)	(0.6,0.6,0.6)	(0.8,0.8,0.8)	(0.6,0.4,0.7)	(0.7,0.7,0.0)
(e_2, p, t_3)	(0.4,0.4,0.0)	(0.3,0.6,0.3)	(0.6,0.7,0.5)	(0.4,0.3,0.3)	(0.2,0.6,0.1)	(0.7,0.5,0.8)
(e_2, r, t_3)	(0.9,0.1,0.2)	(0.8,0.8,0.8)	(0.9,0.2,0.2)	(0.7,0.8,0.3)	(0.0,0.9,0.7)	(0.3,0.8,0.7)
(e_2, q, t_3)	(0.4,0.1,0.5)	(0.7,0.8,0.5)	(0.6,0.2,0.9)	(0.5,0.9,0.5)	(0.1,0.1,0.5)	(0.6,0.8,0.2)
(e_3, p, t_3)	(0.7,0.8,0.0)	(0.2,0.1,0.2)	(0.7,0.4,0.5)	(0.1,0.6,0.6)	(0.9,0.8,0.3)	(0.8,0.8,0.7)
(e_3, r, t_3)	(0.5,0.1,0.3)	(0.5,0.8,0.2)	(0.7,0.5,0.0)	(0.0,0.7,0.3)	(0.7,0.4,0.8)	(0.5,0.2,0.5)
(e_3, q, t_3)	(0.4,0.3,0.8)	(0.0,0.7,0.7)	(0.5,0.5,0.6)	(0.7,0.7,0.3)	(0.3,0.7,0.1)	(0.6,0.3,0.8)

Assume that $\alpha_{t_1} = 0.3, \alpha_{t_2} = 0.5$, and $\alpha_{t_3} = 0.8$, and by using relation (1) compute the $F_Q(E)$ to transform the time -Q neutrosophic soft expert set to Q-neutrosophic soft expert set, to show this step we calculate $F_Q(e_1, p_1)$ for (u_1, s_1) as below:

$$F_Q(e_1, p) = \left\{ \frac{(u_1, s_1)}{\langle T(e_1, p), I(e_1, p), F(e_1, p) \rangle} \right\}$$

Where,

$$T(e_1, p) = \frac{0.3 * 0.1 + 0.5 * 0.8 + 0.8 * 0.9}{3 * \max\{0.3, 0.5, 0.8\}} = \frac{1.15}{3 * 0.8} = 0.48$$

$$I(e_1, p) = \frac{0.3 * 0.1 + 0.5 * 0.7 + 0.8 * 0.2}{3 * \max\{0.3, 0.5, 0.8\}} = \frac{0.54}{3 * 0.8} = 0.23$$

$$F(e_1, p) = \frac{0.3 * 0.3 + 0.5 * 0.5 + 0.8 * 0.6}{3 * \max\{0.3, 0.5, 0.8\}} = \frac{0.82}{3 * 0.8} = 0.34$$

Then we get,

$$F_Q(e_1, p) = \left\{ \frac{(u_1, s_1)}{\langle 0.48, 0.23, 0.34 \rangle} \right\}$$

In a similar way, we are able to transform (u_i, s_i) with all parameters. The result of the transformation is shown in Table 3 and Table 4.

Table 3: Representation of Agree QNSEs $F_Q(E)$

$U \times Q$	(u_1, s_1)	(u_2, s_1)	(u_3, s_1)	(u_1, s_2)	(u_2, s_2)	(u_3, s_2)
(e_1, p)	(0.48, 0.23, 0.34)	(0.43, 0.41, 0.47)	(0.08, 0.29, 0.27)	(0.31, 0.4, 0.28)	(0.38, 0.18, 0.21)	(0.42, 0.42, 0.35)
(e_1, r)	(0.20, 0.16, 0.11)	(0.21, 0.39, 0.24)	(0.39, 0.2, 0.30)	(0.42, 0.45, 0.37)	(0.43, 0.39, 0.33)	(1.28, 0.23, 0.37)
(e_1, q)	(0.44, 0.38, 0.45)	(0.27, 0.38, 0.2)	(0.10, 0.09, 0.26)	(0.43, 0.08, 0.31)	(0.45, 0.15, 0.39)	(0.18, 0.3, 0.22)
(e_2, p)	(0.28, 0.43, 0.42)	(0.28, 0.15, 0.4)	(0.58, 0.46, 0.25)	(0.38, 0.22, 0.53)	(0.3, 0.33, 0.37)	(0.23, 0.46, 0.21)
(e_2, r)	(0.40, 0.43, 0.53)	(0.41, 0.26, 0.28)	(0.26, 0.29, 0.23)	(0.4, 0.46, 0.15)	(0.34, 0.32, 0.3)	(0.25, 0.3, 0.3)
(e_2, q)	(0.41, 0.45, 0.42)	(0.28, 0.27, 0.03)	(0.18, 0.31, 0.29)	(0.35, 0.16, 0.22)	(0.59, 0.3, 0.25)	(0.5, 0.40, 0.36)
(e_3, p)	(0.33, 0.03, 0.25)	(0.53, 0.18, 0.42)	(0.45, 0.48, 0.18)	(0.28, 0.37, 0.21)	(0.43, 0.04, 0.27)	(0.31, 0.12, 0.35)
(e_3, r)	(0.34, 0.06, 0.3)	(0.16, 0.37, 0.34)	(0.5, 0.3, 0.25)	(0.15, 0.39, 0.45)	(0.29, 0.25, 0.36)	(0.33, 0.18, 0.26)
(e_3, q)	(0.2, 0.28, 0.36)	(0.31, 0.28, 0.28)	(0.10, 0.42, 0.29)	(0.14, 0.16, 0.3)	(0.2, 0.38, 0.35)	(0.1, 0.17, 0.23)

Table 4: Representation of Disagree QNSEs $F_Q(E)$

$U \times Q$	(u_1, s_1)	(u_2, s_1)	(u_3, s_1)	(u_1, s_2)	(u_2, s_2)	(u_3, s_2)
(e_1, p)	(0.24, 0.1, 0.3)	(0.1, 0.31, 0.06)	(0.21, 0.37, 0.17)	(0.07, 0.47, 0.5)	(0.40, 0.48, 0.29)	(0.24, 0.42, 0.27)
(e_1, r)	(0.34, 0.33, 0.24)	(0.27, 0.13, 0.05)	(0.18, 0.16, 0.17)	(0.28, 0.2, 0.3)	(0.24, 0.22, 0.3)	(0.21, 0.5, 0.4)
(e_1, q)	(0.11, 0.38, 0.26)	(0.31, 0.22, 0.08)	(0.28, 0.28, 0.43)	(0.45, 0.37, 0.40)	(0.27, 0.17, 0.5)	(0.33, 0.45, 0.06)
(e_2, p)	(0.35, 0.22, 0.15)	(0.17, 0.25, 0.30)	(0.43, 0.4, 0.25)	(0.28, 0.26, 0.38)	(0.24, 0.41, 0.17)	(0.3, 0.37, 0.48)
(e_2, r)	(0.46, 0.21, 0.12)	(0.32, 0.43, 0.33)	(0.41, 0.11, 0.15)	(0.42, 0.55, 0.21)	(0.22, 0.58, 0.4)	(0.24, 0.4, 0.33)
(e_2, q)	(0.32, 0.17, 0.18)	(0.29, 0.33, 0.33)	(0.72, 0.09, 0.34)	(0.33, 0.4, 0.28)	(0.24, 0.15, 0.32)	(0.27, 0.35, 0.21)
(e_3, p)	(0.38, 0.41, 0.05)	(0.19, 0.07, 0.25)	(0.38, 0.37, 0.43)	(0.23, 0.28, 0.39)	(0.49, 0.42, 0.25)	(0.4, 0.46, 0.42)
(e_3, r)	(0.28, 0.26, 0.23)	(0.35, 0.35, 0.26)	(0.31, 0.42, 0.20)	(0.14, 0.44, 0.28)	(0.35, 0.33, 0.35)	(0.23, 0.11, 0.36)
(e_3, q)	(0.36, 0.12, 0.48)	(0.22, 0.3, 0.35)	(0.4, 0.32, 0.38)	(0.32, 0.39, 0.38)	(0.36, 0.35, 0.22)	(0.29, 0.18, 0.38)

Use the Hassan algorithm, to compute the QNSEs $(u_1, s_1) = |\mu - \nu - w|$ as follow: $|0.48 - 0.23 - 0.34| = 0.09$

In a similar way, the transforming of (u_i, s_i) with all parameters can be done. The result of the transformation is shown in Table 5 and Table 6.

Table 5: c_j of Agree QNSEs

$U \times Q$	(u_1, s_1)	(u_2, s_1)	(u_3, s_1)	(u_1, s_2)	(u_2, s_2)	(u_3, s_2)
(e_1, p)	0.09	0.45	0.48	0.37	0.01	0.35
(e_1, r)	0.07	0.42	0.11	0.4	0.29	0.68

(e_1, q)	0.39	0.31	0.25	0.04	0.09	0.34
(e_2, p)	0.57	0.27	0.13	0.37	0.4	0.44
(e_2, r)	0.56	0.13	0.26	0.21	0.28	0.35
(e_2, q)	0.46	0.02	0.42	0.03	0.04	0.26
(e_3, p)	0.05	0.07	0.21	0.3	0.12	0.16
(e_3, r)	0.02	0.55	0.05	0.69	0.32	0.11
(e_3, q)	0.44	0.25	0.61	0.32	0.53	0.3
c_j $= \sum_i (u,s)_{ij}$	$c_1 = 2.65$	$c_2 = 2.47$	$c_3 = 2.52$	$c_4 = 2.73$	$c_5 = 2.08$	$c_6 = 2.99$

Table 6: k_j of Disagree QNSEs

$U \times Q$	(u_1, s_1)	(u_2, s_1)	(u_3, s_1)	(u_1, s_2)	(u_2, s_2)	(u_3, s_2)
(e_1, p)	0.16	0.27	0.33	0.9	0.37	0.45
(e_1, r)	0.23	0.09	0.15	0.22	0.28	0.69
(e_1, q)	0.23	0.01	0.43	0.32	0.4	0.18
(e_2, p)	0.02	0.38	0.22	0.36	0.34	0.55
(e_2, r)	0.13	0.44	0.15	0.34	0.76	0.49
(e_2, q)	0.03	0.37	0.29	0.35	0.23	0.29
(e_3, p)	0.08	0.13	0.42	0.44	0.18	0.48
(e_3, r)	0.21	0.26	0.31	0.58	0.33	0.24
(e_3, q)	0.24	0.43	0.3	0.45	0.21	0.27
k_j $= \sum_i (u,s)_{ij}$	$k_1 = 1.33$	$k_2 = 2.38$	$k_3 = 2.6$	$k_4 = 3.96$	$k_5 = 3.1$	$k_6 = 3.64$

From Table 5 and Table 6, we can calculate $s_j = c_j - k_j$ as in Table 7.

Table 7: Score table $s_j = c_j - k_j$

j	$U \times Q$	c_j	k_j	$s_j = c_j - k_j$
1	(u_1, s_1)	2.65	1.33	1.32
2	(u_2, s_1)	2.47	2.38	0.09
3	(u_3, s_1)	2.52	2.6	-0.08
4	(u_1, s_2)	2.73	3.96	-1.23
5	(u_2, s_2)	2.08	3.1	-1.02
6	(u_3, s_2)	2.99	3.64	-0.65

The maximum score from table 7 is 1.32 for u_1

6. Conclusion

In this article, we define a time on QNSEs and introduce the new concept which is T-QNSEs. We discuss some properties and operations on T-QNSEs studied with illustrative examples. An algorithm of this concept was introduced to solve problems in decision-making.

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