



## Some Results About 2-Cyclic Refined Neutrosophic Complex Numbers

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### Abstract

This paper is dedicated to define for the first time the concept of 2-cyclic complex refined neutrosophic numbers as a direct application of 2-cyclic refined neutrosophic sets. Also, it presents some of their elementary properties such as conjugates, absolute values, invertibility, and algebraic operations.

Also, we illustrate many examples to clarify the validity of our discussion.

**Keywords:** refined neutrosophic complex number ;cyclic refined neutrosophic complex number; refined neutrosophic real number, invertible number.

### 1.Introduction

Neutrosophy is a new branch of philosophy founded by Smarandache [6,36] to study the indeterminacy in the real world problems and science. It has a master effect in many areas such as topology [7,27,29], equations [3,30], decision making [8], abstract algebra [25,26,39,41], and number theory [35].

Neutrosophic algebra began with the definitions of neutrosophic groups [9,17], and rings [13]. The neutrosophic rings and their generalizations such as refined neutrosophic rings [19], and n-refined neutrosophic rings [11,12], were very useful in the study of neutrosophic algebraic structures.

Neutrosophic algebraic structures were defined as new generalizations of classical ones based on neutrosophic rings and fields, where we find many concepts from linear algebra were generalized into neutrosophic systems such as neutrosophic matrices and spaces over neutrosophic fields [1,42], refined neutrosophic spaces and matrices over refined neutrosophic fields [24], n-refined neutrosophic spaces over n-refined neutrosophic fields [21,32], linear modules and ideals [4,5,20,22].

Neutrosophic complex numbers were firstly studied in [43]. Recently, many of their properties were discussed in [44], especially their invertibility, absolute values, and complex functions.

Through this paper, we define refined neutrosophic complex numbers for the first time. On the other hand, we study many related properties of these numbers such as the inverses, conjugates, and absolute values.

## 2. Neutrosophic complex number

### Definition : Neutrosophic Real Number:

Suppose that  $w$  is a neutrosophic number, then it takes the following standard form:  $w = a + bI$  where  $a, b$  are real coefficients, and  $I$  represents the indeterminacy, where  $0.I = 0$  and  $I^n = I$  for all positive integers  $n$ .

For example:

$$w = 1 + 2I, w = 3 = 3 + 0I.$$

### Definition: Neutrosophic Complex Number:

Suppose that  $z$  is a neutrosophic complex number, then it takes the following standard form:  $z = a + bI + i(c + dI)$  where  $a, b, c, d$  are real coefficients, and  $I$  is the indeterminacy element, where  $i^2 = -1$  i.e.  $i = \sqrt{-1}$ .

We recall  $a + bI$  the real part, then it takes the following standard form  $Re(z) = a + bI$ .

We recall  $c + dI$  the imagine part, then it takes the following standard form  $Im(z) = c + dI$ .

For example:

$$z = 4 + I + i(2 + 2I)$$

Note: we can say that any real number can be considered a neutrosophic number.

For example:  $z = 3 = 3 + 0.I + i(0 + 0.I)$

## 2- cyclic refined neutrosophic complex numbers.

### Definition:

We define a 2-cyclic refined neutrosophic complex number by the following form:

$z = (a_0 + a_1I_1 + a_2I_2) + i(b_0 + b_1I_1 + b_2I_2)$ , where  $a_0, a_1, a_2, b_0, b_1, b_2$  are real coefficients. For example:

$$z = (1 - I_1 + 2I_2) + i(3 + 2I_1 - I_2).$$

We recall  $a_0 + a_1I_1 + a_2I_2$  the real part, then it takes the following standard form  $Re(z) = a_0 + a_1I_1 + a_2I_2$ .

We recall  $b_0 + b_1I_1 + b_2I_2$  the image part, then it takes the following standard form  $Im(z) = b_0 + b_1I_1 + b_2I_2$ .

**Remark :** A 2-cyclic refined neutrosophic complex number can be defined as follows:

$z = a + bI_1 + cI_2$  where  $a, b, c$  are complex numbers. For example:

$$z = (1 - i) + (2 + i)I_1 + (3 - 2i)I_2.$$

**Remark:**

$$I_i \times I_j = I_{(i+j \bmod 2)}.$$

**Definition :**

Let  $z = (a_o + a_1I_1 + a_2I_2) + i(b_o + b_1I_1 + b_2I_2)$  a 2-cyclic refined neutrosophic complex number. We denote the conjugate of a 2-cyclic refined neutrosophic complex number by  $\bar{z}$  and define it by the following form:

$$\bar{z} = (a_o + a_1I_1 + a_2I_2) - i(b_o + b_1I_1 + b_2I_2)$$

For example:

$$z = (-1 + I_1 + 2I_2) + i(1 + 2I_1 - I_2), \text{ Then } \bar{z} = (-1 + I_1 + 2I_2) - i(1 + 2I_1 - I_2).$$

**Definition :**

Suppose that  $z = (a_o + a_1I_1 + a_2I_2) + i(b_o + b_1I_1 + b_2I_2)$  is a 2-cyclic refined neutrosophic complex number. The absolute value of  $z$  can be defined by the following form:

$$|z| = \sqrt{(a_o + a_1I_1 + a_2I_2)^2 + (b_o + b_1I_1 + b_2I_2)^2}$$

**Remark :**

$$(1). \overline{(\bar{z})} = z.$$

Proof: Let  $z = (a_o + a_1I_1 + a_2I_2) + i(b_o + b_1I_1 + b_2I_2)$ , then  $\bar{z} = (a_o + a_1I_1 + a_2I_2) - i(b_o + b_1I_1 + b_2I_2)$ .

Now.

$$\overline{(\bar{z})} = \overline{((a_o + a_1I_1 + a_2I_2) - i(b_o + b_1I_1 + b_2I_2))} = (a_o + a_1I_1 + a_2I_2) + i(b_o + b_1I_1 + b_2I_2) = z$$

$$(2). z + \bar{z} = 2\text{Re}(z)$$

Proof: Let  $z = (a_o + a_1I_1 + a_2I_2) + i(b_o + b_1I_1 + b_2I_2)$ , then  $\bar{z} = (a_o + a_1I_1 + a_2I_2) - i(b_o + b_1I_1 + b_2I_2)$ .

Now.

$$z + \bar{z} = (a_o + a_1I_1 + a_2I_2) + i(b_o + b_1I_1 + b_2I_2) + (a_o + a_1I_1 + a_2I_2) - i(b_o + b_1I_1 + b_2I_2)$$

$$z + \bar{z} = 2[(a_o + a_1I_1 + a_2I_2)] = 2\text{Re}(z)$$

$$(3). z - \bar{z} = 2\text{Im}(z)$$

Let  $z = (a_o + a_1I_1 + a_2I_2) + i(b_o + b_1I_1 + b_2I_2)$ , then  $\bar{z} = (a_o + a_1I_1 + a_2I_2) - i(b_o + b_1I_1 + b_2I_2)$ .

Now.

$$z - \bar{z} = (a_o + a_1I_1 + a_2I_2) + i(b_o + b_1I_1 + b_2I_2) - [(a_o + a_1I_1 + a_2I_2) - i(b_o + b_1I_1 + b_2I_2)]$$

$$z - \bar{z} = (a_o + a_1I_1 + a_2I_2) + i(b_o + b_1I_1 + b_2I_2) - (a_o + a_1I_1 + a_2I_2) + i(b_o + b_1I_1 + b_2I_2)$$

$$z - \bar{z} = 2i(b_o + b_1I_1 + b_2I_2) = 2\text{Im}(z)$$

$$(4). \overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$$

Proof:

Let  $z_1 = (a_o + a_1I_1 + a_2I_2) + i(b_o + b_1I_1 + b_2I_2)$ ,  $z_2 = (c_o + c_1I_1 + c_2I_2) + i(d_o + d_1I_1 + d_2I_2)$ .

Now.

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$$z_1 + z_2 = [(a_o + c_o) + (a_1 + c_1)I_1 + (a_2 + c_2)I_2] + i[(b_o + d_o) + (b_1 + d_1)I_1 + (b_2 + d_2)I_2]$$

Then.

$$\overline{z_1 + z_2} = [(a_o + c_o) + (a_1 + c_1)I_1 + (a_2 + c_2)I_2] - i[(b_o + d_o) + (b_1 + d_1)I_1 + (b_2 + d_2)I_2]$$

$$\overline{z_1} + \overline{z_2} = [(a_o + a_1I_1 + a_2I_2) - i(b_o + b_1I_1 + b_2I_2)] + [(c_o + c_1I_1 + c_2I_2) - i(d_o + d_1I_1 + d_2I_2)] = \overline{z_1} + \overline{z_2}$$

**Definition :**

Let  $w_1 = a_o + a_1I_1 + a_2I_2, w_2 = b_o + b_1I_1 + b_2I_2$  are two cyclic refined neutrosophic real numbers, we define the multiplication  $w_1 \cdot w_2$  as follows:

$$w_1 \times w_2 = (a_o + a_1I_1 + a_2I_2) \times (b_o + b_1I_1 + b_2I_2) = (a_o \times b_o) + (a_o \times b_1)I_1 + (a_o \times b_2)I_2 + (a_1 \times b_o)I_1 + (a_1 \times b_1)I_1 \times I_1 + (a_1 \times b_2)I_1 \times I_2 + (a_2 \times b_o)I_2 + (a_2 \times b_1)I_2 \times I_1 + (a_2 \times b_2)I_2 \times I_2.$$

By remark 3.2, we have.

$$I_1 \times I_1 = I_2, I_1 \times I_2 = I_1, I_2 \times I_1 = I_1, I_2 \times I_2 = I_2.$$

Then

$$w_1 \times w_2 = (a_o \times b_o) + [(a_o \times b_1) + (a_1 \times b_o) + (a_1 \times b_2) + (a_2 \times b_1)]I_1 + [(a_1 \times b_1) + (a_o \times b_2) + (a_2 \times b_o) + (a_2 \times b_2)]I_2.$$

**Definition:**

$$\text{Let } z_1 = (a_o + a_1I_1 + a_2I_2) + i(b_o + b_1I_1 + b_2I_2), z_2 = (c_o + c_1I_1 + c_2I_2) + i(d_o + d_1I_1 + d_2I_2).$$

A product  $z_1 \times z_2$  is defined by form:

$$z_1 \times z_2 = [(a_o + a_1I_1 + a_2I_2) + i(b_o + b_1I_1 + b_2I_2)] \times [(c_o + c_1I_1 + c_2I_2) + i(d_o + d_1I_1 + d_2I_2)]$$

$$z_1 \times z_2 = (a_o + a_1I_1 + a_2I_2) \times (c_o + c_1I_1 + c_2I_2) - (b_o + b_1I_1 + b_2I_2) \times (d_o + d_1I_1 + d_2I_2) + i[(a_o + a_1I_1 + a_2I_2) \times (d_o + d_1I_1 + d_2I_2) + (c_o + c_1I_1 + c_2I_2) \times (b_o + b_1I_1 + b_2I_2)].$$

By using Definition 3.4, we get the product  $z_1 \times z_2$ .

**Remark:**

$$(1). \overline{z_1 \times z_2} = \overline{z_1} \times \overline{z_2}$$

$$(1). z \times \bar{z} = |z|^2.$$

**Definition :**

Let  $w = a_o + a_1I_1 + a_2I_2$  be a 2-cyclic refined neutrosophic real number, then the inverse of  $w$  is defined as follows:

$$w^{-1} = \frac{1}{w} = \frac{1}{a_o + a_1I_1 + a_2I_2} = (a_o + a_1I_1 + a_2I_2)^{-1}$$

$$\frac{1}{w} = (a_o)^{-1} + [(a_o + a_1 + a_2)^{-1} - (a_o + a_2)^{-1}]I_1 + [(a_o + a_2)^{-1} - (a_o)^{-1}]I_2$$

Where  $a_o + a_1 + a_2, a_o + a_2, a_o$  are not zero elements.

**Definition :**

Let  $z = (a_o + a_1I_1 + a_2I_2) + i(b_o + b_1I_1 + b_2I_2)$ , then the inverse of  $z$  is defined as follows:

$$z^{-1} = \frac{1}{z} = \frac{1}{(a_o + a_1I_1 + a_2I_2) + i(b_o + b_1I_1 + b_2I_2)} = \frac{\bar{z}}{z \times \bar{z}} = \frac{\bar{z}}{|z|^2}$$

**Example:**

Let  $z = (1 + I_1 - I_2) + i(2 + 2I_1 - I_2)$ ,  $\bar{z} = (1 + I_1 - I_2) - i(2 + 2I_1 - I_2)$ .  
then.

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{(1 + I_1 - I_2) - i(2 + 2I_1 - I_2)}{(1 + I_1 - I_2)^2 + (2 + 2I_1 - I_2)^2} =$$

Now we have.

$$(1 + I_1 - I_2)^2 = (1 + I_1)^2 - 2(1 + I_1) \times I_2 + (I_2 \times I_2) = 1 + 2I_1 + (I_1 \times I_1) - 2I_2 - 2I_1 \times I_2 + I_2 \times I_2$$

$$(1 + I_1 - I_2)^2 = 1 + 2I_1 + I_2 - 2I_2 - 2I_1 + I_2 = 1$$

$$(2 + 2I_1 - I_2)^2 = (2 + 2I_1)^2 - 2(2 + 2I_1) \times I_2 + (I_2 \times I_2) = 4 + 8I_1 + 4(I_1 \times I_1) - 4I_2 - 4I_1 \times I_2 + I_2 \times I_2$$

$$(2 + 2I_1 - I_2)^2 = 4 + 8I_1 + 4I_2 - 4I_2 - 4I_1 + I_2 = 4 + 4I_1 + I_2$$

Then.

$$z^{-1} = \frac{(1 + I_1 - I_2) - i(2 + 2I_1 - I_2)}{1 + 4 + 4I_1 + I_2} = \frac{(1 + I_1 - I_2) - i(2 + 2I_1 - I_2)}{5 + 4I_1 + I_2}$$

$$z^{-1} = [(1 + I_1 - I_2) - i(2 + 2I_1 - I_2)] \times (5 + 4I_1 + I_2)^{-1}$$

$$z^{-1} = (1 + I_1 - I_2) \times (5 + 4I_1 + I_2)^{-1} - i(2 + 2I_1 - I_2) \times (5 + 4I_1 + I_2)^{-1}$$

$$z^{-1} = (1 + I_1 - I_2) \times \left(\frac{1}{5} - \frac{1}{15}I_1 - \frac{1}{30}I_2\right) - i(2 + 2I_1 - I_2) \times \left(\frac{1}{5} - \frac{1}{15}I_1 - \frac{1}{30}I_2\right)$$

$$(1 + I_1 - I_2) \times \left(\frac{1}{5} - \frac{1}{15}I_1 - \frac{1}{30}I_2\right) = \frac{1}{5} + \frac{1}{6}I_1 - \frac{4}{15}I_2$$

$$(2 + 2I_1 - I_2) \times \left(\frac{1}{5} - \frac{1}{15}I_1 - \frac{1}{30}I_2\right) = \frac{2}{5} + \frac{4}{15}I_1 - \frac{11}{30}I_2$$

Hence,

$$z^{-1} = \left(\frac{1}{5} + \frac{1}{6}I_1 - \frac{4}{15}I_2\right) - i\left(\frac{2}{5} + \frac{4}{15}I_1 - \frac{11}{30}I_2\right).$$

**Conclusion**

In this paper, we have defined for the first time the concept of 2-cyclic refined neutrosophic complex numbers. Also, we have discussed some of their elementary properties such as the conjugate, the multiplication, absolute values and other related topics.

As a future research direction, we aim to study the natural generalization of those numbers by n-cyclic refined neutrosophic complex numbers.

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