On Some Results About The Second Order Neutrosophic Differential Equations By Using Neutrosophic Thick Function

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Abstract

In this paper we define a novel neutrosophic differential equation by using neutrosophic thick function. In addition, we present the concept of Laplace transformation on neutrosophic thick function and apply this transformation to solve some neutrosophic differential equations. Also, we illustrate many examples to clarify the methods and algorithms.

Keywords: Neutrosophic Thick Function; Neutrosophic Differential equation; Laplace transform Neutrosophic linear differential equation.

Introduction

Neutrosophic logic since it was released by Smarandache [1,7], has a huge effect in mathematical studies and theorems. We find many great results about neutrosophic algebra, analysis, and number theory [2-6,8-20, 24-35].

Differential equations play a basic role in many problems such as probability, optimization, and real analysis [50-52]. From this point of view, we try to combine the theory of differential equations with neutrosophic logic to study the solution of some problems that have uncertainty in its structure.

On the other hand, we discuss some different cases such as neutrosophic second order homogeneous and non homogeneous differential equations with their Laplace Transformations and the applications of these transformations in finding solutions. Also, we use the neutrosophic thick function to prove some related novel results and algorithms.

2. A second order neutrosophic differential equation.

Definition 2.1.

A second order neutrosophic non-homogeneous differential equation with variable coefficients is defined as follows:

$$[p_1(x), p_2(x)]y'' + [q_1(x), q_2(x)]\acute{y} + [r_1(x), r_2(x)]y = [f_1(x), f_2(x)] \dots \dots \ (1)$$

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Definition 2.2.

A second order neutrosophic homogeneous differential equation with variable coefficients is defined as follows:

$$[p_1(x), p_2(x)]y" + [q_1(x), q_2(x)]\acute{y} + [r_1(x), r_2(x)]y = 0 \dots\dots (2)$$

Definition 2.3.

A second order neutrosophic non-homogeneous differential equation with constant coefficients is defined as follows:

$$a_2 y" + a_1 \acute{y} + a_0 y = [f_1(x), f_2(x)] \dots\dots (3)$$

Definition 2.4.

A second order neutrosophic homogeneous differential equation with constant coefficients is defined as follows:

$$a_2 y" + a_1 \acute{y} + a_0 y = 0 \dots\dots (4)$$

3. Eliminate the first derivative of a homogeneous differential equation with second order and variable coefficients.

Consider the equation:

$$[p_1(x), p_2(x)]y" + [q_1(x), q_2(x)]\acute{y} + [r_1(x), r_2(x)]y = 0 \dots\dots (5)$$

Method of solution.

1- We put the proverbs of $y"$ equal to one, then we have.

$$y" + [\alpha_1(x), \beta_2(x)]\acute{y} + [\alpha_0(x), \beta_0(x)]y = 0 \dots\dots (6)$$

2- We make a change in the variable of the form:

$$y = \left[e^{\frac{-1}{2}\int \alpha_1(x)dx} z_1 \ , e^{\frac{-1}{2}\int \alpha_2(x)dx} z_2 \ \right] \dots\dots (7)$$

3- We calculate the derivatives $\acute{y}$ ، $y"$ from the transformation (7), and substitute in (6), we get a non-homogeneous differential equation with second order variable coefficients, where the first derivative does not contain the function $z = [z_1, z_2]$, and the variable is x.

Example 3.1. Let the equation:

$$y" - \left[\frac{4}{x}, \frac{2}{x}\right]\acute{y} + \left[\frac{6}{x^2} - 1, 1 + \frac{2}{x^2}\right]y = 0 \dots\dots (8)$$

solution.

$$y = \left[e^{\frac{-1}{2}\int\frac{-4}{x}dx} z_1 \ , e^{\frac{-1}{2}\int\frac{-2}{x}dx} z_2 \ \right] = \left[e^{2\int\frac{1}{x}dx} z_1 \ , e^{\int\frac{1}{x}dx} z_2 \ \right] = \left[e^{2\ln x} z_1 \ , e^{\ln x} z_2 \ \right] = \left[e^{\ln x^2} z_1 \ , e^{\ln x} z_2 \ \right]$$
$$= [x^2 z_1 \ , x z_2 \ ]$$

$$y = [x^2 z_1 \ , x z_2 \ ] \dots\dots (9)$$

$$\acute{y} = [x^2 z_1 \ , x z_2 \ ]' = [(x^2 z_1)' , (x z_2)'] = [2x z_1 + x^2 \acute{z}_1 , z_2 + x\acute{z}_2]$$

$$y" = [2x z_1 + x^2 \acute{z}_1 , z_2 + x\acute{z}_2]' = [(2x z_1 + x^2 \acute{z}_1)' , (z_2 + x\acute{z}_2)']$$
$$= [2z_1 + 2x\acute{z}_1 + 2x\acute{z}_1 + x^2 z"_1 , \acute{z}_2 + \acute{z}_2 + x z"_2] = [x^2 z"_1 + 4x\acute{z}_1 + 2z_1 , x z"_2 + 2\acute{z}_2]$$

$$[x^2 z"_1 + 4x\acute{z}_1 + 2z_1 , x z"_2 + 2\acute{z}_2] - \left[\frac{4}{x}, \frac{2}{x}\right][2x z_1 + x^2\acute{z}_1 , z_2 + x\acute{z}_2] + \left[\frac{6}{x^2} - 1, \frac{x^2+2}{x^2}\right][x^2 z_1 \ , x z_2 \ ] = 0$$

$$\Rightarrow [x^2 z"_1 + 4x\acute{z}_1 + 2z_1 , x z"_2 + 2\acute{z}_2] + \left[2x\left(\frac{-4}{x}\right)z_1 + \left(\frac{-4}{x}\right)x^2\acute{z}_1 , \left(\frac{-2}{x}\right)z_2 + \left(\frac{-2}{x}\right)x\acute{z}_2\right]$$
$$+ \left[\left(\frac{6}{x^2}\right)x^2 z_1 - x^2 z_1 \ , x z_2 \ + \left(\frac{2}{x^2}\right)x z_2 \ \right] = 0$$

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$$\Rightarrow [x^2z"_1 + 4x\acute{z}_1 + 2z_1\, , xz"_2 + 2\acute{z}_2] + \left[-8z_1 + -4x\,\acute{z}_1\, , \frac{-2}{x}z_2 + -2\acute{z}_2\right] + \left[6z_1 - x^2z_1\, , xz_2 + \frac{2}{x}z_2\right] = 0$$

$$\Rightarrow \left[x^2z"_1 + 4x\acute{z}_1 + 2z_1 - 8z_1 - 4x\,\acute{z}_1 + 6z_1 - x^2z_1\, , xz"_2 + 2z_2{'} - \frac{2}{x}z_2 + -2\acute{z}_2 + xz_2 + \frac{2}{x}z_2\right] = 0$$

$$[x^2z"_1 - x^2z_1\, , xz"_2 + xz_2] = 0$$

Example 3.2. Consider the equation:

$$y" + [3,2]\acute{y} + \left[-2,1 - \frac{2}{x^2}\right]y = 0 \ldots\ldots (10)$$

solution.

$$y = \left[e^{\frac{-1}{2}\int 3dx}z_1\, , e^{\frac{-1}{2}\int 2dx}z_2\right] = \left[e^{\frac{-3}{2}x}z_1\, , e^{-x}z_2\right]$$

$$y = \left[e^{\frac{-3}{2}x}z_1\, , e^{-x}z_2\right] \ldots\ldots (11)$$

$$\acute{y} = \left[e^{\frac{-3}{2}x}z_1\, , e^{-2x}z_2\right]' = \left[\left(e^{\frac{-3}{2}x}z_1\right)'\, , (e^{-x}z_2)'\right] = \left[\frac{-3}{2}e^{\frac{-3}{2}x}z_1 + e^{\frac{-3}{2}x}\acute{z}_1\, , -e^{-x}z_2 + e^{-x}\acute{z}_2\right]$$

$$y" = \left[\frac{-3}{2}e^{\frac{-3}{2}x}z_1 + e^{\frac{-3}{2}x}\acute{z}_1\, , -e^{-x}z_2 + e^{-x}\acute{z}_2\right]' = \left[\left(\frac{-3}{2}e^{\frac{-3}{2}x}z_1 + e^{\frac{-3}{2}x}\acute{z}_1\right)'\, , (-e^{-x}z_2 + e^{-x}\acute{z}_2)'\right]$$

$$= \left[\frac{9}{4}e^{-3x}z_1 - \frac{3}{2}e^{\frac{-3}{2}x}\acute{z}_1 - \frac{3}{2}e^{\frac{-3}{2}x}\acute{z}_1 + e^{\frac{-3}{2}x}z_1"\, , e^{-x}z_2 - e^{-x}\acute{z}_2 - e^{-x}\acute{z}_2 + e^{-x}z_2"\right]$$

$$= \left[e^{\frac{-3}{2}x}z"_1 - 3e^{\frac{-3}{2}x}\acute{z}_1 + \frac{9}{4}e^{-3x}z_1\, , e^{-x}z"_2 - 2e^{-x}z_2{'} + e^{-x}z_2\right]$$

$$\left[e^{\frac{-3}{2}x}z"_1 - 3e^{\frac{-3}{2}x}\acute{z}_1 + \frac{9}{4}e^{-3x}z_1\, , e^{-x}z"_2 - 2e^{-x}z_2{'} + e^{-x}z_2\right] + [3,2]\left[\frac{-3}{2}e^{\frac{-3}{2}x}z_1 + e^{\frac{-3}{2}x}\acute{z}_1\, , -e^{-x}z_2 + e^{-x}\acute{z}_2\right]$$
$$+ \left[-2,1 - \frac{2}{x^2}\right]\left[e^{\frac{-3}{2}x}z_1\, , e^{-x}z_2\right] = 0$$

$$\Rightarrow \left[e^{\frac{-3}{2}x}z"_1 - 3e^{\frac{-3}{2}x}\acute{z}_1 + \frac{9}{4}e^{-3x}z_1\, , e^{-x}z"_2 - 2e^{-x}z_2{'} + e^{-x}z_2\right]$$
$$+ \left[\frac{-9}{2}e^{\frac{-3}{2}x}z_1 + 3e^{\frac{-3}{2}x}\acute{z}_1\, , -2e^{-x}z_2 + 2e^{-x}\acute{z}_2\right] + \left[-2e^{\frac{-3}{2}x}z_1\, , e^{-x}z_2 - \frac{2}{x^2}e^{-x}z_2\right] = 0$$

$$\Rightarrow \left[e^{\frac{-3}{2}x}z"_1 - 3e^{\frac{-3}{2}x}\acute{z}_1 + \frac{9}{4}e^{-3x}z_1 - \frac{9}{2}e^{\frac{-3}{2}x}z_1 + 3e^{\frac{-3}{2}x}\acute{z}_1 - 2e^{\frac{-3}{2}x}z_1\, , e^{-x}z"_2 - 2e^{-x}z_2{'} + e^{-x}z_2 - 2e^{-x}z_2\right.$$
$$\left. + 2e^{-x}\acute{z}_2 + e^{-x}z_2 - \frac{2}{x^2}e^{-x}z_2\right] = 0$$

$$\Rightarrow \left[e^{\frac{-3}{2}x}z"_1 - \frac{13}{4}e^{\frac{-3}{2}x}z_1\, , e^{-x}z"_2 - \frac{2}{x^2}e^{-x}z_2\right] = 0$$

$$\left[e^{\frac{-3}{2}x}z"_1 - \frac{13}{4}e^{\frac{-3}{2}x}z_1\, , e^{-x}z"_2 - \frac{2}{x^2}e^{-x}z_2\right] = 0$$

4.The general solution to the homogeneous differential equation with second order and variable coefficients.

The Lyovil-Ostrogradsky method.

Consider the equation:

$$y" + [p_1(x), p_2(x)]\acute{y} + [q_1(x), q_2(x)]y = 0 \ldots\ldots (12)$$

Let $y_1 = [\gamma_1(x), \gamma_2(x)]$ be a special solution to equation (12).

The general solution to equation(12) by using Lyovil-Ostrogradskyis is given by form:

$$y_h = [\gamma_1(x), \gamma_2(x)]\left[\int \frac{c_1 e^{-\int p_1(x)dx}}{{p_1}^2}dx\, , \int \frac{c_2 e^{-\int p_2(x)dx}}{{p_2}^2}dx\right]$$

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$$y_h = \left[\gamma_1(x)\int \frac{c_1 e^{-\int p_1(x)dx}}{{p_1}^2}dx\ ,\gamma_2(x)\int \frac{c_2 e^{-\int p_2(x)dx}}{{p_2}^2}dx\right] \dots\dots (13)$$

Where $c_1 = a_1 + Ib_1, c_2 = a_2 + Ib_2$.

Example 4.1. Find the general solution for the equation:

$$y" + \left[\frac{-1}{x-1}, -2\right]\acute{y} + \left[\frac{1}{x(x-1)}, -3\right]y = 0 \dots\dots (14)$$

Where $y_1 = [x, e^{-x}]$ is a special solution to it.

Solution.

$$y_h = \left[x\int \frac{c_1 e^{-\int \frac{-1}{x-1}dx}}{x^2}dx\ ,e^{-x}\int \frac{c_2 e^{-\int -2dx}}{e^{-2x}}dx\right]$$

$$y_h = \left[x\int \frac{c_1 e^{\int \frac{1}{x-1}dx}}{x^2}dx\ \ ,e^{-x}\int \frac{c_2 e^{2\int dx}}{e^{-2x}}dx\right]$$

$$y_h = \left[x\int \frac{c_1 e^{\ln(x-1)}}{x^2}dx\ ,e^{-x}\int \frac{c_2 e^{2x}}{e^{-2x}}dx\right]$$

$$y_h = \left[x\int \frac{c_1 e^{\ln(x-1)}}{x^2}dx\ ,e^{-x}\int \frac{c_2 e^{2x}}{e^{-2x}}dx\right]$$

$$y_h = \left[x\int \frac{c_1 x - 1}{x^2}\ dx\ ,e^{-x}\int \frac{c_2 e^{2x}}{e^{-2x}}dx\right]$$

$$y_h = \left[c_1 x\int \frac{x-1}{x^2}\ dx\ ,c_2 e^{-x}\int e^{4x}dx\right]$$

$$y_h = \left[\frac{c_1}{2}x\left(\ln x + \frac{1}{x}\right)\ ,\frac{c_2}{4}e^{-x}e^{4x}\right]$$

$$y_h = \left[\frac{c_1}{2}(x\ln x + 1)\ ,\frac{c_2}{4}e^{3x}\right]$$

Where $c_1 = a_1 + Ib_1, c_2 = a_2 + Ib_2$.

Definition 4.1.(The second order neutrosophic complete differential equation).

Consider the equation:

$$[p_1(x), p_2(x)]y" + [q_1(x), q_2(x)]\acute{y} + [r_1(x), r_2(x)]y = [f_1(x), f_2(x)] \dots\dots (15)$$

The necessary and sufficient condition for equation (15) to be complete is that the following condition is fulfilled:

$$[p"_1(x), p"_2(x)] - [\acute{q}_1(x), \acute{q}_2(x)] + [r_1(x), r_2(x)] = [0,0] \dots\dots (16)$$

Equation (17)is a primary integration of the equation (15):

$$[B_1(x), B_2(x)]\acute{y} + [M_1(x), M_2(x)]y = [g_1(x), g_2(x)] \dots\dots (17)$$

Where

$$[B_1(x), B_2(x)] = [p_1(x), p_2(x)]$$

$$[M_1(x), M_2(x)] = [q_1(x) - \acute{p}_1(x), q_2(x) - \acute{p}_2(x)]$$

$$[g_1(x), g_2(x)] = \left[\int f_1(x)dx\ ,\int f_2(x)dx\right]$$

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Example 4.2. Prove that the following equation is complete and find its general solution.

$$[x^2+2, sinx]y"+[4x, 3cosx]\acute{y}+[2,-2sinx]y=[sinx, 5cosx] \ldots\ldots \ (18)$$

Solution.

We have.

$$p_1=x^2+2\,, q_1=4x\,, r_1=2, f_1=sinx$$

$$p_2=sinx\,, q_2=3cosx\,, r_2=-2sinx, f_2=5cosx$$

Now:

$$[p"_1(x), p"_2(x)]-[\acute{q}_1(x),\acute{q}_2(x)]+[r_1(x), r_2(x)]=[2,-sinx]-[4,-3sinx]+[2,-2sinx]$$
$$=[2-4+2,-sinx+3sinx-2sinx]=[0,0]$$

The the condition (16) is correct.

Then.

$$[B_1(x), B_2(x)]\acute{y}+[M_1(x), M_2(x)]y=[g_1(x), g_2(x)]$$

$$[B_1(x), B_2(x)]=[p_1(x), p_2(x)]=[x^2+2, sinx]$$

$$[M_1(x), M_2(x)]=[q_1(x)-\acute{p}_1(x), q_2(x)-\acute{p}_2(x)]=[2x, 2cosx]$$

$$[g_1(x), g_2(x)]=\left[\int f_1(x)dx\,,\int f_2(x)dx\right]=[-cosx, 5sinx]$$

$$[x^2+2, sinx]\acute{y}+[2x, 2cosx]y=[-cosx, 5sinx]$$

$$\acute{y}+\left[\frac{2x}{x^2+2},\frac{2cosx}{sinx}\right]y=\left[\frac{-cosx}{x^2+2},\frac{5sinx}{sinx}\right]$$

$$\acute{y}+\left[\frac{2x}{x^2+2},\frac{2cosx}{sinx}\right]y=\left[\frac{-cosx}{x^2+2},5\right]$$

It's a neutrosophic homogeneous differential equation that we solve solution using the complement factor.

$$\mu(x)=[\mu_1(x),\mu_2(x)]=\left[e^{\int\frac{2x}{x^2+2}dx}, e^{\int\frac{2cosx}{sinx}dx}\right]$$

$$\mu(x)=\left[e^{\ln(x^2+2)}, e^{2\ln(sinx)}\right]=\left[e^{\ln(x^2+2)}, e^{\ln(sin^2x)}\right]$$

$$\mu(x)=[x^2+2, sin^2x]$$

then.

$$y=\frac{1}{[x^2+2, sin^2x]}\left(a+bI+\left[\int(x^2+2)\frac{2x}{x^2+2}dx\,,\int(sin^2x)\frac{2cosx}{sinx}dx\right]\right)$$

$$y=\frac{1}{[x^2+2, sin^2x]}\left(a+bI+\left[\int 2xdx\,,\int 2sinxcosxdx\right]\right)$$

$$y=\frac{1}{[x^2+2, sin^2x]}\left(a+bI+\left[\int 2xdx\,,\int sin2xdx\right]\right)$$

$$y=\frac{1}{[x^2+2, sin^2x]}\left(a+bI+\left[x^2\,,\frac{-1}{2}\,cos2x\right]\right)$$

5- Laplace transformation of neutrosophic thick function.

Definition 5.1. Let $[\boldsymbol{f_1(x)}, \boldsymbol{f_2(x)}]$ be a neutrosophic thick function, the Laplace transformaition of the previous function is defined as follows:

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$$F(p) = [F_1(p), F_2(p)] = L[f_1(x), f_2(x)] = \int_0^{-\infty} e^{-px}[f_1(x), f_2(x)]dx = \int_0^{-\infty} [e^{-px}f_1(x), e^{-px}f_2(x)]dx$$
$$= \left[\int_0^{-\infty} e^{-Px}f_1(x)dx , \int_0^{-\infty} e^{-Px}f_2(x)dx\right] \dots\dots (19)$$

We now show the laplace transform table for some analytical functions.

$f(x)$	$F(p) = L[f(x)]$
a	$\frac{a}{p}$
1	$\frac{1}{p}$
x^n	$\frac{n!}{p^{n+1}}; n = 1,2,3, \dots \dots$
$\sqrt{x}$	$\frac{\sqrt{\pi}}{2p^{\frac{3}{2}}}$
$\sin ax$	$\frac{a}{p^2 + a^2}$
$\cos ax$	$\frac{p}{p^2 + a^2}$
$x \sin ax$	$\frac{2ap}{(p^2 + a^2)^2}$
$x \cos ax$	$\frac{p^2 - a^2}{(p^2 + a^2)^2}$
e^{ax}	$\frac{1}{p - a}$
$\sin(ax + b)$	$\frac{p \sin b + a \cos b}{p^2 + a^2}$
$\cos(ax + b)$	$\frac{p \cos b - a \sin b}{p^2 + a^2}$
$e^{ax} \sin bx$	$\frac{b}{(p - a)^2 + b^2}$
$e^{ax} \cos bx$	$\frac{p - a}{(p - a)^2 - b^2}$
$\sinh ax$	$\frac{a}{p^2 - a^2}$
$\cosh ax$	$\frac{p}{p^2 - a^2}$

Properties laplace transform.

1- $L[e^{ax}f(x)] = F(p - a)$
2- $L[x^n f(x)] = (-1)^n \frac{d}{d^n p} F(p)$
3- $L\left[\frac{f(x)}{x}\right] = \int_p^{-\infty} F(p)dp$
4- $L[y'] = pL[y] - y(0)$
5- $L[y''] = p^2L[y] - py(0) - y'(0)$
6- $L[y'''] = p^3L[y] - p^2y(0) - y''(0) + py'(0)$
7- $L\left[y^{(n)}\right] = p^nL[y] - p^{n-1}y(0) - p^{n-2}y'(0) - \cdots py^{(n-2)}(0) - y^{(n-1)}(0)$

Definition 5.2. Solving a neutrosophic differential equation by using the laplace transform.

Consider the equation.

$$y^{(n)} + [a_1, a_2]y^{(n-1)} + \cdots + [b_1, b_2]y' + [c_1, c_2]y = [f_1(x), f_2(x)] \dots\dots (20)$$

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Method of solution.

1- We take the laplace transform of both sides of the equation (20).
2- We substitute the initial conditions.
3- We take the inverselaplace transform.

Example 5.1. find the solution of equation.

$$y" + [16,1]y = [2sin4x, x] \dots\dots \ (21)$$

Where.

$$y'(0) = \left[\frac{-1}{2}, 2\right], y(0) = [0,1] \dots\dots (22)$$

Solution.

$$L[y''] + L\big[[16,1]y\big] = L\big[[2sin4x, x]\big]$$

$$\Rightarrow L[y''] + [16,1]L[y] = [2L(sin4x), L(x)]$$

$$\Rightarrow [p^2, p^2]L[y] - py(0) - y'(0) + [16,1]L[y] = [2L(sin4x), L(x)]$$

$$\Rightarrow [p^2, p^2]L[y] - py(0) - y'(0) + [16,1]L[y] = \left[\frac{8}{p^2+16}, \frac{1}{p^2}\right]$$

$$[p^2, p^2]L[y] - p[0,1] - \left[\frac{-1}{2}, 2\right] + [16,1]L[y] = \left[\frac{8}{p^2+16}, \frac{1}{p^2}\right]$$

$$\Rightarrow [p^2, p^2]L[y] - [0, p] - \left[\frac{-1}{2}, 2\right] + [16,1]L[y] = \left[\frac{8}{p^2+16}, \frac{1}{p^2}\right]$$

$$\Rightarrow [p^2+16, p^2+1]L[y] - \left[\frac{-1}{2}, p+2\right] = \left[\frac{8}{p^2+16}, \frac{1}{p^2}\right]$$

$$\Rightarrow [p^2+16, p^2+1]L[y] = \left[\frac{8}{p^2+16}, \frac{1}{p^2}\right] + \left[\frac{-1}{2}, p+2\right]$$

$$\Rightarrow [p^2+16, p^2+1]L[y] = \left[\frac{8}{p^2+16} - \frac{1}{2}, \frac{1}{p^2} + p + 2\right]$$

$$\Rightarrow [p^2+16, p^2+1]L[y] = \left[\frac{-p^2-16}{2(p^2+16)}, \frac{p^3+2p^2+1}{p^2}\right]$$

$$L[y] = \left[\frac{-1}{2(p^2+16)}, \frac{p^3+2p^2+1}{p^2(p^2+16)}\right]$$

$$\frac{p^3+2p^2+1}{p^2(p^2+16)} = \frac{A}{p} + \frac{B}{p^2} + \frac{Cp+D}{(p^2+16)} \Rightarrow \mathrm{A} = 0, \mathrm{B} = \mathrm{C} = \mathrm{D} = 1$$

$$\Rightarrow \frac{p^3+2p^2+1}{p^2(p^2+16)} = \frac{1}{p^2} + \frac{p}{(p^2+16)} + \frac{1}{(p^2+16)}$$

$$\Rightarrow L[y] = \left[\frac{-1}{2(p^2+16)}, \frac{1}{p^2} + \frac{p}{(p^2+16)} + \frac{1}{(p^2+16)}\right]$$

$$L^{-1}[y] = \left[L^{-1}\left\{\frac{-1}{2(p^2+16)}\right\}, L^{-1}\left\{\frac{1}{p^2}\right\} + L^{-1}\left\{\frac{p}{(p^2+16)}\right\} + L^{-1}\left\{\frac{1}{(p^2+16)}\right\}\right]$$

$$\Rightarrow L^{-1}[y] = \left[\frac{-1}{8}L^{-1}\left\{\frac{-1}{(p^2+16)}\right\}, L^{-1}\left\{\frac{1}{p^2}\right\} + L^{-1}\left\{\frac{p}{(p^2+16)}\right\} + L^{-1}\left\{\frac{1}{(p^2+16)}\right\}\right]$$

$$\Rightarrow y = \left[\frac{-1}{8}sin4x\ , x + cosx + sinx\right]$$

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Example 5.2. find the solution of equation.

$$y'' + [3,2]y' + [2,5]y = [0, e^{-x} sinx] \ldots\ldots (23)$$

Where.

$$y'(0) = [-1,1] , y(0) = [1,0] \ldots\ldots (24)$$

Solution.

$$L[y''] + [3,2]L[y'] + [2,5]L[y] = [L(0), L(e^{-x} sinx)]$$

$$\Rightarrow [p^2, p^2]L[y] - py(0) - y'(0) + [-3,2]pL[y] - [3,2]y(0) + [2,5]L[y] = [L(0), L(e^{-x} sinx)]$$

$$\Rightarrow [p^2, p^2]L[y] - py(0) - y'(0) + [-3,2]pL[y] - [3,2]y(0) + [2,5]L[y] = \left[0, \frac{1}{(p+1)^2+1}\right]$$

$$\Rightarrow [p^2, p^2]L[y] - [p, 0] - [-1,1] + [-3p, 2p]L[y] - [3,2][1,0] + [2,5]L[y] = \left[0, \frac{1}{(p+1)^2+1}\right]$$

$$\Rightarrow [p^2, p^2]L[y] - [p, 0] - [-1,1] + [-3p, 2p]L[y] - [3,0] + [2,5]L[y] = \left[0, \frac{1}{(p+1)^2+1}\right]$$

$$\Rightarrow [p^2+3p+2, p^2+2p+5]L[y] = \left[p+2, 1 + \frac{1}{p^2+2p+3}\right]$$

$$\Rightarrow [p^2+3p+2, p^2+2p+5]L[y] = \left[p+2, \frac{p^2+2p+4}{p^2+2p+3}\right]$$

$$L[y] = \left[\frac{p+2}{p^2+3p+2}, \frac{p^2+2p+4}{(p^2+2p+3)(p^2+2p+5)}\right]$$

$$\Rightarrow L[y] = \left[\frac{p+2}{(p+2)(p+1)}, \frac{p^2+2p+4}{(p^2+2p+3)(p^2+2p+5)}\right]$$

$$\Rightarrow L[y] = \left[\frac{1}{p+1}, \frac{p^2+2p+4}{(p^2+2p+3)(p^2+2p+5)}\right]$$

$$\frac{p^2+2p+4}{(p^2+2p+3)(p^2+2p+5)} = \frac{Ap+B}{p^2+2p+5} + \frac{Cp+D}{p^2+2p+3}$$

$$\Rightarrow \mathrm{A} = 0, \mathrm{B} = \frac{1}{2}, \mathrm{C} = 0, \mathrm{D} = \frac{1}{2}$$

$$\Rightarrow \frac{p^2+2p+4}{(p^2+2p+3)(p^2+2p+5)} = \frac{1}{2}\frac{1}{p^2+2p+5} + \frac{1}{2}\frac{1}{p^2+2p+3}$$

$$\Rightarrow L[y] = \left[\frac{1}{p+1}, \frac{1}{2}\frac{1}{p^2+2p+5} + \frac{1}{2}\frac{1}{p^2+2p+3}\right]$$

$$L^{-1}[y] = \left[L^{-1}\left\{\frac{1}{p+1}\right\}, L^{-1}\left\{\frac{1}{2}\frac{1}{p^2+2p+5}\right\} + L^{-1}\left\{\frac{1}{2}\frac{1}{p^2+2p+3}\right\}\right]$$

$$\Rightarrow L^{-1}[y] = \left[L^{-1}\left\{\frac{1}{p+1}\right\}, \frac{1}{2}L^{-1}\left\{\frac{1}{p^2+2p+5}\right\} + \frac{1}{2}L^{-1}\left\{\frac{1}{p^2+2p+3}\right\}\right]$$

$$\Rightarrow L^{-1}[y] = \left[L^{-1}\left\{\frac{1}{p+1}\right\}, \frac{1}{2}L^{-1}\left\{\frac{1}{(p+1)^2+4}\right\} + \frac{1}{2}L^{-1}\left\{\frac{1}{(p+1)^2+2}\right\}\right]$$

$$\Rightarrow y = \left[sinx , \frac{1}{4}e^{-x}sin2x + \frac{1}{2\sqrt{2}}e^{-x}sin\sqrt{2}x\right]$$

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6. Conclusion

In this paper, we have presented a new concept of neutrosophic differential equation by using a neutrosophic thick function, such as the neutrosophic equation of the second order with its two types, fixed and variable coefficients, and proposed methods for solving them. In addition, we have introduced the concept of the neutrosophic thick function with Laplace transform of some neutrosophic functions, and used this transformation to find solutions for neutrosophic differential equations.

In the future, other types of neutrosophic differential equations can be studied using the neutrosophic thick function especially the equation of higher orders, and Turiyam differential equations, see [42,47,48].

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