



On New Concepts of Weakly Neutrosophic Continuous Functions

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Abstract

In this paper, we intend to utilize the ideas of Neu^α -open and $Neu^{S\alpha}$ -open sets to distinguish several novel conceptions of weakly neutrosophic continuous functions, for instance; $Neu^{\alpha*}$ -continuous, $Neu^{\alpha**}$ -continuous, $Neu^{S\alpha}$ -continuous, $Neu^{S\alpha*}$ -continuous and $Neu^{S\alpha**}$ -continuous functions. Moreover, we will describe the interactions among these thoughts of weakly neutrosophic continuous functions.

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Keywords: $Neu^{S\alpha}$ -open set, Neu^α -continuous, $Neu^{\alpha*}$ -continuous, $Neu^{\alpha**}$ -continuous, $Neu^{S\alpha}$ -continuous, $Neu^{S\alpha*}$ -continuous and $Neu^{S\alpha**}$ -continuous functions.

1. Introduction

F. Smarandache [3,4] initially presented the idea of a "neutrosophic set". A. A. Salama et al. [1] introduced the principles of neutrosophic topological space (fleetingly, Neu^{TS}). Q. H. Imran et al. [6] stated that a class of $Neu^{S\alpha}$ -open sets in neutrosophic topological spaces. A. A. Salama et al. [2] submitted the notion of neutrosophic continuous mappings. Q. H. Imran et al. [7] established and examined the sense of continuity in neutrosophic generalized alpha generalized. The concentration of this article is to demonstrate pioneering perceptions of weakly neutrosophic continuous functions, for example; $Neu^{\alpha*}$ -continuous, $Neu^{\alpha**}$ -continuous, $Neu^{S\alpha}$ -continuous, $Neu^{S\alpha*}$ -continuous and $Neu^{S\alpha**}$ -continuous functions. Additionally, we have in mind to rationalize the associations among these thoughts of weakly neutrosophic continuous functions.

2. Preliminaries

In this article, $(\mathcal{M}, \tau)$, $(\mathcal{N}, \sigma)$, and $(\mathcal{O}, \rho)$ (or simply $\mathcal{M}$, $\mathcal{N}$, and $\mathcal{O}$) always mean Neu^{TS} . A neutrosophic closed set (briefly Neu -closed set) in $(\mathcal{M}, \tau)$ is a complement of a neutrosophic open set (briefly Neu -open set). Let neutrosophic set $\mathcal{D}$ be in a $Neu^{TS}(\mathcal{M}, \tau)$, then the neutrosophic closure of $\mathcal{D}$, the neutrosophic interior of $\mathcal{D}$, and the neutrosophic complement of $\mathcal{D}$ symbolize by $NeuCl(\mathcal{D})$, $NeuInt(\mathcal{D})$ and $\mathcal{D}^c$, correspondingly.

Definition 2.1:

Assume $\mathcal{D}$ is a neutrosophic subset of a $Neu^{TS}(\mathcal{M}, \tau)$, then it is named as:

- (1) A neutrosophic α -open set (in brief Neu^α -open set) [5] if $\mathcal{D} \subseteq NeuInt(NeuCl(NeuInt(\mathcal{D})))$. The collection of every Neu^α -open set of $\mathcal{M}$ is symbolized by $Neu^\alpha O(\mathcal{M})$. A neutrosophic α -closed set (in short Neu^α -closed set) is a complement of Neu^α -open set.
- (2) A neutrosophic semi- α -open set (in short $Neu^{S\alpha}$ -open set) [6] if for any a Neu^α -open set $\mathcal{P}$ in $\mathcal{M}$ where $\mathcal{P} \subseteq \mathcal{D} \subseteq NeuCl(\mathcal{P})$ or equivalently if $\mathcal{D} \subseteq NeuCl(NeuInt(NeuCl(NeuInt(\mathcal{D}))))$. The collection of every $Neu^{S\alpha}$ -open set of $\mathcal{M}$ is symbolized by $Neu^{S\alpha} O(\mathcal{M})$. A neutrosophic semi- α -closed set (in short $Neu^{S\alpha}$ -closed set) is a complement of $Neu^{S\alpha}$ -open set.

Remark 2.2 [6]:

In a $Neu^{TS}(\mathcal{M}, \tau)$, then the succeeding arguments stand, and the opposite of every argument does not hold:

- (1) Every Neu -open set is a Neu^α -open and $Neu^{S\alpha}$ -open.
- (2) Every Neu^α -open set is a $Neu^{S\alpha}$ -open.

Definition 2.3 [2]:

Assume $\mathcal{h}: (\mathcal{M}, \tau) \rightarrow (\mathcal{N}, \sigma)$ is a function, then $\mathcal{h}$ is stated to be neutrosophic continuous (in short Neu -continuous) iff for each $\mathcal{D}$ Neu -open set in $\mathcal{N}$, then $\mathcal{h}^{-1}(\mathcal{D})$ is a Neu -open set in $\mathcal{M}$.

Definition 2.4 [5]:

Assume $\mathcal{h}: (\mathcal{M}, \tau) \rightarrow (\mathcal{N}, \sigma)$ is a function, then $\mathcal{h}$ is stated to be neutrosophic α -continuous (in short Neu^α -continuous) iff for each $\mathcal{D}$ Neu -open set in $\mathcal{N}$, then $\mathcal{h}^{-1}(\mathcal{D})$ is a Neu^α -open set in $\mathcal{M}$.

Theorem 2.5 [2]:

A function $\mathcal{h}: (\mathcal{M}, \tau) \rightarrow (\mathcal{N}, \sigma)$ is Neu -continuous iff $\mathcal{h}^{-1}(NeuInt(\mathcal{D})) \subseteq NeuInt(\mathcal{h}^{-1}(\mathcal{D}))$ for each $\mathcal{D} \subseteq \mathcal{N}$.

Definition 2.6 [2]:

Let $\mathcal{h}: (\mathcal{M}, \tau) \rightarrow (\mathcal{N}, \sigma)$ be a function, then $\mathcal{h}$ is stated to be neutrosophic open (in short Neu -open) iff for each $\mathcal{D}$ Neu -open subset in $\mathcal{M}$, then $\mathcal{h}(\mathcal{D})$ is a Neu -open subset in $\mathcal{N}$.

3. Concepts of Weakly Neutrosophic Continuous Functions**Definition 3.1:**

Assume $\mathcal{h}: (\mathcal{M}, \tau) \rightarrow (\mathcal{N}, \sigma)$ is a function, then $\mathcal{h}$ is stated to be:

- (1) Neutrosophic α^* -continuous (in short Neu^{α^*} -continuous) iff for each $\mathcal{D}$ Neu^α -open subset in $\mathcal{N}$, then $\mathcal{h}^{-1}(\mathcal{D})$ is a Neu^α -open subset in $\mathcal{M}$.
- (2) Neutrosophic α^{**} -continuous (in short $Neu^{\alpha^{**}}$ -continuous) iff for each $\mathcal{D}$ Neu^α -open subset in $\mathcal{N}$, then $\mathcal{h}^{-1}(\mathcal{D})$ is a Neu -open subset in $\mathcal{M}$.

Definition 3.2:

Assume $\mathcal{h}: (\mathcal{M}, \tau) \rightarrow (\mathcal{N}, \sigma)$ is a function, then $\mathcal{h}$ is stated to be:

- (1) Neutrosophic semi- α -continuous (in short $Neu^{S\alpha}$ -continuous) iff for each $\mathcal{D}$ Neu -open set in $\mathcal{N}$, then $\mathcal{h}^{-1}(\mathcal{D})$ is a $Neu^{S\alpha}$ -open set in $\mathcal{M}$.
- (2) Neutrosophic semi- α^* -continuous (in short $Neu^{S\alpha^*}$ -continuous) iff for each $\mathcal{D}$ $Neu^{S\alpha}$ -open set in $\mathcal{N}$, then $\mathcal{h}^{-1}(\mathcal{D})$ is a $Neu^{S\alpha}$ -open set in $\mathcal{M}$.
- (3) Neutrosophic semi- α^{**} -continuous (in short $Neu^{S\alpha^{**}}$ -continuous) iff for each $\mathcal{D}$ $Neu^{S\alpha}$ -open set in $\mathcal{N}$, then $\mathcal{h}^{-1}(\mathcal{D})$ is a Neu -open set in $\mathcal{M}$.

Theorem 3.3:

Assume $\mathcal{h}: (\mathcal{M}, \tau) \rightarrow (\mathcal{N}, \sigma)$ is a function. Then the succeeding arguments are the same:

- (1) $\mathcal{h}$ is a $Neu^{S\alpha}$ -continuous.
- (2) The reverse image of each Neu -closed subset in $\mathcal{N}$ is $Neu^{S\alpha}$ -closed subset in $\mathcal{M}$.
- (3) $\mathcal{h}(NeuInt(NeuCl(NeuInt(NeuCl(\mathcal{C})))) \subseteq NeuCl(\mathcal{h}(\mathcal{C}))$, for each $\mathcal{C} \in \mathcal{M}$.
- (4) $NeuInt(NeuCl(NeuInt(NeuCl(\mathcal{h}^{-1}(\mathcal{D})))) \subseteq \mathcal{h}^{-1}(NeuCl(\mathcal{D}))$, for every $\mathcal{D} \in \mathcal{N}$.

Proof:

(1) $\Rightarrow$ (2). Assume $\mathcal{D}$ is a *Neu*-closed set in $\mathcal{N}$. It indicates that $\mathcal{D}^c$ is a *Neu*-open set. Consequently, $\mathcal{h}^{-1}(\mathcal{D}^c)$ is a *Neu*^{S α} -open set in $\mathcal{M}$. It means $(\mathcal{h}^{-1}(\mathcal{D}))^c$ is a *Neu*^{S α} -open set in $\mathcal{M}$. So, $\mathcal{h}^{-1}(\mathcal{D})$ is a *Neu*^{S α} -closed set in $\mathcal{M}$.

(2) $\Rightarrow$ (3). Assume $\mathcal{C} \in \mathcal{M}$, and we have $\text{NeuCl}(\mathcal{h}(\mathcal{C}))$ is a *Neu*-closed subset in $\mathcal{N}$. Hence, $\mathcal{h}^{-1}(\text{NeuCl}(\mathcal{h}(\mathcal{C})))$ is *Neu*^{S α} -closed set in $\mathcal{M}$.

Consequently, we have $\mathcal{h}^{-1}(\text{NeuCl}(\mathcal{h}(\mathcal{C}))) \supseteq \text{NeuInt}(\text{NeuCl}(\text{NeuInt}(\text{NeuCl}(\mathcal{h}^{-1}(\text{NeuCl}(\mathcal{h}(\mathcal{C})))))) \supseteq \text{NeuInt}(\text{NeuCl}(\text{NeuInt}(\text{NeuCl}(\mathcal{C}))))$. Or $\text{NeuCl}(\mathcal{h}(\mathcal{C})) \supseteq \mathcal{h}(\text{NeuInt}(\text{NeuCl}(\text{NeuInt}(\text{NeuCl}(\mathcal{C}))))$.

(3) $\Rightarrow$ (4). Assume $\mathcal{D} \in \mathcal{N}$, $\mathcal{h}^{-1}(\mathcal{D}) \in \mathcal{M}$. Therefore, by the hypothesis, we obtain $\text{NeuInt}(\text{NeuCl}(\text{NeuInt}(\text{NeuCl}(\mathcal{h}^{-1}(\mathcal{D})))) \subseteq \text{NeuCl}(\mathcal{h}(\mathcal{h}^{-1}(\mathcal{D}))) \subseteq \text{NeuCl}(\mathcal{D})$, that is $\text{NeuInt}(\text{NeuCl}(\text{NeuInt}(\text{NeuCl}(\mathcal{h}^{-1}(\mathcal{D})))) \subseteq \mathcal{h}^{-1}(\text{NeuCl}(\mathcal{D}))$.

(4) $\Rightarrow$ (1). Assume $\mathcal{C}$ is a *Neu*-open subset of $\mathcal{N}$. Suppose $\mathcal{D} = \mathcal{C}^c$ and $\mathcal{C} = \mathcal{h}^{-1}(\mathcal{D})$; so by (iii), we have $\text{NeuInt}(\text{NeuCl}(\text{NeuInt}(\text{NeuCl}(\mathcal{h}^{-1}(\mathcal{D})))) \subseteq \text{NeuCl}(\mathcal{D}) = \mathcal{D}$. It means that the following fact holds $\text{NeuInt}(\text{NeuCl}(\text{NeuInt}(\text{NeuCl}(\mathcal{h}^{-1}(\mathcal{C}^c)))) \subseteq \mathcal{h}^{-1}(\mathcal{C}^c)$. Or $\text{NeuInt}(\text{NeuCl}(\text{NeuInt}(\text{NeuCl}(\mathcal{h}^{-1}(\mathcal{C})))) \supseteq \mathcal{h}^{-1}(\mathcal{C})$. Consequently, $\mathcal{h}^{-1}(\mathcal{C})$ is a *Neu*^{S α} -open set in $\mathcal{M}$ and therefore $\mathcal{h}$ is a *Neu*^{S α} -continuous.

Proposition 3.4:

(1) Every *Neu*-continuous is a *Neu* ^{α} -continuous; as a result, it is *Neu*^{S α} -continuous. Nonetheless, the reverse does not stand.

(2) Every *Neu* ^{α} -continuous is a *Neu*^{S α} -continuous. Nevertheless, the contrary does not stand.

Proof:

(1) Let $\mathcal{h}: (\mathcal{M}, \tau) \rightarrow (\mathcal{N}, \sigma)$ be a *Neu*-continuous function and a *Neu*-open subset $\mathcal{D}$ be in $\mathcal{N}$. Then $\mathcal{h}^{-1}(\mathcal{D})$ is a *Neu*-open subset in $\mathcal{M}$. Since any *Neu*-open set is *Neu*^{S α} -open (*Neu*^{S α} -open), $\mathcal{h}^{-1}(\mathcal{D})$ is a *Neu* ^{α} -open (*Neu*^{S α} -open) set in $\mathcal{M}$. Thus $\mathcal{h}$ is a *Neu* ^{α} -continuous (*Neu*^{S α} -continuous).

(2) Let $\mathcal{h}: (\mathcal{M}, \tau) \rightarrow (\mathcal{N}, \sigma)$ be a *Neu* ^{α} -continuous function and a *Neu*-open set $\mathcal{D}$ be in $\mathcal{N}$. Then $\mathcal{h}^{-1}(\mathcal{D})$ is a *Neu* ^{α} -open set in $\mathcal{M}$. Since any *Neu* ^{α} -open set is *Neu*^{S α} -open, $\mathcal{h}^{-1}(\mathcal{D})$ is a *Neu*^{S α} -open set in $\mathcal{M}$. Thus $\mathcal{h}$ is a *Neu*^{S α} -continuous.

Example 3.5:

Let $\mathcal{M} = \{p, q\}$. Describe the neutrosophic subsets $\mathcal{A}, \mathcal{B}, \mathcal{C}$, and $\mathcal{D}$ in $\mathcal{M}$ in this manner:

$$\mathcal{A} = \langle m, \left(\frac{p}{0.5}, \frac{q}{0.3}\right), \left(\frac{p}{0.5}, \frac{q}{0.3}\right), \left(\frac{p}{0.5}, \frac{q}{0.7}\right) \rangle, \mathcal{B} = \langle m, \left(\frac{p}{0.5}, \frac{q}{0.6}\right), \left(\frac{p}{0.5}, \frac{q}{0.6}\right), \left(\frac{p}{0.5}, \frac{q}{0.4}\right) \rangle,$$

$$\mathcal{C} = \langle m, \left(\frac{p}{0.6}, \frac{q}{0.3}\right), \left(\frac{p}{0.6}, \frac{q}{0.3}\right), \left(\frac{p}{0.4}, \frac{q}{0.7}\right) \rangle \text{ and } \mathcal{D} = \langle m, \left(\frac{p}{0.6}, \frac{q}{0.7}\right), \left(\frac{p}{0.6}, \frac{q}{0.7}\right), \left(\frac{p}{0.4}, \frac{q}{0.3}\right) \rangle.$$

Then the families $\tau = \{0_N, \mathcal{A}, 1_N\}$ and $\sigma = \{0_N, \mathcal{D}, 1_N\}$ are neutrosophic topologies on $\mathcal{M}$. Thus, $(\mathcal{M}, \tau)$ and $(\mathcal{M}, \sigma)$ are *Neu*^{TS}. Define $\mathcal{h}: (\mathcal{M}, \tau) \rightarrow (\mathcal{M}, \sigma)$ as $\mathcal{h}(p) = p$, $\mathcal{h}(q) = q$. Then $\mathcal{D}$ is a *Neu*-open in $(\mathcal{M}, \sigma)$. But, $\mathcal{h}^{-1}(\mathcal{D})$ is not a *Neu*-open subset in $(\mathcal{M}, \tau)$ for $\mathcal{D} \in \sigma$. Hence, $\mathcal{h}$ is *Neu* ^{α} -continuous (*Neu*^{S α} -continuous); however, it is not a *Neu*-continuous.

Example 3.6:

Let $\mathcal{M} = \{q, r\}$. Describe the neutrosophic subsets $\mathcal{A}, \mathcal{B}, \mathcal{C}$, and $\mathcal{D}$ in $\mathcal{M}$ in this manner:

$$\mathcal{A} = \langle m, \left(\frac{q}{0.5}, \frac{r}{0.3}\right), \left(\frac{q}{0.5}, \frac{r}{0.3}\right), \left(\frac{q}{0.5}, \frac{r}{0.7}\right) \rangle, \mathcal{B} = \langle m, \left(\frac{q}{0.5}, \frac{r}{0.6}\right), \left(\frac{q}{0.5}, \frac{r}{0.6}\right), \left(\frac{q}{0.5}, \frac{r}{0.4}\right) \rangle,$$

$$\mathcal{C} = \langle m, \left(\frac{q}{0.6}, \frac{r}{0.3}\right), \left(\frac{q}{0.6}, \frac{r}{0.3}\right), \left(\frac{q}{0.4}, \frac{r}{0.7}\right) \rangle \text{ and } \mathcal{D} = \langle m, \left(\frac{q}{0.6}, \frac{r}{0.7}\right), \left(\frac{q}{0.6}, \frac{r}{0.7}\right), \left(\frac{q}{0.4}, \frac{r}{0.3}\right) \rangle.$$

Then the families $\tau = \{0_N, \mathcal{A}, 1_N\}$ and $\sigma = \{0_N, \mathcal{B}, 1_N\}$ are neutrosophic topologies on $\mathcal{M}$. Thus, $(\mathcal{M}, \tau)$ and $(\mathcal{M}, \sigma)$ are *Neu*^{TS}. Define $\mathcal{h}: (\mathcal{M}, \tau) \rightarrow (\mathcal{M}, \sigma)$ as $\mathcal{h}(q) = q$, $\mathcal{h}(r) = r$. Then $\mathcal{B}$ is a *Neu*-open in $(\mathcal{M}, \sigma)$. But, $\mathcal{h}^{-1}(\mathcal{B})$ is not a *Neu* ^{α} -open subset in $(\mathcal{M}, \tau)$ for $\mathcal{B} \in \sigma$. Hence, $\mathcal{h}$ is a *Neu*^{S α} -continuous; however, it is not a *Neu* ^{α} -continuous.

Remark 3.7:

The ideas of *Neu*-continuous and *Neu* ^{α^*} -continuous are independent.

Theorem 3.8:

- (1) If a function $h: (\mathcal{M}, \tau) \rightarrow (\mathcal{N}, \sigma)$ is *Neu*-open, *Neu*-continuous, and bijective, then h is a Neu^{α^*} -continuous.
- (2) A function $h: (\mathcal{M}, \tau) \rightarrow (\mathcal{N}, \sigma)$ is Neu^{α^*} -continuous iff $h: (\mathcal{M}, Neu^{\alpha}O(\mathcal{M})) \rightarrow (\mathcal{N}, Neu^{\alpha}O(\mathcal{N}))$ is a *Neu*-continuous.

Proof:

- (1) Assume that $\mathcal{D} \in Neu^{\alpha}O(\mathcal{N})$. To show that $h^{-1}(\mathcal{D}) \in Neu^{\alpha}O(\mathcal{M})$ We have to prove that $h^{-1}(\mathcal{D}) \subseteq NeuInt(NeuCl(NeuInt(h^{-1}(\mathcal{D}))))$.
 Assume $r \in h^{-1}(\mathcal{D}) \Rightarrow h(r) \in \mathcal{D}$. Therefore, $h(r) \in NeuInt(NeuCl(NeuInt(\mathcal{D})))$ (because $\mathcal{D} \in Neu^{\alpha}O(\mathcal{N})$). Consequently, for any *Neu*-open set $\mathcal{H}$ in $\mathcal{N}$ where $h(r) \in \mathcal{H} \subseteq NeuCl(NeuInt(\mathcal{D}))$. Then $r \in h^{-1}(\mathcal{H}) \subseteq h^{-1}(NeuCl(NeuInt(\mathcal{D})))$, but $h^{-1}(NeuCl(NeuInt(\mathcal{D}))) \subseteq NeuCl(h^{-1}(NeuInt(\mathcal{D})))$ (because h^{-1} is a *Neu*-continuous, corresponding to h is a *Neu*-open and bijective). Next, $r \in h^{-1}(\mathcal{H}) \subseteq NeuCl(h^{-1}(NeuInt(\mathcal{D})))$.
 Therefore, $r \in h^{-1}(\mathcal{H}) \subseteq NeuCl(h^{-1}(NeuInt(\mathcal{D}))) \subseteq NeuCl(NeuInt(h^{-1}(\mathcal{D})))$ (because h is a *Neu*-continuous). Consequently, $r \in h^{-1}(\mathcal{H}) \subseteq NeuCl(NeuInt(h^{-1}(\mathcal{D})))$, but $h^{-1}(\mathcal{H})$ is a *Neu*-open set in $\mathcal{M}$ (since h is a *Neu*-continuous). Therefore, $r \in NeuInt(NeuCl(NeuInt(h^{-1}(\mathcal{D}))))$. Therefore, $h^{-1}(\mathcal{D}) \subseteq NeuInt(NeuCl(NeuInt(h^{-1}(\mathcal{D})))) \Rightarrow h^{-1}(\mathcal{D}) \in Neu^{\alpha}O(\mathcal{M}) \Rightarrow h$ is a Neu^{α^*} -continuous.
- (2) The proof of (2) is evident.

Theorem 3.9:

A function $h: (\mathcal{M}, \tau) \rightarrow (\mathcal{N}, \sigma)$ is a $Neu^{S\alpha^*}$ -continuous iff $h: (\mathcal{M}, Neu^{S\alpha}O(\mathcal{M})) \rightarrow (\mathcal{N}, Neu^{S\alpha}O(\mathcal{N}))$ is a *Neu*-continuous.

Proof: Comprehensible.

Remark 3.10:

The ideas of *Neu*-continuous and $Neu^{S\alpha^*}$ -continuous are autonomous.

Remark 3.11:

Every Neu^{α^*} -continuous is a Neu^{α} -continuous and $Neu^{S\alpha}$ -continuous. Nevertheless, the contrary does not hold.

Remark 3.12:

The concepts of Neu^{α^*} -continuous and $Neu^{S\alpha^*}$ -continuous are autonomous.

Theorem 3.13:

If a function $h: (\mathcal{M}, \tau) \rightarrow (\mathcal{N}, \sigma)$ is Neu^{α^*} -continuous, *Neu*-open, and bijective, then it is $Neu^{S\alpha^*}$ -continuous.

Proof:

Assume that $h: (\mathcal{M}, \tau) \rightarrow (\mathcal{N}, \sigma)$ is a Neu^{α^*} -continuous, *Neu*-open, and bijective. Let $\mathcal{D}$ be a $Neu^{S\alpha}$ -open set in $\mathcal{N}$. Then any Neu^{α} -open set say $\mathcal{P}$ where $\mathcal{P} \subseteq \mathcal{D} \subseteq NeuCl(\mathcal{P})$. Consequently, $h^{-1}(\mathcal{P}) \subseteq h^{-1}(\mathcal{D}) \subseteq h^{-1}(NeuCl(\mathcal{P})) \subseteq NeuCl(h^{-1}(\mathcal{P}))$ (because h is a *Neu*-open), but $h^{-1}(\mathcal{P}) \in Neu^{\alpha}O(\mathcal{M})$ (since h is a Neu^{α^*} -continuous). Later, $h^{-1}(\mathcal{P}) \subseteq h^{-1}(\mathcal{D}) \subseteq NeuCl(h^{-1}(\mathcal{P}))$. Therefore, $h^{-1}(\mathcal{D}) \in Neu^{S\alpha}O(\mathcal{M})$. Thus, h is a $Neu^{S\alpha^*}$ -continuous.

Remark 3.14:

Let $h_1: (\mathcal{M}, \tau) \rightarrow (\mathcal{N}, \sigma)$ and $h_2: (\mathcal{N}, \sigma) \rightarrow (\mathcal{O}, \rho)$ be two functions, then:

- (1) If h_1 and h_2 are Neu^{α} -continuous, then $h_2 \circ h_1: (\mathcal{M}, \tau) \rightarrow (\mathcal{O}, \rho)$ need not to be a Neu^{α} -continuous.
- (2) If h_1 and h_2 are $Neu^{S\alpha}$ -continuous, then $h_2 \circ h_1: (\mathcal{M}, \tau) \rightarrow (\mathcal{O}, \rho)$ need not to be a $Neu^{S\alpha}$ -continuous.

Theorem 3.15:

Assume $h_1: (\mathcal{M}, \tau) \rightarrow (\mathcal{N}, \sigma)$ and $h_2: (\mathcal{N}, \sigma) \rightarrow (\mathcal{O}, \rho)$ are two functions, then $h_2 \circ h_1: (\mathcal{M}, \tau) \rightarrow (\mathcal{O}, \rho)$ is

- (1) a Neu^{α} -continuous if h_1 is Neu^{α} -continuous and h_2 is *Neu*-continuous.
- (2) a Neu^{α} -continuous if h_1 is Neu^{α^*} -continuous and h_2 is Neu^{α} -continuous.

- (3) a Neu^{α^*} -continuous if $\mathfrak{h}_1$ and $\mathfrak{h}_2$ are Neu^{α^*} -continuous.
- (4) a $Neu^{S\alpha^*}$ -continuous if $\mathfrak{h}_1$ and $\mathfrak{h}_2$ are $Neu^{S\alpha^*}$ -continuous.
- (5) a $Neu^{\alpha^{**}}$ -continuous if $\mathfrak{h}_1$ and $\mathfrak{h}_2$ are $Neu^{\alpha^{**}}$ -continuous.
- (6) a $Neu^{S\alpha^{**}}$ -continuous if $\mathfrak{h}_1$ and $\mathfrak{h}_2$ are $Neu^{S\alpha^{**}}$ -continuous.
- (7) a $Neu^{\alpha^{**}}$ -continuous if $\mathfrak{h}_1$ is $Neu^{\alpha^{**}}$ -continuous and $\mathfrak{h}_2$ is Neu^{α^*} -continuous.
- (8) a Neu -continuous if $\mathfrak{h}_1$ is $Neu^{\alpha^{**}}$ -continuous and $\mathfrak{h}_2$ is Neu^{α} -continuous.
- (9) a Neu^{α^*} -continuous if $\mathfrak{h}_1$ is Neu^{α} -continuous and $\mathfrak{h}_2$ is $Neu^{\alpha^{**}}$ -continuous.
- (10) a $Neu^{\alpha^{**}}$ -continuous if $\mathfrak{h}_1$ is Neu -continuous and $\mathfrak{h}_2$ is $Neu^{\alpha^{**}}$ -continuous.

Proof:

- (1) Assume $\mathcal{F}$ is a Neu -open subset in $\mathcal{O}$. Subsequently, $\mathfrak{h}_2$ is a Neu -continuous, $\mathfrak{h}_2^{-1}(\mathcal{F})$ is a Neu -open subset in $\mathcal{N}$. Later, $\mathfrak{h}_1$ is a Neu^{α} -continuous, $\mathfrak{h}_1^{-1}(\mathfrak{h}_2^{-1}(\mathcal{F})) = (\mathfrak{h}_2 \circ \mathfrak{h}_1)^{-1}(\mathcal{F})$ is a Neu^{α} -open subset in $\mathcal{M}$. Therefore, $\mathfrak{h}_2 \circ \mathfrak{h}_1: (\mathcal{M}, \tau) \rightarrow (\mathcal{O}, \rho)$ is a Neu^{α} -continuous.
- (2) Assume $\mathcal{F}$ is a Neu -open subset in $\mathcal{O}$. Subsequently, $\mathfrak{h}_2$ is a Neu^{α} -continuous, $\mathfrak{h}_2^{-1}(\mathcal{F})$ is a Neu^{α} -open subset in $\mathcal{N}$. Later, $\mathfrak{h}_1$ is a Neu^{α^*} -continuous, $\mathfrak{h}_1^{-1}(\mathfrak{h}_2^{-1}(\mathcal{F})) = (\mathfrak{h}_2 \circ \mathfrak{h}_1)^{-1}(\mathcal{F})$ is a Neu^{α} -open subset in $\mathcal{M}$. Therefore, $\mathfrak{h}_2 \circ \mathfrak{h}_1: (\mathcal{M}, \tau) \rightarrow (\mathcal{O}, \rho)$ is a Neu^{α} -continuous.
- (3) Assume $\mathcal{F}$ is a Neu^{α} -open subset in $\mathcal{O}$. Subsequently, $\mathfrak{h}_2$ is a Neu^{α^*} -continuous, $\mathfrak{h}_2^{-1}(\mathcal{F})$ is a Neu^{α} -open subset in $\mathcal{N}$. Later, $\mathfrak{h}_1$ is a Neu^{α^*} -continuous, $\mathfrak{h}_1^{-1}(\mathfrak{h}_2^{-1}(\mathcal{F})) = (\mathfrak{h}_2 \circ \mathfrak{h}_1)^{-1}(\mathcal{F})$ is a Neu^{α} -open subset in $\mathcal{M}$. Therefore, $\mathfrak{h}_2 \circ \mathfrak{h}_1: (\mathcal{M}, \tau) \rightarrow (\mathcal{O}, \rho)$ is a Neu^{α^*} -continuous.
- (4) Assume $\mathcal{F}$ be a $Neu^{S\alpha}$ -open subset in $\mathcal{O}$. Since $\mathfrak{h}_2$ is a $Neu^{S\alpha^*}$ -continuous, $\mathfrak{h}_2^{-1}(\mathcal{F})$ is a $Neu^{S\alpha}$ -open subset in $\mathcal{N}$. Later, $\mathfrak{h}_1$ is a $Neu^{S\alpha^*}$ -continuous, $\mathfrak{h}_1^{-1}(\mathfrak{h}_2^{-1}(\mathcal{F})) = (\mathfrak{h}_2 \circ \mathfrak{h}_1)^{-1}(\mathcal{F})$ is a $Neu^{S\alpha}$ -open subset in $\mathcal{M}$. Therefore, $\mathfrak{h}_2 \circ \mathfrak{h}_1: (\mathcal{M}, \tau) \rightarrow (\mathcal{O}, \rho)$ is a $Neu^{S\alpha^*}$ -continuous.
- (5) Assume $\mathcal{F}$ be a Neu^{α} -open subset in $\mathcal{O}$. Subsequently, $\mathfrak{h}_2$ is a $Neu^{\alpha^{**}}$ -continuous, $\mathfrak{h}_2^{-1}(\mathcal{F})$ is a Neu -open subset in $\mathcal{N}$. Later, any Neu -open subset is a Neu^{α} -open, $\mathfrak{h}_2^{-1}(\mathcal{F})$ is a Neu^{α} -open subset in $\mathcal{N}$. Since $\mathfrak{h}_1$ is a $Neu^{\alpha^{**}}$ -continuous, $\mathfrak{h}_1^{-1}(\mathfrak{h}_2^{-1}(\mathcal{F})) = (\mathfrak{h}_2 \circ \mathfrak{h}_1)^{-1}(\mathcal{F})$ is a Neu -open subset in $\mathcal{M}$. Therefore, $\mathfrak{h}_2 \circ \mathfrak{h}_1: (\mathcal{M}, \tau) \rightarrow (\mathcal{O}, \rho)$ is a $Neu^{\alpha^{**}}$ -continuous. The proof is evident to others.

Remark 3.16:

The later illustration clarifies the connection among various thoughts of weakly *Neu*-continuous functions:

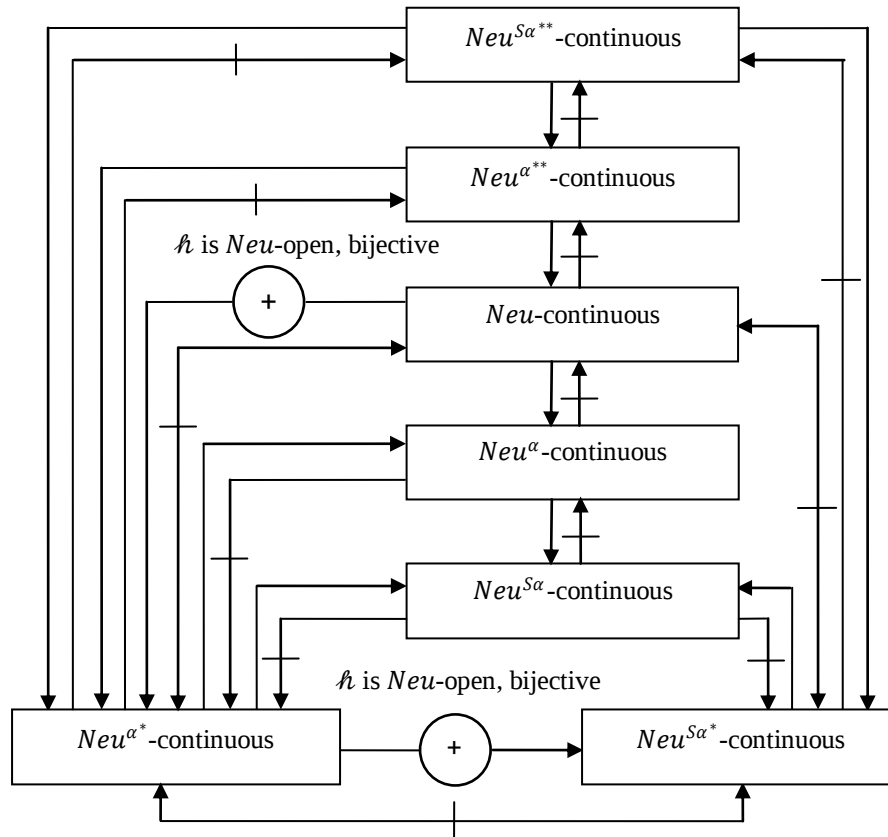


Fig. 1: The later

4. Conclusion

We intend to exercise the notions of Neu^{α} -open and $Neu^{S\alpha}$ -open sets to outline some novel concepts of weakly *Neu*-continuous functions, for example; $Neu^{\alpha^{*}}$ -continuous, $Neu^{\alpha^{**}}$ -continuous, $Neu^{S\alpha}$ -continuous, $Neu^{S\alpha^{*}}$ -continuous and $Neu^{S\alpha^{**}}$ -continuous functions. The Neu^{α} -open and $Neu^{S\alpha}$ -open sets can be employed to arise some novel notions of weakly *Neu*-open (*Neu*-closed) functions and *Neu*-separation axioms.

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