



Shortest Path Problem on Neutrosophic Environment using Modified Circle Breaking Algorithm

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Abstract

Neutrosophic set (NS) is generalization of Intuitionistic Fuzzy Set (IFS) and Fuzzy Set (FS) where Neutrosophic Set (NS) is the collection of Membership, Non-Membership, Indeterminacy Membership of the constituent element. This paper includes the modified circle breaking technique which is used to evaluate the Shortest Path Problem in which edge weight are portrayed in Single Valued Linear Heptagonal Neutrosophic Number (SVLHNN) and an numerical illustration is given for the efficiency of the given algorithm.

Keywords: Shortest Path Problem; Modified Circle Breaking Algorithm; Single Valued Linear Heptagonal Neutrosophic Number

1 Introduction

Neutrosophy theory and its applications has extended around the globe in all direction at an increasing rate. New theories, Methodology, Models, techniques, algorithm have been rapidly developed. The Neutrosophic number contains degree of membership function, degree of uncertainty membership function and degree of non-membership function. This research article deals with the single valued linear heptagonal neutrosophic number which is an extension of the linear single valued pentagonal neutrosophic number which deals with the uncertainty, hesitation, vagueness, also more reliable, logical for decision making, optimization problems, linear programming problem, etc,. The shortest path problem is a emerging in all fields of scientific. The drainage routes to connect the sea in the city makes people to suffer during the flood in monsoon season. The sewer routes that can be expressed become indeterminate when measured accurately. The objective of shortest path problem is to minimize the cost of the path from the beginning to terminal point.

The conception of fuzzy set and membership function was developed by L.A.Zadeh [1] in 1965 to grip the uncertain data. Later Atanassov [2] explored Intuitionistic fuzzy set which deals with membership and non-membership function. Smarandache [3] in 1998 introduced the concept of Neutrosophic Set (NS) which contains truth membership function, Indeterminacy membership function, falsity membership function which is extension of intuitionistic fuzzy sets and general fuzzy sets also researchers throughout the globe developing bipolar neutrosophic set, hesitant neutrosophic set. Chakraborty et.al [4] in 2019 developed De-Neutrosophication technique using area removal method i.e. to convert neutrosophic number into a classical number. Amala S Richard and Rajkumar [5] developed single valued linear heptagonal neutrosophic number along with their Deneutrosophication. Avishek chakraborty [6] in 2020 applied pentagonal neutrosophic number in shortest path

problem also used score and accuracy function in a different aspects. Florentin Smarandache et.al [9] introduced New Theories in neutrosophic theory focusing on algorithm for decision making data mining, e-learning, topology and more. Broumi Said [10] in 2018 introduced the shortest path problem (SPP) in neutrosophic graph and introduced the algorithm to solve SPP under Neutrosophic environment.

2 Preliminaries

“Definition 1: Neutrosophic Set(NS)[3] Let \mathbb{X} be the Universe of discourse $x \in \mathbb{X}$. Then a neutrosophic set α in \mathbb{X} is characterized by a membership function, T_α , an indeterminacy membership function I_α and a non-membership function F_α . The function

$$T_\alpha: \mathbb{X} \rightarrow [0,1]; I_\alpha: \mathbb{X} \rightarrow [0,1]; F_\alpha: \mathbb{X} \rightarrow [0,1]$$

There is no restriction on the sum of $T_\alpha(x), I_\alpha(x), F_\alpha(x)$ i.e.

$$0 \leq T_\alpha(x) + I_\alpha(x) + F_\alpha(x) \leq 3$$

Definition 2: Single Valued Linear Pentagonal Neutrosophic Number: [4]

Let \tilde{S} is defined as follows where \tilde{S} is single valued linear pentagonal neutrosophic number and described as $\tilde{S} = \langle[(\zeta^1, \zeta^2, \zeta^3, \zeta^4, \zeta^5)]$

Where $\rho, \Xi, \omega \in [0,1]$. The truth membership function $(\theta_{\tilde{S}}): R \rightarrow [0, \rho]$, the indeterminacy membership function $(\varnothing_{\tilde{S}}): R \rightarrow [\Xi, 1]$ and the falsity membership function $(\varphi_{\tilde{S}}): R \rightarrow [\omega, 1]$ are given as:

$$\theta_{\tilde{S}}(y) = \begin{cases} \theta_{\tilde{S}\bar{1}}(y), & \zeta^1 \leq y \leq \zeta^2 \\ \theta_{\tilde{S}\bar{2}}(y), & \zeta^2 \leq y \leq \zeta^3 \\ \rho, & y = \zeta^3 \\ \theta_{\tilde{S}\bar{r}1}(y), & \zeta^3 \leq y \leq \zeta^4 \\ \theta_{\tilde{S}\bar{r}2}(y), & \zeta^4 \leq y \leq \zeta^5 \\ 0, & \text{otherwise} \end{cases}$$

$$\varphi_{\tilde{S}}(y) = \begin{cases} \varphi_{\tilde{S}\bar{1}}(y), & \psi^1 \leq y \leq \psi^2 \\ \varphi_{\tilde{S}\bar{2}}(y), & \psi^2 \leq y \leq \psi^3 \\ \Xi, & y = \psi^3 \\ \varphi_{\tilde{S}\bar{r}1}(y), & \psi^3 \leq y \leq \psi^4 \\ \varphi_{\tilde{S}\bar{r}2}(y), & \psi^4 \leq y \leq \psi^5 \\ 1, & \text{otherwise} \end{cases}$$

$$\varphi_{\tilde{S}}(y) = \begin{cases} \varphi_{\tilde{S}\bar{1}}(y), & \sigma^1 \leq y \leq \sigma^2 \\ \varphi_{\tilde{S}\bar{2}}(y), & \sigma^2 \leq y \leq \sigma^3 \\ \omega, & y = \sigma^3 \\ \varphi_{\tilde{S}\bar{r}1}(y), & \sigma^3 \leq y \leq \sigma^4 \\ \varphi_{\tilde{S}\bar{r}2}(y), & \sigma^4 \leq y \leq \sigma^5 \\ 1, & \text{otherwise} \end{cases}$$

Definition 3: Single Valued Linear Heptagonal Neutrosophic Number:[5]

A single valued linear heptagonal neutrosophic number \widetilde{H}_{Neu} is defined and described as

$$\widetilde{H}_{Neu} = \langle [(\Theta_{11}, \Theta_{22}, \Theta_{33}, \Theta_{44}, \Theta_{55}, \Theta_{66}, \Theta_{77}); \rho], [(\varphi_{11}, \varphi_{22}, \varphi_{33}, \varphi_{44}, \varphi_{55}, \varphi_{66}, \varphi_{77}); \sigma], [(\eta_{11}, \eta_{22}, \eta_{33}, \eta_{44}, \eta_{55}, \eta_{66}, \eta_{77}); \omega] \rangle$$

Where $\rho, \sigma, \omega \in [0,1]$ The Membership function $\theta_{\widetilde{H}_{Neu}}: \mathbb{R} \rightarrow [0, \rho]$, the indeterminacy membership function $\varnothing_{\widetilde{H}_{Neu}}: \mathbb{R} \rightarrow [\sigma, 1]$, the Non - membership function $\varphi_{\widetilde{H}_{Neu}}: \mathbb{R} \rightarrow [\omega, 1]$ are given as

$$\theta_{\widetilde{HNeu}}(x) = \begin{cases} \frac{x-\theta_{11}}{\theta_{22}-\theta_{11}} & \theta_{11} \leq x \leq \theta_{22} \\ \frac{x-\theta_{22}}{\theta_{33}-\theta_{22}} & \theta_{22} \leq x \leq \theta_{33} \\ \frac{x-\theta_{33}}{\theta_{44}-\theta_{33}} & \theta_{33} \leq x \leq \theta_{44} \\ 1 & x = \theta_{44} \\ \frac{\theta_{55}-x}{\theta_{55}-\theta_{44}} & \theta_{44} \leq x \leq \theta_{55} \\ \frac{\theta_{66}-x}{\theta_{66}-\theta_{55}} & \theta_{55} \leq x \leq \theta_{66} \\ \frac{\theta_{77}-x}{\theta_{77}-\theta_{66}} & \theta_{66} \leq x \leq \theta_{77} \\ 0 & \text{otherwise} \end{cases}$$

$$\varphi_{\widetilde{HNeu}}(x) = \begin{cases} \frac{x-\varphi_{11}}{\varphi_{22}-\varphi_{11}} & \varphi_{11} \leq x \leq \varphi_{22} \\ \frac{x-\varphi_{22}}{\varphi_{33}-\varphi_{22}} & \varphi_{22} \leq x \leq \varphi_{33} \\ \frac{x-\varphi_{33}}{\varphi_{44}-\varphi_{33}} & \varphi_{33} \leq x \leq \varphi_{44} \\ 0 & x = \varphi_{44} \\ \frac{\varphi_{55}-x}{\varphi_{55}-\varphi_{44}} & \varphi_{44} \leq x \leq \varphi_{55} \\ \frac{\varphi_{66}-x}{\varphi_{66}-\varphi_{55}} & \varphi_{55} \leq x \leq \varphi_{66} \\ \frac{\varphi_{77}-x}{\varphi_{77}-\varphi_{66}} & \varphi_{66} \leq x \leq \varphi_{77} \\ 1 & \text{otherwise} \end{cases}$$

$$\psi_{\widetilde{HNeu}}(x) = \begin{cases} \frac{x-\eta_{11}}{\eta_{22}-\eta_{11}} & \eta_{11} \leq x \leq \eta_{22} \\ \frac{x-\eta_{22}}{\eta_{33}-\eta_{22}} & \eta_{22} \leq x \leq \eta_{33} \\ \frac{x-\eta_{33}}{\eta_{44}-\eta_{33}} & \eta_{33} \leq x \leq \eta_{44} \\ 0 & x = \eta_{44} \\ \frac{\eta_{55}-x}{\eta_{55}-\eta_{44}} & \eta_{44} \leq x \leq \eta_{55} \\ \frac{\eta_{66}-x}{\eta_{66}-\eta_{55}} & \eta_{55} \leq x \leq \eta_{66} \\ \frac{\eta_{77}-x}{\eta_{77}-\eta_{66}} & \eta_{66} \leq x \leq \eta_{77} \\ 1 & \text{otherwise} \end{cases}$$

Where $0 \leq \theta_{\widetilde{HNeu}}(x) + \varphi_{\widetilde{HNeu}}(x) + \psi_{\widetilde{HNeu}}(x) \leq 3$

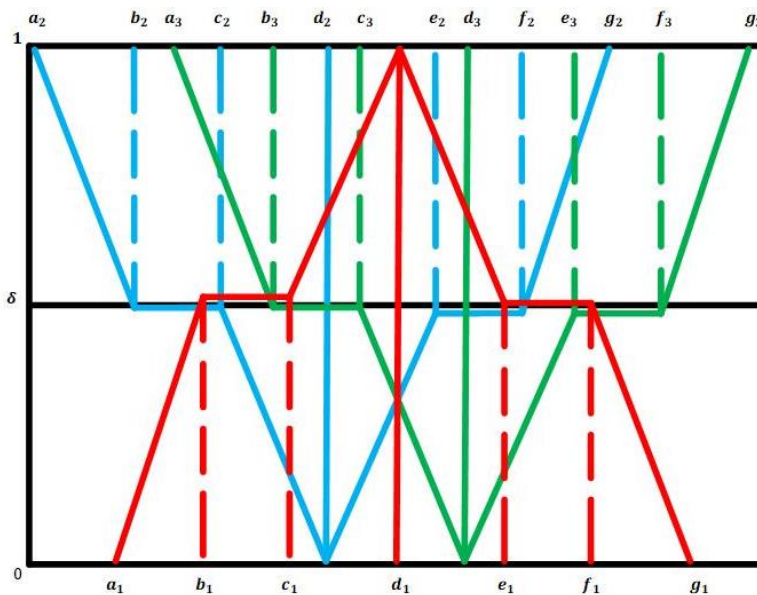


Figure 1: Graphical representation of single valued heptagonal Neutrosophic number

3 Arithmetic operations on Single Valued Linear Heptagonal Neutrosophic Number (SVLHNN)

Let $\widetilde{R}_{Hneu} = \{(\xi^{11}, \xi^{22}, \xi^{33}, \xi^{44}, \xi^{55}, \xi^{66}, \xi^{77}); (\psi^{11}, \psi^{22}, \psi^{33}, \psi^{44}, \psi^{55}, \psi^{66}, \psi^{77})(v^{11}, v^{22}, v^{33}, v^{44}, v^{55}, v^{66}, v^{77})\}$

$$\tilde{S}_{Hneu} = \{(\varphi^{11}, \varphi^{22}, \varphi^{33}, \varphi^{44}, \varphi^{55}, \varphi^{66}, \varphi^{77}); (v^{11}, v^{22}, v^{33}, v^{44}, v^{55}, v^{66}, v^{77}); (\sigma^{11}, \sigma^{22}, \sigma^{33}, \sigma^{44}, \sigma^{55}, \sigma^{66}, \sigma^{77})\}$$

$$1) \tilde{R}_{Hneu} \oplus \tilde{S}_{Hneu} =$$

$$\left[\begin{array}{l} (\xi^{11} + \varphi^{11} - \xi^{11}\varphi^{11}, \xi^{22} + \varphi^{22} - \xi^{22}\varphi^{22}, \xi^{33} + \varphi^{33} - \xi^{33}\varphi^{33}, \xi^{44} + \varphi^{44} - \xi^{44}\varphi^{44}, \\ \xi^{55} + \varphi^{55} - \xi^{55}\varphi^{55}, \xi^{66} + \varphi^{66} - \xi^{66}\varphi^{66}, \xi^{77} + \varphi^{77} - \xi^{77}\varphi^{77}); \\ (\psi^{11}v^{11}, \psi^{22}v^{22}, \psi^{33}v^{33}, \psi^{44}v^{44}, \psi^{55}v^{55}, \psi^{66}v^{66}, \psi^{77}v^{77}); \\ (v^{11}\sigma^{11}, v^{22}\sigma^{22}, v^{33}\sigma^{33}, v^{44}\sigma^{44}, v^{55}\sigma^{55}, v^{66}\sigma^{66}, v^{77}\sigma^{77}); \end{array} \right] \tag{1}$$

$$2) \tilde{R}_{Hneu} \otimes \tilde{S}_{Hneu} =$$

$$\left[\begin{array}{l} (\xi^{11}\varphi^{11}, \xi^{22}\varphi^{22}, \xi^{33}\varphi^{33}, \xi^{44}\varphi^{44}, \xi^{55}\varphi^{55}, \xi^{66}\varphi^{66}, \xi^{77}\varphi^{77}); \\ (\psi^{11} + v^{11} - \psi^{11}v^{11}, \psi^{22} + v^{22} - \psi^{22}v^{22}, \psi^{33} + v^{33} - \psi^{33}v^{33}, \psi^{44} + v^{44} - \psi^{44}v^{44}, \\ \psi^{55} + v^{55} - \psi^{55}v^{55}, \psi^{66} + v^{66} - \psi^{66}v^{66}, \psi^{77} + v^{77} - \psi^{77}v^{77}); \\ (v^{11} + \sigma^{11} - v^{11}\sigma^{11}, v^{22} + \sigma^{22} - v^{22}\sigma^{22}, v^{33} + \sigma^{33} - v^{33}\sigma^{33}, v^{44} + \sigma^{44} - v^{44}\sigma^{44}, \\ v^{55} + \sigma^{55} - v^{55}\sigma^{55}, v^{66} + \sigma^{66} - v^{66}\sigma^{66}, v^{77} + \sigma^{77} - v^{77}\sigma^{77}); \end{array} \right] \tag{2}$$

$$3) \lambda \tilde{R}_{Hneu} =$$

$$\left[\begin{array}{l} (1 - (1 - \xi^{11})^\lambda, 1 - (1 - \xi^{22})^\lambda, 1 - (1 - \xi^{33})^\lambda, 1 - (1 - \xi^{44})^\lambda, \\ 1 - (1 - \xi^{55})^\lambda, 1 - (1 - \xi^{66})^\lambda, 1 - (1 - \xi^{77})^\lambda); \\ (\lambda\psi^{11}, \lambda\psi^{22}, \lambda\psi^{33}, \lambda\psi^{44}, \lambda\psi^{55}, \lambda\psi^{66}, \lambda\psi^{77}); \\ (\lambda v^{11}, \lambda v^{22}, \lambda v^{33}, \lambda v^{44}, \lambda v^{55}, \lambda v^{66}, \lambda v^{77}); \end{array} \right] \tag{3}$$

4 Modified Circle Breaking Algorithm

Step 1: Arbitrarily define a closed circle in the single valued linear heptagonal edge weighted graph and determine the two paths P_1 and P_2 enclosed with the completely closed circle where P_1 & P_2 having a common initial node denoted by T_1 and a common terminal node denoted by T_2

Step 2: Using **equations (1)** all the edges are added to the path. The single valued heptagonal neutrosophic number (SVLHNN) \tilde{H}_{P_1} and \tilde{H}_{P_2} are obtained which represents two paths.

Step 3: Calculate the score function values represented as $S(\tilde{H}_{P_1})$ and $S(\tilde{H}_{P_2})$ and accuracy function value $A(\tilde{H}_{P_1})$ as well as $A(\tilde{H}_{P_2})$

Step 4: Evaluating the size of the \tilde{H}_{P_1} and \tilde{H}_{P_2} and rank the obtained result and Eradicate the edges of the larger path whose vertices in T_1

Step 5: END,if circle is not closed. if the circle is enclosed then go to **step 1**.

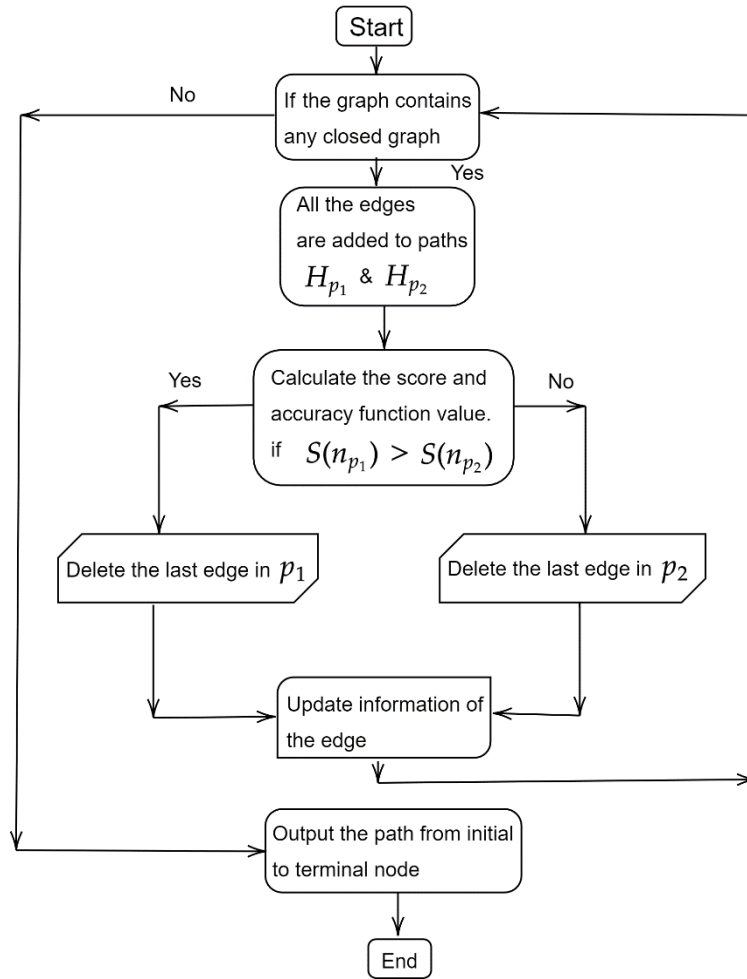


Figure 2: Flow chart of the proposed algorithm

5 Numerical Example

The Numerical analysis that is taken is an case analysis to find the optimal path to rescue people from rainstorms in wuhan city, china. figure 3 depicts the topological structure of the road network during this period and table 1 indicates the neutrosophic distance, the rescue team move from start point 1 to terminal node 6, the main aim of the paper is to find the optimal shortest path from the initial node to terminal node.[8]

Table 1: single valued heptagonal neutrosophic number edge information details

Edges	Single Valued Linear Heptagonal Neutrosophic Number
(1,2)	< (0.1,0.2,0.3,0.5,0.4,0.3,0.2); (0.2,0.3,0.5,0.6,0.2,0.4,0.3); (0.4,0.5,0.6,0.8,0.4,0.2,0.3) >
(1,3)	< (0.2,0.4,0.5,0.7,0.2,0.3,0.4); (0.3,0.5,0.6,0.9,0.4,0.2,0.3); (0.1,0.2,0.3,0.4,0.2,0.4,0.3) >
(2,4)	< (0.3,0.4,0.6,0.7,0.1,0.3,0.5); (0.1,0.2,0.3,0.5,0.3,0.5,0.1); (0.3,0.5,0.7,0.9,0.1,0.5,0.1) >
(2,5)	< (0.1,0.3,0.4,0.5,0.5,0.3,0.1); (0.3,0.4,0.5,0.7,0.5,0.1,0.3); (0.2,0.3,0.6,0.7,0.1,0.3,0.1) >
(3,4)	< (0.2,0.3,0.5,0.6,0.5,0.3,0.1); (0.2,0.5,0.6,0.7,0.1,0.3,0.5); (0.4,0.5,0.6,0.8,0.3,0.2,0.4) >
(3,5)	< (0.3,0.6,0.7,0.8,0.3,0.2,0.1); (0.1,0.2,0.3,0.4,0.5,0.3,0.1); (0.1,0.4,0.5,0.6,0.7,0.3,0.1) >
(4,6)	< (0.4,0.6,0.8,0.9,0.3,0.3,0.1); (0.2,0.4,0.5,0.6,0.3,0.1,0.2); (0.1,0.3,0.4,0.5,0.2,0.1,0.3) >
(5,6)	< (0.2,0.3,0.4,0.5,0.3,0.2,0.5); (0.3,0.4,0.5,0.6,0.3,0.5,0.1); (0.1,0.3,0.5,0.6,0.5,0.7,0.1) >

The Shortest path problem from 1 - 6 is evaluated using Modified Circle Breaking Algorithm. The initial circle is selected from the figure 2 the largest path to find and delete the last edge of the circle so that the circle is broken and this process is repeated until no circle is formed.

Step 1: circle ①②④③① is selected path. $p_1 = \{(1,3)(3,4)\}$ and $p_2 = \{(1,2)(2,4)\}$ which makes the closed circle.

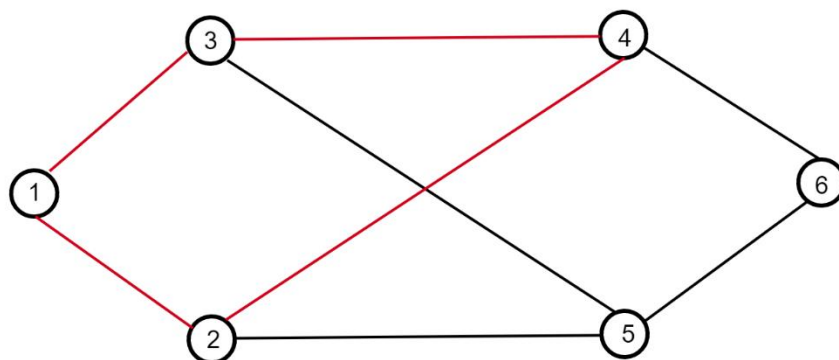


Figure 3: closed edge graph ①②④③①

Step 2: Using the **equation (1)** the neutrosophic edges (1,3)(4,3) and (1,2)(2,4) are added using the derived formula

$$\begin{aligned} \tilde{n}_{p_1} &= \tilde{n}_{(1,3)} \oplus \tilde{n}_{(3,4)} \\ &= \left(\begin{array}{l} (0.2 + 0.2 - 0.2 \cdot 0.2, 0.4 + 0.3 - 0.4 \cdot 0.3, 0.5 + 0.5 - 0.5 \cdot 0.5, 0.7 + 0.6 - 0.7 \cdot 0.6), \\ (0.2 + 0.5 - 0.2 \cdot 0.5, 0.3 + 0.3 - 0.3 \cdot 0.3, 0.4 + 0.1 - 0.4 \cdot 0.1); \\ (0.3 \cdot 0.2, 0.5 \cdot 0.5, 0.6 \cdot 0.6, 0.9 \cdot 0.7, 0.4 \cdot 0.1, 0.3 \cdot 0.2, 0.5 \cdot 0.3); \\ (0.1 \cdot 0.4, 0.2 \cdot 0.5, 0.6 \cdot 0.3, 0.4 \cdot 0.8, 0.2 \cdot 0.3, 0.4 \cdot 0.2, 0.3 \cdot 0.4); \end{array} \right) \\ &= \left(\begin{array}{l} (0.36, 0.58, 0.75, 0.88, 0.6, 0.51, 0.46) \\ (0.06, 0.25, 0.36, 0.63, 0.04, 0.06, 0.15) \\ (0.04, 0.1, 0.18, 0.32, 0.06, 0.08, 0.12) \end{array} \right) \end{aligned}$$

$$\begin{aligned} \tilde{n}_{p_2} &= \tilde{n}_{(1,2)} \oplus \tilde{n}_{(2,4)} \\ &= \left(\begin{array}{l} (0.1 + 0.3 - 0.1 \cdot 0.3, 0.2 + 0.4 - 0.2 \cdot 0.4, 0.3 + 0.6 - 0.3 \cdot 0.6, 0.5 + 0.7 - 0.5 \cdot 0.7), \\ (0.4 + 0.1 - 0.4 \cdot 0.1, 0.3 + 0.3 - 0.3 \cdot 0.3, 0.2 + 0.5 - 0.2 \cdot 0.5); \\ (0.2 \cdot 0.1, 0.3 \cdot 0.2, 0.5 \cdot 0.3, 0.6 \cdot 0.5, 0.2 \cdot 0.3, 0.4 \cdot 0.5, 0.3 \cdot 0.1); \\ (0.4 \cdot 0.3, 0.5 \cdot 0.5, 0.6 \cdot 0.7, 0.8 \cdot 0.9, 0.4 \cdot 0.1, 0.2 \cdot 0.5, 0.3 \cdot 0.1); \end{array} \right) \\ &= \left(\begin{array}{l} (0.37, 0.52, 0.72, 0.85, 0.46, 0.51, 0.6) \\ (0.02, 0.06, 0.15, 0.30, 0.06, 0.20, 0.03) \\ (0.12, 0.25, 0.42, 0.72, 0.04, 0.10, 0.03) \end{array} \right) \end{aligned}$$

Step 3: Obtain the score function value and accuracy function value given in [5]

$$S(\tilde{A}) = \frac{1}{18} [(\xi^{11} + 2\xi^{22} + \xi^{33} + \xi^{55} + 2\xi^{66} + \xi^{77} + \psi^{33} + 2\psi^{44} + \psi^{55} + \nu^{33} + 2\nu^{44} + \nu^{55})\delta + (\xi^{33} + 2\xi^{44} + \xi^{55} + \psi^{11} + 2\psi^{22} + \psi^{33} + \psi^{55} + 2\psi^{66} + \psi^{77} + \nu^{11} + 2\nu^{22} + \nu^{33} + \nu^{55} + 2\nu^{66} + \nu^{77})(1 - \delta)]$$

here $\delta = 0.5$

$$H(\tilde{A}) = \frac{1}{7} [(\xi^{11} + \xi^{22} + \xi^{33} + \xi^{44} + \xi^{55} + \xi^{66} + \xi^{77}) - (\nu^{11} + \nu^{22} + \nu^{33} + \nu^{44} + \nu^{55} + \nu^{66} + \nu^{77})]$$

calculating the score and accuracy function value we get

$$S(\tilde{n}_{p_1}) = 0.3330$$

$$S(\tilde{n}_{p_2}) = 0.3272$$

$$H(\tilde{n}_{p_1}) = 0.4628$$

$$H(\tilde{n}_{p_2}) = 0.3357$$

Step 4: $S(\tilde{n}_{p_1}) > S(\tilde{n}_{p_2})$ $\tilde{n}_{p_1} > \tilde{n}_{p_2}$ delete the terminal edge (3,4) in p_1 then the figure is given below

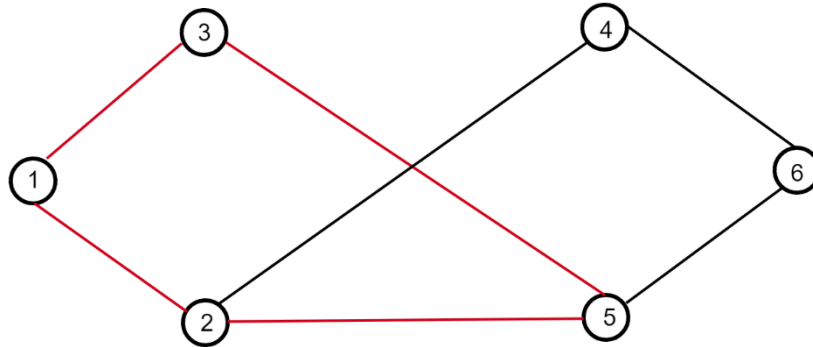


Figure 4: Removal of (3,4) & closed edge graph ①③⑤②①

Step 5: Consider another closed circle ①③⑤②①

$$p_3 = \{(1,3)(3,5)\} \text{ and } p_4 = \{(1,2)(2,5)\}$$

Step 2: Again by **equation (1)** all the neutrosophic edges of each path are summed,

$$\tilde{n}_{p_3} = \tilde{n}_{(1,3)} \oplus \tilde{n}_{(3,5)}$$

$$= \left\langle \begin{matrix} (0.44, 0.76, 0.85, 0.94, 0.44, 0.44, 0.46); \\ (0.03, 0.10, 0.18, 0.36, 0.20, 0.06, 0.03); \\ (0.01, 0.08, 0.15, 0.24, 0.14, 0.12, 0.03) \end{matrix} \right\rangle$$

$$\tilde{n}_{p_4} = \tilde{n}_{(1,2)} \oplus \tilde{n}_{(2,5)}$$

$$= \left\langle \begin{matrix} (0.19, 0.44, 0.58, 0.75, 0.7, 0.51, 0.28); \\ (0.06, 0.12, 0.25, 0.42, 0.10, 0.04, 0.03); \\ (0.08, 0.15, 0.12, 0.56, 0.04, 0.06, 0.03) \end{matrix} \right\rangle$$

Step 3: Computing Score and accuracy values of the paths \tilde{n}_{p_3} and \tilde{n}_{p_4} we get

$$S(\tilde{n}_{p_3}) = 0.3014$$

$$S(\tilde{n}_{p_4}) = 0.2875$$

$$H(\tilde{n}_{p_3}) = 0.5085$$

$$H(\tilde{n}_{p_4}) = 0.3442$$

Step 4: since $S(\tilde{n}_{p_3}) > S(\tilde{n}_{p_4})$ $\tilde{n}_{p_3} > \tilde{n}_{p_4}$ eliminate the last edge from $S(\tilde{n}_{p_3})$ then the graph is shown

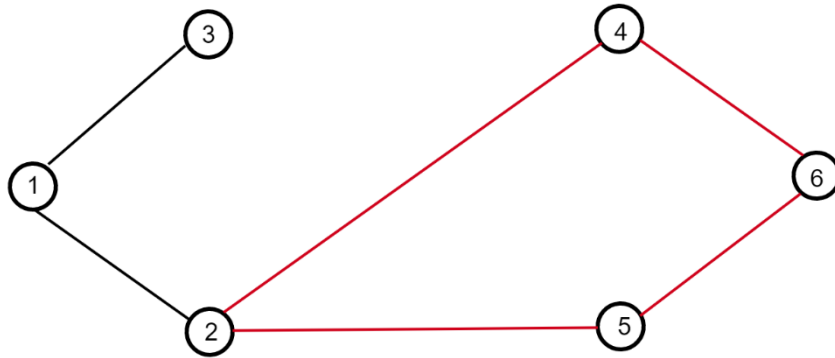


Figure 5: Removal of (3,5) edge & closed edge graph ②④⑥⑤②

Step 5: Again choose the closed circle ②④⑥⑤② such that the paths

$p_5 = \{(2,4)(4,6)\}$ and $p_6 = \{(2,5)(5,6)\}$ are chosen

Step 2: Again with the help of **equation (1)** all the neutrosophic edge weight of each path are added and we got \tilde{n}_{p_5} and \tilde{n}_{p_6} by computing.

$$\tilde{n}_{p_5} = \tilde{n}_{(2,4)} \oplus \tilde{n}_{(4,6)}$$

$$= \left\langle \begin{matrix} (0.58, 0.76, 0.92, 0.97, 0.37, 0.51, 0.55); \\ (0.02, 0.08, 0.15, 0.30, 0.09, 0.05, 0.02); \\ (0.03, 0.15, 0.28, 0.45, 0.02, 0.05, 0.03) \end{matrix} \right\rangle$$

$$\tilde{n}_{p_6} = \tilde{n}_{(2,5)} \oplus \tilde{n}_{(5,6)}$$

$$= \left\langle \begin{matrix} (0.28, 0.51, 0.64, 0.75, 0.65, 0.44, 0.55); \\ (0.09, 0.16, 0.25, 0.42, 0.15, 0.05, 0.03); \\ (0.02, 0.19, 0.30, 0.42, 0.05, 0.21, 0.01) \end{matrix} \right\rangle$$

Step 3: Aggregate the score and accuracy function values of \tilde{n}_{p_5} and \tilde{n}_{p_6}

$$S(\tilde{n}_{p_5}) = 0.32027$$

$$S(\tilde{n}_{p_6}) = 0.31$$

$$H(\tilde{n}_{p_5}) = 0.3828$$

$$H(\tilde{n}_{p_6}) = 0.3886$$

Step 4: since $S(\tilde{n}_{p_5}) > S(\tilde{n}_{p_6})$ $\tilde{n}_{p_5} > \tilde{n}_{p_6}$ eliminate the last edge from i.e. (4,6) the graph is shown as:

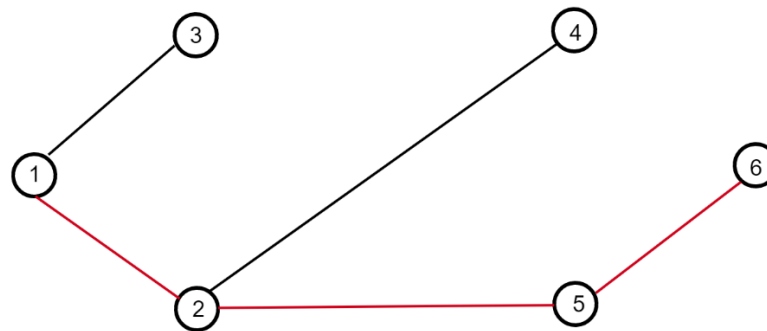


Figure 6: Removal of edge (4,6) and No closed edge graph

Step 5: We find no closed circle in the above graph so, the algorithm terminates and the shortest path of the given problem is from ①②⑤⑥

6 Conclusion

In this Research article we solved the shortest path problem (SSP) using modified circle breaking algorithm using single valued linear heptagonal neutrosophic number (SVLHNN) and an example is illustrated for the better understanding of the algorithm which is more feasible and easy to understand the shortest path of the given problem is ①②⑤⑥. In Future developing the neutrosophic number and apply it in decision making, optimization problems etc, which is much needed in the upcoming scientific fields.

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