More on Neutrosophic Nano Open Sets

A. Vadivel[1,*], C. John Sundar[2], K. Kirubadevi[3], S. Tamilselvan[4]
[1]PG and Research Department of Mathematics, Government Arts College (Autonomous), Karur - 639 005, India.
[1,2]Department of Mathematics, Annamalai University, Annamalai Nagar - 608 002, India.
[3]Department of Mathematics, Thiruvalluvar Government Arts College, Rasipuram, Namakkal - 637 401, India.
[4]Mathematics Section (FEAT), Annamalai University, Annamalai Nagar - 608 002, India.
Emails: avmaths@gmail.com[1]; johnphdau@hotmail.com[2]; kirubadevi08@gmail.com[3]; tamil_au@yahoo.com[4]

Abstract

In this paper, we introduce the concepts of neutrosophic nano δ-open sets and some stronger and weaker forms of neutrosophic nano open sets in neutrosophic nano topological spaces. And, show that the set of all neutrosophic nano δ-open sets are also a neutrosophic nano topology, which is called the neutrosophic nano δ topology. Further, we dealt with the concepts of neutrosphic nano δ-interior and neutrosophic nano δ-closure operators. Moreover, we define the product related neutrosophic nano topological spaces and proved some theorems related to this.

Keywords: neutrosophic nano open, neutrosophic nano δ-open; neutrosophic nano δ-α open; neutrosophic nano δ-$\mathcal{S}$ open; neutrosophic nano δ-$\mathcal{P}$ open; neutrosophic nano δ-γ open and neutrosophic nano δ-β open
AMS (2000) subject classification: 03E72; 54A05; 54A40.

1 Introduction

In 1965, Zadeh[23] made his significant theory on fuzzy sets. Later, fuzzy topology was introduced by Chang.[4] Generalization of fuzzy sets called as intuitionistic fuzzy sets introduced by Atanassov[2,3] in 1986 and its topological framework by Coker.[5] In 1995, Smarandache[17,18] introduced Neutrosophic logic and its topological spaces by Salama et.al[16] in 2012. The neutrosophic concept have wide range of real time applications for the fields by many authors.[1,6,9,12]

Pawlak[13] introduced Rough set theory by handling vagueness and uncertainty. This can be often defined by means of topological operations, interior and closure, called approximations. In 2013, Lellis Thivagar[7] introduced an extension of rough set theory called Nano topology and defined its topological spaces in terms of approximations and boundary region of a subset of a universe using an equivalence relation on it.

S. Saha[14] defined δ-open sets in fuzzy topological spaces and nano topological space by Pankajam et al.[10] In a neutrosophic topological space by Vadivel et al.[19–22] Recently, Lellis Thivagar et al.[8] explored a new concept of neutrosophic nano topology, intuitionistic nano topology and fuzzy nano topology.

In this paper, we introduce the concept of neutrosophic nano δ-interior, neutrosophic nano δ-closure operators and show that the set of all $Neu\mathcal{N}\delta o$ sets are also a neutrosophic nano topology, which is called the neutrosophic nano δ topology. It also established $Neu\mathcal{N}\delta o$, $Neu\mathcal{N}\delta\alpha o$, $Neu\mathcal{N}\delta\mathcal{S}o$, $Neu\mathcal{N}\delta\mathcal{P}o$, $Neu\mathcal{N}\delta\gamma o$,

https://doi.org/10.54216/IJNS.180419
Received: March 18, 2022 Accepted: June 28, 2022

$Neu\mathcal{N}\delta\beta o$ sets and studied some of their properties. Also, we dealt with the concepts of neutrosophic nano interior and neutrosophic nano closure operators in various nano open and closed sets. Finally, we define the product related neutrosophic nano topological spaces and proved some theorems based on this.

2 Preliminaries

Definition 2.1. [15] Let U be a universe of discourse with a generic element in U denoted by s, the neutrosophic set (briefly, $Neut\ s$)is an object having the form $S = \{\langle s, \mu_S(s), \sigma_S(s), \nu_S(s)\rangle : x \in U\}$, where $\mu_S, \sigma_S, \nu_S : U \to [0,1]$ denote the degree of membership, indeterminacy and non-membership functions respectively of each element $s \in U$ to the set S and $0 \leq \mu_S(s) + \sigma_S(s) + \nu_S(s) \leq 3$ for each $s \in U$.

Definition 2.2. [8] Let U be a non-empty set and Re be an equivalence relation on U.

1. Let F be a $Neut\ s$ in U with μ_F, σ_F and ν_F. The neutrosophic nano lower (resp. upper) approximations and neutrosophic nano boundary of F in the approximation (U, Re) denoted by $\underline{Neu\mathcal{N}}(F), \overline{Neu\mathcal{N}}(F)$ & $B_{Neu\mathcal{N}}(F)$ are

   (i) $\underline{Neu\mathcal{N}}(F) = \left\{\langle s, \mu_{\underline{Re}(J)}(s), \sigma_{\underline{Re}(J)}(s), \nu_{\underline{Re}(J)}(s)\rangle / t \in [s]_{Re}, s \in U\right\}$

   (ii) $\overline{Neu\mathcal{N}}(F) = \left\{\langle s, \mu_{\overline{Re}(J)}(s), \sigma_{\overline{Re}(J)}(s), \nu_{\overline{Re}(J)}(s)\rangle / t \in [s]_{Re}, s \in U\right\}$

   (iii) $B_{Neu\mathcal{N}}(F) = \overline{Neu\mathcal{N}}(F) - \underline{Neu\mathcal{N}}(F)$

   where $\mu_{\underline{Re}(J)}(s) = \bigwedge_{t\in[s]_{Re}} \mu_J(t), \sigma_{\underline{Re}(J)}(s) = \bigwedge_{t\in[s]_{Re}} \sigma_J(t), \nu_{\underline{Re}(J)}(s) = \bigvee_{t\in[s]_{Re}} \nu_J(t)$.

   $\mu_{\overline{Re}(J)}(s) = \bigvee_{t\in[s]_{Re}} \mu_J(t), \sigma_{\overline{Re}(J)}(s) = \bigvee_{t\in[s]_{Re}} \sigma_J(t), \nu_{\overline{Re}(J)}(s) = \bigwedge_{t\in[s]_{Re}} \nu_J(t)$.

   The collection $\tau_N(F) = \left\{0_N, 1_N, \underline{Neu\mathcal{N}}(F), \overline{Neu\mathcal{N}}(F), B_{Neu\mathcal{N}}(F)\right\}$ forms a topology called as neutrosophic nano topology and $(U, \tau_N(F))$ as neutrosophic nano topological space (briefly, $Neut$ $\mathcal{N}ts$). The elements of $\tau_N(F)$ are called neutrosophic nano open (briefly, $Neu\mathcal{N}o$) sets. Elements of $[\tau_N(F)]^c$ are called neutrosophic nano closed (briefly, $Neu\mathcal{N}c$) sets.

2. Let F be an intuitionistic set (briefly, $Int\ s$) in U with μ_F and ν_F. The intuitionistic nano lower (resp. upper) approximations and intuitionistic nano boundary of F in the approximation (U, Re) denoted by $\underline{Int\mathcal{N}}(F), \overline{Int\mathcal{N}}(F)$ & $B_{Int\mathcal{N}}(F)$ are

   (i) $\underline{Int\mathcal{N}}(F) = \left\{\langle s, \mu_{\underline{Re}(J)}(s), \nu_{\underline{Re}(J)}(s)\rangle / t \in [s]_{Re}, s \in U\right\}$

   (ii) $\overline{Int\mathcal{N}}(F) = \left\{\langle s, \mu_{\overline{Re}(J)}(s), \nu_{\overline{Re}(J)}(s)\rangle / t \in [s]_{Re}, s \in U\right\}$

   (iii) $B_{Int\mathcal{N}}(F) = \overline{Int\mathcal{N}}(F) - \underline{Int\mathcal{N}}(F)$

   where $\mu_{\underline{Re}(J)}(s) = \bigwedge_{t\in[s]_{Re}} \mu_J(t), \nu_{\underline{Re}(J)}(s) = \bigvee_{t\in[s]_{Re}} \nu_J(t)$.

   $\mu_{\overline{Re}(J)}(s) = \bigvee_{t\in[s]_{Re}} \mu_J(t), \nu_{\overline{Re}(J)}(s) = \bigwedge_{t\in[s]_{Re}} \nu_J(t)$.

   The collection $\tau_I(F) = \left\{0_I, 1_I, \underline{Int\mathcal{N}}(F), \overline{Int\mathcal{N}}(F), B_{Int\mathcal{N}}(F)\right\}$ forms a topology called as an intuitionistic nano topology and $(U, \tau_I(F))$ as an intuitionistic nano topological space (briefly, $Int\mathcal{N}ts$). The elements of $\tau_I(F)$ are called intuitionistic nano open (briefly, $Int\mathcal{N}o$) sets. Elements of $[\tau_I(F)]^c$ are called intuitionistic nano closed (briefly, $Neu\mathcal{N}c$) sets.

3. Let F be a fuzzy set (briefly, $\mathcal{F}\ s$) in U with μ_F. The fuzzy nano lower (resp. upper) approximations and fuzzy nano boundary of F in the approximation (U, Re) denoted by $\underline{\mathcal{FN}}(F), \overline{\mathcal{FN}}(F)$ & $B_{\mathcal{FN}}(F)$ are

   (i) $\underline{\mathcal{FN}}(F) = \left\{\langle s, \mu_{\underline{Re}(J)}(s)\rangle / t \in [s]_{Re}, s \in U\right\}$

   (ii) $\overline{\mathcal{FN}}(F) = \left\{\langle s, \mu_{\overline{Re}(J)}(s)\rangle / t \in [s]_{Re}, s \in U\right\}$

   (iii) $B_{\mathcal{FN}}(F) = \overline{\mathcal{FN}}(F) - \underline{\mathcal{FN}}(F)$

https://doi.org/10.54216/IJNS.180419
Received: March 18, 2022 Accepted: June 28, 2022

where $\mu_{\underline{Re}(J)}(s) = \bigwedge_{t\in[s]_{Re}} \mu_J(t)$. $\mu_{\overline{Re}(J)}(s) = \bigvee_{t\in[s]_{Re}} \mu_J(t)$.

The collection $\tau_{\mathcal{F}}(F) = \{0_{\mathcal{F}}, 1_{\mathcal{F}}, \underline{\mathcal{FN}}(F), \overline{\mathcal{FN}}(F), B_{\mathcal{FN}}(F)\}$ forms a topology called as fuzzy nano topology and $(U, \tau_{\mathcal{F}}(F))$ as fuzzy nano topological space (briefly, $\mathcal{FN}ts$). The elements of $\tau_{\mathcal{F}}(F)$ are called fuzzy nano open (briefly, $\mathcal{FN}o$) sets. Elements of $[\tau_{\mathcal{F}}(F)]^c$ are called fuzzy nano closed (briefly, $Neu\mathcal{N}c$) sets.

Definition 2.3. [8] Let U be a nonempty set and the neutrosophic subsets (briefly, $Neut\ subs$'s) S and T in the form $S = \{\langle s : \mu_S(s), \sigma_S(s), \nu_S(s)\rangle, s \in U\}, T = \{\langle s : \mu_T(s), \sigma_T(s), \nu_T(s)\rangle, s \in U\}$. Then the statements are hold:

(i) $0_N = \{\langle s, 0, 0, 1\rangle : s \in U\}$.

(ii) $1_N = \{\langle s, 1, 1, 0\rangle : s \in U\}$.

(iii) $S \subseteq T$ iff $\mu_S(s) \leq \mu_T(s), \sigma_S(s) \leq \sigma_T(s), \nu_S(s) \geq \nu_T(s) \ \forall\ s \in U\}$.

(iv) $S = T$ iff $S \subseteq T$ and $T \subseteq S$

(v) $S^c = \{\langle s, \nu_S(s), 1 - \sigma_S(s), \mu_S(s)\rangle : s \in U\}$

(vi) $S \cap T = \{s, \mu_S(s) \wedge \mu_T(s), \sigma_S(s) \wedge \sigma_T(s), \nu_S(s) \vee \nu_T(s) \ \forall\ s \in U\}$.

(vii) $S \cup T = \{s, \mu_S(s) \vee \mu_T(s), \sigma_S(s) \vee \sigma_T(s), \nu_S(s) \wedge \nu_T(s) \ \forall\ s \in U\}$.

Definition 2.4. Let $(U, \tau_N(F))$ be a $Neu\mathcal{N}ts$. Let S be a $Neut\ subs$ of U. Then neutrosophic nano

(i) interior of S[8] (briefly, $Neu\mathcal{N}int(S)$) is described as $Neu\mathcal{N}int(S) = \bigcup\{C : C \subseteq S\ \&\ C$ is a $Neut\ \mathcal{N}o\}$.

(ii) closure of S[8] (briefly, $Neu\mathcal{N}cl(S)$) is described as $Neu\mathcal{N}cl(S) = \bigcap\{C : S \subseteq C\ \&\ C$ is a $Neut\ \mathcal{N}c\}$.

(iii) regular open[11] (briefly, $Neu\mathcal{N}ro$) set if $S = Neu\mathcal{N}int(Neu\mathcal{N}cl(S))$.

(iv) regular closed[11] (briefly, $Neu\mathcal{N}rc$) set if $S = Neu\mathcal{N}cl(Neu\mathcal{N}int(S))$.

3 More on neutrosophic nano open sets via nano δ-open sets

Definition 3.1. Let $(U, \tau_N(F))$ be a $Neu\mathcal{N}ts$. Let S be a $Neut\ subs$ of U. The neutrosophic nano

(i) δ interior of S (briefly, $Neu\mathcal{N}\delta int(S)$) is described as $Neu\mathcal{N}\delta int(S) = \bigcup\{C : C \subseteq S\ \&\ C$ is a $Neu\mathcal{N}ro\}$.

(ii) δ closure of S (briefly, $Neu\mathcal{N}\delta cl(S)$) is described as $Neu\mathcal{N}\delta cl(S) = \bigcap\{C : S \subseteq C\ \&\ C$ is a $Neu\mathcal{N}rc\}$.

(iii) δ-open (briefly, $Neu\mathcal{N}\delta o$) set if $S = Neu\mathcal{N}\delta int(S)$.

(iv) a-open (or) $\delta\ \alpha$-open (briefly, $Neu\mathcal{N}ao$ (or) $Neu\mathcal{N}\delta\alpha o$) set if $S \subseteq Neu\mathcal{N}int(Neu\mathcal{N}cl(Neut\mathcal{N}\delta\ int(S)))$.

(v) δ-semi open (briefly, $Neu\mathcal{N}\delta\mathcal{S}o$) set if $S \subseteq Neu\mathcal{N}cl(Neu\mathcal{N}\delta int(S))$.

(vi) δ-pre open (briefly, $Neu\mathcal{N}\delta\mathcal{P}o$) set if $S \subseteq Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S))$.

(vii) e-open (or) $\delta\ \gamma$-open (briefly, $Neu\mathcal{N}eo$ (or) $Neu\mathcal{N}\delta\gamma o$) set if $S \subseteq Neu\mathcal{N}cl(Neu\mathcal{N}\delta\ int(S)) \cup Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S))$.

(viii) e^*-open (or) $\delta\ \beta$-open (briefly, $Neu\mathcal{N}e^*o$ (or) $Neu\mathcal{N}\delta\beta o$) set if $S \subseteq Neu\mathcal{N}cl(Neu\mathcal{N}int(Neu\mathcal{N}\ \delta cl(S)))$.

https://doi.org/10.54216/IJNS.180419
Received: March 18, 2022 Accepted: June 28, 2022

The complement of an $Neu\mathcal{N}\delta o$ (resp. $Neu\mathcal{N}\delta\alpha o$, $Neu\mathcal{N}\delta\mathcal{S}o$, $Neu\mathcal{N}\delta\mathcal{P}o$, $Neu\mathcal{N}\delta\gamma o$ & $Neu\mathcal{N}\delta\beta o$) set is called a neutrosophic nano δ (resp. neutrosophic nano δ-α, neutrosophic nano δ-semi, neutrosophic nano δ-pre, neutrosophic nano δ-γ & neutrosophic nano δ-β) closed (briefly, $Neu\mathcal{N}\delta c$ (resp. $Neu\mathcal{N}\delta\alpha c$, $Neu\mathcal{N}\delta\mathcal{S}c$, $Neu\mathcal{N}\delta\mathcal{P}c$, $Neu\mathcal{N}\delta\gamma c$ & $Neu\mathcal{N}\delta\beta c$)) in U.

Remark 3.2. (i) Since the union of $Neu\mathcal{N}\delta o$ sets in U is also $Neu\mathcal{N}\delta o$ set in U, $Neu\mathcal{N}\delta int(S)$ is $Neu\mathcal{N}\delta o$ set in U.

(ii) Since the intersection of $Neu\mathcal{N}\delta c$ sets in U is also $Neu\mathcal{N}\delta c$ set in U, $Neu\mathcal{N}\delta cl(S)$ is $Neu\mathcal{N}\delta c$ set in U.

Definition 3.3. Let $(U, \tau_I(F))$ be a $Int\mathcal{N}ts$. Let S be an intuitionistic subset of U. An intuitionistic nano

(i) interior of S (briefly, $Int\mathcal{N}int(S)$) is defined by $Int\mathcal{N}int(S) = \bigcup\{C : C \subseteq S \,\&\, C$ is a $Int\mathcal{N}o\}$.

(ii) closure of S (briefly, $Int\mathcal{N}cl(S)$) is defined by $Int\mathcal{N}cl(S) = \bigcap\{C : S \subseteq C \,\&\, C$ is a $Int\mathcal{N}c\}$.

(iii) regular open (briefly, $Int\mathcal{N}ro$) set if $S = Int\mathcal{N}int(Int\mathcal{N}cl(S))$.

(iv) regular closed (briefly, $Int\mathcal{N}rc$) set if $S = Int\mathcal{N}cl(Int\mathcal{N}int(S))$.

(v) δ interior of S (briefly, $Int\mathcal{N}\delta int(S)$) is described as $Int\mathcal{N}\delta int(S) = \bigcup\{C : C \subseteq S \,\&\, C$ is a $Int\mathcal{N}$ $ro\}$.

(vi) δ closure of S (briefly, $Int\mathcal{N}\delta cl(S)$) is described as $Int\mathcal{N}\delta cl(S) = \bigcap\{C : S \subseteq C \,\&\, C$ is a $Int\mathcal{N}rc\}$.

(vii) δ-open (briefly, $Int\mathcal{N}\delta o$) set if $S = Int\mathcal{N}\delta int(S)$.

(viii) a-open (or) δ α-open (briefly, $Int\mathcal{N}ao$ (or) $Int\mathcal{N}\delta\alpha o$) set if $S \subseteq Int\mathcal{N}int(Int\mathcal{N}cl\ (Int\mathcal{N}\delta int(S)))$.

(ix) δ-semi open (briefly, $Int\mathcal{N}\delta\mathcal{S}o$) set if $S \subseteq Int\mathcal{N}cl(Int\mathcal{N}\delta int(S))$.

(x) δ-pre open (briefly, $Int\mathcal{N}\delta\mathcal{P}o$) set if $S \subseteq Int\mathcal{N}int(Int\mathcal{N}\delta cl(S))$.

(xi) e-open (or) δ γ-open (briefly, $Int\mathcal{N}eo$ (or) $Int\mathcal{N}\delta\gamma o$) set if $S \subseteq Int\mathcal{N}cl(Int\mathcal{N}\delta int(S)) \cup Int\mathcal{N}int(Int\mathcal{N}\delta cl(S))$.

(xii) e^*-open (or) δ β-open (briefly, $Int\mathcal{N}e^*o$ (or) $Int\mathcal{N}\delta\beta o$) set if $S \subseteq Int\mathcal{N}cl(Int\mathcal{N}int\ (Int\mathcal{N}\delta cl(S)))$.

The complement of an $Int\mathcal{N}\delta o$ (resp. $Int\mathcal{N}\delta\alpha o$, $Int\mathcal{N}\delta\mathcal{S}o$, $Int\mathcal{N}\delta\mathcal{P}o$, $Int\mathcal{N}\delta\gamma o$ & $Int\mathcal{N}\delta\beta o$) set is called a intuitionistic nano δ (resp. intuitionistic nano δ-α, intuitionistic nano δ-semi, intuitionistic nano δ-pre, intuitionistic nano δ-γ & intuitionistic nano δ-β) closed (briefly, $Int\mathcal{N}\delta c$ (resp. $Int\mathcal{N}\delta\alpha c$, $Int\mathcal{N}\delta\mathcal{S}c$, $Int\mathcal{N}\delta\mathcal{P}c$, $Int\mathcal{N}\delta\gamma c$ & $Int\mathcal{N}\delta\beta c$)) in U.

Definition 3.4. Let $(U, \tau_{\mathcal{F}}(F))$ be a $\mathcal{FN}ts$. Let S be a fuzzy subset of U. The fuzzy nano

(i) interior of S (briefly, $\mathcal{FN}int(S)$) is defined by $\mathcal{FN}int(S) = \bigvee\{C : C \leq S \,\&\, C$ is a $\mathcal{FN}o\}$.

(ii) closure of S (briefly, $\mathcal{FN}cl(S)$) is defined by $\mathcal{FN}cl(S) = \bigwedge\{C : S \leq C \,\&\, C$ is a $\mathcal{FN}c\}$.

(iii) regular open (briefly, $\mathcal{FN}ro$) set if $S = \mathcal{FN}int(\mathcal{FN}cl(S))$.

(iv) regular closed (briefly, $\mathcal{FN}rc$) set if $S = \mathcal{FN}cl(\mathcal{FN}int(S))$.

(v) δ interior of S (briefly, $\mathcal{FN}\delta int(S)$) is described as $\mathcal{FN}\delta int(S) = \bigvee\{C : C \leq S \,\&\, C$ is a $\mathcal{FN}ro\}$.

(vi) δ closure of S (briefly, $\mathcal{FN}\delta cl(S)$) is described as $\mathcal{FN}\delta cl(S) = \bigwedge\{C : S \leq C \,\&\, C$ is a $\mathcal{FN}rc\}$.

(vii) δ-open (briefly, $\mathcal{FN}\delta o$) set if $S = \mathcal{FN}\delta int(S)$.

(viii) a-open (or) δ α-open (briefly, $\mathcal{FN}ao$ (or) $\mathcal{FN}\delta\alpha o$) set if $S \leq \mathcal{FN}int(\mathcal{FN}cl\ (\mathcal{FN}\delta int(S)))$.

(ix) δ-semi open (briefly, $\mathcal{FN}\delta\mathcal{S}o$) set if $S \leq \mathcal{FN}cl(\mathcal{FN}\delta int(S))$.

(x) δ-pre open (briefly, $\mathcal{FN}\delta\mathcal{P}o$) set if $S \leq \mathcal{FN}int(\mathcal{FN}\delta cl(S))$.

https://doi.org/10.54216/IJNS.180419

Received: March 18, 2022 Accepted: June 28, 2022

(xi) e-open (or) $\delta\ \gamma$-open (briefly, $\mathcal{FN}eo$ (or) $\mathcal{FN}\delta\gamma o$) set if $S \leq \mathcal{FN}cl(\mathcal{FN}\delta\ int(S)) \vee \mathcal{FN}int(\mathcal{FN}\ \delta cl(S))$.

(xii) e^*-open (or) $\delta\ \beta$-open (briefly, $\mathcal{FN}e^*o$ (or) $\mathcal{FN}\delta\beta o$) set if $S \leq \mathcal{FN}cl(\mathcal{FN}int\ (\mathcal{FN}\delta cl(S)))$.

The complement of an $\mathcal{FN}\delta o$ (resp. $\mathcal{FN}\delta\alpha o$, $\mathcal{FN}\delta\mathcal{S}o$, $\mathcal{FN}\delta\mathcal{P}o$, $\mathcal{FN}\delta\gamma o$ & $\mathcal{FN}\delta\beta o$) set is called a fuzzy nano δ (resp. fuzzy nano δ-α, fuzzy nano δ-semi, fuzzy nano δ-pre, fuzzy nano δ-γ & fuzzy nano δ-β) closed (briefly, $\mathcal{FN}\delta c$ (resp. $\mathcal{FN}\delta\alpha c$, $\mathcal{FN}\delta\mathcal{S}c$, $\mathcal{FN}\delta\mathcal{P}c$, $\mathcal{FN}\delta\gamma c$ & $\mathcal{FN}\delta\beta c$)) in U.

Definition 3.5. The set of all $Neu\mathcal{N}\delta o$ (resp. $Int\mathcal{N}\delta o$ and $\mathcal{FN}\delta o$) sets of $(U, \tau_N(F))$ (resp. $(U, \tau_I(F))$ and $(U, \tau_{\mathcal{F}}(F))$) is also a neutrosophic (resp. intuitionistic and fuzzy) nano topology on U. We denote it by $\tau_N^\delta(F)$ (resp. $\tau_I^\delta(F)$ and $\tau_{\mathcal{F}}^\delta(F)$) and it is called a neutrosophic (resp. intuitionistic and fuzzy) nano δ-topology on U. An ordered pair $(U, \tau_N^\delta(F))$ (resp. $\tau_I^\delta(F)$ and $\tau_{\mathcal{F}}^\delta(F)$) is called a neutrosophic (resp. intuitionistic and fuzzy) nano δ-topological space.

Example 3.6. Assume $U = \{l_1, l_2, l_3, l_4, l_5\}$ and $U/Re = \{\{l_1, l_4\}, \{l_2, l_5\}, \{l_3\}\}$.

Let $L = \left\{\left\langle\frac{l_1}{(0.20,0.50,0.80)}\right\rangle, \left\langle\frac{l_2}{(0.30,0.50,0.70)}\right\rangle, \left\langle\frac{l_3}{(0.40,0.50,0.60)}\right\rangle, \left\langle\frac{l_4}{(0.10,0.50,0.90)}\right\rangle, \left\langle\frac{l_5}{(0.10,0.50,0.80)}\right\rangle\right\}$ be a $Neut\ subs$ of U.

$$\underline{Neu\mathcal{N}}(L) = \left\{\left\langle\frac{l_1, l_4}{(0.20, 0.50, 0.80)}\right\rangle, \left\langle\frac{l_2, l_5}{(0.30, 0.50, 0.70)}\right\rangle, \left\langle\frac{l_3}{(0.40, 0.50, 0.60)}\right\rangle\right\},$$
$$\overline{Neu\mathcal{N}}(L) = \left\{\left\langle\frac{l_1, l_4}{(0.10, 0.50, 0.90)}\right\rangle, \left\langle\frac{l_2, l_5}{(0.10, 0.50, 0.80)}\right\rangle, \left\langle\frac{l_3}{(0.40, 0.50, 0.60)}\right\rangle\right\},$$
$$B_{Neu\mathcal{N}}(L) = \left\{\left\langle\frac{l_1, l_4}{(0.10, 0.50, 0.90)}\right\rangle, \left\langle\frac{l_2, l_5}{(0.10, 0.50, 0.80)}\right\rangle, \left\langle\frac{l_3}{(0.40, 0.50, 0.60)}\right\rangle\right\}.$$

Thus $\tau_N(L) = \{0_N, 1_N, \underline{Neu\mathcal{N}}(L), \overline{Neu\mathcal{N}}(L) = B_{Neu\mathcal{N}}(L)\}$.

Then $Neu\mathcal{N}\delta o$ sets are $\left\{\left\{\left\langle\frac{l_1,l_4}{(0,0,1)}\right\rangle, \left\langle\frac{l_2,l_5}{(0,0,1)}\right\rangle, \left\langle\frac{l_3}{(0,0,1)}\right\rangle\right\}, \left\{\left\langle\frac{l_1,l_4}{(1,1,0)}\right\rangle, \left\langle\frac{l_2,l_5}{(1,1,0)}\right\rangle, \left\langle\frac{l_3}{(1,1,0)}\right\rangle\right\}, \left\{\left\langle\frac{l_1,l_4}{(0.20,0.50,0.80)}\right\rangle, \left\langle\frac{l_2,l_5}{(0.30,0.50,0.70)}\right\rangle, \left\langle\frac{l_3}{(0.40,0.50,0.60)}\right\rangle\right\}\right\}$.

Hence, $\tau_N^\delta(L) = \left\{\left\{\left\langle\frac{l_1,l_4}{(0,0,1)}\right\rangle, \left\langle\frac{l_2,l_5}{(0,0,1)}\right\rangle, \left\langle\frac{l_3}{(0,0,1)}\right\rangle\right\}, \left\{\left\langle\frac{l_1,l_4}{(1,1,0)}\right\rangle, \left\langle\frac{l_2,l_5}{(1,1,0)}\right\rangle, \left\langle\frac{l_3}{(1,1,0)}\right\rangle\right\}, \left\{\left\langle\frac{l_1,l_4}{(0.20,0.50,0.80)}\right\rangle, \left\langle\frac{l_2,l_5}{(0.30,0.50,0.70)}\right\rangle, \left\langle\frac{l_3}{(0.40,0.50,0.60)}\right\rangle\right\}\right\}$.

Thus $\tau_I(L) = \{0_I, 1_I, \underline{Int\mathcal{N}}(L), \overline{Int\mathcal{N}}(L) = B_{Int\mathcal{N}}(L)\}$.

Then $Int\mathcal{N}\delta o$ sets are $\left\{\left\{\left\langle\frac{l_1,l_4}{(0,1)}\right\rangle, \left\langle\frac{l_2,l_5}{(0,1)}\right\rangle, \left\langle\frac{l_3}{(0,1)}\right\rangle\right\}, \left\{\left\langle\frac{l_1,l_4}{(1,0)}\right\rangle, \left\langle\frac{l_2,l_5}{(1,0)}\right\rangle, \left\langle\frac{l_3}{(1,0)}\right\rangle\right\}, \left\{\left\langle\frac{l_1,l_4}{(0.20,0.80)}\right\rangle, \left\langle\frac{l_2,l_5}{(0.30,0.70)}\right\rangle, \left\langle\frac{l_3}{(0.40,0.60)}\right\rangle\right\}\right\}$.

Hence, $\tau_I^\delta(L) = \left\{\left\{\left\langle\frac{l_1,l_4}{(0,1)}\right\rangle, \left\langle\frac{l_2,l_5}{(0,1)}\right\rangle, \left\langle\frac{l_3}{(0,1)}\right\rangle\right\}, \left\{\left\langle\frac{l_1,l_4}{(1,0)}\right\rangle, \left\langle\frac{l_2,l_5}{(1,0)}\right\rangle, \left\langle\frac{l_3}{(1,0)}\right\rangle\right\}, \left\{\left\langle\frac{l_1,l_4}{(0.20,0.80)}\right\rangle, \left\langle\frac{l_2,l_5}{(0.30,0.70)}\right\rangle, \left\langle\frac{l_3}{(0.40,0.60)}\right\rangle\right\}\right\}$.

Thus $\tau_{\mathcal{F}}(L) = \{0_{\mathcal{F}}, 1_{\mathcal{F}}, \underline{\mathcal{FN}}(L), \overline{\mathcal{FN}}(L) = B_{\mathcal{FN}}(L)\}$. Then $\mathcal{FN}\delta o$ sets are $\left\{\left\{\left\langle\frac{l_1,l_4}{0}\right\rangle, \left\langle\frac{l_2,l_5}{0}\right\rangle, \left\langle\frac{l_3}{0}\right\rangle\right\}, \left\{\left\langle\frac{l_1,l_4}{1}\right\rangle, \left\langle\frac{l_2,l_5}{1}\right\rangle,\right.\right.$

Hence, $\tau_{\mathcal{F}}^\delta(L) = \left\{\left\{\left\langle\frac{l_1,l_4}{0}\right\rangle, \left\langle\frac{l_2,l_5}{0}\right\rangle, \left\langle\frac{l_3}{0}\right\rangle\right\}, \left\{\left\langle\frac{l_1,l_4}{1}\right\rangle, \left\langle\frac{l_2,l_5}{1}\right\rangle, \left\langle\frac{l_3}{1}\right\rangle\right\}, \left\{\left\langle\frac{l_1,l_4}{0.20}\right\rangle, \left\langle\frac{l_2,l_5}{0.30}\right\rangle, \left\langle\frac{l_3}{0.40}\right\rangle\right\}\right\}$.

Remark 3.7. Thus from the definitions of intuitionistic and fuzzy nano δ topologies we can assure that throughout this paper all the properties and examples also holds good when it is possible for neutrosophic nano δ topology.

Hereafter throughout this section, let $(U, \tau_N(F))$ be a $Neu\mathcal{N}ts$ with respect to F where F is a $Neut\ subs$ of U.

https://doi.org/10.54216/IJNS.180419
Received: March 18, 2022 Accepted: June 28, 2022

Proposition 3.8. In $Neu\mathcal{N}ts$, the statements are hold.

(i) Every $Neu\mathcal{N}\delta o$ set (resp. $Neu\mathcal{N}\delta c$ set) is a $Neu\mathcal{N}o$ set (resp. $Neu\mathcal{N}c$ set).

(ii) Every $Neu\mathcal{N}o$ set (resp. $Neu\mathcal{N}c$ set) is a $Neu\mathcal{N}\delta\mathcal{S}o$ set (resp. $Neu\mathcal{N}\delta\mathcal{S}c$ set).

(iii) Every $Neu\mathcal{N}o$ set (resp. $Neu\mathcal{N}c$ set) is a $Neu\mathcal{N}\delta\mathcal{P}o$ set (resp. $Neu\mathcal{N}\delta\mathcal{P}c$ set).

(iv) Every $Neu\mathcal{N}\delta\mathcal{S}o$ set (resp. $Neu\mathcal{N}\delta\mathcal{S}c$ set) is a $Neu\mathcal{N}\delta\gamma o$ set (resp. $Neu\mathcal{N}\delta\gamma c$ set).

(v) Every $Neu\mathcal{N}\delta\mathcal{P}o$ set (resp. $Neu\mathcal{N}\delta\mathcal{P}c$ set) is a $Neu\mathcal{N}\delta\gamma o$ set (resp. $Neu\mathcal{N}\delta\gamma c$ set).

(vi) Every $Neu\mathcal{N}\delta\gamma o$ set (resp. $Neu\mathcal{N}\delta\gamma c$ set) is a $Neu\mathcal{N}\delta\beta o$ set (resp. $Neu\mathcal{N}\delta\beta c$ set).

But not converse.

Proof. (i) Let S_o is a $Neu\mathcal{N}\delta o$, then $S_o = Neu\mathcal{N}\delta int(S_o) \subseteq Neu\mathcal{N}int(S_o)$. Therefore S_o is a $Neu\mathcal{N}o$.

(ii) Let S_o is a $Neu\mathcal{N}o$, then $S_o = Neu\mathcal{N}int(S_o) \subseteq Neu\mathcal{N}cl(Neu\mathcal{N}\delta int(S_o))$. Therefore S_o is a $Neu\mathcal{N}\delta\mathcal{S}_l o$.

(iii) Let S_o is a $Neu\mathcal{N}o$, then $S_o = Neu\mathcal{N}int(S_o) \subseteq Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S_o))$. Therefore S_o is a $Neu\mathcal{N}\delta\mathcal{P}o$.

(iv) Let S_o is a $Neu\mathcal{N}\delta\mathcal{S}_l o$, then $S_o \subseteq Neu\mathcal{N}cl(Neu\mathcal{N}\delta int(S_o)) \subseteq Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S_o)) \cup Neu\mathcal{N}cl(Neu\mathcal{N}\delta int(S_o))$. Therefore S_o is a $Neu\mathcal{N}\delta\gamma o$.

(v) Let S_o is a $Neu\mathcal{N}\delta\mathcal{P}os$, then $S_o \subseteq Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S_o)) \subseteq Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S_o)) \cup Neu\mathcal{N}cl(Neu\mathcal{N}\delta int(S_o))$. Therefore S_o is a $Neu\mathcal{N}\delta\gamma o$.

(vi) Let S_o is a $Neu\mathcal{N}\delta\gamma o$, then $S_o \subseteq Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S_o)) \cup Neu\mathcal{N}cl(Neu\mathcal{N}\delta int(S_o)) \subseteq Neu\mathcal{N}cl(Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S_o)))$. Therefore S_o is a $Neu\mathcal{N}\delta\beta o$.

Proof of the closed sets are also in a similar way. □

[arrows] $Neu\mathcal{N}\delta\mathcal{S}o$ [dr]
$Neu\mathcal{N}\delta o$ [r] $Neu\mathcal{N}o$ [ur] [dr] $Neu\mathcal{N}\delta\gamma o$ [r] $Neu\mathcal{N}\delta\beta o$
$Neu\mathcal{N}\delta\mathcal{P}o$ [ur]

Figure 1: $Neu\mathcal{N}\delta os$'s in $Neu\mathcal{N}ts$.

Example 3.9. In Example, 3.6, the sets

(i) $A = \left\{\left\langle \frac{l_1,l_4}{(0.10,0.50,0.90)} \right\rangle, \left\langle \frac{l_2,l_5}{(0.10,0.50,0.80)} \right\rangle, \left\langle \frac{l_3}{(0.40,0.50,0.60)} \right\rangle\right\}$ is a $Neu\mathcal{N}o$ set but not $Neu\mathcal{N}\delta o$ set.

(ii) $B = \left\{\left\langle \frac{l_1,l_4}{(0.20,0.50,0.80)} \right\rangle, \left\langle \frac{l_2,l_5}{(0.30,0.50,0.70)} \right\rangle, \left\langle \frac{l_3}{(0.60,0.50,0.40)} \right\rangle\right\}$ is a $Neu\mathcal{N}\delta\mathcal{S}o$ set but not $Neu\mathcal{N}o$ set.

(iii) B is a $Neu\mathcal{N}\delta\gamma o$ set but not $Neu\mathcal{N}\delta\mathcal{P}o$ set.

(iv) $C = \left\{\left\langle \frac{l_1,l_4}{(0.10,0.50,0.90)} \right\rangle, \left\langle \frac{l_2,l_5}{(0.10,0.50,0.90)} \right\rangle, \left\langle \frac{l_3}{(0.60,0.50,0.40)} \right\rangle\right\}$ is a $Neu\mathcal{N}\delta\beta o$ set but not $Neu\mathcal{N}\delta\gamma o$ set.

https://doi.org/10.54216/IJNS.180419
Received: March 18, 2022 Accepted: June 28, 2022

Example 3.10. Assume $U = \{l_1, l_2, l_3\}$ and $U/Re = \{\{l_1\}, \{l_2, l_3\}\}$.

Let $L = \left\{\left\langle \frac{l_1}{(0.30,0.50,0.50)} \right\rangle, \left\langle \frac{l_2}{(0.40,0.20,0.60)} \right\rangle, \left\langle \frac{l_3}{(0.40,0.50,0.60)} \right\rangle\right\}$ be a $Neut\ subs$ of U.

$$\underline{Neu\mathcal{N}}(L) = \left\{\left\langle \frac{l_1}{(0.30, 0.50, 0.50)} \right\rangle, \left\langle \frac{l_2, l_3}{(0.40, 0.20, 0.60)} \right\rangle\right\},$$
$$\overline{Neu\mathcal{N}}(L) = \left\{\left\langle \frac{l_1}{(0.30, 0.50, 0.50)} \right\rangle, \left\langle \frac{l_2, l_3}{(0.40, 0.50, 0.60)} \right\rangle\right\},$$
$$B_{Neu\mathcal{N}}(L) = \left\{\left\langle \frac{l_1}{(0.30, 0.50, 0.50)} \right\rangle, \left\langle \frac{l_2, l_3}{(0.40, 0.50, 0.60)} \right\rangle\right\}.$$

Let $\tau_N(L) = \{0_N, 1_N, \underline{Neu\mathcal{N}}(L), \overline{Neu\mathcal{N}}(L) = B_{Neu\mathcal{N}}(L)\}$. Then

(i) $A = \left\{\left\langle \frac{l_1}{(0.30,0.50,0.60)} \right\rangle, \left\langle \frac{l_2,l_3}{(0.40,0.50,0.60)} \right\rangle\right\}$ is a $Neu\mathcal{N}\delta\mathcal{P}o$ set but not $Neu\mathcal{N}o$ set.

(ii) A is a $Neu\mathcal{N}\delta\gamma o$ set but not $Neu\mathcal{N}\delta\mathcal{S}o$ set.

Theorem 3.11. Let S be a $Neut\ subs$ of U. Then

(i) $1_N - Neu\mathcal{N}\delta int(S) = Neu\mathcal{N}\delta cl(1_N - S)$ (or) $(Neu\mathcal{N}\delta int(S))^c = Neu\mathcal{N}\delta cl(S^c)$.

(ii) $1_N - Neu\mathcal{N}\delta cl(S) = Neu\mathcal{N}\delta int(1_N - S)$ (or) $(Neu\mathcal{N}\delta cl(S))^c = Neu\mathcal{N}\delta int(S^c)$.

Proof. (i) By Definition 3.1, $Neu\mathcal{N}\delta int(S) = \bigcup\{D : D \text{ is a } Neu\mathcal{N}\delta o \text{ set in } U \ \&\ D \subseteq S\}$. Taking complement on both sides, $(Neu\mathcal{N}\delta int(S))^c = (\bigcup\{D : D \text{ is a } Neu\mathcal{N}\delta o \text{ set in } U \ \&\ D \subseteq S\})^c = \bigcap\{D^c : D^c \text{ is a } Neu\mathcal{N}\delta c \text{ set in } U \ \&\ S^c \subseteq C^c\}$. Replacing D^c by L, we get $(Neu\mathcal{N}\delta int(S))^c = \bigcap\{L : L \text{ is a } Neu\mathcal{N}\delta c \text{ set in } U \ \&\ S^c \subseteq L\}$. By Definition 3.1, $(Neu\mathcal{N}\delta int(S))^c = Neu\mathcal{N}\delta cl(S^c)$. This proves (i).

(ii) By using (i), $(Neu\mathcal{N}\delta int(S^c))^c = Neu\mathcal{N}\delta cl((S^c)^c) = Neu\mathcal{N}\delta cl(S)$. Taking complement on both sides, we get $Neu\mathcal{N}\delta\ int(S^c) = (Neu\mathcal{N}\delta cl(S))^c$. Hence proved (ii). □

Remark 3.12. Taking complements on either side of (i) and (ii) of Theorem 3.11, we get $Neu\mathcal{N}\delta int(S) = 1_N - Neu\mathcal{N}\delta cl(1_N - S)$ & $Neu\mathcal{N}\delta cl(S) = 1_N - Neu\mathcal{N}\delta int(1_N - S)$.

Example 3.13. In Example 3.6, let $L = \left\{\left\langle \frac{l_1,l_4}{(0.20,0.50,0.80)} \right\rangle, \left\langle \frac{l_2,l_5}{(0.30,0.50,0.70)} \right\rangle, \left\langle \frac{l_3}{(0.10,0.50,0.90)} \right\rangle\right\}$

(i) $Neu\mathcal{N}\delta int(L) = 1_N - Neu\mathcal{N}\delta cl(1_N - L)$,

$$Neu\mathcal{N}\delta int(L) = \left\{\left\langle \frac{l_1, l_4}{(0.20, 0.50, 0.80)} \right\rangle, \left\langle \frac{l_2, l_5}{(0.30, 0.50, 0.70)} \right\rangle, \left\langle \frac{l_3}{(0.40, 0.50, 0.60)} \right\rangle\right\}$$

$$\begin{aligned} 1_N - Neu\mathcal{N}\delta cl(1_N - L) &= 1_N - Neu\mathcal{N}\delta cl\left(\left\{\left\langle \frac{l_1, l_4}{(0.80, 0.50, 0.20)} \right\rangle, \left\langle \frac{l_2, l_5}{(0.70, 0.50, 0.30)} \right\rangle, \left\langle \frac{l_3}{(0.60, 0.50, 0.40)} \right\rangle\right\}\right) \\ &= 1_N - \left\{\left\langle \frac{l_1, l_4}{(0.80, 0.50, 0.20)} \right\rangle, \left\langle \frac{l_2, l_5}{(0.70, 0.50, 0.30)} \right\rangle, \left\langle \frac{l_3}{(0.60, 0.50, 0.40)} \right\rangle\right\} \\ &= \left\{\left\langle \frac{l_1, l_4}{(0.20, 0.50, 0.80)} \right\rangle, \left\langle \frac{l_2, l_5}{(0.30, 0.50, 0.70)} \right\rangle, \left\langle \frac{l_3}{(0.40, 0.50, 0.60)} \right\rangle\right\}. \end{aligned}$$

(ii) $Neu\mathcal{N}\delta cl(L) = 1_N - Neu\mathcal{N}\delta int(1_N - L)$,

$$Neu\mathcal{N}\delta cl(L) = \left\{\left\langle \frac{l_1, l_4}{(0.80, 0.50, 0.20)} \right\rangle, \left\langle \frac{l_2, l_5}{(0.70, 0.50, 0.30)} \right\rangle, \left\langle \frac{l_3}{(0.60, 0.50, 0.40)} \right\rangle\right\}$$

$$\begin{aligned} 1_N - Neu\mathcal{N}\delta int(1_N - L) &= 1_N - Neu\mathcal{N}\delta int\left(\left\{\left\langle \frac{l_1, l_4}{(0.80, 0.50, 0.20)} \right\rangle, \left\langle \frac{l_2, l_5}{(0.70, 0.50, 0.30)} \right\rangle, \left\langle \frac{l_3}{(0.60, 0.50, 0.40)} \right\rangle\right\}\right) \\ &= 1_N - \left\{\left\langle \frac{l_1, l_4}{(0.20, 0.50, 0.80)} \right\rangle, \left\langle \frac{l_2, l_5}{(0.30, 0.50, 0.70)} \right\rangle, \left\langle \frac{l_3}{(0.40, 0.50, 0.60)} \right\rangle\right\} \\ &= \left\{\left\langle \frac{l_1, l_4}{(0.80, 0.50, 0.20)} \right\rangle, \left\langle \frac{l_2, l_5}{(0.70, 0.50, 0.30)} \right\rangle, \left\langle \frac{l_3}{(0.60, 0.50, 0.40)} \right\rangle\right\}. \end{aligned}$$

https://doi.org/10.54216/IJNS.180419
Received: March 18, 2022 Accepted: June 28, 2022

Remark 3.14. By Definition of $Neu\mathcal{N}\delta cl(S_o)$, it is clear that for any $Neu\ s\ S_o$, $Neu\mathcal{N}cl(Neu\mathcal{N}\ \delta cl(S_o)) = Neu\mathcal{N}\delta cl(S_o)$ and we have the following equality:

$$\begin{aligned} Neu\mathcal{N}\delta int(S_o) &= 1_N - Neu\mathcal{N}\delta cl(1_N - S_o) \\ &= 1_N - \bigcap\{D_o : 1_N - S_o \subseteq D_o, D_o = Neu\mathcal{N}cl(Neu\mathcal{N}int(F))\} \\ &= \bigcup\{1_N - D_o : 1_N - D_o \subseteq S_o, 1_N - D_o = 1_N - Neu\mathcal{N}cl(Neu\mathcal{N}int(F))\} \\ &= \bigcup\{L_o : L_o \subseteq S_o, L_o = Neu\mathcal{N}int(Neu\mathcal{N}cl(L_o))\} \end{aligned}$$

That is, $Neu\mathcal{N}\delta int(S_o)$ is the union of all $Neu\mathcal{N}ro$ subsets of S_o. Since any $Neu\mathcal{N}\delta o$ set is the complement of a $Neu\mathcal{N}\delta c$ set, G_o is a $Neu\mathcal{N}\delta o$ set iff $G_o = Neu\mathcal{N}\delta int(G_o)$.

Proposition 3.15. If S and T are two $Neu\mathcal{N}\delta o$ (resp. $Neu\mathcal{N}\delta\mathcal{S}o$, $Neu\mathcal{N}\delta\mathcal{P}o$, $Neu\mathcal{N}\delta\gamma o$ & $Neu\mathcal{N}\delta\ \beta o$) sets, then $S \cup T$ is $Neu\mathcal{N}\delta o$ (resp. $Neu\mathcal{N}\delta\mathcal{S}o$, $Neu\mathcal{N}\delta\mathcal{P}o$, $Neu\mathcal{N}\delta\gamma o$ & $Neu\mathcal{N}\delta\beta o$) set.

Proof. If S & T are two $Neu\mathcal{N}\delta o$ sets. Then by definition $S = Neu\mathcal{N}\delta int(S)$ and $T = Neu\mathcal{N}\delta int(T)$. Now $S \cup T = Neu\mathcal{N}\delta int(S) \cup Neu\mathcal{N}\delta int(T) \subseteq Neu\mathcal{N}\delta int(S \cup T)$. Since, $Neu\mathcal{N}\delta int(S \cup T) \subseteq S \cup T$. So $S \cup T = Neu\mathcal{N}\delta int(S \cup T)$. Hence, $S \cup T$ is $Neu\mathcal{N}\delta o$ set.

The others are similar. □

Proposition 3.16. Arbitrary union of $Neu\mathcal{N}\delta o$ (resp. $Neu\mathcal{N}\delta\mathcal{S}o$, $Neu\mathcal{N}\delta\mathcal{P}o$, $Neu\mathcal{N}\delta\gamma o$ & $Neu\mathcal{N}\delta\ \beta o$) sets is a $Neu\mathcal{N}\delta o$ (resp. $Neu\mathcal{N}\delta\mathcal{S}o$, $Neu\mathcal{N}\delta\mathcal{P}o$, $Neu\mathcal{N}\delta\gamma o$ & $Neu\mathcal{N}\delta\beta o$) set.

Proof. Let $\{S_k\}$ be a collection of $Neu\mathcal{N}\delta o$ sets of a $Neu\mathcal{N}ts$ $(U, \tau_N(F))$. Then by definition $S_k = Neu\mathcal{N}\delta int(S_k)$ for each k. Now $\bigcup S_k = \bigcup Neu\mathcal{N}\delta int(S_k) \subseteq Neu\mathcal{N}\delta int(\bigcup S_k)$.

Since, $Neu\mathcal{N}\delta int(\bigcup S_k) \subseteq \bigcup S_k$. So $\bigcup S_k = Neu\mathcal{N}\delta int(\bigcup S_k)$. Hence, $\bigcup S_k$ is $Neu\mathcal{N}\delta o$ set.

The others are similar. □

Proposition 3.17. Finite intersection of $Neu\mathcal{N}\delta o$ sets is a $Neu\mathcal{N}\delta o$ set.

Remark 3.18. Intersection of any two $Neu\mathcal{N}\delta\mathcal{S}o$ (resp. $Neu\mathcal{N}\delta\mathcal{P}o$, $Neu\mathcal{N}\delta\gamma o$ & $Neu\mathcal{N}\delta\beta o$) sets need not be $Neu\mathcal{N}\delta\mathcal{S}o$ (resp. $Neu\mathcal{N}\delta\mathcal{P}o$, $Neu\mathcal{N}\delta\gamma o$ & $Neu\mathcal{N}\delta\beta o$) set as shown by the following example.

Example 3.19. Assume $U = \{l_1, l_2, l_3\}$ and $U/Re = \{\{l_1, l_3\}, \{l_2\}\}$. Let

(i) $L = \left\{\left\langle \frac{l_1}{(0.20,0.50,0.70)} \right\rangle, \left\langle \frac{l_2}{(0.10,0.50,0.50)} \right\rangle, \left\langle \frac{l_3}{(0.20,0.50,0.70)} \right\rangle\right\}$ be a $Neut\ subs$ of U.

$$\underline{Neu\mathcal{N}}(L) = \left\{\left\langle \frac{l_1, l_3}{(0.20, 0.50, 0.70)} \right\rangle, \left\langle \frac{l_2}{(0.10, 0.50, 0.50)} \right\rangle\right\},$$
$$\overline{Neu\mathcal{N}}(L) = \left\{\left\langle \frac{l_1, l_3}{(0.20, 0.50, 0.70)} \right\rangle, \left\langle \frac{l_2}{(0.10, 0.50, 0.50)} \right\rangle\right\},$$
$$B_{Neu\mathcal{N}}(L) = \left\{\left\langle \frac{l_1, l_3}{(0.20, 0.50, 0.70)} \right\rangle, \left\langle \frac{l_2}{(0.10, 0.50, 0.50)} \right\rangle\right\}.$$

Let $\tau_N(L) = \{0_N, 1_N, \underline{Neu\mathcal{N}}(L) = \overline{Neu\mathcal{N}}(L) = B_{Neu\mathcal{N}}(L)\}$. Then let $A = \left\{\left\langle \frac{l_1,l_3}{(0.3,0.50,0.70)} \right\rangle, \left\langle \frac{l_2}{(0.50,0.50,0.20)} \right\rangle\right\}$ and $B = \left\{\left\langle \frac{l_1,l_3}{(0.10,0.50,0.10)} \right\rangle, \left\langle \frac{l_2}{(0.20,0.50,0.10)} \right\rangle\right\}$ are $Neu\mathcal{N}\delta\gamma o$ sets but $A \cap B = \left\{\left\langle \frac{l_1,l_3}{(0.10,0.50,0.70)} \right\rangle, \left\langle \frac{l_2}{(0.20,0.50,0.20)} \right\rangle\right\}$ is not $Neu\mathcal{N}\delta\gamma o$ set.

Received: March 18, 2022 Accepted: June 28, 2022

(ii) $L = \left\{ \left\langle \frac{l_1}{(0.4,0.50,0.6)} \right\rangle, \left\langle \frac{l_2}{(0.50,0.50,0.50)} \right\rangle, \left\langle \frac{l_3}{(0.4,0.50,0.6)} \right\rangle \right\}$ be a $Neut\ subs$ of U.

$$\underline{Neu\mathcal{N}}(L) = \left\{ \left\langle \frac{l_1, l_3}{(0.4, 0.50, 0.6)} \right\rangle, \left\langle \frac{l_2}{(0.50, 0.50, 0.50)} \right\rangle \right\},$$
$$\overline{Neu\mathcal{N}}(L) = \left\{ \left\langle \frac{l_1, l_3}{(0.4, 0.50, 0.6)} \right\rangle, \left\langle \frac{l_2}{(0.50, 0.50, 0.50)} \right\rangle \right\},$$
$$B_{Neu\mathcal{N}}(L) = \left\{ \left\langle \frac{l_1, l_3}{(0.4, 0.50, 0.6)} \right\rangle, \left\langle \frac{l_2}{(0.50, 0.50, 0.50)} \right\rangle \right\}.$$

Let $\tau_N(L) = \{0_N, 1_N, \underline{Neu\mathcal{N}}(L) = \overline{Neu\mathcal{N}}(L) = B_{Neu\mathcal{N}}(L)\}$. Then let $A = \left\{ \left\langle \frac{l_1,l_3}{(0.50,0.50,0.50)} \right\rangle, \left\langle \frac{l_2}{(0.6,0.50,0.4)} \right\rangle \right\}$ and $B = \left\{ \left\langle \frac{l_1,l_3}{(0.70,0.50,0.3)} \right\rangle, \left\langle \frac{l_2}{(0.10,0.50,0.9)} \right\rangle \right\}$ are $Neu\mathcal{N}$ $\delta\mathcal{P}o$ sets but $A \cap B = \left\{ \left\langle \frac{l_1,l_3}{(0.50,0.50,0.50)} \right\rangle, \left\langle \frac{l_2}{(0.10,0.50,0.9)} \right\rangle \right\}$ is not $Neu\mathcal{N}\delta\mathcal{P}o$ set.

Definition 3.20. A space with the property that every pair of disjoint $Neu\mathcal{N}o$ sets have disjoint boundaries is said to be a neutrosophic nano extremal space (briefly, $Neu\mathcal{N}ES$). The neutrosophic nano extremal Hausdorf space is said to be neutrosophic nano extremally disconnected space (briefly, $Neu\mathcal{N}EDS$).

Theorem 3.21. The following statements of F are equivalent:

(i) F is $Neu\mathcal{N}ES$;

(ii) $Neu\mathcal{N}cl(F)$ of any $Neu\mathcal{N}o$ set is $Neu\mathcal{N}o$;

(iii) $Neu\mathcal{N}int(F)$ of any $Neu\mathcal{N}c$ set is $Neu\mathcal{N}c$;

(iv) every pair of disjoint $Neu\mathcal{N}o$ sets are contained in disjoint $Neu\mathcal{N}c$ sets;

(v) the $Neu\mathcal{N}cl(F)$ of disjoint $Neu\mathcal{N}o$ sets are disjoint.

Proposition 3.22. Finite intersection of $Neu\mathcal{N}\delta\mathcal{S}o$ (resp. $Neu\mathcal{N}\delta\mathcal{P}o$, $Neu\mathcal{N}\delta\gamma o$ & $Neu\mathcal{N}\delta\beta o$) sets in a $Neu\mathcal{N}EDS$ is a $Neu\mathcal{N}\delta\mathcal{S}o$ (resp. $Neu\mathcal{N}\delta\mathcal{P}o$, $Neu\mathcal{N}\delta\gamma o$ & $Neu\mathcal{N}\delta\beta o$) set.

Example 3.23. Assume $U = \{l_1, l_2, l_3\}$ and $U/Re = \{\{l_1, l_3\}, \{l_2\}\}$.

Let $L = \left\{ \left\langle \frac{l_1}{(0.30,0.50,0.70)} \right\rangle, \left\langle \frac{l_2}{(0.50,0.50,0.50)} \right\rangle, \left\langle \frac{l_3}{(0.70,0.50,0.30)} \right\rangle \right\}$ be a $Neut\ subs$ of U.

$$\underline{Neu\mathcal{N}}(L) = \left\{ \left\langle \frac{l_1, l_3}{(0.30, 0.50, 0.70)} \right\rangle, \left\langle \frac{l_2}{(0.50, 0.50, 0.50)} \right\rangle \right\},$$
$$\overline{Neu\mathcal{N}}(L) = \left\{ \left\langle \frac{l_1, l_3}{(0.70, 0.50, 0.30)} \right\rangle, \left\langle \frac{l_2}{(0.50, 0.50, 0.50)} \right\rangle \right\},$$
$$B_{Neu\mathcal{N}}(L) = \left\{ \left\langle \frac{l_1, l_3}{(0.70, 0.50, 0.30)} \right\rangle, \left\langle \frac{l_2}{(0.50, 0.50, 0.50)} \right\rangle \right\}.$$

Let $\tau_N(L) = \{0_N, 1_N, \underline{Neu\mathcal{N}}(L), \overline{Neu\mathcal{N}}(L) = B_{Neu\mathcal{N}}(L)\}$. It's an example of $Neu\mathcal{N}EDS$ and also an example of Proposition 3.22.

Proposition 3.24. Arbitrary intersection of $Neu\mathcal{N}\delta c$ (resp. $Neu\mathcal{N}\delta\mathcal{S}c$, $Neu\mathcal{N}\delta\mathcal{P}c$, $Neu\mathcal{N}\delta\gamma c$ & $Neut$ $\mathcal{N}\delta\beta c$) sets is a $Neu\mathcal{N}\delta c$ (resp. $Neu\mathcal{N}\delta\mathcal{S}c$, $Neu\mathcal{N}\delta\mathcal{P}c$, $Neu\mathcal{N}\delta\gamma c$ & $Neu\mathcal{N}\delta\beta c$) set.

Proposition 3.25. Finite union of $Neu\mathcal{N}\delta c$ (resp. $Neu\mathcal{N}\delta\mathcal{S}c$, $Neu\mathcal{N}\delta\mathcal{P}c$, $Neu\mathcal{N}\delta\gamma c$ & $Neu\mathcal{N}\delta\beta c$) sets is a $Neu\mathcal{N}\delta c$ (resp. $Neu\mathcal{N}\delta\mathcal{S}c$, $Neu\mathcal{N}\delta\mathcal{P}c$, $Neu\mathcal{N}\delta\gamma c$ & $Neu\mathcal{N}\delta\beta c$) set.

Remark 3.26. Union of any two $Neu\mathcal{N}\delta c$ (resp. $Neu\mathcal{N}\delta\mathcal{S}c$, $Neu\mathcal{N}\delta\mathcal{P}c$, $Neu\mathcal{N}\delta\gamma c$ & $Neu\mathcal{N}\delta\beta c$) sets need not be $Neu\mathcal{N}\delta c$ (resp. $Neu\mathcal{N}\delta\mathcal{S}c$, $Neu\mathcal{N}\delta\mathcal{P}c$, $Neu\mathcal{N}\delta\gamma c$ & $Neu\mathcal{N}\delta\beta c$) set as shown by the following example.

Example 3.27. Assume $U = \{l_1, l_2, l_3\}$ and $U/Re = \{\{l_1, l_3\}, \{l_2\}\}$. Let

https://doi.org/10.54216/IJNS.180419

Received: March 18, 2022 Accepted: June 28, 2022

(i) $L = \left\{ \left\langle \frac{l_1}{(0.20,0.50,0.70)} \right\rangle, \left\langle \frac{l_2}{(0.10,0.50,0.50)} \right\rangle, \left\langle \frac{l_3}{(0.20,0.50,0.70)} \right\rangle \right\}$ be a $Neut\ subs$ of U.

$$\underline{Neu\mathcal{N}}(L) = \left\{ \left\langle \frac{l_1, l_3}{(0.20, 0.50, 0.70)} \right\rangle, \left\langle \frac{l_2}{(0.10, 0.50, 0.50)} \right\rangle \right\},$$
$$\overline{Neu\mathcal{N}}(L) = \left\{ \left\langle \frac{l_1, l_3}{(0.20, 0.50, 0.70)} \right\rangle, \left\langle \frac{l_2}{(0.10, 0.50, 0.50)} \right\rangle \right\},$$
$$B_{Neu\mathcal{N}}(L) = \left\{ \left\langle \frac{l_1, l_3}{(0.20, 0.50, 0.70)} \right\rangle, \left\langle \frac{l_2}{(0.10, 0.50, 0.50)} \right\rangle \right\}.$$

Let $\tau_N(L) = \{0_N, 1_N, \underline{Neu\mathcal{N}}(L) = \overline{Neu\mathcal{N}}(L) = B_{Neu\mathcal{N}}(L)\}$. Then let $A = \left\{ \left\langle \frac{l_1,l_3}{(0.70,0.50,0.30)} \right\rangle, \left\langle \frac{l_2}{(0.20,0.50,0.50)} \right\rangle \right\}$ and $B = \left\{ \left\langle \frac{l_1,l_3}{(0.10,0.50,0.10)} \right\rangle, \left\langle \frac{l_2}{(0.10,0.50,0.20)} \right\rangle \right\}$ are $Neu\mathcal{N}\delta\gamma c$ sets but $A \cup B = \left\{ \left\langle \frac{l_1,l_3}{(0.70,0.50,0.10)} \right\rangle, \left\langle \frac{l_2}{(0.20,0.50,0.20)} \right\rangle \right\}$ is not $Neu\mathcal{N}\delta\gamma c$ set.

(ii) $L = \left\{ \left\langle \frac{l_1}{(0.4,0.50,0.6)} \right\rangle, \left\langle \frac{l_2}{(0.50,0.50,0.50)} \right\rangle, \left\langle \frac{l_3}{(0.4,0.50,0.6)} \right\rangle \right\}$ be a $Neut\ subs$ of U.

$$\underline{Neu\mathcal{N}}(L) = \left\{ \left\langle \frac{l_1, l_3}{(0.4, 0.50, 0.6)} \right\rangle, \left\langle \frac{l_2}{(0.50, 0.50, 0.50)} \right\rangle \right\},$$
$$\overline{Neu\mathcal{N}}(L) = \left\{ \left\langle \frac{l_1, l_3}{(0.4, 0.50, 0.6)} \right\rangle, \left\langle \frac{l_2}{(0.50, 0.50, 0.50)} \right\rangle \right\},$$
$$B_{Neu\mathcal{N}}(L) = \left\{ \left\langle \frac{l_1, l_3}{(0.4, 0.50, 0.6)} \right\rangle, \left\langle \frac{l_2}{(0.50, 0.50, 0.50)} \right\rangle \right\}.$$

Let $\tau_N(L) = \{0_N, 1_N, \underline{Neu\mathcal{N}}(L) = \overline{Neu\mathcal{N}}(L) = B_{Neu\mathcal{N}}(L)\}$. Then let $A = \left\{ \left\langle \frac{l_1,l_3}{(0.50,0.50,0.50)} \right\rangle, \left\langle \frac{l_2}{(0.4,0.50,0.6)} \right\rangle \right\}$ and $B = \left\{ \left\langle \frac{l_1,l_3}{(0.30,0.50,0.70)} \right\rangle, \left\langle \frac{l_2}{(0.9,0.50,0.10)} \right\rangle \right\}$ are $Neu\mathcal{N}$ $\delta\mathcal{P}c$ sets but $A \cup B = \left\{ \left\langle \frac{l_1,l_3}{(0.50,0.50,0.50)} \right\rangle, \left\langle \frac{l_2}{(0.9,0.50,0.10)} \right\rangle \right\}$ is not $Neu\mathcal{N}\delta\mathcal{P}c$ set.

Proposition 3.28. Finite union of $Neu\mathcal{N}\delta\mathcal{S}c$ (resp. $Neu\mathcal{N}\delta\mathcal{P}c$, $Neu\mathcal{N}\delta\gamma c$ & $Neu\mathcal{N}\delta\beta c$) sets in a $Neu\mathcal{N}$ EDS is a $Neu\mathcal{N}\delta\mathcal{S}c$ (resp. $Neu\mathcal{N}\delta\mathcal{P}c$, $Neu\mathcal{N}\delta\gamma c$ & $Neu\mathcal{N}\delta\beta c$) set.

Example 3.29. Assume $U = \{l_1, l_2, l_3\}$ and $U/Re = \{\{l_1, l_3\}, \{l_2\}\}$.

Let $L = \left\{ \left\langle \frac{l_1}{(0.40,0.50,0.60)} \right\rangle, \left\langle \frac{l_2}{(0.50,0.50,0.50)} \right\rangle, \left\langle \frac{l_3}{(0.60,0.50,0.40)} \right\rangle \right\}$ be a $Neut\ subs$ of U.

$$\underline{Neu\mathcal{N}}(L) = \left\{ \left\langle \frac{l_1, l_3}{(0.40, 0.50, 0.60)} \right\rangle, \left\langle \frac{l_2}{(0.50, 0.50, 0.50)} \right\rangle \right\},$$
$$\overline{Neu\mathcal{N}}(L) = \left\{ \left\langle \frac{l_1, l_3}{(0.60, 0.50, 0.40)} \right\rangle, \left\langle \frac{l_2}{(0.50, 0.50, 0.50)} \right\rangle \right\},$$
$$B_{Neu\mathcal{N}}(L) = \left\{ \left\langle \frac{l_1, l_3}{(0.60, 0.50, 0.40)} \right\rangle, \left\langle \frac{l_2}{(0.50, 0.50, 0.50)} \right\rangle \right\}.$$

Let $\tau_N(L) = \{0_N, 1_N, \underline{Neu\mathcal{N}}(L), \overline{Neu\mathcal{N}}(L) = B_{Neu\mathcal{N}}(L)\}$. It's an $Neu\mathcal{N}EDS$ and also an example of Proposition 3.28.

4 More on neutrosophic nano interior and closure operators

Throughout this section, let $(U, \tau_N(F))$ be a $Neu\mathcal{N}ts$ with respect to F where F is a $Neut\ subs$ of U.

Definition 4.1. A set S is said to be a neutrosophic nano δ-semi (resp. neutrosophic nano δ-pre, neutrosophic nano δ-γ & neutrosophic nano δ-β) interior of S (briefly, $Neu\mathcal{N}\delta\mathcal{S}int(S)$ (resp. $Neu\mathcal{N}\delta\mathcal{P}int(S)$, $Neu\mathcal{N}\delta\gamma int(S)$ & $Neu\mathcal{N}\delta\beta int(S)$)) is the union of all $Neu\mathcal{N}\delta\mathcal{S}o$ (resp. $Neu\mathcal{N}\delta\mathcal{P}o$, $Neu\mathcal{N}\delta\gamma o$ & $Neu\mathcal{N}$ $\delta\beta o$) set contained in S.

https://doi.org/10.54216/IJNS.180419
Received: March 18, 2022 Accepted: June 28, 2022

Definition 4.2. A set S is said to be a neutrosophic nano δ-semi (resp. neutrosophic nano δ-pre, neutrosophic nano δ-γ & neutrosophic nano δ-β) closure of S (briefly, $Neu\mathcal{N}\delta\mathcal{S}cl(S)$ (resp. $Neu\mathcal{N}\delta\mathcal{P}cl(S)$, $Neu\mathcal{N}\delta\gamma cl(S)$ & $Neu\mathcal{N}\delta\beta cl(S)$)) is the intersection of all $Neu\mathcal{N}\delta\mathcal{S}c$ (resp. $Neu\mathcal{N}\delta\mathcal{P}c$, $Neu\mathcal{N}\delta\gamma c$ & $Neu\mathcal{N}\delta\beta c$) set containing S.

Theorem 4.3. Let S and T be $Neut\ subs$'s of U, then the statements are hold.

(i) $Neu\mathcal{N}\delta int(S_o) \subseteq S_o$.

(ii) S_o is $Neu\mathcal{N}\delta o$ iff $Neu\mathcal{N}\delta int(S_o) = S_o$.

(iii) $Neu\mathcal{N}\delta int(Neu\mathcal{N}\delta int(S_o)) = Neu\mathcal{N}\delta int(S_o)$.

(iv) $S_o \subseteq T_o \Rightarrow Neu\mathcal{N}\delta int(S_o) \subseteq Neu\mathcal{N}\delta int(T_o)$.

(v) $Neu\mathcal{N}\delta int(S_o \cap T_o) = Neu\mathcal{N}\delta int(S_o) \cap Neu\mathcal{N}\delta int(T_o)$.

(vi) $Neu\mathcal{N}\delta int(S_o \cup T_o) \supseteq Neu\mathcal{N}\delta int(S_o) \cup Neu\mathcal{N}\delta int(T_o)$.

(vii) $Neu\mathcal{N}\delta int(0_N) = 0_N$ & $Neu\mathcal{N}\delta int(1_N) = 1_N$.

Proof. (i) Follows from definition, $Neu\mathcal{N}\delta int(S_o)$.

(ii) S_o is $Neu\mathcal{N}\delta o$ iff $1_N - S_o$ is $Neu\mathcal{N}\delta c$, iff $Neu\mathcal{N}\delta cl(1_N - S_o) = 1_N - S_o$, iff $1_N - Neu\mathcal{N}\delta cl(1_N - S_o) = S_o$ iff $Neu\mathcal{N}\delta int(S_o) = S_o$, by Remark 3.12.

(iii) By using (ii) and Remark 3.2, $Neu\mathcal{N}\delta int(Neu\mathcal{N}\delta int(S_o)) = Neu\mathcal{N}\delta int(S_o)$. This proves (iii).

(iv) $S_o \subseteq T_o \Rightarrow 1_N - T_o \subseteq 1_N - S_o$. Therefore, $Neu\mathcal{N}\delta cl(1_N - T_o) \subseteq Neu\mathcal{N}\delta cl(1_N - S_o)$. That is, $1_N - Neu\mathcal{N}\delta cl(1_N - S_o) \subseteq 1_N - Neu\mathcal{N}\delta cl(1_N - T_o)$. That is, $Neu\mathcal{N}\delta int(S_o) \subseteq Neu\mathcal{N}\delta int(T_o)$.

(v) Since $S_o \cap T_o \subseteq S_o$ and $S_o \cap T_o \subseteq T_o$, by using (iv), $Neu\mathcal{N}\delta int(S_o \cap T_o) \subseteq Neu\mathcal{N}\delta int(S_o)$ and $Neu\mathcal{N}\delta int(S_o \cap T_o) \subseteq Neu\mathcal{N}\delta int(T_o)$. This implies that $Neu\mathcal{N}\delta int(S_o \cap T_o) \subseteq Neu\mathcal{N}\delta int(S_o) \cap Neu\mathcal{N}\delta int(T_o)$. Now $Neu\mathcal{N}\delta int(S_o) \subseteq S_o$ and $Neu\mathcal{N}\delta int(T_o) \subseteq T_o$, we get, $Neu\mathcal{N}\delta int(S_o) \cap Neu\mathcal{N}\delta\ int(T_o) \subseteq S_o \cap T_o$.

$\Rightarrow Neu\mathcal{N}\delta int(Neu\mathcal{N}\delta int(S_o) \cap Neu\mathcal{N}\delta int(T_o)) \subseteq Neu\mathcal{N}\delta int(S_o \cap T_o)$, which implies $Neu\mathcal{N}\delta\ int(Neu\mathcal{N}\delta int(S_o)) \cap Neu\mathcal{N}\delta int(Neu\mathcal{N}\delta\ int(T_o)) \subseteq Neu\mathcal{N}\delta int(S_o \cap T_o)$.

$\Rightarrow Neu\mathcal{N}\delta int(S_o) \cap Neu\mathcal{N}\delta int(T_o) \subseteq Neu\mathcal{N}\delta int(S_o \cap T_o)$. Hence, $Neu\mathcal{N}\delta int(S_o) \cap Neu\mathcal{N}\delta int(T_o) = Neu\mathcal{N}\delta int(S_o \cap T_o)$.

(vi) Since $S_o \subseteq S_o \cup T_o$ and $T_o \subseteq S_o \cup T_o$, by using (iv), $Neu\mathcal{N}\delta int(S_o) \subseteq Neu\mathcal{N}\delta int(S_o \cup T_o)$ and $Neu\mathcal{N}\delta int(T_o) \subseteq Neu\mathcal{N}\delta int(S_o \cup T_o)$. This implies that, $Neu\mathcal{N}\delta int(S_o) \cup Neu\mathcal{N}\delta int(T_o) \subseteq Neu\mathcal{N}\delta\ int(S_o \cup T_o)$.

(vii) Since 0_N and 1_N are $Neu\mathcal{N}\delta o$, $Neu\mathcal{N}\delta int(0_N) = 0_N$ and $Neu\mathcal{N}\delta int(1_N) = 1_N$. □

Theorem 4.4. Let S and T be $Neut\ subs$'s of U, then the following statements hold.

(i) $S \subseteq Neu\mathcal{N}\delta cl(S)$.

(ii) S is $Neu\mathcal{N}\delta c$ iff $Neu\mathcal{N}\delta cl(S) = S$.

(iii) $Neu\mathcal{N}\delta cl(Neu\mathcal{N}\delta cl(S)) = Neu\mathcal{N}\delta cl(S)$.

(iv) $S \subseteq T \Rightarrow Neu\mathcal{N}\delta cl(S) \subseteq Neu\mathcal{N}\delta cl(T)$.

(v) $Neu\mathcal{N}\delta cl(S \cap T) \subseteq Neu\mathcal{N}\delta cl(S) \cap Neu\mathcal{N}\delta cl(T)$.

(vi) $Neu\mathcal{N}\delta cl(S \cup T) = Neu\mathcal{N}\delta cl(S) \cup Neu\mathcal{N}\delta cl(T)$.

https://doi.org/10.54216/IJNS.180419
Received: March 18, 2022 Accepted: June 28, 2022

(vii) $Neu\mathcal{N}\delta cl(0_N) = 0_N$ & $Neu\mathcal{N}\delta cl(1_N) = 1_N$.

Proof. (i) Follows from definition, $Neu\mathcal{N}\delta cl(S)$.

(ii) Let S be $Neu\mathcal{N}\delta c$ set in U. By using Definition 3.1, S^c is a $Neu\mathcal{N}\delta o$ set in U. By Theorem 4.3 and Proposition 3.11, $Neu\mathcal{N}\delta int(S^c) = S^c \Leftrightarrow (Neu\mathcal{N}\delta cl(S))^c = S^c \Leftrightarrow Neu\mathcal{N}\delta cl(S) = S$. This proved (ii).

(iii) By using (ii) and Remark 3.2, $Neu\mathcal{N}\delta cl(Neu\mathcal{N}\delta cl(S)) = Neu\mathcal{N}\delta cl(S)$. This proves (iii).

(iv) $S \subseteq T$, $T^c \subseteq S^c$. By using Theorem 4.3 (iv), $Neu\mathcal{N}\delta int(T^c) \subseteq Neu\mathcal{N}\delta in(S^c)$. Taking complement on both sides, $(Neu\mathcal{N}\delta int(T^c))^c \supseteq (Neu\mathcal{N}\delta int(S^c))^c$. By proposition 3.11 (ii), $Neu\mathcal{N}\delta cl(S) \subseteq Neu\mathcal{N}\delta cl(T)$. This proves (iv).

(v) Since $S \cap T \subseteq S$ and $S \cap T \subseteq T$, by using (iv), $Neu\mathcal{N}\delta cl(S \cap T) \subseteq Neu\mathcal{N}\delta cl(S)$ and $Neu\mathcal{N}\delta cl(S \cap T) \subseteq Neu\mathcal{N}\delta cl(T)$. This implies that $Neu\mathcal{N}\delta cl(S \cap T) \subseteq Neu\mathcal{N}\delta cl(S) \cap Neu\mathcal{N}\delta cl(T)$. This proves (v).

(vi) Since $S \subseteq S \cup T$ and $T \subseteq S \cup T$, by using (iv), $Neu\mathcal{N}\delta cl(S) \subseteq Neu\mathcal{N}\delta cl(S \cup T)$ and $Neu\mathcal{N}\delta cl(T) \subseteq Neu\mathcal{N}\delta cl(S \cup T)$. This implies that, $Neu\mathcal{N}\delta cl(S) \cup Neu\mathcal{N}\delta cl(T) \subseteq Neu\mathcal{N}\delta cl(S \cup T)$.

Now $S \subseteq Neu\mathcal{N}\delta cl(S)$ and $T \subseteq Neu\mathcal{N}\delta cl(T)$, we get, $S \cup T \subseteq Neu\mathcal{N}\delta cl(S) \cup Neu\mathcal{N}\delta cl(T)$.

$\Rightarrow Neu\mathcal{N}\delta cl(S \cup T) \subseteq Neu\mathcal{N}\delta cl(Neu\mathcal{N}\delta cl(S) \cup Neu\mathcal{N}\delta cl(T))$, which implies $Neu\mathcal{N}\delta cl(S \cup T) \subseteq Neu\mathcal{N}\delta cl(Neu\mathcal{N}\delta\ cl(S)) \cup Neu\mathcal{N}\delta cl\ (Neu\mathcal{N}\delta cl(T))$.

$\Rightarrow Neu\mathcal{N}\delta cl(S \cup T) \subseteq Neu\mathcal{N}\delta cl(S) \cup Neu\mathcal{N}\delta cl(T)$. Hence, $Neu\mathcal{N}\delta cl(S) \cup Neu\mathcal{N}\delta cl(T) = Neu\mathcal{N}\delta cl(S \cup T)$.

(vii) Since 0_N and 1_N are $Neu\mathcal{N}\delta c$, $Neu\mathcal{N}\delta cl(0_N) = 0_N$ and $Neu\mathcal{N}\delta cl(1_N) = 1_N$. □

The operators $Neu\mathcal{N}\delta\mathcal{S}int(.)$ (resp. $Neu\mathcal{N}\delta\mathcal{P}int(.)$, $Neu\mathcal{N}\delta\gamma int(.)$, $Neu\mathcal{N}\delta\beta int(.)$) and their respective closure operators are also satisfy the Proposition 3.11, Theorem 4.3 & Theorem 4.4.

Proposition 4.5. For any $Neut\ subs\ S_o$ of U,

(i) $Neu\mathcal{N}\delta int(S_o) \subseteq Neu\mathcal{N}int(S_o) \subseteq Neu\mathcal{N}\delta\mathcal{P}int(S_o) \subseteq Neu\mathcal{N}\delta\gamma int(S_o) \subseteq Neu\mathcal{N}\delta\beta int(S_o)$.

(ii) $Neu\mathcal{N}int(S_o) \subseteq Neu\mathcal{N}\delta\mathcal{S}_l int(S_o) \subseteq Neu\mathcal{N}\delta\gamma int(S_o) \subseteq Neu\mathcal{N}\delta\beta int(S_o)$.

(iii) $Neu\mathcal{N}\delta cl(S_o) \supseteq Neu\mathcal{N}cl(S_o) \supseteq Neu\mathcal{N}\delta\mathcal{P}cl(S_o) \supseteq Neu\mathcal{N}\delta\gamma cl(S_o) \supseteq Neu\mathcal{N}\delta\beta cl(S_o)$.

(iv) $Neu\mathcal{N}cl(S_o) \supseteq Neu\mathcal{N}\delta\mathcal{S}_l cl(S_o) \supseteq Neu\mathcal{N}\delta\gamma cl(S_o) \supseteq Neu\mathcal{N}\delta\beta cl(S_o)$.

Lemma 4.6. If S is $Neu\mathcal{N}o$, then $Neu\mathcal{N}cl(S)$ is $Neu\mathcal{N}rc$.

Proof. We know that $S \subseteq Neu\mathcal{N}cl(S)$. Thus $S = Neu\mathcal{N}int(S) \subseteq Neu\mathcal{N}int(Neu\mathcal{N}cl(S))$ and hence $Neu\mathcal{N}cl(S) \subseteq Neu\mathcal{N}cl(Neu\mathcal{N}\ int(Neu\mathcal{N}cl(S)))$. Conversely, we know that $Neu\mathcal{N}int(Neu\mathcal{N}\ cl(S)) \subseteq Neu\mathcal{N}cl(S)$. Thus $Neu\mathcal{N}cl(Neu\mathcal{N}\ int(Neu\mathcal{N}cl(S))) \subseteq Neu\mathcal{N}cl\ (Neu\mathcal{N}cl(S)) = Neu\mathcal{N}cl(S)$. Hence $Neu\mathcal{N}cl(S) = Neu\mathcal{N}cl(Neu\mathcal{N}int(Neu\mathcal{N}cl(S)))$. □

Lemma 4.7. The $\{Neu\mathcal{N}cl(S)|S \in \tau_N(F)\} = \{T : T \text{ is } Neu\mathcal{N}rc \text{ in } U\}$.

Proof. We know that for any $Neu\mathcal{N}o$ set S in U, $Neu\mathcal{N}cl(S)$ is $Neu\mathcal{N}rc$. Conversely, take any $Neu\mathcal{N}rc$ set T in U. Then $T = Neu\mathcal{N}cl(Neu\mathcal{N}int(T)) = Neu\mathcal{N}cl(\bigcup\{S|S \subseteq T, S \in \tau_N(F)\}) \in \{Neu\mathcal{N}\ cl(S)|S \in \tau_N(F)\}$. □

We may have a difficulty in finding the $Neu\mathcal{N}\delta cl$ of any Neutrosophic set. But by the above lemmas we have the clue to find it.

Theorem 4.8. For any $Neut\ sub\ S$ in a $Neu\mathcal{N}ts$ $(U, \tau_N(F))$, then

$$Neu\mathcal{N}\delta cl(S) = \bigcap\{Neu\mathcal{N}cl(G)|S \subseteq Neu\mathcal{N}cl(G), G \in \tau_N(F)\}.$$

Proof. The proof is straightforward. □

Proposition 4.9. For every $Neu\mathcal{N}ts$ $(U, \tau_N(F))$, if S is a $Neu\mathcal{N}\delta\mathcal{P}os$ and T is a $Neu\mathcal{N}\delta\mathcal{S}os$, then $S \cup T$ is a $Neu\mathcal{N}\delta\gamma os$. But converse not true in Examples 3.6 and 3.10.

Proof. Follows from the Proposition 3.8 (v) and (vi). □

Proposition 4.10. (i) if S is $Neu\mathcal{N}\delta o$ and T is $Neu\mathcal{N}\delta\mathcal{S}o$, $Neu\mathcal{N}\delta\mathcal{P}o$, $Neu\mathcal{N}\delta\beta o$, then $S \cap T$ is $Neu\mathcal{N}\delta\mathcal{S}o$, $Neu\mathcal{N}\delta\mathcal{P}o$, $Neu\mathcal{N}\delta\beta o$.

(ii) S is $Neu\mathcal{N}\delta\gamma o$, iff S is the union of a $Neu\mathcal{N}\delta\mathcal{S}o$ and $Neu\mathcal{N}\delta\mathcal{P}o$.

Proof. S is $Neu\mathcal{N}\delta o \Rightarrow S = Neu\mathcal{N}\delta int(S)$.

T is $Neu\mathcal{N}\delta\mathcal{S}o \Rightarrow T \subseteq Neu\mathcal{N}cl(Neu\mathcal{N}\delta int(T))$.

$S \cap T = Neu\mathcal{N}\delta int(S) \cap Neu\mathcal{N}cl(Neu\mathcal{N}\delta int(T)) \subseteq Neu\mathcal{N}cl(Neu\mathcal{N}\delta int(S) \cap Neu\mathcal{N}cl(Neut\,\mathcal{N}\delta int(T))) \subseteq Neu\mathcal{N}cl(Neu\mathcal{N}\delta int(S) \cap Neu\mathcal{N}\delta int(T)) \subseteq Neu\mathcal{N}cl(Neu\mathcal{N}\delta int(S \cap T))$. Similarly other results also proved.

(ii) If S is $Neu\mathcal{N}\delta\gamma o$ iff $S \subseteq Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S)) \cup Neu\mathcal{N}cl(Neu\mathcal{N}\delta int(S))$ iff S is the union of a $Neu\mathcal{N}\delta\mathcal{P}o$ and $Neu\mathcal{N}\delta\mathcal{S}o$. □

Proposition 4.11. The intersection of a $Neu\mathcal{N}\delta\mathcal{P}o$ & $Neu\mathcal{N}\delta\alpha o$ is $Neu\mathcal{N}\delta\mathcal{P}o$.

Proof. Let S_o be $Neu\mathcal{N}\delta\mathcal{P}o$ and T_o be $Neu\mathcal{N}\delta\alpha o$, then $S_o \subseteq Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S_o))$, $T_o \subseteq Neu\mathcal{N}int(Neu\mathcal{N}cl(Neu\mathcal{N}\delta\ int(T_o)))$. So,

$$\begin{aligned}S_o \cap T_o &\subseteq Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S_o)) \cap Neu\mathcal{N}int(Neu\mathcal{N}cl(Neu\mathcal{N}\delta int(T_o)))\\ &\subseteq Neu\mathcal{N}int(Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S_o)) \cap Neu\mathcal{N}cl(Neu\mathcal{N}\delta int(T_o)))\\ &\subseteq Neu\mathcal{N}int(Neu\mathcal{N}cl(Neu\mathcal{N}\delta cl(S_o)) \cap Neu\mathcal{N}cl(Neu\mathcal{N}\delta int(T_o)))\\ &\subseteq Neu\mathcal{N}int(Neu\mathcal{N}cl(Neu\mathcal{N}\delta cl(S_o)) \cap Neu\mathcal{N}\delta int(T_o))\\ &\subseteq Neu\mathcal{N}int(Neu\mathcal{N}cl(Neu\mathcal{N}\delta cl(S_o \cap T_o)))\\ &= Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S_o \cap T_o)).\end{aligned}$$

Hence, $S_o \cap T_o$ is $Neu\mathcal{N}\delta\mathcal{P}o$ set. □

Corollary 4.12. The union of a $Neu\mathcal{N}\delta\mathcal{P}c$ set and a $Neu\mathcal{N}\delta\alpha c$ set is $Neu\mathcal{N}\delta\mathcal{P}c$.

Proposition 4.13. Each $Neu\mathcal{N}\delta\beta o$ and $Neu\mathcal{N}\delta\mathcal{S}c$ is $Neu\mathcal{N}\delta\mathcal{S}o$.

Proof. Let S be $Neu\mathcal{N}\delta\beta o$ set and $Neu\mathcal{N}\delta\mathcal{S}c$, then $S \subseteq Neu\mathcal{N}cl(Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S)))$ and $Neu\mathcal{N}int(Neu\mathcal{N}\delta\ cl(S)) \subseteq S$. Therefore, $Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S)) \subseteq Neu\mathcal{N}\delta int(S)$ and so, $Neut\ \mathcal{N}cl(Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S))) \subseteq Neu\mathcal{N}cl(Neu\mathcal{N}\delta int(S))$. Hence, $S \subseteq Neu\mathcal{N}cl(Neu\mathcal{N}int(Neut\ \mathcal{N}\delta cl(S))) \subseteq Neu\mathcal{N}cl(Neu\mathcal{N}\delta int(S))$. Hence S is $Neu\mathcal{N}\delta\mathcal{S}o$. □

Proposition 4.14. If S is $Neu\mathcal{N}\delta\beta c$ and $Neu\mathcal{N}\delta\mathcal{S}o$, then it is $Neu\mathcal{N}\delta\mathcal{S}c$.

Proof. Since S is $Neu\mathcal{N}\delta\beta c$ and $Neu\mathcal{N}\delta\mathcal{S}o$. Then S^c is $Neu\mathcal{N}\delta\beta o$ and $Neu\mathcal{N}\delta\mathcal{S}c$ and so by Proposition 4.13, S^c is $Neu\mathcal{N}\delta\mathcal{S}o$. Therefore S is $Neu\mathcal{N}\delta\mathcal{S}c$. □

Proposition 4.15. Each $Neu\mathcal{N}\delta\beta o$ set and $Neu\mathcal{N}\delta\alpha c$ set is $Neu\mathcal{N}\delta rc$.

https://doi.org/10.54216/IJNS.180419

Received: March 18, 2022 Accepted: June 28, 2022

Proof. Let S be $Neu\mathcal{N}\delta\beta o$ set and $Neu\mathcal{N}\delta\alpha c$ set. Then $S \subseteq Neu\mathcal{N}cl(Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S)))$ and $Neu\mathcal{N}cl(Neu\mathcal{N}int\ (Neu\mathcal{N}\delta cl(S))) \subseteq S$, which implies that $Neu\mathcal{N}cl(Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S))) \subseteq S \subseteq Neu\mathcal{N}cl(Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S)))$. So, $S = Neu\mathcal{N}cl(Neu\mathcal{N}int(Neu\mathcal{N}\delta\ cl(S)))$. Hence S is $Neu\mathcal{N}\delta c$, and so it is $Neu\mathcal{N}\delta rc$. □

Corollary 4.16. Each $Neu\mathcal{N}\delta\beta c$ set and $Neu\mathcal{N}\delta\alpha o$ set is $Neu\mathcal{N}\delta rc$.

Proposition 4.17. If S is a $Neu\mathcal{N}\delta os$ & T is a $Neu\mathcal{N}\delta\beta os$, then $S \cap T$ is a $Neu\mathcal{N}\delta\beta os$.

Proof. $S \cap T \subseteq S \cap Neu\mathcal{N}cl(Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(T))) \subseteq Neu\mathcal{N}cl(S \cap Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(T))) \subseteq Neu\mathcal{N}cl(Neu\mathcal{N}int\ (Neu\mathcal{N}\delta cl(S \cap T)))$. Therefore, $S \cap T$ is a $Neu\mathcal{N}\delta\beta os$. □

Remark 4.18. The Proposition 4.17 is also true if T is a $Neu\mathcal{N}\delta\mathcal{S}os$, $Neu\mathcal{N}\delta\mathcal{P}os$ and $Neu\mathcal{N}\delta\alpha os$.

Proposition 4.19. If S_o is a $Neu\mathcal{N}\delta\mathcal{P}os$ & T_o is a $Neu\mathcal{N}\delta\alpha os$, then $S_o \cap T_o$ is a $Neu\mathcal{N}\delta\mathcal{P}os$.

Proof.

$$\begin{aligned} S_o \cap T_o \subseteq & Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S_o)) \cap Neu\mathcal{N}int(Neu\mathcal{N}cl(Neu\mathcal{N}\delta int(T_o))) \\ \subseteq & Neu\mathcal{N}int(Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S_o))) \cap Neu\mathcal{N}cl(Neu\mathcal{N}\delta int(T_o)) \\ \subseteq & Neu\mathcal{N}int(Neu\mathcal{N}cl(Neu\mathcal{N}\delta cl(S_o))) \cap Neu\mathcal{N}cl(Neu\mathcal{N}\delta int(T_o)) \\ \subseteq & Neu\mathcal{N}int(Neu\mathcal{N}cl(Neu\mathcal{N}\delta cl(S_o))) \cap Neu\mathcal{N}\delta int(T_o) \\ \subseteq & Neu\mathcal{N}int(Neu\mathcal{N}cl(Neu\mathcal{N}\delta cl(S_o \cap T_o))) \\ = & Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S_o \cap T_o)). \end{aligned}$$

Therefore, $S_o \cap T_o$ is a $Neu\mathcal{N}\delta\mathcal{P}os$. □

Corollary 4.20. If S is a $Neu\mathcal{N}\delta\mathcal{P}cs$ and T is a $Neu\mathcal{N}\delta\alpha os$, then $S \cup T$ is a $Neu\mathcal{N}\delta\mathcal{P}cs$.

Proposition 4.21. T is a $Neut\ subs$ of a $Neu\mathcal{N}ts\ (U, \tau_N(F))$ and S is a $Neu\mathcal{N}\delta\mathcal{P}os$ such that $S \subseteq T \subseteq Neu\mathcal{N}cl(Neu\mathcal{N}\delta int(S))$. Then T is a $Neu\mathcal{N}\delta\beta os$.

Proof. Since S is a $Neu\mathcal{N}\delta\mathcal{P}os$, $S \subseteq Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S))$. Now $T \subseteq Neu\mathcal{N}cl(Neu\mathcal{N}\delta int(S)) \subseteq Neu\mathcal{N}cl(Neu\mathcal{N}\ \delta int(Neu\mathcal{N}int(Neu\mathcal{N}\delta cl\ (S)))) = Neu\mathcal{N}cl(Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(T)))$.

Hence $T \subseteq Neu\mathcal{N}cl(Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(T)))$. Therefore, T is a $Neu\mathcal{N}\delta\beta os$. □

Proposition 4.22. If each S is a $Neu\mathcal{N}\delta\beta os$ which is a $Neu\mathcal{N}\delta\mathcal{S}cs$ is also a $Neu\mathcal{N}\delta\mathcal{S}os$.

Proof. Let S be a $Neu\mathcal{N}\delta\beta os$ & $Neu\mathcal{N}\delta\mathcal{S}cs$. Then, $S \subseteq Neu\mathcal{N}cl(Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S)))$ & $Neu\mathcal{N}\ int(Neu\mathcal{N}\delta cl(S)) \subseteq S$. Therefore, $Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S)) \subseteq S$ and so, $Neu\mathcal{N}cl(Neu\mathcal{N}int(Neu\mathcal{N}\delta\ cl(S))) \subseteq Neu\mathcal{N}cl(Neu\mathcal{N}\delta\ int(S))$. Hence, $S \subseteq Neu\mathcal{N}cl(Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S))) \subseteq Neu\mathcal{N}cl(Neu\ \mathcal{N}\delta int(S))$. Therefore, S is a $Neu\mathcal{N}\delta\mathcal{S}os$. □

Proposition 4.23. If S is a $Neu\mathcal{N}\delta\beta cs$ & $Neu\mathcal{N}\delta\mathcal{S}os$, then it is a $Neu\mathcal{N}\delta\mathcal{S}cs$.

Proof. Similar to Proposition 4.22. □

Proposition 4.24. If each S_o is a $Neu\mathcal{N}\delta\beta os$ which is a $Neu\mathcal{N}\delta\alpha cs$ is also a $Neu\mathcal{N}\delta cs$.

Proof. Let S_o be a $Neu\mathcal{N}\delta\beta os$ & $Neu\mathcal{N}\delta\alpha cs$. Then, $S_o \subseteq Neu\mathcal{N}cl(Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S_o)))$ & $Neu\ \mathcal{N}cl(Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S_o))) \subseteq S_o$. Thus, $Neu\mathcal{N}cl(Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S_o))) \subseteq S_o \subseteq Neu\mathcal{N}cl(Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S_o)))$. So, $S_o = Neu\mathcal{N}cl(Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S_o)))$. Thus, S_o is a $Neu\mathcal{N}\delta cs$. □

Corollary 4.25. If each T is a $Neu\mathcal{N}\delta\beta cs$ which is $Neu\mathcal{N}\delta\alpha os$ is also a $Neu\mathcal{N}\delta os$.

Received: March 18, 2022 Accepted: June 28, 2022

5 Product related neutrosophic nano topological spaces

Definition 5.1. Let $(U_1, \tau_N(F_1))$ and $(U_2, \tau_N(F_2))$ be a $Neu\mathcal{N}ts$'s with respect to F_1 and F_2, where F_1 and F_2 are a $Neut\ subs$'s of U_1 and U_2. Let $S = \{\langle s, \mu_S(s), \sigma_S(s), \nu_S(s)\rangle : s \in U_1\}$ and $T = \{\langle t, \mu_T(t), \sigma_T(t), \nu_T(t)\rangle : t \in U_2\}$ be $Neut\ subs$'s of U_1 and U_2 respectively. Then $S \times T$ is a $Neut\ subs$ of $U_1 \times U_2$ is defined by

(P_1) $(S \times T)(s,t) = \langle (s,t), \min(\mu_S(s), \mu_T(t)), \min(\sigma_S(s), \sigma_T(t)), \max(\nu_S(s), \nu_T(t))\rangle$.

(P_2) $(S \times T)(s,t) = \langle (s,t), \min(\mu_S(s), \mu_T(t)), \max(\sigma_S(s), \sigma_T(t)), \max(\nu_S(s), \nu_T(t))\rangle$.

$({P_1}^c)$ $((S \times T)(s,t))^c = \langle (s,t), \max(\mu_S(s), \mu_T(t)), \max(\sigma_S(s), \sigma_T(t)), \min(\nu_S(s), \nu_T(t))\rangle$.

$({P_2}^c)$ $((S \times T)(s,t))^c = \langle (s,t), \max(\mu_S(s), \mu_T(t)), \min(\sigma_S(s), \sigma_T(t)), \min(\nu_S(s), \nu_T(t))\rangle$.

Lemma 5.2. Let $(U_1, \tau_N(F_1))$ and $(U_2, \tau_N(F_2))$ be a $Neu\mathcal{N}ts$'s with respect to F_1 and F_2, where F_1 and F_2 are a $Neut\ subs$'s of U_1 and U_2. If S and T be $Neut\ subs$'s of U_1 and U_2, then

(i) $(S \times 1_N) \cap (1_N \times T) = S \times T$,

(ii) $(S \times 1_N) \cup (1_N \times T) = (S^c \times T^c)^c$,

(iii) $(S \times T)^c = (S^c \times 1_N) \cup (1_N \times T^c)$.

Proof. Let $S = \{\langle s, \mu_S(s), \sigma_S(s), \nu_S(s)\rangle : s \in U_1\}$ & $T = \{\langle t, \mu_T(t), \sigma_T(t), \nu_T(t)\rangle : t \in U_2\}$.

(i) Since $S \times 1_N = \langle s, \min(\mu_S(s), 1_N), \min(\sigma_S(s), 1_N), \max(\nu_S(s), 0_N)\rangle = \langle s, \mu_S(s), \sigma_S(s), \nu_S(s)\rangle = S$ and similarly $1_N \times T = \langle t, \min(1_N, \mu_T(t)), \min(1_N, \sigma_T(t)), \max(0_N, \nu_T(t))\rangle = \langle t, \mu_T(t), \sigma_T(t), \nu_T(t)\rangle = T$, we have $(S\times 1_N)\cap(1_N\times T) = S(s)\cap T(t) = \langle (s,t), \mu_S(s)\wedge\mu_T(t), \sigma_S(s)\wedge\sigma_T(t), \nu_S(s)\vee\nu_T(t)\rangle = S\times T$.

(ii) Similarly to (i).

(iii) Obvious by putting S, T instead of S^c, T^c in (ii). □

Definition 5.3. Let $(U_1, \tau_N(F_1))$ and $(U_2, \tau_N(F_2))$ be a $Neu\mathcal{N}ts$'s with respect to F_1 and F_2, where F_1 and F_2 are a $Neut\ subs$'s of U_1 and U_2 and let $f : (U_1, \tau_N(F_1)) \to (U_2, \tau_N(F_2))$ be a neutrosophic function.

(i) If $T = \{\langle t, \mu_T(t), \sigma_T(t), \nu_T(t)\rangle : t \in U_2\}$ is a $Neut\ s$ in U_2, then the pre image of T under f is defined by $f^{-1}(T) = \{\langle s, f^{-1}(\mu_T)(s), f^{-1}(\sigma_T)(s), f^{-1}(\nu_T)(s)\rangle : s \in U_1\}$.

(ii) If $S = \{\langle s, \mu_S(s), \sigma_S(s), \nu_S(s)\rangle : s \in U_1\}$ is a $Neut\ s$ in U_1, then the image of S under f is defined by $f(S) = \{\langle t, f(\mu_S)(t), f(\sigma_S)(t), f_-(\nu_S)(t)\rangle : t \in U_2\}$ where $f_-(\nu_S) = (f(S^c))^c$.

In (i), (ii), since $\mu_S, \sigma_S, \nu_S, \mu_T, \sigma_T$ and ν_T are neutrosophic sets, we explain that $f^{-1}(\mu_T)(s) = \mu_T(f(s))$ and

$$f(\mu_S)(t) = \begin{cases} \sup \mu_S(s) & \text{if } s \in f^{-1}(t) \\ 0 & \text{Otherwise} \end{cases}.$$

Definition 5.4. let $(U_1, \tau_N(F_1))$ and $(U_2, \tau_N(F_2))$ be a $Neu\mathcal{N}ts$'s. The neutrosophic nano product topological space [$Neu\mathcal{N}Pts$ for short] of $(U_1, \tau_N(F_1))$ and $(U_2, \tau_N(F_2))$ is the cartesian product $U_1 \times U_2$ of neutrosophic sets U_1 and U_2 together with the neutrosophic nano topology $\tau_N(\xi)$ of $U_1 \times U_2$ which is generated by the family $\{ P_1^{-1}(S_i), P_2^{-1}(T_j) : S_i \in \tau_N(F_1), T_j \in \tau_N(F_2)$ and P_1, P_2 are projections of $U_1 \times U_2$ onto U_1 and U_2 respectively $\}$ (i.e. the family $\{P_1^{-1}(S_i), P_2^{-1}(T_j) : S_i \in \tau_N(F_1), T_j \in \tau_N(F_2)\}$ is a subbase for neutrosophic nano topology $\tau_N(\xi)$ of $U_1 \times U_2$).

Received: March 18, 2022 Accepted: June 28, 2022

Remark 5.5. In the Definition 5.4, since $P_1^{-1}(S_i) = S_i \times 1_N$ and $P_2^{-1}(T_j) = 1_N \times T_j$ and $S_i \times 1_N \cap 1_N \times T_j = S_i \times T_j$, the family $\chi = \{S_i \times T_j : S_i \in \tau_N(F_1), T_j \in \tau_N(F_2)\}$ forms a base for $Neu\mathcal{N}Pts\ \tau_N(\xi)$ of $U_1 \times U_2$.

Lemma 5.6. Let $(U, \tau_N(F))$ be a $Neu\mathcal{N}ts$. If S_1, S_2, T_1 and T_2 be $Neut\ subs$'s of U, then $S_1 \subseteq T_1$, $S_2 \subseteq T_2 \Rightarrow S_1 \times S_2 \subseteq T_1 \times T_2$.

Proof. Let $S_1 = \langle s, \mu_{S_1}(s), \sigma_{S_1}(s), \nu_{S_1}(s)\rangle$, $S_2 = \langle s, \mu_{S_2}(s), \sigma_{S_2}(s), \nu_{S_2}(s)\rangle$, $T_1 = \langle s, \mu_{T_1}(s), \sigma_{T_1}(s), \nu_{T_1}(s)\rangle$ and $T_2 = \langle s, \mu_{T_2}(s), \sigma_{T_2}(s), \nu_{T_2}(s)\rangle$ be neutrosophic sets. Since $S_1 \subseteq T_1 \Rightarrow \mu_{S_1} \leq \mu_{T_1}, \sigma_{S_1} \leq \sigma_{T_1}, \nu_{S_1} \leq \nu_{T_1}$ and also $S_2 \subseteq T_2 \Rightarrow \mu_{S_2} \leq \mu_{T_2}, \sigma_{S_2} \leq \sigma_{T_2}, \nu_{S_2} \leq \nu_{T_2}$, we have $\min(\mu_{S_1}, \mu_{S_2}) \leq \min(\mu_{T_1}, \mu_{T_2})$, $\min(\sigma_{S_1}, \sigma_{S_2}) \leq \min(\sigma_{T_1}, \sigma_{T_2})$ and $\max(\nu_{S_1}, \nu_{S_2}) \geq \max(\nu_{T_1}, \nu_{T_2})$. Hence the result. □

Lemma 5.7. Let $(U_1, \tau_N(F_1))$ and $(U_2, \tau_N(F_2))$ be a $Neu\mathcal{N}ts$'s with respect to F_1 and F_2, where F_1 and F_2 are a $Neut\ subs$'s of U_1 and U_2 such that U_1 is neutrosophic product relative to U_2. Let S and T be $Neu\mathcal{N}\delta cs$'s of U_1 and U_2 respectively. Then $S \times T$ is the $Neu\mathcal{N}\delta cs$ in the $Neu\mathcal{N}Pts$ of $U_1 \times U_2$.

Proof. Let $S = \langle s, \mu_S(s), \sigma_S(s), \nu_S(s)\rangle$, $T = \langle t, \mu_T(t), \sigma_T(t), \nu_T(t)\rangle$.

From Lemma 5.2, $((S \times T)(s,t))^c = (S^c \times 1_N) \cup (1_N \times T^c)(s,t)$. Since $S^c \times 1_N$ and $1_N \times T^c$ are $Neu\mathcal{N}\delta os$'s in U_1 and U_2 respectively. Hence $S^c \times 1_N \cup 1_N \times T^c$ is $Neu\mathcal{N}\delta os$ of $U_1 \times U_2$. Hence $(S \times T)^c$ is a $Neu\mathcal{N}\delta os$ of $U_1 \times U_2$ and consequently $S \times T$ is the $Neu\mathcal{N}cs$ of $U_1 \times U_2$. □

Theorem 5.8. If S and T are $Neut\ set$'s of $Neu\mathcal{N}ts$'s $(U_1, \tau_N(F_1))$ and $(U_2, \tau_N(F_2))$ respectively, then

(i) $Neu\mathcal{N}\delta cl(S) \times Neu\mathcal{N}\delta cl(T) \supseteq Neu\mathcal{N}\delta cl(S \times T)$,

(ii) $Neu\mathcal{N}\delta int(S) \times Neu\mathcal{N}\delta int(T) \subseteq Neu\mathcal{N}\delta int(S \times T)$.

Proof. (i) Since $S \subseteq Neu\mathcal{N}\delta cl(S)$ and $T \subseteq Neu\mathcal{N}\delta cl(T)$, hence $S \times T \subseteq Neu\mathcal{N}\delta cl(S) \times Neu\mathcal{N}\delta\ cl(T)$. This implies that $Neu\mathcal{N}\delta cl(S \times T) \subseteq Neu\mathcal{N}\delta cl(Neu\mathcal{N}\delta cl(S) \times Neu\mathcal{N}\delta cl(T))$ and from Lemma 5.7, $Neu\mathcal{N}\delta cl(S \times T) \subseteq Neu\mathcal{N}\delta cl(S) \times Neu\mathcal{N}\delta cl(T)$.

(ii) Follows from (i) and the fact that $Neu\mathcal{N}\delta int(S^c) = (Neu\mathcal{N}\delta cl(S))^c$. □

Definition 5.9. Let $(U_1, \tau_N(F_1))$ and $(U_2, \tau_N(F_2))$ be $Neu\mathcal{N}ts$'s and $S \in \tau_N(F_1)$, $T \in \tau_N(F_2)$. We say that $(U_1, \tau_N(F_1))$ is neutrosophic nano product related to $(U_2, \tau_N(F_2))$ if for any neutrosophic sets P of U_1 and Q of U_2, whenever $S^c \not\supseteq P$ and $T^c \not\supseteq Q \Rightarrow S^c \times 1_N \cup 1_N \times T^c \supseteq P \times Q$, there exist $S_1 \in \tau_N(F_1)$, $T_1 \in \tau_N(F_2)$ such that $S_1^c \supseteq P$ or $P(T_1) \supseteq Q$ and $P(S_1) \times 1_N \cup 1_N \times P(T_1) = P(S) \times 1_N \cup 1_N \times P(T)$.

Lemma 5.10. For NS's S_i's and T_j's of $Neu\mathcal{N}ts$'s U_1 and U_2 respectively, we have

(i) $\cap\{S_i, T_j\} = \min(\cap S_i, \cap T_j)$; $\cup\{S_i, T_j\} = \max(\cup S_i, \cup T_j)$.

(ii) $\cap\{S_i, 1_N\} = (\cap S_i) \times 1_N$; $\cup\{S_i, 1_N\} = (\cup S_i) \times 1_N$.

(iii) $\cap\{1_N \times T_j\} = 1_N \times (\cap T_j)$; $\cup\{1_N \times T_j\} = 1_N \times (\cup T_j)$.

Proof. Obvious. □

Theorem 5.11. Let $(U_1, \tau_N(F_1))$ and $(U_2, \tau_N(F_2))$ be $Neu\mathcal{N}ts$'s such that U_1 is neutrosophic product related to U_2. Then for $Neut\ subs$'s S of U_1 and T of U_2, we have

(i) $Neu\mathcal{N}\delta cl(S \times T) = Neu\mathcal{N}\delta cl(S) \times Neu\mathcal{N}\delta cl(T)$,

(ii) $Neu\mathcal{N}\delta int(S \times T) = Neu\mathcal{N}\delta int(S) \times Neu\mathcal{N}\delta int(T)$.

https://doi.org/10.54216/IJNS.180419

Received: March 18, 2022 Accepted: June 28, 2022

Proof. (i) Since $Neu\mathcal{N}\delta cl(S \times T) \subseteq Neu\mathcal{N}\delta cl(S) \times Neu\mathcal{N}\delta cl(T)$ (By Theorem 5.8), it is sufficient to show that $Neu\mathcal{N}\delta cl(S \times T) \supseteq Neu\mathcal{N}\delta cl(S) \times Neu\mathcal{N}\delta cl(T)$. Let $S_i \in \tau_N(F_1)$ and $T_j \in \tau_N(F_2)$. Then $Neu\mathcal{N}\delta cl(S \times T) = \langle (s,t), \cap(\{S_i \times T_j\})^c : (\{S_i \times T_j\})^c \supseteq S \times T, \cup\{S_i \times T_j\} : \{S_i \times T_j\} \subseteq S \times T\rangle = \langle (s,t), \cap((S_i)^c \times 1_N \cup 1_N \times (T_j)^c) : (S_i)^c \times 1_N \cup 1_N \times (T_j)^c \supseteq S \times T, \cup(S_i \times 1_N \cap 1_N \times T_j) : S_i \times 1_N \cap 1_N \times T_j \subseteq S \times T\rangle = \langle (s,t), \cap((S_i)^c \times 1_N \cup 1_N \times (T_j)^c) : (S_i)^c \supseteq S$ or $(T_j)^c \supseteq T, \cup(S_i \times 1_N \cap 1_N \times T_j) : S_i \subseteq S$ and $T_j \subseteq T\rangle = \langle (s,t), \min(\cap\{(S_i)^c \times 1_N \cup 1_N \times (T_j)^c : (S_i)^c \supseteq S\}, \cap\{(S_i)^c \times 1_N \cup 1_N \times (T_j)^c : (T_j)^c \supseteq T\}), \max(\cup\{S_i \times 1_N \cap 1_N \times T_j : S_i \subseteq S\}, \cup\{S_i \times 1_N \cap 1_N \times T_j : T_j \subseteq T\})\rangle$. Since $\langle (s,t), \cap\{(S_i)^c \times 1_N \cup 1_N \times (T_j)^c : (S_i)^c \supseteq S\}, \cap\{(S_i)^c \times 1_N \cup 1_N \times (T_j)^c : (T_j)^c \supseteq T\}\rangle \supseteq \langle (s,t), \cap\{(S_i)^c \times 1_N : (S_i)^c \supseteq S\}, \cap\{1_N \times (T_j)^c : (T_j)^c \supseteq T\}\rangle = \langle (s,t), \cap\{(S_i)^c : (S_i)^c \supseteq S\} \times 1_N, 1_N \times \cap\{(T_j)^c : (T_j)^c \supseteq T\}\rangle = \langle (s,t), Neu\mathcal{N}\delta cl(S) \times 1_N, 1_N \times Neu\mathcal{N}\delta cl(T)\rangle$ and $\langle (s,t), \cup\{S_i \times 1_N \cap 1_N \times T_j : S_i \subseteq S\}, \cup\{S_i \times 1_N \cap 1_N \times T_j : T_j \subseteq T\}\rangle \subseteq \langle (s,t), \cup\{S_i \times 1_N : S_i \subseteq S\}, \cup\{1_N \times T_j : T_j \subseteq T\}\rangle = \langle (s,t), \cup\{S_i : S_i \subseteq S\} \times 1_N, 1_N \times \cup\{T_j : T_j \subseteq T\}\rangle = \langle (s,t), Neu\mathcal{N}\delta int(S) \times 1_N, 1_N \times Neu\mathcal{N}\delta int(T)\rangle$, we have $Neu\mathcal{N}\delta cl(S \times T) \supseteq \langle (s,t), \min(Neu\mathcal{N}\delta cl(S) \times 1_N, 1_N \times Neu\mathcal{N}\delta cl(T)), \max(Neu\mathcal{N}\delta int(S) \times 1_N, 1_N \times Neu\mathcal{N}\delta int(T))\rangle = \langle (s,t), \min(Neu\mathcal{N}\delta cl(S), Neu\mathcal{N}\delta cl(T)), \max(Neu\mathcal{N}\delta int(S), Neu\mathcal{N}\ \delta int(T))\rangle = Neu\mathcal{N}\delta cl(S) \times Neu\mathcal{N}\delta cl(T)$.

(ii) follows from (i). □

Theorem 5.12. Let $(U, \tau_N(F))$ be a $Neu\mathcal{N}ts$. Then for a $Neut\ subs$'s S_o and T_o of U we have,

(i) $Neu\mathcal{N}\delta cl(S_o) \supseteq S_o \cup Neu\mathcal{N}\delta cl(Neu\mathcal{N}\delta int(S_o))$,

(ii) $Neu\mathcal{N}\delta int(S_o) \subseteq S_o \cap Neu\mathcal{N}\delta int(Neu\mathcal{N}\delta cl(S_o))$,

(iii) $Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S_o)) \subseteq Neu\mathcal{N}int(Neu\mathcal{N}cl(S_o))$,

(iv) $Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S_o)) \supseteq Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(Neu\mathcal{N}\delta int(S_o)))$.

Proof. By Theorem 4.4 (i),

$$S_o \subseteq Neu\mathcal{N}\delta cl(S_o). \tag{1}$$

Again using Theorem 4.3 (i), $Neu\mathcal{N}\delta int(S_o) \subseteq S_o$. Then

$$Neu\mathcal{N}\delta cl(Neu\mathcal{N}\delta int(S_o)) \subseteq Neu\mathcal{N}\delta cl(S_o). \tag{2}$$

By (1) & (2) we have, $S_o \cup Neu\mathcal{N}\delta cl(Neu\mathcal{N}\delta int(S_o)) \subseteq Neu\mathcal{N}\delta cl(S_o)$. This proves (i).

By Theorem 4.3 (i),

$$Neu\mathcal{N}\delta int(S_o) \subseteq S_o. \tag{3}$$

Again using Theorem 4.4 (i), $S_o \subseteq Neu\mathcal{N}\delta cl(S_o)$. Then

$$Neu\mathcal{N}\delta int(S_o) \subseteq Neu\mathcal{N}\delta int(Neu\mathcal{N}\delta cl(S_o)). \tag{4}$$

From (3)& (4), we have $Neu\mathcal{N}\delta int(S_o) \subseteq S_o \cap Neu\mathcal{N}\delta int(Neu\mathcal{N}\delta cl(S_o))$. This proves(ii).

By Proposition 4.5, $Neu\mathcal{N}\delta cl(S_o) \subseteq Neu\mathcal{N}cl(S_o)$. We get $Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S_o)) \subseteq Neu\mathcal{N}\ int(Neu\mathcal{N}cl(S_o))$. Hence (iii).

By (i), $Neu\mathcal{N}\delta cl(S_o) \supseteq S_o \cup Neu\mathcal{N}\delta cl(Neu\mathcal{N}\delta int(S_o))$. We have $Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S_o) \supseteq Neu\mathcal{N}int(S_o \cup Neu\mathcal{N}\delta cl(N\ \mathcal{N}\delta int(S_o)))$. Since $Neu\mathcal{N}int(S_o \cup T_o) \supseteq Neu\mathcal{N}int(S_o) \cup Neu\mathcal{N}int(T_o)$, $Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(S_o) \supseteq Neu\mathcal{N}int(S_o)) \cup Neu\mathcal{N}int(Neu\mathcal{N}\delta cl(Neu\mathcal{N}\delta int(S_o))) \supseteq Neu\mathcal{N}int(Neu\ \mathcal{N}\delta cl(Neu\mathcal{N}\delta int(S_o)))$. Hence (iv). □

The operators $Neu\mathcal{N}\delta\mathcal{S}int(.)$ (resp. $Neu\mathcal{N}\delta\mathcal{P}int(.)$, $Neu\mathcal{N}\delta\gamma int(.)$, $Neu\mathcal{N}\delta\beta int(.)$) and their respective closure is satisfy Lemma 5.7 and Theorems 5.8, 5.11 and 5.12.

https://doi.org/10.54216/IJNS.180419
Received: March 18, 2022 Accepted: June 28, 2022

6 Conclusions

We introduced the concept of $NeuN\delta o$, $NeuN\delta So$, $NeuN\delta \mathcal{P}o$, $NeuN\delta\gamma o$ and $NeuN\delta\beta o$ sets and some of their properties were discussed in this paper. This can be extended to several functions using neutrosophic nano open sets such as continuous, irresolute, open and closed mappings.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

References

[1] Al-Hamido, R.K. On neutrosophic crisp supra bi-topological spaces. International Journal of Neutrosophic Sciences 2020, Vol 4 (1), pp 36-46.

[2] Atanassov, K.; Stoeva, S. Intuitionistic fuzzy sets. In Proceedings of the Polish Symposium on Interval and Fuzzy Mathematics 1983, Poznan, Poland, pp 23-26.

[3] Atanassov, K. Intuitionistic fuzzy sets. Fuzzy Sets and Systems 1986, Vol 20, pp 87-96.

[4] Chang, C.L. Fuzzy topological spaces. J. Math. Anal. Appl. 1968, Vol 24, pp 182-190.

[5] Coker, D. An introduction to intuitionistic fuzzy topological spaces. Fuzzy sets and systems 1997, Vol 88, pp 81-89.

[6] Das, S.K.; Edalatpanah, S.A. A new ranking function of triangular neutrosophic number and its application in inter programming. International Journal of Neutrosophic Sciences 2020, Vol 4 (2), pp 82-92.

[7] Lellis Thivagar, M.; Richard, C. On nano forms of weekly open sets. International journal of mathematics and statistics invention 2013, Vol 1 (1), pp 31-37.

[8] Lellis Thivagar, M.; Jafari, S.; Sutha Devi, V.; Antonysamy, V. A novel approach to nano topology via neutrosophic sets. Neutrosophic Sets and Systems 2018, Vol 20, pp 86-94.

[9] Mullai, M.; Sangeetha, K.; Surya, R.; Madhan Kumar, G.; Jayabalan, R.; Broumi, S. A single valued neutrosophic inventory model with neutrosophic random variable. International Journal of Neutrosophic Sciences 2020, Vol 1 (2), pp 52-63.

[10] Pankajam, V.; Kavitha, K. δ-open sets and δ-nano continuity in δ-nano topological spaces. International Journal of Innovative Science and Research Technology 2017, Vol 2 (12), pp 110-118.

[11] Parimala, M.; Jeevitha, R.; Jafari, S.; Samarandache, F.; Karthika, M. Neutrosophic nano $\alpha\psi$-closed sets in neutrosophic nano topological spaces. Journal of advance Research in Dynamical and Control Systems 2018, Vol 10, pp 523-531.

[12] Parimala, M.; Karthika, K.; Smarandache, F.; Broumi, S. On $\alpha\omega$-closed sets and its connectedness in terms of neutrosophic topological spaces. International Journal of Neutrosophic Sciences 2020, Vol 2, pp 82-88.

[13] Pawlak, Z. Rough sets. International Journal of Computer and Information Sciences 1982, Vol 11, pp 341-356.

[14] S. Saha, "Fuzzy δ-continuous mappings", Journal of Mathematical Analysis and Applications, Vol 126, pp 130-142, 1987.

[15] Salama, A.A.; Alblowi, S.A. Generalized neutrosophic set and generalized neutrosophic topological spaces. Journal of Computer Sci. Engineering 2012, Vol 2 (7), pp 129-132.

[16] Salama, A.A.; Alblowi, S.A. Neutrosophic Set and Neutrosophic Topological Spaces. IOSR Journal of Mathematics 2012, Vol 3 (4), pp 31-35.

Received: March 18, 2022 Accepted: June 28, 2022

[17] Smarandache, F. A unifying field in logics: neutrosophic logic. neutrosophy, neutrosophic set, neutrosophic probability, American Research Press, Rehoboth, NM, 1999.

[18] Smarandache, F. Neutrosophy and neutrosophic logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics, University of New Mexico, Gallup, NM 87301, USA 2002.

[19] Vadivel, A.; Seenivasan, M.; John Sundar, C. An Introduction to δ-open sets in a Neutrosophic Topological Spaces. Journal of Physics: Conference Series 2021, 1724, 012011.

[20] Vadivel, A.; John Sundar, C. Neutrosophic δ-Open Maps and Neutrosophic δ-Closed Maps. International Journal of Neutrosophic Science (IJNS) 2021, Vol 13 (2), pp 66-74.

[21] Vadivel, A.; Thangaraja, P.; John Sundar, C. Neutrosophic e-continuous maps and neutrosophic e-irresolute maps. Turkish Journal of Computer and Mathematics Education 2021, Vol 12 (1S), pp 369-375.

[22] Vadivel, A.; Thangaraja, P.; John Sundar, C. Neutrosophic e-Open Maps, Neutrosophic e-Closed Maps and Neutrosophic e-Homeomorphisms in Neutrosophic Topological Spaces. AIP Conference Proceedings 2021, 2364, 020016.

[23] Zadeh, L.A. Fuzzy sets. Information and control 1965, Vol 8, pp 338-353.

https://doi.org/10.54216/IJNS.180419

Received: March 18, 2022 Accepted: June 28, 2022