Some Perfectly Continuous Functions via Fuzzy Neutrosophic Topological Spaces

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Abstract

Our purpose in this paper, is to introduce some of new continuous functions, called fuzzy neutrosophic perfectly continuous, fuzzy neutrosophic almost perfectly continuous and fuzzy neutrosophic strongly continuous in the fuzzy neutrosophic topological spaces. We give some theorems, propositions and some necessary examples related to presented definitions. Then, we discuss the relations among the new defined functions.

Keywords: Fuzzy Neutrosophic Sets; Fuzzy Neutrosophic Topology; Fuzzy Neutrosophic Perfectly Continuous; Fuzzy Neutrosophic Almost Perfectly Continuous; Fuzzy Neutrosophic Strongly Continuous.

1.Introduction

L. Zadeh [1], was showing the concept of fuzzy set "FS" where the membership of any object restricted by the a unitary standard interval [0,1]. K. Atanassov [2-4] generalized the "FS" and introduced the concept of intuitionstic fuzzy sets "IFS" whose the elements had the the non-membership values in addition to the membership ones with the same unitary interval [0,1]. Then, Smarsndache [5,6] introduced another new concept named, neutrosophic theory "NSs" where another components have been added, these components belonging to the indeterminacy membership function, regarded as new logic differs from both fuzzy logic and intuitionistic fuzzy logic, then in 2012, A. A. Salama et.al. [7] studied the term of neutrosophic topology "NT" in which the nonstandard interval]0-,1+[made many important consequences and theorems. Y.Veereswari [8] gave an introduction of fuzzy neutrosophic topological spaces "FNTSs".

In this manuscript, the concept of some new continuous functions called, perfectly continuous, almost perfectly continuous and strongly continuous via fuzzy neutrosophic topological spaces were introduced and studied as

Doi : https://doi.org/10.54216/IJNS.180416
Received: March 15, 2022 Accepted: June 28, 2022

generalization of F. Mohammed [9-12], and T.M. Nour et al. [13-24]. Finally, discussed some relationships among them.

2. Preliminaries:

This section has been devoted to recalling fundamental notions, definitions, theorems, and results that will be necessary for originating the remaining sections.

Definition 2.1 [7]: Let $\mathbf{V}_s$ be a non-empty fixed set. The fuzzy neutrosophic set (FNS) μ_N is an object having the form $\mu_N=\{<v_s, \lambda_{\mu N}(v_s), ɣ_{\mu N}(v_s), V_{\mu N}(\mathbf{V}_s)>: v_s \in \mathbf{V}_s\}$ where the functions $\lambda_{\mu N}(v_s), ɣ_{\mu N}(v_s), V_{\mu N}(v_s): \mathbf{V}_s \to [0,1]$ denote the degree of membership function (namely $\lambda_{\mu N}(v_s)$), the degree of indeterminacy function (namely $ɣ_{\mu N}(v_s)$) and the degree of non-membership function (namely $V_{\mu N}(v_s)$) respectively of each element $v_s \in \mathbf{V}_s$ to the set μ_N and $0 \leq \lambda_{\mu N}(v_s)+ ɣ_{\mu N}(v_s) +V_{\mu N}(v_s) \leq 3$, for each $v_s \in \mathbf{V}_s$.

Remark 2.2: FNS $\mu_N = \{< v_s, \lambda_{\mu N}(v_s), \sigma_{\mu N}(v_s), V_{\mu N}(v_s) >: v_s \in \mathbf{V}_s\}$ can be identified to an ordered triple $< v_s, \lambda_{\mu N}, \sigma_{\mu N}, V_{\mu N} >$ in [0,1] on $\mathbf{V}_s$.

Lemma 2.3 [8]: Let $\mathbf{V}_s$ be a non-empty set and the FNSs μ_N and $ɣ_N$ be in the form:

$\mu_N = \{< v_s, \lambda_{\mu N}, \sigma_{\mu N}, V_{\mu N}> \}$ and $ɣ_N=\{< v_s, \lambda_{ɣN}, \sigma_{ɣN}, V_{ɣN}>\}$ on $\mathbf{V}_s$. Then,

i. $\mu_N \subseteq ɣ_N$ iff $\lambda_{\mu N} \leq \lambda_{ɣN}$, $\sigma_{\mu N} \leq \sigma_{ɣN}$ and $V_{\mu N} \geq V_{ɣN}$,
ii. $\mu_N = ɣ_N$ iff $\mu_N \subseteq ɣ_N$ and $ɣ_N \subseteq \mu_N$,
iii. $(\mu_N)^c =\{< v_s, V_{\mu N}, 1-\sigma_{\mu N}, \lambda_{\mu N} >\}$,
iv. $\mu_N \cup ɣ_N =\{< v_s, Max(\lambda_{\mu N}, \lambda_{ɣN}), Max(\sigma_{\mu N}, \sigma_{ɣN}), Min(V_{\mu N}, V_{ɣN}) >\}$,
v. $\mu_N \cap ɣ_N =\{< v_s, Min(\lambda_{\mu N}, \lambda_{ɣN}), Min(\sigma_{\mu N}, \sigma_{ɣN}), Max(V_{\mu N}, V_{ɣN}) >\}$.

Definition 2.4 [8]:A fuzzy neutrosophic topology (briefly, FNT) on a non-empty set $\mathbf{V}_s$ is a family $\mathbf{T}_N$ of fuzzy neutrosophic subset in $\mathbf{V}_s$ satisfying the following axioms.

i. $0_N, 1_N \in \mathbf{T}_N$,
ii. $\mu_{N1} \cap \mu_{N2} \in \mathbf{T}_N \ \forall \ \mu_{N1}, \mu_{N2} \in \mathbf{T}_N$,
iii. $\cup \mu_{Nj} \in \mathbf{T}_N, \forall \{ \mu_{Nj} : j \in J\} \subseteq \mathbf{T}_N$.

In this case the pair $(\mathbf{V}_s, \mathbf{T}_N)$ is called fuzzy neutrosophic topological space (briefly, FNTS). The elements of $\mathbf{T}_N$ are called fuzzy neutrosophic-open sets (briefly, FN-OS). The complement of FN-OS in the FNTS $(\mathbf{V}_s, \mathbf{T}_N)$ is called fuzzy neutrosophic- closed set (briefly, FN-CS).

Definition 2.5 [8]: Let $(\mathbf{V}_s, \mathbf{T}_N)$ is FNTS and $\mu_N =< v_s, \lambda_{\mu N}, \sigma_{\mu N}, V_{\mu N} >$ is FNS in $\mathbf{V}_s$. Then the fuzzy neutrosophic-closure (for shortly, FN-Cl) and the fuzzy neutrosophic -interior (briefly, FN-In) of μ_N are defined by:

$$\text{FN-Cl}(\mu_N) = \cap \{ ɣ_N : ɣ_N \text{ is FN-CS set in } \mathbf{V}_s \text{ and } \mu_N \subseteq ɣ_N \},$$

Doi : https://doi.org/10.54216/IJNS.180416
Received: March 15, 2022 Accepted: June 28, 2022

$$\text{FN-In}(\mu_N) = \cup \{ \gamma_N : \gamma_N \text{ is FN-OS set in } \mathbf{V}_s \text{ and } \gamma_N \subseteq \mu_N \}.$$

Now, the FN-Cl (μ_N) is FN-CS set and FN-In(μ_N) is FN-OS. set in $\mathbf{V}_s$.

Further,

i. μ_N is FN-closd. set in X iff FN-Cl(μ_N) = μ_N,

ii. μ_N is FN-open set in X iff FN-In(μ_N) = μ_N..

Definition 2.6 [9]:The FNS μ_N in FNTS ($\mathbf{V}_s$, $\mathbf{T}_N$) is called:

i. Fuzzy neutrosophic regular-open set (briefly, FNR-OS) iff μ_N =FN-In(FN-Cl(μ_N)),
ii. Fuzzy neutrosophic regular-closed set (briefly, FNR-CS.) iff μ_N = FN-Cl(FN-In(μ_N)).

Note: If $\mu_N =< \mathbf{v}_s, \lambda_{\mu N}, \sigma_{\mu N}, V_{\mu N} >$ so, $\mu_N{}^c = 1_N - \mu_N = \{< \mathbf{v}_s, V_{\mu N}, 1-\sigma_{\mu N}, \lambda_{\mu N} >: \mathbf{v}_s \in \mathbf{V}_s\}$,

3. Fuzzy Neutrosophic Perfectly Continuous in Fuzzy Neutrosophic Topological Spaces

In this part, the original work has been presented in this section and contains new properties of neutrosophic perfectly continuous function in fuzzy neutrosophic topological spaces. As well as, some of its relationships with the other kinds of continuous functions have been proposed in the same space.

Definition 3.1: A function φ: ($\mathbf{V}_s$, $\mathbf{T}_{\mathbf{V}s}$) → (Y, $\mathbf{T}_y$) is said to be:

1. Fuzzy neutrosophic perfectly continuous (briefly, FN-P-con.) if $\varphi^{-1}(\lambda)$ is both FNOS and FNCS in ($\mathbf{V}_s$, $\mathbf{T}_{\mathbf{V}s}$) for each FNOS λ in (Y, $\mathbf{T}_y$).
2. Fuzzy neutrosophic strongly continuous (briefly, FN-S-con.) if φ (FN-Cl (λ)) $\subset$ φ (λ) for each subset λ of ($\mathbf{V}_s$, $\mathbf{T}_{\mathbf{V}s}$).
3. Fuzzy neutrosophic almost perfectly continuous (briefly, FNA-P-con.) if φ^{-1}(V) is closed and open (briefly, clopen) for every FNR-OS (V) in (Y, $\mathbf{T}_y$).

Example 3.2: Let $\mathbf{V}_s$ = Y= { a, b }, and φ : ($\mathbf{V}_s$, $\mathbf{T}_{\mathbf{V}s}$) → (Y, $\mathbf{T}_y$) be a function. Define FNSs λ_s in a FNTS ($\mathbf{V}_s$, $\mathbf{T}_{\mathbf{V}s}$) and β_s in a FNTS (Y, $\mathbf{T}_y$) as follows:

$$\lambda_s = < \mathbf{v}_s, (0.3, 0.5, 0.3), (0.4, 0.5, 0.4) >$$

in FNTS ($\mathbf{V}_s$, $\mathbf{T}_{\mathbf{V}s}$) , with $\mathbf{T}_{\mathbf{V}s} = \{ 0_s, 1_s, \lambda_s \}$.

And FNS, β_s = < y, (0.4, 0.5, 0.4), (0.3, 0.5, 0.3) > in FNTS (Y, $\mathbf{T}_y$) with $\mathbf{T}_y = \{ 0_s, 1_s, \beta_s \}$.

Doi : https://doi.org/10.54216/IJNS.180416
Received: March 15, 2022 Accepted: June 28, 2022

Now, define φ: ($\mathbf{V}_s$, $\mathbf{T}_{\mathbf{V}s}$) → (Y, $\mathbf{T}_y$) as follows:

$$\varphi (a) = b \text{ and } \varphi (b) = a, \text{ so}$$

$$\varphi^{-1} (a) = b \text{ and } \varphi^{-1}(b) = a.$$

Then,

$\varphi^{-1}(\beta_s)$ = < $_{\mathbf{V}s}$, (0.3, 0.5, 0.3), (0.4, 0.5, 0.4) > = $\lambda_s = \lambda_s^c$, so

$\varphi^{-1}(\beta_s)$ is FN-clopen set in ($\mathbf{V}_s$, $\mathbf{T}_{\mathbf{V}s}$) for some FN-open set β_s in (Y, $\mathbf{T}_y$).

This implies φ: ($\mathbf{V}_s$, $\mathbf{T}_{\mathbf{V}s}$) → (Y, $\mathbf{T}_y$) is FNP-con. function.

Also, φ is FNS-con. function because, $\varphi(\lambda_s)$ = { < y, (0.4, 0.5, 0.4) (0.3, 0.5, 0.3) > } = β_s , and since,

FN-Cl (λ_s) = λ_s ,so φ(FN-Cl (λ_s)) ⊂ $\varphi(\lambda_s) = \beta_s$.

Again, β_s is a FNR-OS in (Y, $\mathbf{T}_y$) because, β_s = FN-In (FN-Cl (β_s))

= FN-In (β_s^c) = β_s.

and, $\varphi^{-1}(\beta_s)$ is clopen in ($\mathbf{V}$ s, $\mathbf{T}_{\mathbf{V}s}$) so, φ is FNAP- con. function.

Definition 3.3: A function φ: ($\mathbf{V}_s$, $\mathbf{T}_{\mathbf{V}s}$) → (Y, $\mathbf{T}_y$) is said to be fuzzy neutrosophic almost clopen (briefly, FNA-clopen) if for each $_{\mathbf{V}s}$ ∈ $\mathbf{V}$ s and each FNOS (V) in Y containing $\varphi(_{\mathbf{V}s})$, there exists a FN-clopen set U in $\mathbf{V}$ s containing $_{\mathbf{V}s}$ such that φ(U) ⊆ FN-In (FN-Cl (V)).

Note: The function φ: ($\mathbf{V}$ s, $\mathbf{T}_{\mathbf{V}s}$) → (Y, $\mathbf{T}_y$) is FNA-P-con. iff φ is FNA-clopen function.

Theorem 3.4: If the function φ: ($\mathbf{V}$ s, $\mathbf{T}_{\mathbf{V}s}$) → (Y, $\mathbf{T}_y$) is FN-P-con. and A ⊆ $\mathbf{V}$ s, then, $\varphi|_A$: A → Y is FN-P-con.F function.

Proof: Let V be any fuzzy neutrosophic open subset of (Y, $\mathbf{T}_y$). Then, φ^{-1} (V) is clopen in ($\mathbf{V}$ s, $\mathbf{T}_{\mathbf{V}s}$). Since, φ is FN-P-con. function consequently, φ^{-1} (V) ∩ A = ($\varphi_{|A}$) $^{-1}$ (V) is clopen in A. Hence, $\varphi_{|A}$ is FN-P-con. function.

Definition 3.5: A set G is said to be fuzzy neutrosophic δ-open set (briefly, FNδ-OS) if for each $_{\mathbf{V}s}$ ∈ G, there exists a FNR-OS (H) such that $_{\mathbf{V}s}$ ∈ H ⊂ G, or G is an arbitrary union of fuzzy neutrosophic regular open sets. The complement of a FNδ-OS is a fuzzy neutrosophic δ-closed set (briefly, FNδ-CS).

Note: If we take the Example 3.2 so, the set λ_s is FNδ-OS because λ_s= FN-In (FN-Cl (λ)).

Definition 3.6: A function φ: ($\mathbf{V}$ s, $\mathbf{T}_{\mathbf{V}s}$) → (Y, $\mathbf{T}_y$) is defined to be:

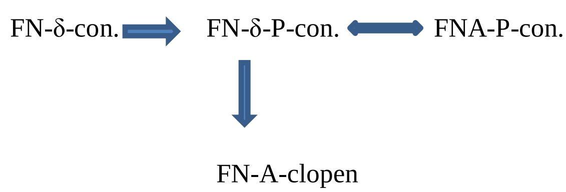
Doi : https://doi.org/10.54216/IJNS.180416
Received: March 15, 2022 Accepted: June 28, 2022

1. Fuzzy neutrosophic-δ- perfectly continuous (briefly, FN-δ-P-con.) if $\varphi^{-1}(V)$ is a FN-clopen set in $(\mathbf{V}\, s, \mathbf{T}_{V\, s})$ for each FNδ-OS say V in (Y, $\mathbf{T}_y$).
2. Fuzzy neutrosophic-δ-continuous (briefly, FN-δ-con.) if for each $_{V\,s} \in \mathbf{V}$ s and for each FNR-OS say V containing $\varphi\,(_{V\,s})$ there is a FNR-OS (U) containing $_{V\,s}$ such that φ (U) ⊂ V.
3. Fuzzy neutrosophic almost -continuous (briefly, FNA-con.) if for each $_{V\,s} \in \mathbf{V}$ s and for each FNR-OS (V) containing $\varphi\,(_{V\,s})$, there exists a FNOS (U) containing $_{V\,s}$ such that φ (U) ⊂ V.

Example 3.7: 1. In Example 3.2, then the function φ: $(\mathbf{V}\, s, \mathbf{T}_{V\, s}) \rightarrow (Y, \mathbf{T}_y)$ is FN-δ-P-con. because β_s is FNδ-OS in (Y, $\mathbf{T}_y$) and $\varphi^{-1}(\beta_s) = \lambda_s$ is FN-clopen in $(\mathbf{V}\, s, \mathbf{T}_{V\, s})$.

2. Again in Example 3.2 we have the function φ: $(\mathbf{V}\, s, \mathbf{T}_{V\, s}) \rightarrow (Y, \mathbf{T}_y)$ is FN-δ-con. function since β_s is FNR-OS in (Y, $\mathbf{T}_y$) because β_s = FN-In (FN-Cl (β_s)). Also, $\varphi^{-1}(\beta_s) = \lambda_s$ is FNR-OS in $(\mathbf{V}\, s, \mathbf{T}_{V\, s})$ because λ_s = FN-In (FN-Cl (λ_s))

3. As in (1) and (2) we have, φ is FNA-con. function because λ_s is a FNOS in $(\mathbf{V}\, s, \mathbf{T}_{V\, s})$ and β_s is a FNR-OS in (Y, $\mathbf{T}_y$), whenever $\varphi(\lambda_s) = \beta_s$.



(Figure 1)

Figure 1: Illustration of the relations between the studied continuous functions.

Corollary 3.8: For the function φ: $(\mathbf{V}\, s, \mathbf{T}_{V\, s}) \rightarrow (Y, \mathbf{T}_y)$.The following statements are equivalent:

i. φ is fuzzy neutrosophic-δ-continuous,
ii. For each $_{V\,s} \in \mathbf{V}$ s and each FNR-OS (V) containing $\varphi\,(_{V\,s})$, there exists a FNR-OS (U) containing $_{V\,s}$ such that φ (U) ⊂ V,
iii. For every FNR-CS F of Y, $\varphi^{-1}(F)$ is FNδ-CS in **V** s.
iv. For every FNR-OS (V) of Y, $\varphi^{-1}(V)$ is FNδ-OS in **V** s.
v. For every FNδ-CS F of Y, $\varphi^{-1}(F)$ is FNδ-CS in **V** s.
vi. For every FNδ-OS (V) of Y, $\varphi^{-1}(V)$ is FNδ-OS in **V** s.

The prove of above is direct.

Definition 3.9: The intersection of any two FNR-OSs is also FNR-OS, then the collection of all $\mathbf{T}_{V\, s}$ FNR-OSs forms a base for a smaller topology $\mathbf{T}_s$ on **V** s is called the fuzzy neutrosophic semi-regularization of $\mathbf{T}_{V\, s}$. The space $(\mathbf{V}\, s, \mathbf{T}_{V\, s})$ is called semi-regular if $\mathbf{T}_s = \mathbf{T}_{V\, s}$.

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Note: Any fuzzy neutrosophic regular space is fuzzy neutrosophic semi-regular space, where the space is said to be fuzzy neutrosophic reqular iff each fuzzy neutrosophic open set λ of **V** s is a union of fuzzy neutrosophic open sets λ_α's of **V** s such that FN-Cl(λ_α) ≤ λ, for each α. but the converse is not true in general.

Example 3.10: If we take Example 3.2, we will see that (**V** s, $\mathbf{T}_{\mathbf{V}\ s}$) and (Y, $\mathbf{T}_{y}$) are fuzzy neutrosophic semi-regularization spaces because every FNOS in them is FNR-OS, the collection of all $\mathbf{T}_{\mathbf{V}\ s}$ forms a base for (**V** s, $\mathbf{T}_{\mathbf{V}\ s}$) and the collection of $\mathbf{T}_{y}$ forms a base for (Y, $\mathbf{T}_{y}$).

Theorem 3.11: A function φ: (**V** s, $\mathbf{T}_{\mathbf{V}\ s}$) → (Y, $\mathbf{T}_{y}$) is FN-δ-P-con. if and only if φ: (**V** s, $\mathbf{T}_{\mathbf{V}\ s}$) →(Y, $\mathbf{T}_{y(s)}$) is FN-P-con.function.

Proof: Let φ: (**V** s, $\mathbf{T}_{\mathbf{V}\ s}$) → (Y, $\mathbf{T}_{y}$) be FN-δ-P-con. function, so φ^{-1}(V) is a FN-clopen set in (**V** s, $\mathbf{T}_{\mathbf{V}\ s}$) for each FNδ-OS (V) in (Y, $\mathbf{T}_{y}$). But, V is FNδ-OS so it is FNR-OS, and since every FNR-OS is FNOS therefore, φ^{-1} (V) is FN-clopen set in (**V** s, $\mathbf{T}_{\mathbf{V}\ s}$) for each FNOS (V) in (Y, $\mathbf{T}_{y}$) which implies φ is FN-P-con. function.

Conversely, Let φ: (**V** s, $\mathbf{T}_{\mathbf{V}\ s}$) → (Y, $\mathbf{T}_{y(s)}$) be FN-P-con. function, so φ^{-1}(V) is FN-clopen in (**V** s, $\mathbf{T}_{\mathbf{V}\ s}$) for each FNOS (V) in (Y, $\mathbf{T}_{y(s)}$), we have (Y, $\mathbf{T}_{y(s)}$) is fuzzy neutrosophic regular space, so it is the collection of all FNR-OSs, therefore, V is FNR-OS, implies V is FNδ-OS, so φ is FN-δ-P-con. because V is FNδ-OS in (Y, $\mathbf{T}_{y}$) and φ^{-1} (V) is FN-clopen set in (**V** s, $\mathbf{T}_{\mathbf{V}\ s}$).

Corollary 3.12: Let (Y, $\mathbf{T}_{y}$) be fuzzy neutrosophic semi-regularization space, the function φ: ($\mathbf{V}_s$, $\mathbf{T}_{\mathbf{V}\ s}$) → (Y, $\mathbf{T}_{y}$) is FN-δ-P-con. if and only if φ: (**V** s, $\mathbf{T}_{\mathbf{V}\ s}$) → (Y, $\mathbf{T}_{y}$) is FN-P- con. Therefore, in the case of fuzzy neutrosophic semi-regularization co-domain, no distinction can be made between FN-δ-P-con. and FN-P- con. functions.

The prove of above is direct

Lemma 3.13: The function φ: (**V** s, $\mathbf{T}_{\mathbf{V}\ s}$) → (Y, $\mathbf{T}_{y}$) is FN-δ-con. if and only if φ : (**V** s, $\mathbf{T}_{\mathbf{V}\ s}$) → (Y, $\mathbf{T}_{y(s)}$) is FN-con. function.

Proof: Let φ: (**V** s, $\mathbf{T}_{\mathbf{V}\ s}$) → (Y, $\mathbf{T}_{y}$) be FN-δ-con. function, then there is a FNR-OS (V) in (Y, $\mathbf{T}_{y}$) and there is a FNR-OS (U) in (**V** s, $\mathbf{T}_{\mathbf{V}\ s}$) such that φ (U) ⊂ V, φ^{-1}(y) = $_{\mathbf{V}\ s}$ for each $_{\mathbf{V}\ s}$ ∈ U, y ∈ V, so φ is FN-con. function because of every FNR-OS is FNOS, so V is FNOS in (Y, $\mathbf{T}_{y}$) and φ^{-1}(V) = U is FNOS in

($\mathbf{V}_s$, $\mathbf{T}_{\mathbf{V}\ s}$).

Now, if φ: (**V** s, $\mathbf{T}_{\mathbf{V}\ s}$) → (Y, $\mathbf{T}_{y(s)}$) is FN-con. function and (**V** s, $\mathbf{T}_{\mathbf{V}\ s}$) also (Y, $\mathbf{T}_{y}$) are fuzzy neutrosophic semi-regularization spaces, there is a collection of FNR-OSs forms regarded as a base for (**V** s, $\mathbf{T}_{x\ \mathbf{V}\ s}$) and a collection of FNR-OSs forms regarded as a base for (Y, $\mathbf{T}_{y}$), so there is a FNδ-OS U in (**V** s, $\mathbf{T}_{\mathbf{V}\ s}$) and there is a FNδ-OS V in (Y, $\mathbf{T}_{y}$) such that φ^{-1}(V) = U, therefore φ: (**V** s, $\mathbf{T}_{\mathbf{V}\ s}$) → (Y, $\mathbf{T}_{y}$) is FN-δ-con. function.

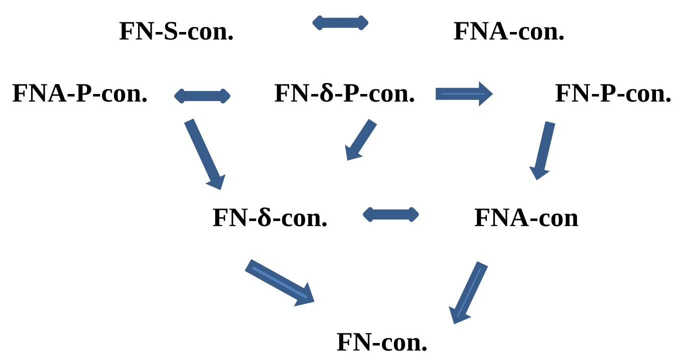
Doi : https://doi.org/10.54216/IJNS.180416
Received: March 15, 2022 Accepted: June 28, 2022

Theorem 3.14: If φ: ($\mathbf{V}$ s, $\mathbf{T}_{\mathbf{V}\,s}$) → (Y, $\mathbf{T}_y$) is FN-δ-P-con. functions and ξ: (Y, $\mathbf{T}_y$) → (Z, $\mathbf{T}_z$) is FN-δ-con. function then, ($\varphi \circ \xi$) is FN-δ-P-con. function.

Proof: Let φ: ($\mathbf{V}$ s, $\mathbf{T}_{\mathbf{V}\,s}$) → (Y, $\mathbf{T}_y$) be FN-δ-P-con. function and ξ: (Y, $\mathbf{T}_Y$) → (Z, $\mathbf{T}_Z$) be FN-δ-con. function, then, to prove $\xi \circ \varphi$ is FN-δ-P-con. function, let W be a FNδ-OS in Z. Since ξ is FN-δ-con., so

ξ^{-1} (W) is FNδ-OS in Y. In view of FN-δ-P-con. of φ, $\varphi^{-1}(\xi^{-1}(W))$ is FN-clopen in $\mathbf{V}$ s.

Since $(\xi \circ \varphi)^{-1}(W) = \varphi^{-1}(\xi^{-1}(W))$, $\xi \circ \varphi$ is FN-δ-P-con.



(Figure 2)

Figure 2: Illustrates the interrelations among FN-δ-P-con. and other types of continuity.

Definition 3.15: A function φ: ($\mathbf{V}$ s, $\mathbf{T}_{\mathbf{V}\,s}$) → (Y, $\mathbf{T}_y$) is called FN-contra continuous (briefly, FN-c.con.) if the invers image of every FNOS in (Y, $\mathbf{T}_y$) is a FNCS in ($\mathbf{V}$ s, $\mathbf{T}_{\mathbf{V}\,s}$).

Equivalently if the invers image of every FNCS in (Y, $\mathbf{T}_{y,}$) is a FNOS in ($\mathbf{V}$ s, $\mathbf{T}_{\mathbf{V}\,s}$).

Example 3.16: If we take the Example 3.2, we get the function φ: ($\mathbf{V}$ s, $\mathbf{T}_{\mathbf{V}\,s}$) → (Y, $\mathbf{T}_y$) is FN-c.con. function, since β_s is FNOS in (Y, $\mathbf{T}_y$) and $\varphi^{-1}(\beta_s) = \lambda_s$ is FNCS in ($\mathbf{V}$ s, $\mathbf{T}_{\mathbf{V}\,s}$).

Theorem 3.17: A function φ: ($\mathbf{V}$ s, $\mathbf{T}_{\mathbf{V}\,s}$) → (Y, $\mathbf{T}_y$) is FN-P-con. if it is FN-con. and FN-c.con. function.

Proof: Let φ: ($\mathbf{V}$ s, $\mathbf{T}_{\mathbf{V}\,s}$) → (Y, $\mathbf{T}_y$) be FN-con. and FN-c.con. function, then take an FNOS β_s in (Y, $\mathbf{T}_y$), so, $\varphi^{-1}(\beta_s)$ is FNOS in ($\mathbf{V}$ s, $\mathbf{T}_{\mathbf{V}\,s}$) because of φ is FN-con. function, and $\varphi^{-1}(\beta_s)$ is FNCS in ($\mathbf{V}$ s, $\mathbf{T}_{\mathbf{V}\,s}$). Since, φ : (X, $\mathbf{T}_x$) → (Y, $\mathbf{T}_y$) is FN-c.con. function, so φ is FN-P-con. function.

Hence, β_s is FNOS in (Y, $\mathbf{T}_y$) and $\varphi^{-1}(\beta_s)$ is FN-clopen set in ($\mathbf{V}$ s, $\mathbf{T}_{\mathbf{V}\,s}$).

Corollary 3.18: A function φ: ($\mathbf{V}$ s, $\mathbf{T}_{\mathbf{V}\,s}$) → (Y, $\mathbf{T}_{y(s)}$) is FN-δ-P-con. if and only if it is FN-con. and FN-c.con. function.

Doi : https://doi.org/10.54216/IJNS.180416
Received: March 15, 2022 Accepted: June 28, 2022

Proof: Let φ: (**V** s, **T** $_{V\,s}$) → (Y, **T** $_{y(s)}$) be FN-δ-P-con. function, so $\varphi^{-1}(\beta_s)$ is FN-clopen set in (**V** s, **T** $_{V\,s}$) for some FNδ-OS (β_s) in (Y, **T** $_y$) , this implies (β_s) is FNR-OS.

But, every FNR-OS is FNOS, so φ is FN-con. function because β_s is FNOS in (Y, T $_y$) and $\varphi^{-1}(\beta)$ is FNOS in (**V** s, **T** $_{V\,s}$) and in the same time it is FN-c.con. function because φ^{-1} (β_s) is FNCS in (**V** s, **T** $_{V\,s}$).

Conversely, let φ: (**V** s, **T** $_{V\,s}$) → (Y, **T** $_y$) be FN-con. and FN-c.con function, so $\varphi^{-1}(\beta_s)$ is FNOS and FNCS in (**V** s, **T** $_{V\,s}$) for each FNOS (β_s) in (Y, **T** $_y$) and we have (Y, **T** $_y$) is fuzzy neutrosophic semi-regular space therefore (β_s) and every set in (Y, **T** $_y$) is FNR-OS which implies, φ is FN-δ-P-con. function.

4.Conclusions

In this manuscript, introduced the notion of a new class of continuous functions whose is including fuzzy neutrosophic perfectly continuous, fuzzy neutrosophic strongly continuous and fuzzy neutrosophic almost perfectly continuous via fuzzy neutrosophic topological spaces as generalized of classical fuzzy topological spaces and intuitionistic fuzzy topological spaces. Furthermore, properties are obtained and relationships among them compared and studied are investigated.

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Received: March 15, 2022 Accepted: June 28, 2022

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Doi : https://doi.org/10.54216/IJNS.180416
Received: March 15, 2022 Accepted: June 28, 2022