On $Ng^{\#}-$ Nowhere Dense Sets in Neutrosophic Topological Spaces

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Abstract

In this article we introduce new class of Neutrosophic sets called $Ng^{\#}-$ dense sets and $Ng^{\#}-$ nowhere dense sets. Also furnishes some interesting characterizations in Neutrosophic topological space with necessary examples.
Keywords: $\mathcal{N}g^{\#}-$ closed set, $\mathcal{N}g^{\#}-$ Open sets, $\mathcal{N}g^{\#}-$ dense sets, $\mathcal{N}g^{\#}-$ nowhere dense sets.

1 Introduction

F. Smarandache[6] introduced and studied the concept of Neutrosophic set as an approach for solving issues that cover unreliable, indeterminacy and persistent data. The concept of neutrosophic set overcomes the inherent difficulties that existed in fuzzy sets and intuitionistic fuzzy sets. Neutrosophic logic was developed to represent mathematical model of uncertainty, vagueness, ambiguity, imprecision, undefined incompleteness, inconsistency, redundancy, contradiction. The theory of fuzzy topological spaces was introduced and developed by Chang.[2] Since then, various notions in classical topology have been extended to fuzzy topological space. Later topological structures in fuzzy topological spaces were generalized to intuitionistic fuzzy topological spaces by Coker.[3] A.A.Salama[12] extended the neutrosophic set into neutrosophic topological spaces. R. Dhavaseelan et.al.[5] introduced Neutrosophic generalized closed sets. Pious Missier et.al.,[9] introduced the concept of $\mathcal{N}g^{\#}-$ closed and open sets in Neutrosophic Topological Spaces. In this article we derived $\mathcal{N}g^{\#}-$ dense sets and $\mathcal{N}g^{\#}-$ nowhere dense sets and discussed their properties .

2 Preliminaries

Definition 2.1. [6]
A Neutrosophic set $\mathcal{NS}$ $\mathcal{A_N}$ is an object having the form $\mathcal{A_N} = \{\langle\lambda, \mu_{\mathcal{A_N}}(\lambda\lambda), \sigma_{\mathcal{A_N}}(\lambda), \gamma_{\mathcal{A_N}}(\lambda)\rangle : \lambda \in \mathcal{X_N}\}$. Here

1. $\mu_{\mathcal{A_N}}(\lambda)-$ degree of membership
2. $\sigma_{\mathcal{A_N}}(\lambda)-$ degree of indeterminacy
3. $\gamma_{\mathcal{A_N}}(\lambda)-$ degree of non-membership

A Neutrosophic set $\mathcal{A_N} = \{\langle\lambda, \mu_{\mathcal{A_N}}(\lambda), \sigma_{\mathcal{A_N}}(\lambda), \gamma_{\mathcal{A_N}}(\lambda)\rangle : \lambda \in \mathcal{X_N}\}$ can be identified as an ordered triple $\langle\mu_{\mathcal{A_N}}(\lambda), \sigma_{\mathcal{A_N}}(\lambda), \gamma_{\mathcal{A_N}}(\lambda)\rangle$ in $]-0, 1+[$ on $\mathcal{X_N}$.

Definition 2.2. [12] For any two Neutrosophic sets $\mathcal{A}_{\mathcal{N}} = \{\langle \lambda, \mu_{\mathcal{A}_{\mathcal{N}}}(\lambda), \sigma_{\mathcal{A}_{\mathcal{N}}}(\lambda), \gamma_{\mathcal{A}_{\mathcal{N}}}(\lambda)\rangle : \lambda \in \mathcal{X}_{\mathcal{N}}\}$ and $\mathcal{B}_{\mathcal{N}} = \{\langle \lambda, \mu_{\mathcal{B}_{\mathcal{N}}}(\lambda), \sigma_{\mathcal{B}_{\mathcal{N}}}(\lambda), \gamma_{\mathcal{B}_{\mathcal{N}}}(\lambda)\rangle : \lambda \in \mathcal{X}_{\mathcal{N}}\}$ we have

1. $\mathcal{A}_{\mathcal{N}} \subseteq \mathcal{B}_{\mathcal{N}} \Longleftrightarrow \mu_{\mathcal{A}_{\mathcal{N}}}(\lambda) \leq \mu_{\mathcal{B}_{\mathcal{N}}}(\lambda), \sigma_{\mathcal{A}_{\mathcal{N}}}(\lambda) \leq \sigma_{\mathcal{B}_{\mathcal{N}}}(\lambda)$ and $\gamma_{\mathcal{A}_{\mathcal{N}}}(\lambda) \geq \gamma_{\mathcal{B}_{\mathcal{N}}}(\lambda)$
2. $\mathcal{A}_{\mathcal{N}} \cap \mathcal{B}_{\mathcal{N}} = \langle \lambda, \mu_{\mathcal{A}_{\mathcal{N}}}(\lambda) \wedge \mu_{\mathcal{B}_{\mathcal{N}}}(\lambda), \sigma_{\mathcal{A}_{\mathcal{N}}}(\lambda) \wedge \sigma_{\mathcal{B}_{\mathcal{N}}}(\lambda)$ and $\gamma_{\mathcal{A}_{\mathcal{N}}}(\lambda) \vee \gamma_{\mathcal{B}_{\mathcal{N}}}(\lambda)\rangle$
3. $\mathcal{A}_{\mathcal{N}} \cup \mathcal{B}_{\mathcal{N}} = \langle \lambda, \mu_{\mathcal{A}_{\mathcal{N}}}(\lambda) \vee \mu_{\mathcal{B}_{\mathcal{N}}}(\lambda), \sigma_{\mathcal{A}_{\mathcal{N}}}(\lambda) \vee \sigma_{\mathcal{B}_{\mathcal{N}}}(\lambda)$ and $\gamma_{\mathcal{A}_{\mathcal{N}}}(\lambda) \wedge \gamma_{\mathcal{B}_{\mathcal{N}}}(\lambda)\rangle$

Definition 2.3. [12] Let $\mathcal{A}_{\mathcal{N}} = \langle \mu_{\mathcal{A}_{\mathcal{N}}}(\lambda), \sigma_{\mathcal{A}_{\mathcal{N}}}(\lambda), \gamma_{\mathcal{A}_{\mathcal{N}}}(\lambda)\rangle$ be a $\mathcal{N}S$ on $\mathcal{X}_{\mathcal{N}}$, then the complement ${\mathcal{A}_{\mathcal{N}}}^c$ defined as

- ${\mathcal{A}_{\mathcal{N}}}^c = \{\langle \lambda, \gamma_{\mathcal{A}_{\mathcal{N}}}(\lambda), 1 - \sigma_{\mathcal{A}_{\mathcal{N}}}(\lambda), \mu_{\mathcal{A}_{\mathcal{N}}}(\lambda)\rangle : \lambda \in \mathcal{X}_{\mathcal{N}}\}$

Note that for any two Neutrosophic sets $\mathcal{A}_{\mathcal{N}}$ and $\mathcal{B}_{\mathcal{N}}$,

- $(\mathcal{A}_{\mathcal{N}} \cup \mathcal{B}_{\mathcal{N}})^c = {\mathcal{A}_{\mathcal{N}}}^c \cap {\mathcal{B}_{\mathcal{N}}}^c$
- $(\mathcal{A}_{\mathcal{N}} \cap \mathcal{B}_{\mathcal{N}})^c = {\mathcal{A}_{\mathcal{N}}}^c \cup {\mathcal{B}_{\mathcal{N}}}^c$.

Definition 2.4. [12] A Neutrosophic topology $(\mathcal{NT})$ on a non-empty set $\mathcal{X}_{\mathcal{N}}$ is a family τ_N of Neutrosophic subsets in $\mathcal{X}_{\mathcal{N}}$ satisfies the following axioms:

1. $\mathbf{0}_{\mathcal{N}}, \mathbf{1}_{\mathcal{N}} \in \tau_N$
2. $\mathcal{R}_{N_1} \cap \mathcal{R}_{N_2} \in \tau_N$ for any $\mathcal{R}_{N_1}, R_{N_2} \in \tau_N$
3. $\bigcup \mathcal{R}_{N_i} \in \tau_N \quad \forall \quad \mathcal{R}_{N_i} : i \in I \subseteq \tau_N$

Here the empty set $\mathbf{0}_{\mathcal{N}}$ and the whole set $\mathbf{1}_{\mathcal{N}}$ may be defined as follows:

1. $\mathbf{0}_{\mathcal{N}} = \{\langle \lambda, 0, 0, 1\rangle : \lambda \in \mathcal{X}_{\mathcal{N}}\}$
2. $\mathbf{1}_{\mathcal{N}} = \{\langle \lambda, 1, 1, 0\rangle : \lambda \in \mathcal{X}_{\mathcal{N}}\}$

Definition 2.5. [12] Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{NS}$ in $\mathcal{NTS}$ $\mathcal{X}_{\mathcal{N}}$. Then

1. $\mathcal{N}int(\mathcal{A}_{\mathcal{N}}) = \bigcup\{\mathcal{G}_{\mathcal{N}} : \mathcal{G}_{\mathcal{N}}$ is a $\mathcal{NOS}$ in $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{G}_{\mathcal{N}} \subseteq \mathcal{A}_{\mathcal{N}}\}$ is called a Neutrosophic interior of $\mathcal{A}_{\mathcal{N}}$.
2. $\mathcal{N}cl(\mathcal{A}_{\mathcal{N}}) = \bigcap\{\mathcal{K}_{\mathcal{N}} : \mathcal{K}_{\mathcal{N}}$ is a $\mathcal{NCS}$ in $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{A}_{\mathcal{N}} \subseteq \mathcal{K}_{\mathcal{N}}\}$ is called Neutrosophic closure of $\mathcal{A}_{\mathcal{N}}$.

Definition 2.6. [7] A Neutrosophic set $\mathcal{A}_{\mathcal{N}}$ of a $\mathcal{NTS}$ $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is called a neutrosophic $\mathcal{N}\alpha gCS$ if $\mathcal{N}\alpha cl(\mathcal{A}_{\mathcal{N}}) \subseteq \mathcal{U}_{\mathcal{N}}$, whenever $\mathcal{A}_{\mathcal{N}} \subseteq \mathcal{U}_{\mathcal{N}}$ and $\mathcal{U}_{\mathcal{N}}$ is a $\mathcal{NOS}$ in $\mathcal{X}_{\mathcal{N}}$. The complement of $\mathcal{N}\alpha gCS$ is $\mathcal{N}\alpha gOS$.

Definition 2.7. [9]
A Neutrosophic set $\mathcal{A}_{\mathcal{N}}$ of a $\mathcal{NTS}$ $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is called a Neutrosophic $g^{\#}-$ closed $(\mathcal{N}g^{\#}CS)$ if $\mathcal{N}cl(\mathcal{A}_{\mathcal{N}}) \subseteq \mathcal{Q}_{\mathcal{N}}$ whenever $\mathcal{A}_{\mathcal{N}} \subseteq \mathcal{Q}_{\mathcal{N}}$ and $\mathcal{Q}_{\mathcal{N}}$ is $\mathcal{N}\alpha gOS$ in $\mathcal{X}_{\mathcal{N}}$. The complement of $\mathcal{N}g^{\#}CS$ is $\mathcal{N}g^{\#}OS$.

Definition 2.8. [11] Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{NS}$ in $\mathcal{NTS}$ $\mathcal{X}_{\mathcal{N}}$. Then

1. $\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}) = \bigcup\{\mathcal{G}_{\mathcal{N}} : \mathcal{G}_{\mathcal{N}}$ is a $\mathcal{N}g^{\#}OS$ in $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{G}_{\mathcal{N}} \subseteq \mathcal{A}_{\mathcal{N}}\}$ is called a Neutrosophic $g^{\#}-$ interior of $\mathcal{A}_{\mathcal{N}}$.
2. $\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}) = \bigcap\{\mathcal{K}_{\mathcal{N}} : \mathcal{K}_{\mathcal{N}}$ is a $\mathcal{N}g^{\#}CS$ in $\mathcal{X}_{\mathcal{N}}$ and $\mathcal{A}_{\mathcal{N}} \subseteq \mathcal{K}_{\mathcal{N}}\}$ is called Neutrosophic $g^{\#}-$ closure of $\mathcal{A}_{\mathcal{N}}$.

Definition 2.9. [10] A function $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ is said to be $\mathcal{N}g^{\#}-$ continuous function if $f_{\mathcal{N}}^{-1}(\mathcal{V}_{\mathcal{N}})$ is a $\mathcal{N}g^{\#}-$ closed set of $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ for every neutrosophic closed set $\mathcal{V}_{\mathcal{N}}$ of $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$.

Definition 2.10. [10] A function $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ is said to be Neutrosophic $g^{\#}-$ irresolute function if $f_{\mathcal{N}}^{-1}(\mathcal{V}_{\mathcal{N}})$ is a $\mathcal{N}g^{\#}CS$ of $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ for every $\mathcal{N}g^{\#}CS$ $\mathcal{V}_{\mathcal{N}}$ of $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$.

Definition 2.11. [11] A Neutrosophic Topological space $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is called a $T_{\mathcal{N}}g^{\#}-$ space if every $\mathcal{N}g^{\#}CS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is $\mathcal{NCS}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.

Definition 2.12. [13] A function $f_N : (X, \tau_N) \longrightarrow (Y, \zeta_N)$ is called

1. Neutrosophic closed mapping (NCM) if $f_N(V_N)$ is a $\mathcal{NCS}$ of $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ for every $\mathcal{NCS}$ $\mathcal{V}_{\mathcal{N}}$ of $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.
2. Neutrosophic open mapping (NOM) if $f_N(V_N)$ is a $\mathcal{NOS}$ of $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ for every $\mathcal{NOS}$ $\mathcal{V}_{\mathcal{N}}$ of $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.

Definition 2.13. [4] A Neutrosophic set $\mathcal{A}_{\mathcal{N}}$ in a Neutrosophic topological space $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is called Neutrosophic dense set if there exists no $\mathcal{NCS}$ $\mathcal{B}_{\mathcal{N}}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ such that $\mathcal{A}_{\mathcal{N}} \subset \mathcal{B}_{\mathcal{N}} \subset \mathbf{1}_{\mathcal{N}}$.

Definition 2.14. [4] A Neutrosophic set $\mathcal{A}_{\mathcal{N}}$ in a Neutrosophic topological space $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is called Neutrosophic nowhere dense set if there exists no non- zero $\mathcal{NOS}$ $\mathcal{B}_{\mathcal{N}}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ such that $\mathcal{B}_{\mathcal{N}} \subset \mathcal{N}cl(\mathcal{A}_{\mathcal{N}})$. That is $\mathcal{N}int(\mathcal{N}cl(\mathcal{A}_{\mathcal{N}})) = \mathbf{0}_{\mathcal{N}}$.

3 $\mathcal{N}g^{\#}-$ Nowhere Dense Sets

Definition 3.1. A Neutrosophic set $\mathcal{A}_{\mathcal{N}}$ in a Neutrosophic topological space $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is called $\mathcal{N}g^{\#}-$ dense set if there exists no $\mathcal{N}g^{\#}CS$ $\mathcal{B}_{\mathcal{N}}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ such that $\mathcal{A}_{\mathcal{N}} \subset \mathcal{B}_{\mathcal{N}} \subset \mathbf{1}_{\mathcal{N}}$. That is $\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}) = \mathbf{1}_{\mathcal{N}}$.

Example 3.2. Let $\mathcal{X}_{\mathcal{N}} = \{p, q\}$. Consider the Neutrosophic sets $\mathcal{M}_{\mathcal{N}_1} = \langle(0.4, 0.5, 0.6), (0.4, 0.3, 0.5)\rangle$, $\mathcal{M}_{\mathcal{N}_2} = \langle(0.6, 0.5, 0.4), (0.5, 0.7, 0.4)\rangle$, $\mathcal{M}_{\mathcal{N}_3} = \langle(0.6, 0.5, 0.3), (0.6, 0.7, 0.3)\rangle$. Now $(\mathcal{X}_{\mathcal{N}}, \tau_N) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathcal{M}_{\mathcal{N}_2}, \mathcal{M}_{\mathcal{N}_3}, \mathbf{1}_{\mathcal{N}}\}$ is Neutrosophic topological space. Then $\tau_N = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathbf{1}_{\mathcal{N}}\}$ is $\mathcal{NT}$ on $\mathcal{X}_{\mathcal{N}}$. Here $\mathcal{N}g^{\#}CS(\mathcal{X}_{\mathcal{N}}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_2}, \mathcal{M}_{\mathcal{N}_3}, \mathbf{1}_{\mathcal{N}}\}$. Here, $\mathcal{M}_{\mathcal{N}_3}$ is a $\mathcal{NS}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ there is no $\mathcal{N}g^{\#}CS$ $\mathcal{A}_{\mathcal{N}}$ such that $\mathcal{M}_{\mathcal{N}_3} \subset \mathcal{A}_{\mathcal{N}} \subset \mathbf{1}_{\mathcal{N}}$. Therefore, $\mathcal{M}_{\mathcal{N}_3}$ is a $\mathcal{N}g^{\#}-$ dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.

Definition 3.3. A Neutrosophic set $\mathcal{A}_{\mathcal{N}}$ in a Neutrosophic topological space $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is called $\mathcal{N}g^{\#}-$ nowhere dense set if there exists no non- zero $\mathcal{N}g^{\#}OS$ $\mathcal{B}_{\mathcal{N}}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ such that $\mathcal{B}_{\mathcal{N}} \subset \mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}})$. That is $\mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}})) = \mathbf{0}_{\mathcal{N}}$.

Example 3.4. Let $\mathcal{X}_{\mathcal{N}} = \{p, q\}$. Consider the Neutrosophic sets $\mathcal{M}_{\mathcal{N}_1} = \langle(0.6, 0.6, 0.4), (0.7, 0.7, 0.3)\rangle$, $\mathcal{M}_{\mathcal{N}_2} = \langle(0.4, 0.4, 0.6), (0.3, 0.3, 0.7)\rangle$, $\mathcal{M}_{\mathcal{N}_3} = \langle(0.3, 0.3, 0.7), (0.2, 0.2, 0.8)\rangle$, $\mathcal{M}_{\mathcal{N}_4} = \langle(0.7, 0.7, 0.3), (0.8, 0.8, 0.2)\rangle$. Now $(\mathcal{X}_{\mathcal{N}}, \tau_N) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathcal{M}_{\mathcal{N}_2}, \mathcal{M}_{\mathcal{N}_3}, \mathcal{M}_{\mathcal{N}_4}, \mathbf{1}_{\mathcal{N}}\}$ is Neutrosophic topological space. Then $\tau_N = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathbf{1}_{\mathcal{N}}\}$ is $\mathcal{NT}$ on $\mathcal{X}_{\mathcal{N}}$. Here, $\mathcal{N}g^{\#}OS(\mathcal{X}_{\mathcal{N}}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathbf{1}_{\mathcal{N}}\}$. Here, $\mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{M}_{\mathcal{N}_3})) = \mathbf{0}_{\mathcal{N}}$. Therefore, $\mathcal{M}_{\mathcal{N}_3}$ is a $\mathcal{N}g^{\#}-$ nowhere dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.

Theorem 3.5. *If $\mathcal{A}_{\mathcal{N}}$ is a $\mathcal{N}g^{\#}CS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$, then $\mathcal{A}_{\mathcal{N}}$ is $\mathcal{N}g^{\#}-$ nowhere dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ iff $\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}) = \mathbf{0}_{\mathcal{N}}$.*

Proof. Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{N}g^{\#}CS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ with $\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}) = \mathbf{0}_{\mathcal{N}}$. Then $\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}) = \mathcal{A}_{\mathcal{N}}$. Now $\mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}})) = \mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}) = \mathbf{0}_{\mathcal{N}}$. Hence $\mathcal{A}_{\mathcal{N}}$ is a $\mathcal{N}g^{\#}-$ nowhere dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.

Conversely, Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{N}g^{\#}-$ nowhere dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Then $\mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}})) = \mathbf{0}_{\mathcal{N}}$. Since $\mathcal{A}_{\mathcal{N}}$ is $\mathcal{N}g^{\#}CS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$, $\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}) = \mathcal{A}_{\mathcal{N}}$. Which implies that $\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}) = \mathbf{0}_{\mathcal{N}}$. Hence proved. □

Theorem 3.6. *If $\mathcal{A}_{\mathcal{N}}$ is a $\mathcal{N}g^{\#}-$ nowhere dense set in a Neutrosophic topological space $(\mathcal{X}_{\mathcal{N}}, \tau_N)$, then $\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}) = \mathbf{0}_{\mathcal{N}}$.*

Proof. Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{N}g^{\#}-$ nowhere dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Now $\mathcal{A}_{\mathcal{N}} \subseteq \mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}})$ which implies that $\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}) \subseteq \mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}})) = \mathbf{0}_{\mathcal{N}} \implies \mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}) \subseteq \mathbf{0}_{\mathcal{N}}$. Clearly, $\mathbf{0}_{\mathcal{N}} \subseteq \mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}})$. Hence $\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}) = \mathbf{0}_{\mathcal{N}}$ □

Remark 3.7. Counter example is given to prove the converse of above theorem is not true.

Example 3.8. Let $\mathcal{X}_{\mathcal{N}} = \{p\}$. Consider the Neutrosophic sets $\mathcal{M}_{\mathcal{N}_1} = \langle(0.6, 0.7, 0.4)\rangle$, $\mathcal{M}_{\mathcal{N}_2} = \langle(0.4, 0.3, 0.6)\rangle$, $\mathcal{M}_{\mathcal{N}_3} = \langle(0.3, 0.2, 0.8)\rangle$. Now $(\mathcal{X}_{\mathcal{N}}, \tau_N) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathcal{M}_{\mathcal{N}_2}, \mathcal{M}_{\mathcal{N}_3}, \mathbf{1}_{\mathcal{N}}\}$ is Neutrosophic topological space. Then $\tau_N = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathbf{1}_{\mathcal{N}}\}$ is $\mathcal{NT}$ on $\mathcal{X}_{\mathcal{N}}$. Here $\mathcal{N}g^{\#}OS(\mathcal{X}_{\mathcal{N}}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathbf{1}_{\mathcal{N}}\}$,$\mathcal{N}g^{\#}CS(\mathcal{X}_{\mathcal{N}}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_2}, \mathbf{1}_{\mathcal{N}}\}$. Now $\mathcal{N}g^{\#}int(\mathcal{M}_{\mathcal{N}_3}) = \mathbf{0}_{\mathcal{N}}$. But $\mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{M}_{\mathcal{N}_3})) = \mathbf{1}_{\mathcal{N}} \neq \mathbf{0}_{\mathcal{N}}$. Therefore, $\mathcal{M}_{\mathcal{N}_3}$ is not a $\mathcal{N}g^{\#}-$ nowhere dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.

Theorem 3.9. *If $\mathcal{A}_{\mathcal{N}}$ is a $\mathcal{N}g^{\#}-$ nowhere dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Then ${\mathcal{A}_{\mathcal{N}}}^c$ is a $\mathcal{N}g^{\#}-$ dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.*

Proof. Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{N}g^{\#}-$ nowhere dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Then by theorem 3.6 we have $\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}) = \mathbf{0}_{\mathcal{N}}$. Now $(\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}))^c = \mathbf{0}_{\mathcal{N}}{}^c$ which implies $\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}{}^c) = \mathbf{1}_{\mathcal{N}}$. Therefore, $\mathcal{A}_{\mathcal{N}}{}^c$ is $\mathcal{N}g^{\#}-$ dense set. □

Theorem 3.10. *If $\mathcal{A}_{\mathcal{N}}$ is a $\mathcal{N}g^{\#}-$ dense set and $\mathcal{N}g^{\#}OS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$, such that $\mathcal{B}_{\mathcal{N}} \subseteq \mathcal{A}_{\mathcal{N}}{}^c$, then $\mathcal{B}_{\mathcal{N}}$ is $\mathcal{N}g^{\#}-$ nowhere dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.*

Proof. Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{N}g^{\#}OS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Since $\mathcal{A}_{\mathcal{N}}$ is a $\mathcal{N}g^{\#}-$ dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N), \mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}) = \mathbf{1}_{\mathcal{N}}$. Now $\mathcal{B}_{\mathcal{N}} \subseteq \mathcal{A}_{\mathcal{N}}{}^c$, which implies that $\mathcal{N}g^{\#}cl(\mathcal{B}_{\mathcal{N}}) \subseteq \mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}{}^c) = \mathcal{A}_{\mathcal{N}}{}^c$. Then we have, $\mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{B}_{\mathcal{N}})) \subseteq \mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}{}^c) = (\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}))^c = \mathbf{1}_{\mathcal{N}}{}^c = \mathbf{0}_{\mathcal{N}}$. That is, $\mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{B}_{\mathcal{N}})) = \mathbf{0}_{\mathcal{N}}$. Therefore, $\mathcal{B}_{\mathcal{N}}$ is $\mathcal{N}g^{\#}-$ nowhere dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. □

Theorem 3.11. *If $\mathcal{A}_{\mathcal{N}}$ is a $\mathcal{N}g^{\#}-$ dense set and $\mathcal{N}g^{\#}OS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Then $\mathcal{A}_{\mathcal{N}}{}^c$ is a $\mathcal{N}g^{\#}-$ nowhere dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.*

Proof. Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{N}g^{\#}OS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Since $\mathcal{A}_{\mathcal{N}}$ is $\mathcal{N}g^{\#}-$ dense set, $\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}) = \mathbf{1}_{\mathcal{N}}$. Now $\mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}{}^c)) = \mathcal{N}g^{\#}int(\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}))^c = (\mathcal{N}g^{\#}cl(\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}})))^c = (\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}))^c = \mathbf{1}_{\mathcal{N}}{}^c = \mathbf{0}_{\mathcal{N}}$. Which implies $\mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}{}^c)) = \mathbf{0}_{\mathcal{N}}$. Therefore, $\mathcal{A}_{\mathcal{N}}{}^c$ is a $\mathcal{N}g^{\#}-$ nowhere dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. □

Theorem 3.12. *If $\mathcal{A}_{\mathcal{N}}$ is a $\mathcal{N}g^{\#}-$ nowhere dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Then $\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}})$ is $\mathcal{N}g^{\#}-$ nowhere dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.*

Proof. Let $\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}) = \mathcal{B}_{\mathcal{N}}$. Now $\mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{B}_{\mathcal{N}})) = \mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}))) = \mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}})) = \mathbf{0}_{\mathcal{N}}$. Hence $\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}})$ is a $\mathcal{N}g^{\#}-$ nowhere dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. □

Theorem 3.13. *If $\mathcal{A}_{\mathcal{N}}$ is a $\mathcal{N}g^{\#}-$ nowhere dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Then $(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}))^c$ is a $\mathcal{N}g^{\#}-$ dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.*

Proof. By Theorem 3.12 $\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}})$ is $\mathcal{N}g^{\#}-$ nowhere dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Now by Theorem 3.9, $(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}))^c$ is a $\mathcal{N}g^{\#}-$ dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. □

Theorem 3.14. *Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{N}g^{\#}-$ dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. If $\mathcal{B}_{\mathcal{N}}$ is a Neutrosophic set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ then $\mathcal{B}_{\mathcal{N}}$ is a $\mathcal{N}g^{\#}-$ nowhere dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ if and only if $\mathcal{A}_{\mathcal{N}} \cap \mathcal{B}_{\mathcal{N}}$ is a $\mathcal{N}g^{\#}-$ nowhere dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.*

Proof. Let $\mathcal{B}_{\mathcal{N}}$ be a $\mathcal{N}g^{\#}-$ nowhere dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Now,

$$\begin{aligned}
\mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}} \cap \mathcal{B}_{\mathcal{N}})) &= \mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}) \cap \mathcal{N}g^{\#}cl(\mathcal{B}_{\mathcal{N}})) \\
&= \mathcal{N}g^{\#}int(\mathbf{1}_{\mathcal{N}} \cap \mathcal{N}g^{\#}cl(\mathcal{B}_{\mathcal{N}})) \\
&= \mathcal{N}g^{\#}int(\mathbf{1}_{\mathcal{N}}) \cap \mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{B}_{\mathcal{N}})) \\
&= \mathbf{1}_{\mathcal{N}} \cap \mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{B}_{\mathcal{N}}) \\
&= \mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{B}_{\mathcal{N}}) \\
&= \mathbf{0}_{\mathcal{N}} \text{ (since } \mathcal{B}_{\mathcal{N}} \text{ is } \mathcal{N}g^{\#} - \text{nowhere dense set)}
\end{aligned}$$

Which implies $\mathcal{A}_{\mathcal{N}} \cap \mathcal{B}_{\mathcal{N}}$ is $\mathcal{N}g^{\#}-$ nowhere dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.
Conversely, Let $\mathcal{A}_{\mathcal{N}} \cap \mathcal{B}_{\mathcal{N}}$ be a $\mathcal{N}g^{\#}-$ nowhere dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.
Then $\mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}} \cap \mathcal{B}_{\mathcal{N}})) = \mathbf{0}_{\mathcal{N}}$

$$\begin{aligned}
&\Longrightarrow \mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}) \cap \mathcal{N}g^{\#}cl(\mathcal{B}_{\mathcal{N}})) = \mathbf{0}_{\mathcal{N}} \\
&\Longrightarrow \mathcal{N}g^{\#}int(\mathbf{1}_{\mathcal{N}}) \cap \mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{B}_{\mathcal{N}})) = \mathbf{0}_{\mathcal{N}} \\
&\Longrightarrow \mathbf{1}_{\mathcal{N}} \cap \mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{B}_{\mathcal{N}}) = \mathbf{0}_{\mathcal{N}} \\
&\Longrightarrow \mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{B}_{\mathcal{N}}) = \mathbf{0}_{\mathcal{N}} \\
&\Longrightarrow \mathcal{B}_{\mathcal{N}} \text{ is a } \mathcal{N}g^{\#} - \text{nowhere dense set } (\mathcal{X}_{\mathcal{N}}, \tau_N).
\end{aligned}$$

□

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