

NEUTROSOPHIC MICRO TOPOLOGICAL SPACES

¹S. GANESAN AND ²S. JAFARI

¹Assistant Professor, PG & Research Department of Mathematics,
Raja Doraisingam Government Arts College, Sivagangai-630561, Tamil Nadu, India.

(Affiliated to Alagappa University, Karaikudi, Tamil Nadu, India)

e-mail : ¹sgsgsgsg77@gmail.com; Orcid : 0000-0002-7728-8941

²College of Vestsjaelland South & Mathematical and Physical Science Foundation,
4200 Slagelse, Denmark.

E-mail : saeidjafari@topositus.com; Orcid: 0000-0002-6845-7539

ABSTRACT. The main objective of this study is to introduce a new hybrid intelligent structure called neutrosophic micro topology. Some properties like neutrosophic micro interior and neutrosophic micro closure are derived.

1. INTRODUCTION

Zadehs [17] Fuzzy set laid the foundation of many theories such as intuitionistic fuzzy set and neutrosophic set, rough sets etc. Later, researchers developed K. T. Atanassovs [2] intuitionistic fuzzy set theory in many fields such as differential equations, topology, computerscience and so on. F. Smarandache [15, 16] found that some objects have indeterminacy or neutral other than membership and non-membership. So he coined the notion of neutrosophy. R. Dhavaseelan et al [1, 4, 5, 6, 7, 8, 9, 13]

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studied in various concept covered in neutrosophy. M. Lellis Thivagar et al [12] introduced and studied neutrosophic nano topology. S. Chandrasekar [3] introduced and studied micro topology like micro interior, micro closure, micro continuous respectively. The main objective of this study is to introduce a new hybrid intelligent structure called neutrosophic micro topology. The significance of introducing hybrid structures is that the computational techniques, based on any one of these structures alone, will not always yield the best results but a fusion of two or more of them can often give better results. The rest of this article is organized as follows. Some preliminary concepts required in our work are briefly recalled in section 2. In section 3, the concept of neutrosophic micro topology is investigated with some properties on neutrosophic micro interior, neutrosophic micro closure and neutrosophic micro continuous.

2. PRELIMINARIES

Definition 2.1. [15, 16] *A neutrosophic set (in short ns) K on a set $X \neq \emptyset$ is defined by $K = \{ \prec a, P_K(a), Q_K(a), R_K(a) \succ : a \in X \}$ where $P_K : X \rightarrow [0,1]$, $Q_K : X \rightarrow [0,1]$ and $R_K : X \rightarrow [0,1]$ denotes the membership of an object, indeterminacy and non-membership of an object, for each $a \in X$ to K , respectively and $0 \leq P_K(a) + Q_K(a) + R_K(a) \leq 3$ for each $a \in X$.*

Definition 2.2. [14] *Let $K = \{ \prec a, P_K(a), Q_K(a), R_K(a) \succ : a \in X \}$ be a ns. We must introduce the ns 0_{\sim} and 1_{\sim} in X as follows:*

0_{\sim} may be defined as:

- (1) $0_{\sim} = \{ \prec x, 0, 0, 1 \succ : x \in X \}$
- (2) $1_{\sim} = \{ \prec x, 0, 1, 1 \succ : x \in X \}$

$$(3) 0_{\sim} = \{\prec x, 0, 1, 0 \succ : x \in X\}$$

$$(4) 0_{\sim} = \{\prec x, 0, 0, 0 \succ : x \in X\}$$

1_{\sim} may be defined as:

$$(a) 1_{\sim} = \{\prec x, 1, 0, 0 \succ : x \in X\}$$

$$(b) 1_{\sim} = \{\prec x, 1, 0, 1 \succ : x \in X\}$$

$$(c) 1_{\sim} = \{\prec x, 1, 1, 0 \succ : x \in X\}$$

$$(d) 1_{\sim} = \{\prec x, 1, 1, 1 \succ : x \in X\}$$

Proposition 2.3. [14] *For any ns S , then the following conditions are satisfied:*

$$(1) 0_{\sim} \leq S, 0_{\sim} \leq 0_{\sim}.$$

$$(2) S \leq 1_{\sim}, 1_{\sim} \leq 1_{\sim}.$$

Definition 2.4. [14] *Let $K = \{\prec a, P_K(a), Q_K(a), R_K(a) \succ : a \in X\}$ be a ns.*

(1) *A ns K is an empty set i.e., $K = 0_{\sim}$ if 0 is membership of an object and 0 is an indeterminacy and 1 is a non-membership of an object respectively. i.e.,*
 $0_{\sim} = \{x, (0, 0, 1) : x \in X\}$

(2) *A ns K is a universal set i.e., $K = 1_{\sim}$ if 1 is membership of an object and 1 is an indeterminacy and 0 is a non-membership of an object respectively. 1_{\sim}*
 $= \{x, (1, 1, 0) : x \in X\}$

(3) $K_1 \cup K_2 = \{a, \max \{P_{K_1}(a), P_{K_2}(a)\}, \max \{Q_{K_1}(a), Q_{K_2}(a)\}, \min \{R_{K_1}(a), R_{K_2}(a)\} : a \in X\}$

(4) $K_1 \cap K_2 = \{a, \min \{P_{K_1}(a), P_{K_2}(a)\}, \min \{Q_{K_1}(a), Q_{K_2}(a)\}, \max \{R_{K_1}(a), R_{K_2}(a)\} : a \in X\}$

(5) $K_1^C = \{\prec a, R_K(a), 1 - Q_K(a), P_K(a) \succ : a \in X\}$

Definition 2.5. [14] *A neutrosophic topology (nt) in Salamas sense on a nonempty set X is a family τ of ns in X satisfying three axioms:*

- (1) *Empty set (0_\sim) and universal set (1_\sim) are members of τ .*
- (2) *$K_1 \cap K_2 \in \tau$ where $K_1, K_2 \in \tau$.*
- (3) *$\cup K_\delta \in \tau$ for every $\{K_\delta : \delta \in \Delta\} \leq \tau$.*

Each ns in nt are called neutrosophic open sets. Its complements are called neutrosophic closed sets.

Definition 2.6. [11]

Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

- (1) *The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$.
 $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$ where $R(x)$ denotes the equivalence class determined by X .*
- (2) *The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$.
 $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \phi\}$*
- (3) *The boundary region of X with respect to R is the set of all objects, which can be neither in nor as not- X with respect to R and it is denoted by $B_R(X)$ and
 $B_R(X) = U_R(X) - L_R(X)$*

Proposition 2.7. [11] *If (U, R) is an approximation space and $X, Y \subseteq U$, then*

- (1) $L_R(X) \subseteq X \subseteq U_R(X)$.

- (2) $L_R(\phi) = U_R(\phi) = \phi$, $L_R(U) = U_R(U) = U$.
- (3) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$.
- (4) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$.
- (5) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$.
- (6) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$.
- (7) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$, whenever $X \subseteq Y$.
- (8) $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$.
- (9) $U_R(U_R(X)) = L_R(U_R(X)) = U_R(X)$.
- (10) $L_R(L_R(X)) = U_R(L_R(X)) = L_R(X)$.

Definition 2.8. [11] Let U be a universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$, where $X \subseteq U$. Then by Proposition 2.7, $\tau_R(X)$ satisfies the following axioms

- (1) $U, \phi \in \tau_R(X)$.
- (2) The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.
- (3) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $\tau_R(X)$ is called the nano topology on U with respect to X .

The space $(U, \tau_R(X))$ is the nano topological space. The elements of are called nano open sets.

Definition 2.9. [11]

If $(U, \tau_R(X))$ is the nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

- (1) The nano interior of the set A is defined as the union of all nano open subsets contained in A and it is denoted by $nint(A)$. That is, $nint(A)$ is the largest nano open subset of A .
- (2) The nano closure of the set A is defined as the intersection of all nano closed sets containing A and it is denoted by $ncl(A)$. That is, $ncl(A)$ is the smallest nano closed set containing A .

Definition 2.10. [3] Let $(U, \tau_R(X))$ be a nano topological space. Then, $\mu_R(X) = \{N \cup (\acute{N} \cap \mu) : N, \acute{N} \in \tau_R(X) \text{ and } \mu \notin \tau_R(X)\}$ is called the Micro topology on U with respect to X . The triplet $(U, \tau_R(X), \mu_R(X))$ is called Micro topological space.

Definition 2.11. [3] The Micro topology $\mu_R(X)$ satisfies the following axioms

- (1) $U, \phi \in \mu_R(X)$.
- (2) The union of the elements of any sub-collection of $\mu_R(X)$ is in $\mu_R(X)$.
- (3) The intersection of the elements of any finite sub collection of $\mu_R(X)$ is in $\mu_R(X)$.

Then $\mu_R(X)$ is called the Micro topology on U with respect to X . The triplet $(U, \tau_R(X), \mu_R(X))$ is called Micro topological spaces and the elements of $\mu_R(X)$ are called Micro open sets and the complement of a Micro open set is called a Micro closed set.

Definition 2.12. [3] For any two Micro sets A and B in a Micro topological space $(U, \tau_R(X), \mu_R(X))$,

- (1) A is a Micro closed set if and only if $Mic-cl(A) = A$.
- (2) A is a Micro open set if and only if $Mic-int(A) = (A)$.
- (3) $A \subseteq B$ implies $Mic-int(A) \subseteq Mic-int(B)$ and $Mic-cl(A) \subseteq Mic-cl(B)$.
- (4) $Mic-cl(Mic-cl(A)) = Mic-cl(A)$ and $Mic-int(Mic-int(A)) = Mic-int(A)$.

- (5) $Mic-cl(A \cup B) \supseteq Mic-cl(A) \cup Mic-cl(B)$.
- (6) $Mic-cl(A \cap B) \subseteq Mic-cl(A) \cap Mic-cl(B)$.
- (7) $Mic-int(A \cup B) \supseteq Mic-int(A) \cup Mic-int(B)$.
- (8) $Mic-int(A \cap B) \subseteq Mic-int(A) \cap Mic-int(B)$.
- (9) $Mic-cl(\mathcal{A}^C) = [Mic - int(A)]^C$.
- (10) $Mic-int(\mathcal{A}^C) = [Mic - cl(A)]^C$.

Definition 2.13. [12] Let U be a non-empty set and R be an equivalence relation on U . Let F be a neutrosophic set in U with the membership function μ_F , the indeterminancy function σ_F and the non-membership function ν_F . The neutrosophic nano lower, neutrosophic nano upper approximation and neutrosophic nano boundary of F in the approximation (U, R) denoted by $\underline{N}(F)$, $\overline{N}(F)$ and $BN(F)$ are respectively defined as follows:

- (1) $\underline{N}(F) = \{ \prec x, \mu_{\underline{RA}}(x) \sigma_{\underline{RA}}(x) \nu_{\underline{RA}}(x) \succ : y \in [X]_R, x \in U \}$
- (2) $\overline{N}(F) = \{ \prec x, \mu_{\overline{RA}}(x) \sigma_{\overline{RA}}(x) \nu_{\overline{RA}}(x) \succ : y \in [X]_R, x \in U \}$
- (3) $BN(F) = \overline{N}(F) - \underline{N}(F)$

where $\mu_{\underline{RA}}(x) = \bigwedge_{y \in [X]_R} \mu A(y)$, $\sigma_{\underline{RA}}(x) = \bigwedge_{y \in [X]_R} \sigma A(y)$, $\nu_{\underline{RA}}(x) = \bigwedge_{y \in [X]_R} \nu A(y)$.
 $\mu_{\overline{RA}}(x) = \bigwedge_{y \in [X]_R} \mu A(y)$, $\sigma_{\overline{RA}}(x) = \bigwedge_{y \in [X]_R} \sigma A(y)$, $\nu_{\overline{RA}}(x) = \bigwedge_{y \in [X]_R} \nu A(y)$.

Definition 2.14. [12] Let U be a universe, R an equivalence relation on U and F a neutrosophic set in U and if the collection $\tau_N(F) = \{0_N, 1_N, \underline{N}(F), \overline{N}(F), BN(F)\}$ forms a topology then it is said to be a neutrosophic nano topology. We call $(U, \tau_N(F))$ as the neutrosophic nano topological space. The elements of $\tau_N(F)$ are called neutrosophic nano open sets.

Definition 2.15. [12] Let U be a nonempty set and the neutrosophic sets A and B in the form $A = \{ \prec x : \mu A(x), \sigma A(x), \nu A(x) \succ, x \in U \}$, $B = \{ \prec x : \mu B(x), \sigma B(x), \nu B(x) \succ, x \in U \}$. Then the following statements hold

- (1) $0_N = \{x, (0, 0, 1) : x \in U\}$.
- (2) $1_N = \{x, (1, 1, 0) : x \in U\}$.
- (3) $A \cup B = \{a, \max \{ \mu A(x), \mu B(x) \}, \max \{ \sigma A(x), \sigma B(x) \}, \min \{ \nu A(x), \nu B(x) \} : x \in U\}$.
- (4) $A \cap B = \{a, \min \{ \mu A(x), \mu B(x) \}, \min \{ \sigma A(x), \sigma B(x) \}, \max \{ \nu A(x), \nu B(x) \} : x \in U\}$.
- (5) $A_1^C = \{ \prec a, \nu A(x), 1 - \sigma A(x), \mu A(x) \succ : x \in U \}$.
- (6) $A \subseteq B$ iff $\mu A(x) \leq \mu B(x)$, $\sigma A(x) \leq \sigma B(x)$, $\nu A(x) \leq \nu B(x)$ for all $x \in U$.
- (7) $A = B$ iff $A \subseteq B$, $B \subseteq A$

Definition 2.16. [12] If $(U, \tau_N(F))$ is a neutrosophic nano topological space with respect to neutrosophic subset of U and if A be any neutrosophic subset of U , then the neutrosophic nano interior of A is defined as the union of all neutrosophic nano open subsets of A and it is denoted by $N_F \text{int}(A)$. That is, $N_F \text{int}(A)$ is the largest neutrosophic nano open subset of A . The neutrosophic nano closure of A is defined as the intersection of all neutrosophic nano closed sets containing A and it is denoted by $N_F \text{cl}(A)$. That is, $N_F \text{cl}(A)$ is the smallest neutrosophic nano closed set containing A .

Definition 2.17. [3] A map $f : (U, \tau_R(X), \mu_R(X)) \rightarrow (L, \tau'_R(Y), \mu'_R(Y))$ is called Micro-continuous if $f^{-1}(V)$ is a Micro closed set of $(U, \tau_R(X), \mu_R(X))$ for every Micro closed set V of $(L, \tau'_R(Y), \mu'_R(Y))$.

3. NEUTROSOPHIC MICRO TOPOLOGICAL SPACES

Definition 3.1. Let $(U, \tau_N(F))$ be a neutrosophic nano topological space. Then, $\lambda_N(F) = \{S \cup (\acute{S} \cap \lambda) : S, \acute{S} \in \tau_N(F) \text{ and } \lambda \notin \tau_N(F)\}$ is called the neutrosophic micro topology on U with respect to F . The triplet $(U, \tau_N(F), \lambda_N(F))$ is called neutrosophic micro topological space.

Definition 3.2. The neutrosophic micro topology $\lambda_N(F)$ satisfies the following axioms

- (1) $U, \phi \in \lambda_N(F)$.
- (2) The union of the elements of any sub-collection of $\lambda_N(F)$ is in $\lambda_N(F)$.
- (3) The intersection of the elements of any finite sub collection of $\lambda_N(F)$ is in $\lambda_N(F)$.

Then $\lambda_N(F)$ is called the neutrosophic micro topology on U with respect to F . The triplet $(U, \tau_N(F), \lambda_N(F))$ is called neutrosophic micro topological spaces and the elements of $\lambda_N(F)$ are called neutrosophic micro open sets and the complement of a neutrosophic micro open set is called a neutrosophic micro closed set.

Definition 3.3. If $(U, \tau_N(F), \lambda_N(F))$ is a neutrosophic micro topological space with respect to neutrosophic subset of U and if A is any neutrosophic subset of U , then the neutrosophic nano interior of A is defined as the union of all neutrosophic micro open subsets of A and it is denoted by $N_{mi}\text{-int}(A)$. That is, $N_{mi}\text{-int}(A)$ is the largest neutrosophic micro open subset of A . The neutrosophic micro closure of A is defined as the intersection of all neutrosophic micro closed sets containing A and it is denoted by $N_{mi}\text{-cl}(A)$. That is, $N_{mi}\text{-cl}(A)$ is the smallest neutrosophic micro closed set containing A .

Theorem 3.4. *Let $(U, \tau_N(F), \lambda_N(F))$ be a neutrosophic micro topological space with respect to F where F is a neutrosophic subset of U . Let A and B be neutrosophic subsets of U . Then the following statements hold :*

- (1) $A \subseteq N_{mi}\text{-cl}(A)$.
- (2) A is a neutrosophic micro closed set if and only if $N_{mi}\text{-cl}(A) = A$.
- (3) $N_{mi}\text{-cl}(0_\sim) = 0_\sim$ and $N_{mi}\text{-cl}(1_\sim) = 1_\sim$.
- (4) $A \subseteq B$ implies $N_{mi}\text{-cl}(A) \subseteq N_{mi}\text{-cl}(B)$.
- (5) $N_{mi}\text{-cl}(A \cup B) \subseteq N_{mi}\text{-cl}(A) \cup N_{mi}\text{-cl}(B)$.
- (6) $N_{mi}\text{-cl}(A \cap B) \subseteq N_{mi}\text{-cl}(A) \cap N_{mi}\text{-cl}(B)$.
- (7) $N_{mi}(N_{mi}\text{-cl}(A)) = N_{mi}\text{-cl}(A)$.
- (8) $N_{mi}\text{-int}(U - A) = X - N_{mi}\text{-cl}(A)$.

Proof. (1) By definition of neutrosophic micro closure, $A \subseteq N_{mi}\text{-cl}(A)$.

- (2) If A is neutrosophic micro closed, then A is the smallest neutrosophic micro closed set containing itself and hence $N_{mi}\text{-cl}(A) = A$. Conversely, if $N_{mi}\text{-cl}(A) = A$, then A is the smallest neutrosophic micro closed set containing itself and hence A is neutrosophic micro closed.
- (3) Since 0_\sim and 1_\sim are neutrosophic micro closed in $(U, \tau_N(F), \lambda_N(F))$, $N_{mi}\text{-cl}(0_\sim) = 0_\sim$ and $N_{mi}\text{-cl}(1_\sim) = 1_\sim$.
- (4) If $A \subseteq B$, since $B \subseteq N_{mi}\text{-cl}(B)$, then $A \subseteq N_{mi}\text{-cl}(B)$. That is, $N_{mi}\text{-cl}(B)$ is a Neutrosophic micro closed set containing A . But $N_{mi}\text{-cl}(A)$ is the smallest Neutrosophic micro closed set containing A . Therefore, $N_{mi}\text{-cl}(A) \subseteq N_{mi}\text{-cl}(B)$.
- (5) Since $A \subseteq A \cup B$ and $B \subseteq A \cup B$, $N_{mi}\text{-cl}(A) \subseteq N_{mi}\text{-cl}(A \cup B)$ and $N_{mi}\text{-cl}(B) \subseteq N_{mi}\text{-cl}(A \cup B)$. Therefore, $N_{mi}\text{-cl}(A) \cup N_{mi}\text{-cl}(B) \subseteq N_{mi}\text{-cl}(A \cup B)$. By the fact that $A \cup B \subseteq N_{mi}\text{-cl}(A) \cup N_{mi}\text{-cl}(B)$, and since $N_{mi}\text{-cl}(A \cup B)$ is the

smallest micro closed set containing $A \cup B$, so $N_{mi}\text{-cl}(A \cup B) \subseteq N_{mi}\text{-cl}(A) \cup N_{mi}\text{-cl}(B)$. Thus, $N_{mi}\text{-cl}(A \cup B) = N_{mi}\text{-cl}(A) \cup N_{mi}\text{-cl}(B)$.

- (6) Since $A \cap B \subseteq A$ and $A \cap B \subseteq B$, $N_{mi}\text{-cl}(A \cap B) \subseteq N_{mi}\text{-cl}(A) \cap N_{mi}\text{-cl}(B)$.
- (7) Since $N_{mi}\text{-cl}(A)$ is micro closed, $N_{mi}\text{-cl}(N_{mi}\text{-cl}(A)) = N_{mi}\text{-cl}(A)$.
- (8) For $A \leq U$, $U - N_{mi}\text{-cl}(A) = U - \min \{L : L \geq A, L \text{ is neutrosophic micro closed}\} = \max \{U - L : L \geq A, L \text{ is neutrosophic micro closed}\} = \max \{U - L : U - A \geq U - L, U \text{ is neutrosophic micro closed}\} = N_{mi}\text{-int}(U - A)$.

□

Theorem 3.5. *Let $(U, \tau_N(F), \lambda_N(F))$ be a neutrosophic micro topological space with respect to F where F is a neutrosophic subset of U . Let A and B be neutrosophic subsets of U . Then the following statements hold :*

- (1) $A \subseteq N_{mi}\text{-int}(A)$.
- (2) A is a neutrosophic micro open set if and only if $N_{mi}\text{-int}(A) = A$.
- (3) $N_{mi}\text{-int}(0_{\sim}) = 0_{\sim}$ and $N_{mi}\text{-int}(1_{\sim}) = 1_{\sim}$.
- (4) $A \subseteq B$ implies $N_{mi}\text{-int}(A) \subseteq N_{mi}\text{-int}(B)$.
- (5) $N_{mi}\text{-int}(A \cup B) \subseteq N_{mi}\text{-int}(A) \cup N_{mi}\text{-int}(B)$.
- (6) $N_{mi}\text{-int}(A \cap B) \subseteq N_{mi}\text{-int}(A) \cap N_{mi}\text{-int}(B)$.
- (7) $N_{mi}(N_{mi}\text{-int}(A)) = N_{mi}\text{-int}(A)$.
- (8) $N_{mi}\text{-cl}(U - A) = X - N_{mi}\text{-int}(A)$.

Proof. It is similar to the proof of Theorem 3.5. □

Example 3.6. *Let $U = \{P\}$ be the universe of discourse. Let $U / R = \{P\}$ be an equivalence relation on U and $A = \prec (0.6, 0.5, 0.5) \succ$ be a neutrosophic set on U then $\underline{N}(A) = \{\prec (0.4, 0.5, 0.5) \succ\}$, $\overline{N}(A) = \{\prec (0.6, 0.5, 0.5) \succ\}$, $B(A) = \{\prec (0.5, 0.5, 0.5) \succ\}$. Then the collection $\tau_N(A) = \{0_{\sim}, \prec (0.4, 0.5, 0.5) \succ, \prec (0.6, 0.5, 0.5) \succ,$*

$\prec (0.5, 0.5, 0.5)\succ, 1_{\sim}\}$. Then $\lambda = \prec (0.2, 0.3, 0.4)\succ$. Neutrosophic micro open $= \lambda_N(A) = \{0_{\sim}, \prec (0.2, 1, 0.4)\succ, \prec (0.2, 1, 0.5)\succ, \prec (1, 0.3, 0)\succ, \prec (0.4, 0.5, 0.5)\succ, \prec (0.4, 0.5, 0.4)\succ, \prec (0.6, 0.5, 0.5)\succ, \prec (0.6, 0.5, 0.4)\succ, \prec (0.5, 0.5, 0.5)\succ, \prec (0.5, 0.5, 0.4)\succ, 1_{\sim}\}$ is a neutrosophic micro topology on U and neutrosophic micro closed $= (\lambda_N(A))^C = \{1_{\sim}^C, \prec (0.4, 0, 0.2)\succ, \prec (0.5, 0, 0.2)\succ, \prec (0, 0.7, 1)\succ, \prec (0.5, 0.5, 0.4)\succ, \prec (0.4, 0.5, 0.4)\succ, \prec (0.5, 0.5, 0.6)\succ, \prec (0.4, 0.5, 0.6)\succ, \prec (0.5, 0.5, 0.5)\succ, \prec (0.4, 0.5, 0.5)\succ, 0_{\sim}^C\}$ is also a neutrosophic micro topology on U .

We know that $0_{\sim} = \{\prec x, 0, 0, 1 \succ : x \in U\}$, $1_{\sim} = \{\prec x, 1, 1, 0 \succ : x \in U\}$ and $0_{\sim}^C = \{\prec x, 1, 1, 0 \succ : x \in U\}$, $1_{\sim}^C = \{\prec x, 0, 0, 1 \succ : x \in U\}$.

Here $A = \prec (0.2, 1, 0.4)\succ$. $N_{mi-int}(A) = \prec (0, 1, 1)\succ \vee \prec (0.2, 0, 0.4)\succ \vee \prec (0.2, 1, 0.4)\succ \vee \prec (0.2, 1, 0.5)\succ \vee \prec (0.2, 0.3, 0.4)\succ \vee \prec (0.2, 0.5, 0.5)\succ \vee \prec (0.2, 0.5, 0.4)\succ \vee \prec (0.2, 0.5, 0.5)\succ \vee \prec (0.2, 0.5, 0.4)\succ \vee \prec (0.2, 0.5, 0.5)\succ \vee \prec (0.2, 0.5, 0.4)\succ = \prec (0.2, 1, 0.4)\succ \vee \prec (0.2, 0.5, 0.4)\succ \vee \prec (0.2, 0.5, 0.4)\succ = \prec (0.2, 1, 0.4)\succ \vee \prec (0.2, 0.5, 0.4)\succ = \prec (0.2, 1, 0.4)\succ$.

Therefore $N_{mi-int}(A) = \prec (0.2, 1, 0.4)\succ$.

Hence A is a neutrosophic micro open set if and only if $N_{mi-int}(A) = A$.

Similarly, A is a neutrosophic micro closed set if and only if $N_{mi-cl}(A) = A$.

Theorem 3.7. $(U, \tau_N(F), \lambda_N(F))$ be a neutrosophic micro topological space with respect to F where F is a neutrosophic subset of U . Let A be a neutrosophic subset of U . Then

- (1) $1_{\sim} - N_{mi-int}(A) = N_{mi-cl}(1_{\sim} - A)$.
- (2) $1_{\sim} - N_{mi-cl}(A) = N_{mi-int}(1_{\sim} - A)$.

Remark 3.8. Taking complements on either side of (1) and (2) of Theorem 3.7, we get $(N_{mi-int}(A)) = 1_{\sim} - N_{mi-cl}(1_{\sim} - A)$ and $(N_{mi-cl}(A)) = 1_{\sim} - (N_{mi-int}(1_{\sim} - A))$.

Definition 3.9. A map $f : (U, \tau_N(F), \lambda_N(F)) \rightarrow (L, \tau_N'(F), \lambda_N'(F))$ is called neutrosophic micro-continuous if $f^{-1}(V)$ is a neutrosophic micro closed set of $(U, \tau_N(F), \lambda_N(F))$ for every neutrosophic micro closed set V of $(L, \tau_N'(F), \lambda_N'(F))$.

Theorem 3.10. A map $f : (U, \tau_N(F), \lambda_N(F))$ and $(L, \tau_N'(F), \lambda_N'(F))$ are neutrosophic micro topological spaces. Then

- (1) Identity map from $(U, \tau_N(F), \lambda_N(F))$ to $(L, \tau_N'(F), \lambda_N'(F))$ is a neutrosophic micro continuous function.
- (2) Any constant function which maps from $(U, \tau_N(F), \lambda_N(F))$ to $(L, \tau_N'(F), \lambda_N'(F))$ is a neutrosophic micro continuous function.

Proof. The proof is obvious. □

Theorem 3.11. Let $f : U \rightarrow L$ be a map on two nms $(U, \tau_N(F), \lambda_N(F))$ and $(L, \tau_N'(F), \lambda_N'(F))$. Then the following statements are equivalent:

- (1) f is neutrosophic micro continuous function.
- (2) $f^{-1}(V)$ is an neutrosophic micro closed set for each neutrosophic micro closed set V in L .
- (3) $N_{mi-cl}(f^{-1}(V)) \leq f^{-1}(N_{mi-cl}(V))$ for $V \in L$.
- (4) $N_{mi-cl}(f(A)) \geq f(N_{mi-cl}(A))$ for $A \in U$.
- (5) $N_{mi-int}(f^{-1}(A)) \geq f^{-1}(N_{mi-int}(A))$ for $A \in U$.

Proof. (1) \Rightarrow (2) Let V be a neutrosophic micro closed in L . Then $f^{-1}(V)^C = f^{-1}(V^C)$ neutrosophic micro closed in U .

(2) \Rightarrow (3) $N_{mi-cl}(f^{-1}(V)) = \min \{G : f^{-1}(V) \leq G, G^C \text{ is neutrosophic micro closed}\} \leq \min \{f^{-1}(G) : V \leq G, G^C \text{ is neutrosophic micro closed in } L\} = f^{-1}(\{G : V \leq G^C \text{ is neutrosophic micro closed in } L\}) = f^{-1}(N_{mi-cl}(V))$.

(3) \Rightarrow (4) Since $A \leq f^{-1}(f(A))$, then $N_{mi}\text{-cl}(f(A)) \leq N_{mi}\text{-cl}(f^{-1}(f(A))) \leq f^{-1}(N_{mi}\text{-cl}(f(A)))$. Therefore $f(N_{mi}\text{-cl}(A)) \leq N_{mi}\text{-cl}(f(A))$.

(4) \Rightarrow (5) $f(N_{mi}\text{-int}(f^{-1}(A)))^C = f(N_{mi}\text{-cl}(f^{-1}(A))^C) = f(N_{mi}\text{-cl}(f(A)^C)) \leq N_{mi}\text{-cl}(f(f^{-1}(A^C))) \leq N_{mi}\text{-cl}(A^C) = (N_{mi}\text{-int}(A))^C$. This implies that $N_{mi}\text{-int}(f^{-1}(B))^C \leq f^{-1}(N_{mi}\text{-int}(A))^C = (f^{-1}(N_{mi}\text{-int}(A)))^C$. Taking complement on both sides, $f^{-1}(N_{mi}\text{-int}(A)) \leq N_{mi}\text{-int}(f^{-1}(A))$. \square

CONCLUSION

Neutrosophic set is a general formal framework, which generalizes the concept of classic set, fuzzy set, interval valued fuzzy set, intuitionistic fuzzy set, interval intuitionistic fuzzy set, nano set, interval valued nano set, intuitionistic nano set, and interval intuitionistic nano set. Since the world is full of indeterminacy, the neutrosophic micro topology found its place into contemporary research world. Hence this neutrosophic micro topological spaces can also be extended to a neutrosophic spatial region.

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