A Note on Basic Proof of Some Famous Mathematical Theorem and Its Illustration

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Abstract

There are several mathematical theorem and other equation which is used frequently. However many researchers or scholar unable to prove them mathematically. One of the famous example is Pythagrous theorem, Budhayana, Pingala, Fibonacci series or even $(a+b)^2=a^2+b^2+2ab$. It is indeed requirement to understand the basic proof of thiese mathematical theorem and its contradictory. This paper tried to provide some basic proof for these famous theorem and its relations with existing approaches for various applications.

Keywords: Geometry; Knowledge representation; Theorerm; Proof; Mathematics.

1. Introduction

It is well known that Scopus does not contain linguistic diverse papers which are published before 1800 [1]. Hence the research done before the Scopus indexed tradinional analysis rather than time based [2]. In this regard the inventions done around the world and its proof is another issue for the research communities. The Pingala can be considered as one of the founding researcher for binary number system based on laghu and guru concept. It is based based on music as well as regional division of given geometry decirbed via circle [3]. The Budhayana already given the concept of rope and its extension which later discover as PythaGarous Theoream. However the issue arises when researcher do not the applications of 0 and its algebraic properties. It is invented by Aryabhat and used by Brahamgupta. The same was used by Nagrajuna for dealing human concisousness as Sunyata Theory. These all need mathematical revision and proof for basic understanding [4]. The reason is many researchers came to know about these geometry and its complement when Lobachevsky and Russel Bertrand objected the Euclid geometry concept [5-7]. It is totally based on human turiyam awareness that which mathematical logic is invented as first with proof rather than knowledge [8]. It can be observed that the 1+1=2 proof given by Russel in 300 pages nearly 19th century at Principla mathematician [9]. It means how Indian

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mathematician like AryaBhatt or other researchers engaged people without knowing the Proof [10]. These questions was recently addressed by Raju [11]. It is discussed that the researcher who discover the idea can be considered as founder or the one who provided the proof of theorem. To understand the issue author revisited some of the mathematical concept and its proof for further applications.

The linguistic word Geometry came from Sanskrit Jamiti. The Baudhayana Shulba sutra provided the construction of geometric shapes such as squares and rectangles in 1st-millennium BCE [12]. The Pythagoras (570 – c. 495 BC) given the equation which related the length of legs (a) and (b) with the hypotenuse (c) i.e. $a^2+b^2=c^2$ without proof. It is discussed that the Pythagoras theorem is similar to Baudhayana rope geometry [13-14]. Same time Baudhayana given several sutra for Geometry in Shulba Sutras or Śulbasūtras (Sanskrit: शुल्बसूत्र; śulba: "string, cord, rope"). It become more useful when the Pingala started concept of laghu and Guru in 3rd and 2nd century BC [15-16]. The Pingala given calculating the number of combinations of short (laghu) and long (guru) sounds (or syllable patterns) in a given poetical composition in Chandaḥśāstra. The music of sound variations (varnasangita) introduced Pingala series when laghu and guru combination. These all given long back to Euclid which considered as "founder of geometry" based on five postulates which failed in case of hyberbolic and other Geometry as discussed by Lobachevsky. The concept of geometry and its numerical system become strong when the zero is defined [17-18]. The zero word which is used today derived from Sanskrit word Sunya and Arabic word sifr (meaning 'vacuum' or equivalently 'nothing'). It is noticed that the Chandah-sutra of Pingala uses zero or śūnya symbol rather than its algebraic uses. It become more interesting when the Nāgārjuna established the Sunyata theory and its possibility as:

(i) All things (dharma) exist: affirmation of being, negation of non-being,

(ii) All things (dharma) do not exist: affirmation of non-being, negation of being,

(iii) All things (dharma) both exist and do not exist: both affirmation and negation, and

(iv) All things (dharma) neither exist nor do not exist: neither affirmation nor negation.

This theory given the concept of non-dualism which later motivated the fuzzy or vague concepts. Same time the Sanskrit is considered as one of the prominent language for knowledge representation[19]. It is later discovered by Aryabhata as numerical representation of 0 in 4th CE [20-21]. The BrhamaGupta in 6th century uses the algebra of 0 and its addition. Dasagitka describes an alphabetic scheme in Sanskrit for representing the numerals based on distinguishing between classified Mathematics (varga), unclassified (avarga) consonants and vowels [22-23]. The vargas fall into five phonetic groups: ka-varga (guttural), ca-varga (palatal), ta-varga (lingual), ta-varga (dental) and pa-varga (labial). Each group has five letters associated to it where the letters run from k to m in the Sanskrit alphabet [24-25]. In this way the numbers from 1 to $5 \times 5 = 25$ numbers represented. The Sanskrit letters y to h consist of seven avargas of semi-vowels and sibilants representing numerical values 30, 40, 50,..., 90; and the eighth avarga used as a means of extending to the next place value [26-28]. The 10 vowels denoted successive integral powers of 10 from 100 onwards. This provided mathematical representation of any number as described in Upanishads as well as discussed in Bakhshali text [29-30]. It represents the addition as 'plus' (+)

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symbol used to denote addition or a positive quantity. It means Purna+Purna=Purna, infinity+Infinity=Infinity, a+0=a etc. The problem arises while representation of negative or subtraction like Purna-a, infinity − infinity etc. The Brahmagupta initiated a discussion on the four fundamental operations with zero, gave rules for operations of negative numbers ("debts") as well as surds. It is represented as Purna − Purna = Purna i.e. 0-0=0, Purna-a=-a, or infinity − infinity = infinity. This given a way to represent the concept of linear equation and quadratic equation. The problem arises while investigating the proof for some of these mathematical equations. Lobachevsky and Bolyai proprosed that there are many data sets which cannot be represented by Euclidean Postulates. Same time Russel opposed several postulates of Euclid as well as proof of Pythagoras. Same time Russel claim the proof of 1+1=2 in 300 pages. It tooks almost 2000 years to challenage the mathematical equations and its proof. To resolve this issue Cantor started first conference in mathematics in 1890. This paper tried to provide some basic proof of famous mathematical equations.

The paper is organized as follows: The Section 2 provides some of the problem addressed in this paper. The Section 3 provides the illustrations and Proof for the selected problem followed by Conclusions, Acknowledgements and references.

2. Problem Descritpion

There are several basic problem which need to revisit and require a Proof as given below:

(i) What is time and its measurement?

(ii) What is Shunya or Zero?

(iii) What is infinite Series?

(iv) Pingala Series or Fibonacci?

(v) Proof of Pythagoras or Budhayana Rope stretch $a^2+b^2=c^2$?

(vi) $(a+b)^2=a^2+ b^2- 2ab$.

(vii) $(a-b)^2=a^2+ b^2- 2ab$.

(viii) Quadratic equation Proof?

(ix) Geometrical proof of $\sqrt{2}$ is rational or irraltion.

(x) Euclid geometry failure?

(xi) 1+1=2 Proof?

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In the next section each of the problem and its solution is given.

3. The mathematical of geometrical Proof for mentioned problems

In this section, proof of all the problem shown in section 2 is given as below:

(i) What is time and its measurement?

Answer: It is defined in Siddhantasiromani by Bhaskara II who defines the time as blink of eyes. The blink of eyes is same for all the human being. It is called as nimesha by Siddhantasiromani. In this way everyone took 1/972000 blink almost in a day. It can be computed as 89 milliseconds. In similar way , divide it further till truti. It is a unit of time equal to 1/2916000000 th of a day!. The author of Surya Siddhanta defines time as of two types: the first which is continuous and endless, destroys all animate and inanimate objects and second is time which can be known. This latter type is further defined as having two types: the first is Murta (Measureable) and Amurta (immeasureable because it is too small or too big). The time Amurta is a time that begins with an infinitesimal portion of time (Truti) and Murta is a time that begins with 4-second time pulses called Prana. In this way the division of time further generated the concept of Shunyata or Shunya or Zero.

(ii) What is Shunya or Zero?

Answer: The philosophical interpretation by Nagrajuna or Budhism concept is that the condition when the positive conscious and negative consciousness meet each other at a point. It is like Purna consciousness. It is the case nothing exists, no positive, no negative and one can extract infinite number of purna for the given consciousness can be called as Shunya conditions. It is nothing, void, or maximual level of consciousness. This given the concept of infinity to define.

(iii) What is infinite Series?

Answer: The crucial step in Nilakantha's argument for the derivation of this infinite convergent geometric series is using regions of circle as shown in Figure 1

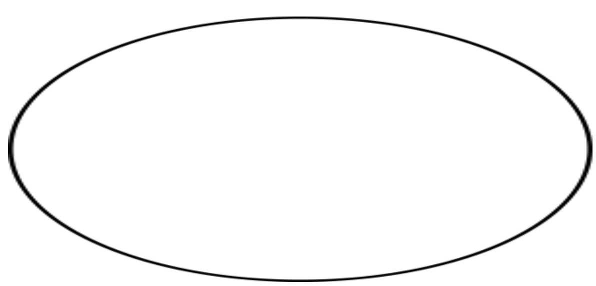

Figure 1: The circle with region one as r^1

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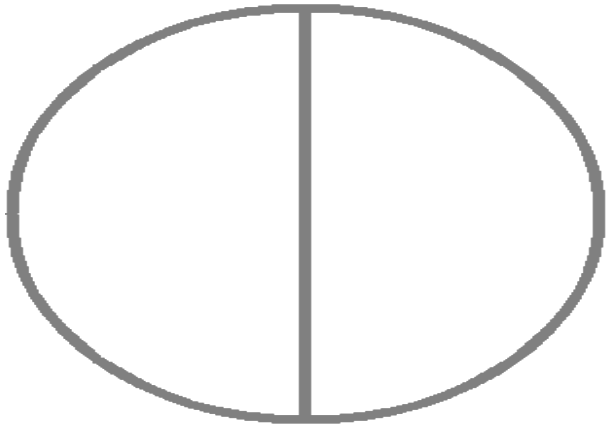

Figure 2: The circle with two region as r^2

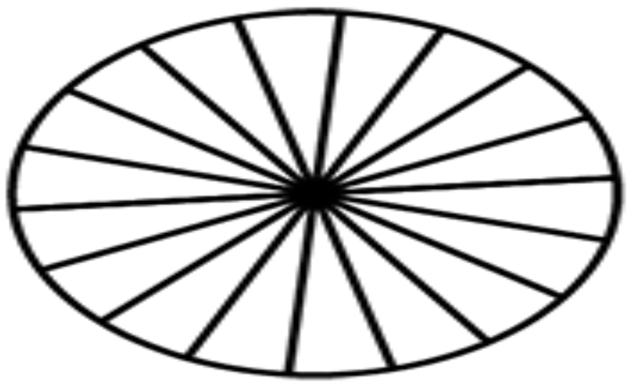

Figure 3: The circle with n finite regions can be drawn as r^k

$1 + r + r^2 + r^3 + \cdots + r^k$ which can be computed for infinite series as discussed in Yuktibhasa.

(iv) **Pingala Series or Fibonacci:**

The *Chandaḥśāstra* provided the first evidence about binary numeral system. It provides a systematic enumeration of patterns using the short (laghu) and long (guru) syllables. The *meruprastāra* and *mātrāmeru* contains the Pingala's work which represents the series for computing the binomial expansion $(a+b)^n$ as shown in Figure 4.

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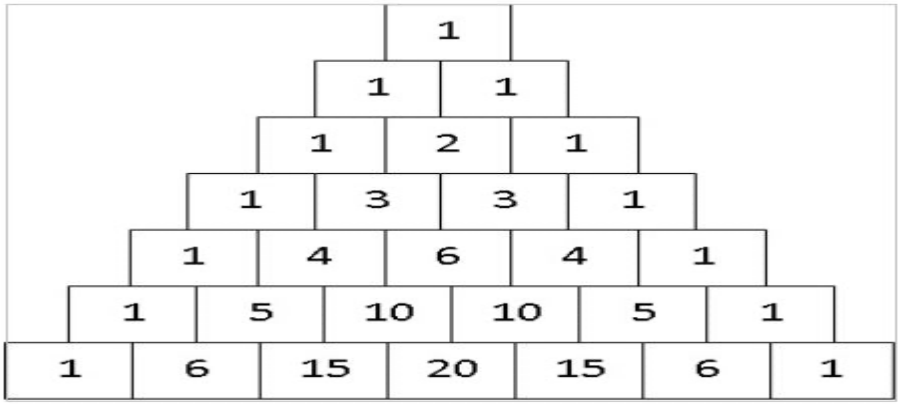

Figure 4: The Pingala Series decribed in mātrāmeru

It is like adding same square of given length to build a similar square as shown in Figure 5. It given a way to prove the binomial expansion as discussed next.

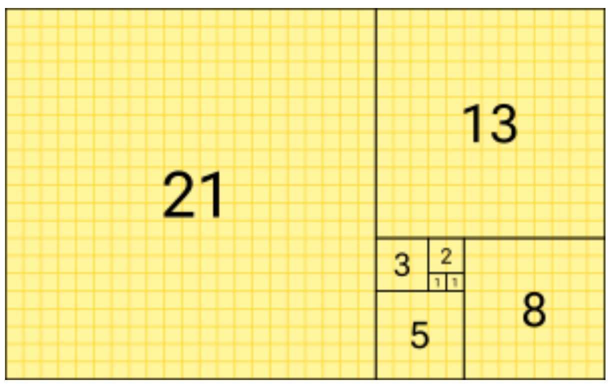

Figure 5: The Pingala Series in Geometry

(v) The proof for binomial expression $(a+b)^2=a^2+b^2+2ab$

It can be proved via Pingala Series considering a as guru and b as laghu. It can be repreented via different combination of the binomial expansion as $(a+b)^2=a^2+2ab+b^2$

Or $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.

It can be written for a four-syllabic metre as different combinations of the two sounds i.e. $(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

It can be easily proved via connecting the pingala series shown in Figure 4 like (1, 2,1) means can be used for $(a+b)^2=a^2+2ab+b^2$. Similarly (1, 3, 3, 1) of Pingala Series shown in Figure 4 can be used for $(a + b)^3 = a^3 +$

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$3a^2b + 3ab^2 + b^3$ and (1, 4, 6, 4, 1) can be used to represent $(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$. In Similar way other can be represented. However it can be proved via using the Square also as shown in Figure 6.

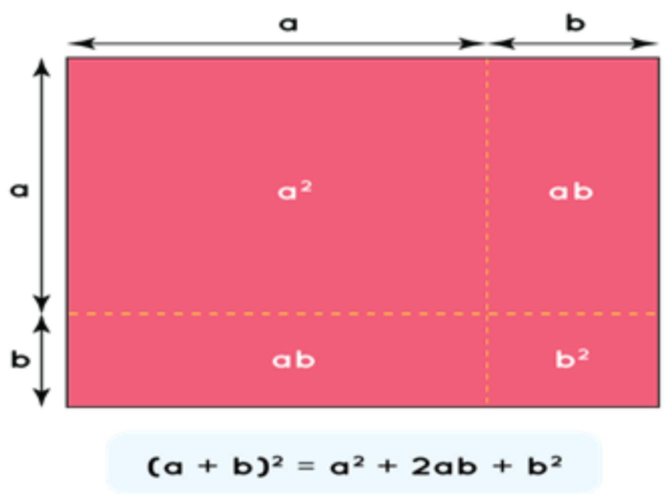

Figure 6: The geometrical proof of $(a+b)^2=a^2+2ab+b^2$

The binomial expansion given a way to think about $a^2+b^2=c^2$

(vi) Proof of $a^2+b^2=c^2$ Budhayana Shulba Sastra Rope Geometry or Pythagoras ?

The *Baudhāyana Sulba Sūtra* given a way to use the formula $a^2+b^2=c^2$ before 1800 BC. It is used in many Indian Homes as Rope Geometry. It can be understood by Sanskrit Poem:

दीर्घचतुरश्रस्याक्ष्णया रज्जु: पार्श्र्वमानी तिर्यग् मानी च यत् पृथग् भूते कुरूतस्तदुभयं करोति ॥

dīrghachatursrasyākṣaṇayā rajjuḥ pārśvamānī, tiryagmānī, cha yatpṛthagbhūte kurutastadubhayāṅ karoti.

It represents that a rope stretched along the length of the diagonal produces an area which the vertical and horizontal sides make together. Similarly, Apastamba's rules for constructing right angles in fire-altars use the following triples: (3, 4, 5) (5, 12, 13), (8, 15, 17) etc. It is almost equivalent to the Pythagorean theorem as Shown in Figure 7.

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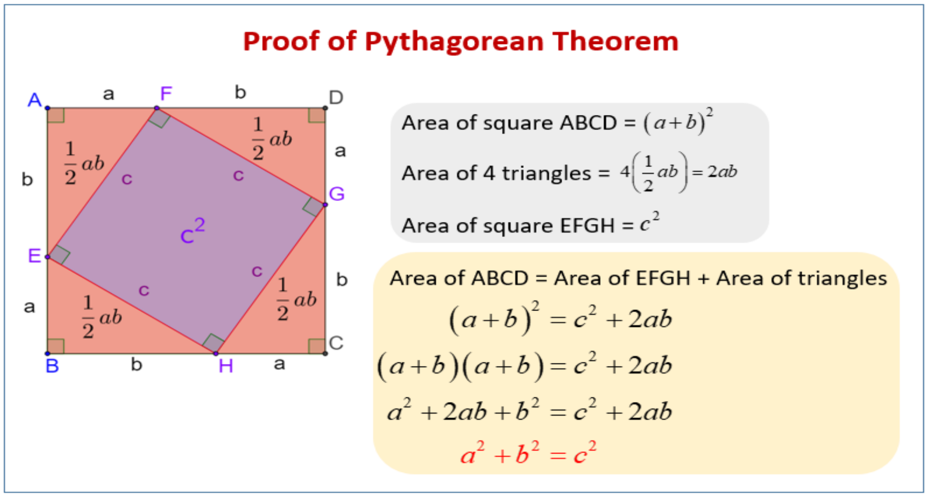

Figure 7: The geometrical proof $a^2+b^2=c^2$

(vii) **$(a-b)^2=a^2+b^2-2ab$. It can be proved geometrically as shown in Figure 8.**

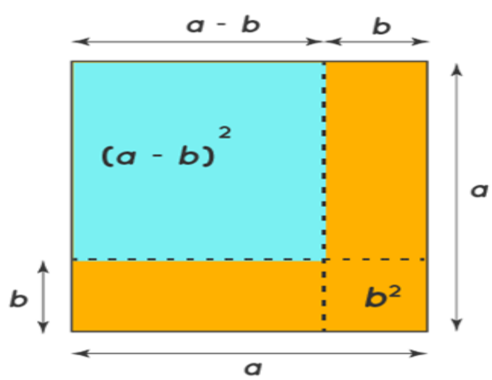

Figure 8: The geometrical proof of $(a-b)^2=a^2+b^2-2ab$

(viii) **Quadratic equation i.e. $ax^2+bx+c=y$ Proof ?**

Bhaskaracharya's Bijaganita contains problems on determining unknown quantities, evaluating surds (i.e. square roots that cannot be reduced to whole numbers), solving simple and quadratic equations, and some general rules which went beyond Sridhara in dealing with the solution of indeterminate equations of the second degree and even equations of the third and fourth degree. It is discussed that any quadratic equation can be written in form of Sqaure as shown in Figure 9.

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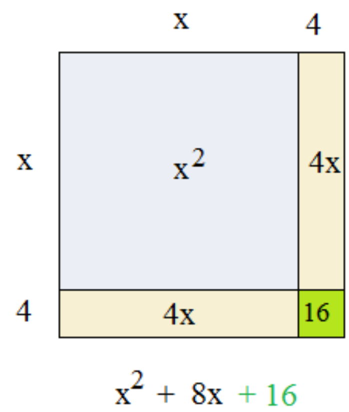

Figure 9: The geometrical representation of $x^2 + 8x + 16$

Its troot can be investigated using the geometry shown in Figure 10 as per Bhaskaracharya. It can be solved via intermediate steps as shown in Figure 11 and Figure 12.

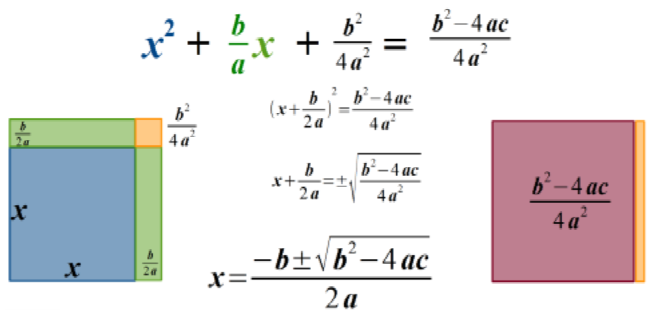

Figure 10: The geometrical representation of $ax^2 + bx + c$ and its solution

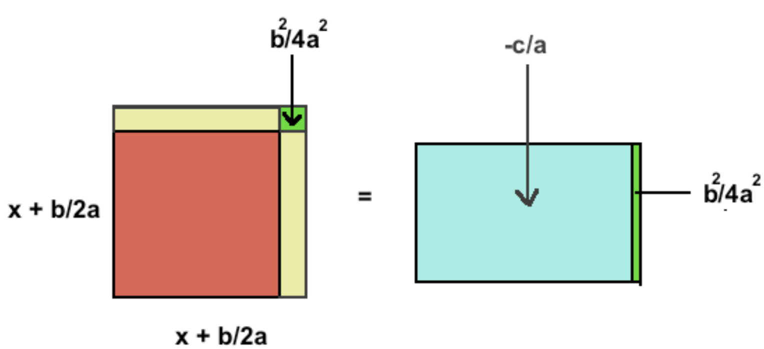

Figure 11: The first Intermediate Steps for solving the quadratic equation $ax^2 + bx + c$

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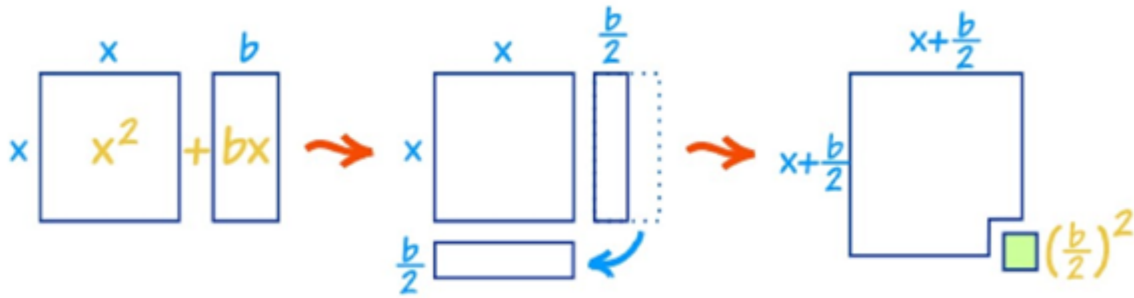

Figure 12: The Second Intermediate Steps for solving the quadratic equation $ax^2+ bx+ c$

This give a new area to find root of x or $\sqrt{2}$ is rational or irrational. It is proved in the next step.

(ix) $\sqrt{2}$ is rational or irrational number prove it?

It can be proved via contradiction. Let us suppose that, there exists some positive integer a and b such that $\sqrt{2}=a/b$ which provides $a^2=2b^2=b^2+b^2$. In geometric way this means that there is an integer-by-integer square i.e. a^2whose area is twice the area of another integer-by-integer b^2 square as shown in Figure 13. It is assumed that a is smallest integer which satisfy this condition.

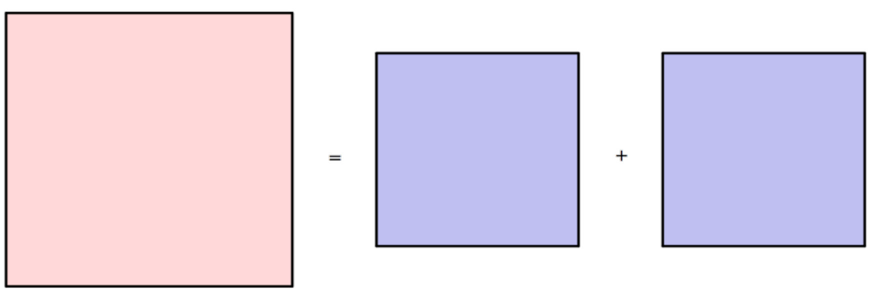

Figure 13: The Integer by Integer Square left Side a^2 and right side b^2

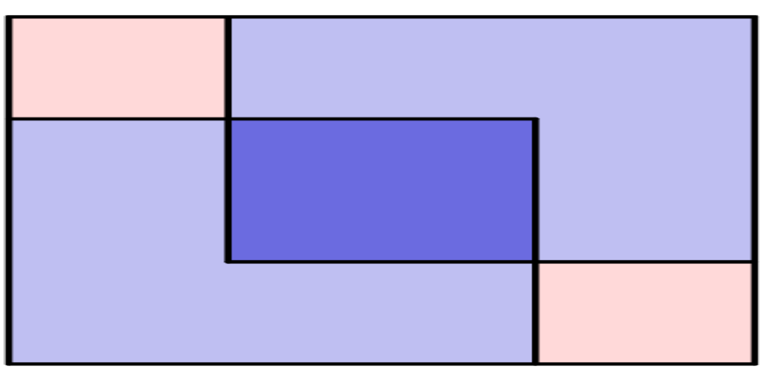

Figure 14: The b^2 square insertion in large square a^2

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It can be observed that while keeping small squate b^2 in a^2 they will overlap as shown in Figure 14. This overlap can be represented geometrtically as shown in Figure 15.

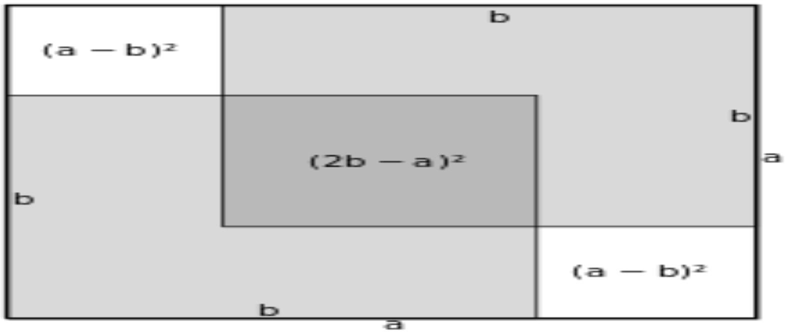

Figure 15: The geometrical insertion of a^2 and b^2

It is assumed in Figure 15 that the sum of the areas of the two b^2 squares is the area of the large square a^2. It means that the overlap area $(2b\text{-}a)^2$ in the center must have the same area as the two uncovered squares. However the overlap square and the small squares have integer sides. It contradicts the assumption that a^2 is smallest such square. It means that $\sqrt{2}$ is irrational. In this way it gives approval for Geometry, Sqaure and other shape to explore.

(x) Euclid geometry failure?

There are no axiomatic proofs for the Euclid**.** The problem raised when there is zero unknown by the world. Howcome the geometry and Cartesian product postulates started. This fact is admitted publically by Bertrand Russell as well as David Hilbert in eighteen century. However the Sulbsutras contain instructions for the construction of sacrificial altars (vedi) and for the location of sacred fire (agni) that had to conform to clearly laid down requirements regarding their shapes and areas if they were to be effective instruments of sacrifice. Square and circular altars were usually sufficient for household rituals, while more elaborate altars of shapes consisting of combinations of rectangles, triangles, and trapeze were required for public worship. A more elaborate public altar was shaped like a giant falcon about to take flight (vakrapaksa-syenacit). The first failure of Euclid is given by Lobachevsky and Bolyai called as Non-Euclidean Geometry as shown in Figure 16. This given an attention to find the Proof of 1+1=2 also which is discussed in next step.

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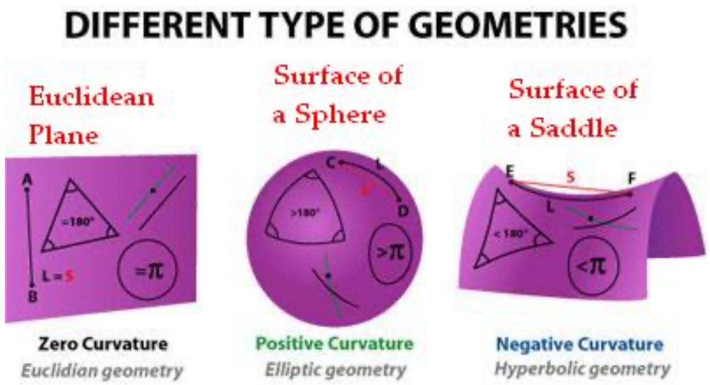

Figure 16: The different type of Non-Euclidean Geometry

(xii) 1+1=2 Proof?

The natural number one and its representation done via Ishango bone as shown in Figure 17 which is believed to have been used 20,000 years ago. It is shown on exhibition at the Royal Belgian Institute of Natural Sciences. The problem arises when its Arithmetic like Addition 1+1?

Figure 17: The representation of natural number via Ishango bone

It took many years to prove 1+1=2. The Principle Mathematia provides 300 pages to prove this equality as shown in Figure 18.

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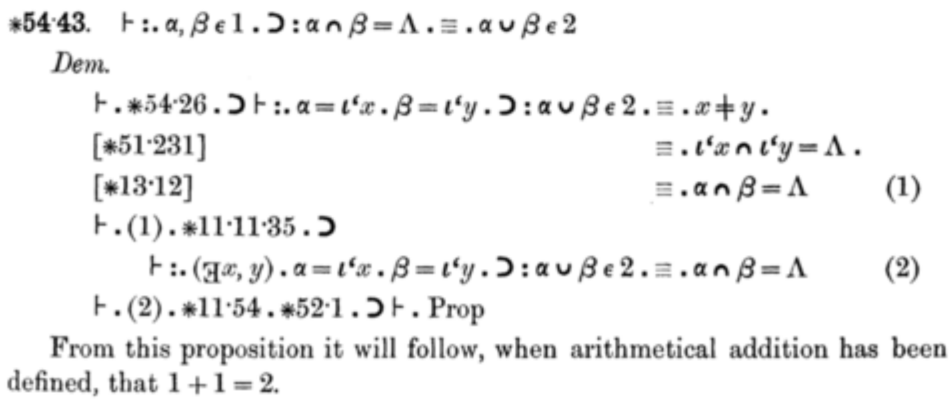

***54·43.** $\vdash :. \alpha, \beta \in 1 . \supset : \alpha \cap \beta = \Lambda . \equiv . \alpha \cup \beta \in 2$

Dem.

$\vdash . *54{\cdot}26 . \supset \vdash :. \alpha = \iota`x . \beta = \iota`y . \supset : \alpha \cup \beta \in 2 . \equiv . x \neq y .$

$[*51{\cdot}231] \qquad \equiv . \iota`x \cap \iota`y = \Lambda .$

$[*13{\cdot}12] \qquad \equiv . \alpha \cap \beta = \Lambda \qquad (1)$

$\vdash . (1) . *11{\cdot}11{\cdot}35 . \supset$

$\vdash :. (\exists x, y) . \alpha = \iota`x . \beta = \iota`y . \supset : \alpha \cup \beta \in 2 . \equiv . \alpha \cap \beta = \Lambda \qquad (2)$

$\vdash . (2) . *11{\cdot}54 . *52{\cdot}1 . \supset \vdash .$ Prop

From this proposition it will follow, when arithmetical addition has been defined, that $1 + 1 = 2$.

Figure 18: The Principle Mathematia proof of 1+1=2.

It is totally vague that to Proof 1+1=2 took more than 2000 years i.e. almost eighteen century. Same time the proof is given in 300 pages which is totally hard to understand. Another solution given at early of nineteen century as using the successor function. Let us suppose, the set of natural numbers (a) and define a successor function. The successor function (S) provides a next natural number for any given natural number. Now define the addition of natural numbers in recursive way using 0 as $a + 0 = a$ and $a + S(b) = S(a + b)$ for all natural number a, b .

Now natural number 1 is defined as per Figure 1 which can be represented by Successor function as $S(0)$, then $b + 1 = b + S(0) = S(b + 0) = S(b)$. It means the $b + 1$ is simply the successor of b. In this way it provide 1+1=2. It can be observed that the ($\mathbb{N}$, +) forms a commutative monoid having the identity element 0. It is a free monoid on one generator. This commutative monoid satisfies the cancellation property. Hence it can be embedded in a group. The smallest group containing the natural numbers is the integers.

5. CONCLUSIONS

This paper provides proof of some basic mathematical formula and its parallel investigation by various researchers around the world. It is believed that the current paper will be helpful for the young researchers while starting their research problem. In near future the author will try to explore some other mathematical problems and its proof.

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