Introduction to Intuitionistic Semigraph

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Abstract

In this paper, basic concepts of semigraph is introduced based on intuitionistic set. Definition of Intuitionistic Semigraph is introduced and Union, intersection, and complement of intuitionistic semigraph is studied with graph.

Keywords: Fuzzy set; Intuitionistic Set; Semigraph; Intuitionistic Semigraph.

1. Introduction

As a generalisation of crisp set, Zadeh introduces the fuzzy set. The concept of partial truth between absolute true and absolute false was established with the introduction of the fuzzy set. His proposal inspired a large number of scholars who worked in a variety of scientific and technological fields.

The link between vertices and edges is described by a graph, which is a mathematical description of a network. Graph theory is used to model real-life occurrences, yet many phenomena are not adequately represented by graphs due to the inherent uncertainty of certain system properties. The fuzzy graphs were inspired by a variety of real-world occurrences. Using Zadeh's [1] fuzzy relation, Kauffman [4] introduced fuzzy graphs (FG). Networking, communication, data mining, clustering, picture capturing, image segmentation, planning, and scheduling are just a few of the many applications of FG theory. Rosenfeld [3] proposed a fuzzy analogy for various graph theoretical concepts. Bhattacharya [7] went on to explain a few points about FGs later on. The complement of a FG was established by Sunitha and Vijayakumar[8, 9], as well as certain fuzzy tree characterisation.

Mordeson and Nair [10] came up with a set of FG. Bhutani and Battou [11] proposed M-strong FGs with certain properties. Types of arcs in a FG were defined by Mathew and Sunitha [12]. Operations FGs were defined by Mordeson and Chang-Shyh [13]. Nagoor Gani and Radha[14 - 17] discussed the properties of FGs in conjunction, regular FGs, FG sequences, and the degree of vertex in some FGs. Soft graphs, bipolar FGs, and hypergraphs are all concepts defined by Akram et al.[18].

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Intuitionistic Fuzzy Set (IFS) was first introduced by Atanassov [25]. Later on, he proposed the Intuitionistic FG theory. Karunambigai and Parvathi [26] introduced the intuitionistic FG as a particular example of Atanassov's IFG in their paper.

Arc analysis isn't very relevant in graph theory because all arcs are strong in the sense of [26]. However, in IFG, it is critical to understand the nature of arcs, and there is no such analysis in the literature.

Sampath Kumar [2] introduces the semigraph. The concept of semigraph is a broadening of the concept of graph. Many authors investigated semigraph after it was first introduced. Graphoidal Covers and Graphoidal Covering Number of a Graph were investigated by Acharya and E. Sampathkuma [29]. Graphoidal Covers of a Graph was examined by Armugam [30].

2. Preliminaires

Definition 2.1: [28]
Let X be a non-empty set. Then A is called an intuitionisticset (in short, IS) of X, if it is an object having the form $A = (A_T; A_F)$; such that $A_T \cap A_F = \phi$ where A_T is called the set of members and A_F *is* non-members of A.

Definition 2.2: [4]
A FG G = (σ, μ) is a pair of function $\sigma : V \to [0, 1]$, $\mu : V \times V \to [0, 1]$ with

$\mu(x, y) \leq \min\{\sigma(x), \sigma(y)\}$, $\forall$ x, y $\in$ V; $V \neq \emptyset$ and σ , μ are fuzzy subset of V and symmetric fuzzy relation on σ.

Definition 2.3: [2]
Let G = < V, E >be an IFG, where

(i) $V = \{v_1, v_2, \ldots, v_n\}$, where $\mu_1$and γ_1 are the degree of membership and non-membership functions from V to [0, 1]of the element $v_i$in V, and $0 \leq \mu_1 (v_1) + \gamma_1 (v_1) \leq 1$ $v_i \epsilon V$, $(i = 1,2, \ldots \ldots n)$,
(ii) $E \subseteq V \times V$ where$\mu_2 : V \times V \longrightarrow [0,1]$ and$\gamma_2 : V \times V \longrightarrow [0,1]$such that $\mu_2 (v_i, v_j) \leq min [\mu_1 (v_i), \mu_1 (v_j)], \gamma_2(v_i, v_j) \leq max [\gamma_1 (v_i), \gamma_1 (v_j)]$,
and $0 \leq \mu_2 (v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$ for every $(v_i, v_j) \epsilon E$, $(i,j = 1,2, \ldots \ldots n)$

Definition 2.4: [2]
A semigraph G is an order pair (V, X) where V is a nonempty set whose elements are called vertices of G, and X is a set of n-tuples, called edges of G, of distinct vertices, for various $n \geq 2$, satisfying the following conditions:

A semigraph is a type of graph G is an order pair (V, X) in which V is a nonempty set whose elements are termed vertices of G and X is a set of n-tuples, called edges of G, containing distinct vertices, for various $n \geq 2$, that satisfy the following conditions.

i. Any two edges have at most one vertex in common.
ii. Two edges $(x_1, x_2, x_3, \ldots\ldots, x_p)$ and $(y_1, y_2, y_3, \ldots\ldots, y_q)$ are considered to be equal iff a. $p = q$
and b. either $x_i = y_i$ for $1 \leq i \leq p$, or $x_i = y_{p-i+1}$ for $1 \leq i \leq p$.

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Thus the edge $E = (x_1, x_2, x_3, ..., x_p)$ is same as the edge $(x_p, x_{p-1}, ..., x_1)$ where x_1 and x_p are called the end vertices which are identified as dark point and $x_2, x_3, ..., x_{p-1}$ are called the middle vertices for E, represented as hollow circles, while the middle-end vertex is denoted by hollow circle with a small tangent drawn to it marking the end of its adjacent edge.

Definition 2.5: [2]
Adjacent vertices: [2]Two vertices in a semigraph G are said to be adjacent if they belong to the same edge and are said to be consecutively adjacent if in addition they are consecutive in order as well.
In a semigraph G, two vertices are said to be adjacent if they belong to the same edge, and sequentially adjacent if they are also adjacent in order.
Adjacent Edges:[2] Two edges E_i and E_j in a semigraph G are said to be adjacent if they have a common vertex.
When two edges E and M of a semigraph G have a common vertex, then they are said to be adjacent.

3. Main Results

3.1. Definition

An Intuitionistic Semigraph (I-Semigraph) $G_{IS} = (V, X, X^c)$ corresponding to a Semigraph $G = (V, X)$, where $V(\neq \phi)$ is the set of vertices and X, X^c are the sets of *n*-tuples, called edges of G_{IS} of distinct vertices $n \geq 2$, satisfying the following conditions:

IS(i): Any two edges have at most one vertex in common.

IS(ii): The edge $(v_1, v_2, ..., v_{n-1}, v_n)$ and $(v_n, v_{n-1}, ..., v_2, v_1)$ are the same edge of G_{IS}.

IS(iii): $(v_1, v_2, ..., v_n)^c \in X^c$, $\forall (v_1, v_2, ..., v_n) \in X$. By $(v_1, v_2, ..., v_n)^c$ we mean it is a complement of the edge $(v_1, v_2, ..., v_n)$. i.e. no two of v_k's $(1 \leq k \leq n)$ are adjacent.

The above definition is clarified with an example given below:

3.1. Example

Consider a Semigraph $G = (V, X)$ where $V = \{,2,3,4,5,6,7,8,9\}$ and $X = \{(1,2,3), (4,5,6,7), (7,8,3), (8,9)\}$, figure (Figure 1) is given below.

G_{SG}:

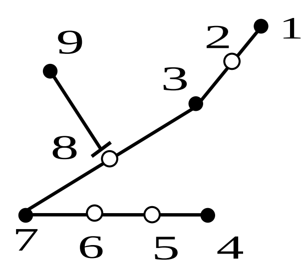

Figure 1

Then the I-Semigraph G_{IS} of *G* is describe as follows, the vertex set of G_{IS} is the same as of *G* and the edge set is

$\{(1,2,3), (1,2,3)^c, (4,5,6,7), (4,5,6,7)^c, (8,9), (8,9)^c\}$

$= \{(1,2,3), (1,9,7), (4,5,6,7), (2,4), (7,8,3), (7,2), (8,9), (8,5)\}$.

The figure (Figure 2) is given below.

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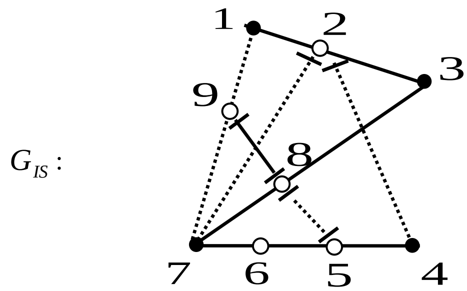

Figure 2

Note that I-Semigraph G_{IS} corresponding to a Semigraph G is not unique.

3.2. Definition

Complement of I-Semigraphs: The complement of I-Semigraph $G_{Is} = (V, X, X^c)$ is an I-Semigraph G_{Is}^C, whose vertex set is the same set V and the edge set is the combination of the sets X^c and $(X^c)^c = X$. Thus $G_{IS}^C = (V, X^c, X)$. The following example (**Figure 3**) will illustrate this fact.

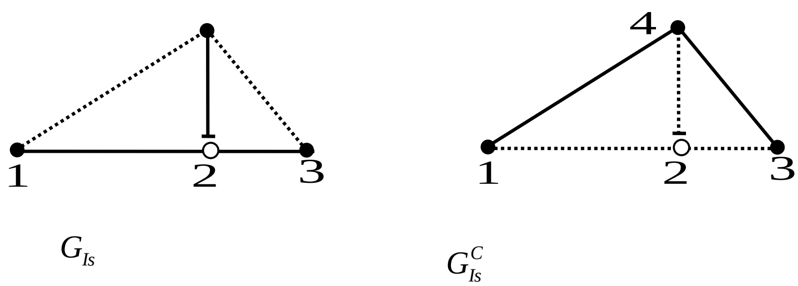

Figure 3

3.3. Definition

Union of two I-Semigraphs: The union of two I-Semigraphs $G_{Is}^1 = (V_1, X_1, X_1{}^c)$ and $G_{Is}^2 = (V_2, X_2, X_2{}^c)$ corresponding to the Semigraphs $G^1 = (V_1, X_1)$ and $G^2 = (V_2, X_2)$ respectively is the I-Semigraph $G_{IS}^1 \cup G_{IS}^2$, whose vertex set is $V_1 \cup V_2$ and the edge set is the combination of the sets $X_1 \cup X_2$ and $X_1^c \cup X_2^c$. An example is given below (**Figure 4**)

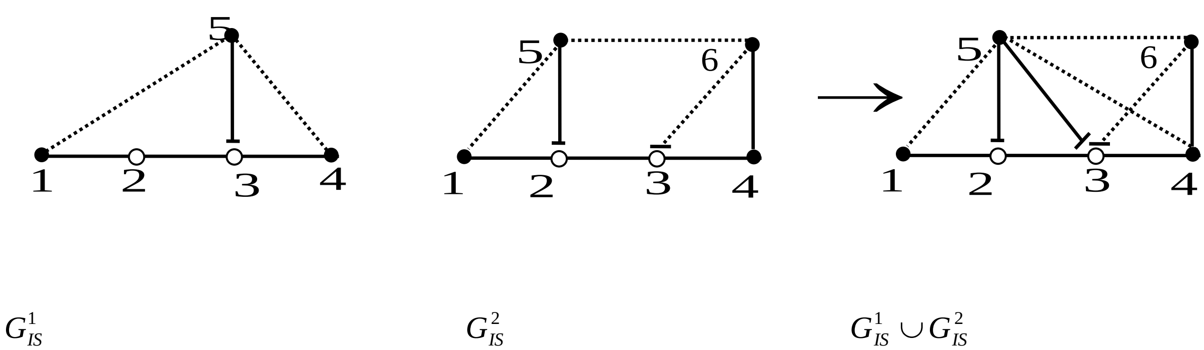

Figure 4

3.1. Proposition

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Let $G=(V,X)$, $G^1=(V_1,X_1)$ and $G^1=(V^2,X^2)$ be any three Semigraphs. Then, G_{IS}, G_{IS}^1 and G_{IS}^2 are I-Semigraphs corresponding to G, G^1 and G^2 respectively. We have,

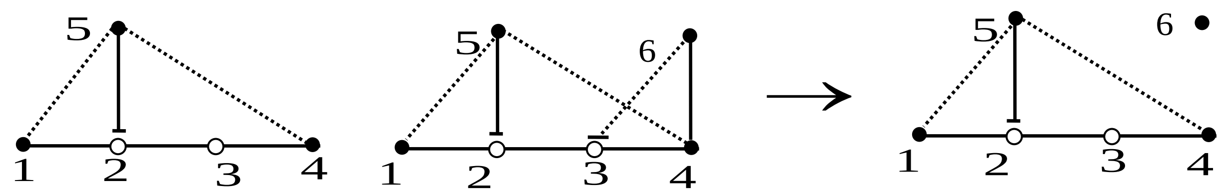
(i) For any I-Semigraph G_{IS}, $G_{IS} \cup G_{IS} = G_{IS}$

(ii) For any two I-Semigraphs G_{IS}^1 and G_{IS}^2, $G_{IS}^1 \cup G_{IS}^2 = G_{IS}^2 \cup G_{IS}^1$

(iii) For any three I-Semigraphs G_{IS}^1, G_{IS}^2 and G_{IS}^3, $G_{IS}^1 \cup (G_{IS}^2 \cup G_{IS}^3) = (G_{IS}^1 \cup G_{IS}^2) \cup G_{IS}^3$

3.4. Definition

Intersection of two I-Semigraphs: The intersection of two I-Semigraphs $G_{Is}^1 = (V_1, X_1, X_1{}^c)$ and $G_{Is}^2 = (V_2, X_2, X_2{}^c)$ corresponding to the Semigraphs $G^1=(V_1,X_1)$ and $G^2=(V_2,X_2)$ respectively is the I-Semigraph $G_{IS}^1 \cap G_{IS}^2$, whose vertex set is $V_1 \cap V_2 (\neq \phi)$ and the edge set is the combination of the sets $X_1 \cap X_2$ and $X_1^c \cap X_2^c$ An example is given below.



G_{IS}^1 G_{IS}^2 $G_{IS}^1 \cap G_{IS}^2$

Figure 5

3.2. Proposition

Let $G=(V,X)$, $G^1=(V_1,X_1)$ and $G^1=(V^2,X^2)$ be any three Semigraphs. Then, G_{IS}, G_{IS}^1 and G_{IS}^2 are I-Semigraphs corresponding to G, G^1 and G^2 respectively. We have,

(i) For any I-Semigraph G_{IS}, $G_{IS} \cap G_{IS} = G_{IS}$

(ii) For any two I-Semigraphs G_{IS}^1 and G_{IS}^2, $G_{IS}^1 \cap G_{IS}^2 = G_{IS}^2 \cap G_{IS}^1$

For any three I-Semigraphs G_{IS}^1, G_{IS}^2 and G_{IS}^3, $G_{IS}^1 \cap (G_{IS}^2 \cap G_{IS}^3) = (G_{IS}^1 \cap G_{IS}^2) \cap G_{IS}^3$

3.5. Definition

Difference of two I-Semigraphs: Before define the difference of any two I-Semigraphs we define the difference of any two Semigraphs.

The difference of two I-Semigraphs $G_{Is}^1 = (V_1, X_1, X_1{}^c)$ and $G_{Is}^2 = (V_2, X_2, X_2{}^c)$ corresponding to the Semigraphs $G^1=(V_1,X_1)$ and $G^2=(V_2,X_2)$ respectively is the I-Semigraph $G_{IS}^1 - G_{IS}^2$, whose vertex set is $V_1 - V_2 (\neq \phi)$ and the edge set is the combination of the sets $X_1 - X_2$ and $X_1^c - X_2^c$. An example is given below.

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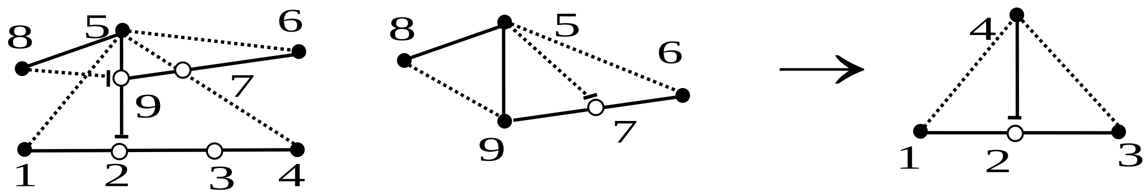


G_{IS}^1 G_{IS}^2 $G_{IS}^1 - G_{IS}^2$

Figure 6

3.3. Proposition

Let $G=(V,X)$, $G^1=(V_1,X_1)$ and $G^1=(V^2,X^2)$ be any three Semigraphs. Then, G_{IS}, G_{IS}^1 and G_{IS}^2 are I-Semigraphs corresponding to G, G^1 and G^2 respectively. We have,

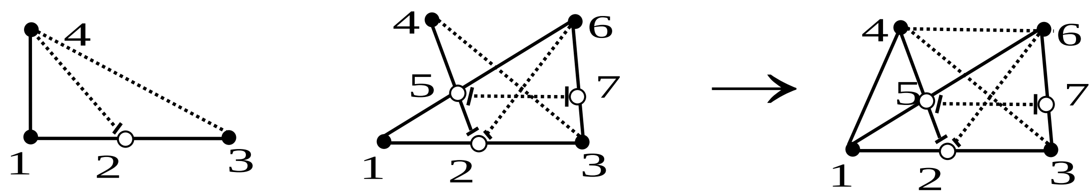
(i) For any two I-Semigraphs G_{IS}^1 and G_{IS}^2, $G_{IS}^1 - G_{IS}^2 \neq G_{IS}^2 - G_{IS}^1$ if $G_{IS}^1 \neq G_{IS}^2$

(ii) For any two I-Semigraphs G_{IS}^1 and G_{IS}^2, $G_{IS}^2 - G_{IS}^1 = G_{IS}^2 \cap (G_{IS}^1)^C$

(iii) For any three I-Semigraphs G_{IS}^1, G_{IS}^2 and G_{IS}^3, $G_{IS}^1 \cap (G_{IS}^2 \cap G_{IS}^3) = (G_{IS}^1 \cap G_{IS}^2) \cap G_{IS}^3$

3.6. Definition

Ring Sum of two I-Semigraphs: The ring sum of two I-Semigraphs $G_{IS}^1 = (V_1, X_1, X_1{}^c)$ and $G_{IS}^2 = (V_2, X_2, X_2{}^c)$ corresponding to the Semigraphs $G^1=(V_1,X_1)$ and $G^2=(V_2,X_2)$ respectively is the I-Semigraph $G_{IS}^1 \oplus G_{IS}^2 = (V_1 \cup V_2, |X_1 - X_2|, |X_1 - X_2|^c)$, where $|X_1 - X_2| = \{(X_1 \cup X_2) - (X_1 \cap X_2)\}$ *i.e.* the symmetric difference of the edges sets X_1 and X_2. And $|X_1 - X_2|^c$ is the complement of the set $|X_1 - X_2|$. An example is given below.



G_{IS}^1 G_{IS}^2 $G_{IS}^1 \oplus G_{IS}^2$

Figure 7

4. Conclusion:

In this paper basic concepts of intuitionistic semi graph is introduced. Union, intersection, complement of intuitionsticsemigraph is observed with examples. Also, some basic properties of intuitionistic semigraph is studied.

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