Neutrosophical dynamic programming

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Abstract:

The great development that science has witnessed in all fields has reduced the risks and losses resulting from undertaking any business or projects. Since the emergence of the science of operations research, many life issues have been addressed by relying on it, and by using its methods, we have been able to establish projects and businesses and use the available capabilities in an ideal manner. Which achieved great success in all areas and reduced the losses of all kinds, whether material or human, that we were exposed to because of carrying out these works or projects without prior study. We are now able to model, analyze and solve a wide range of problems that can be broken down into a set of partial problems using dynamic programming. Programming that is used to find the optimal solution in a multi-step situation that involves a set of related decisions. In this research, we study one of the operations research problems that are solved using dynamic programming. It is the problem of creating an expressway between two cities, using the neutrosophic logic. The logic that takes into account all the specific and non-specific data and takes into account all the circumstances that can face us during the implementation of the project. The goal of studying this issue is to determine the optimal total cost, which is related to the partial costs presented by the study prepared for this project. In order to avoid losses we will take the partial costs neutrosophic values of the form $[\lambda_{1i} , \lambda_{2i}]$, where λ_{1i} represents the minimum partial cost in stage i and λ_{2i} represents the upper limit of the partial cost in stage i . Through the indeterminacy offered by neutrosophic logic, we are able to find the ideal solution that will bring us the lowest possible cost for constructing this expressway. It takes into account all the circumstances that may encounter us in our study, and we will present an applied example that illustrates the study.

Keywords: Dynamic programming; neutrosophic logic; neutrosophic dynamic programming; the problem of constructing a expressway between two cities

1.Introduction

Dynamic programming is the method that helps in selecting the right decisions and approving the best program for independent activities, taking into consideration the available resources. We will use the concept of dimension, which is the criterion that affects the decision-making process at a certain stage of obtaining the optimal solution. If the criterion is the only one in making the decision, then the problem is called one-dimensional, otherwise the problem is multi-dimensional. As well as the concept of influencer

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which is the target follower. That is, if the problem contains more than one goal, then we call it a multi-factor problem [19,20,21].

In our study, we present the issue of creating an expressway between two cities based on dynamic programming within the framework of the neutrosophic logic, the logic that takes into account all possible cases and enables us to deal with data, whether specific or undefined, and taking into account all the circumstances that can face us during the implementation of the study.

It should be noted that the neutrosophic logic saw the light through its founder, the American Mathematical Professor Florentin Smarandache in 1995, where he presented it as a generalization of fuzzy logic, especially the intuitive fuzzy logic [1,2,3]. This interesting logic began to spread widely among researchers of various specializations with the passage of time [4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,22].

Our goal in this study is to determine the optimum total cost, which is related to the partial costs presented by the study prepared for this project in a neutrosophic manner, so that we avoid falling into any losses and complete the project at the lowest possible cost.

2. Discussion:

To formulate the dynamic program for a particular problem, we break it down into partial problems with a specific criterion according to the nature of the problem under study. In each issue we identify a set of possibilities from which a set of possible decisions branch.We define the measure of effectiveness in the form of cost, profit, time, or any other measure. We call the scale the goal function. We point out that the optimal decision in each partial problem is related to the decision that achieves the optimum value (the greatest profit - the least cost - the least time period) for the goal function in the previous problem. We consider the stage and condition resulting from the first problem as initial conditions.

In classical logic, when the problem of dynamic programming has one dimension and one influence, then we represent the dynamic programming method with the following relationships:

$$Z_i = F_i(x_i) \qquad i = 1,2,\ldots,n$$

where:

Z_i is the state function that expresses the state the problem has reached at a particular partial stage.

x_i is the decision variable in stage i.

The relationship between the case function Z_i (in a certain stage) and the case Z_{i-1} (in the previous stage) and the decision in stage i. It takes the following form:

$$Z_i = g_i(x_i, z_{i-1})$$

- The value of the influencer that we are looking for is the goal function G. We calculate it from the following relationship:

$$G_i(Z_i) = F_i\,(x_i, z_{i-1}, z_i)$$

We note that it is calculated according to the partial functions of the goal in the previous stages.

- This function should take an optimal value at each stage.

We calculate the optimal value by the relationship:

$$G_n^*(Z_n) = \underset{x_1 \to x_n}{opt}\left[F_1(x_1, z_0, z_1) + \ldots + F_1(x_n, z_{n-1}, z_n)\right]$$

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In addition to the general relationship of constraints:

$$Z_i = g_i\,(x_i, z_i)$$

Which can be expressed as follows:

$$G_n^*(z_n) = \underset{n}{\text{opt}}\left[F_n(x_n, z_n, z_{n-1}) + G_{n-1}^*(z_{n-1})\right]$$

Where:

$G_n^*(z_n)$: The ideal goal function at stage n.

$G_{n-1}^*(z_{n-1})$: The ideal goal function at stage n-1.

The previous relations take the following detailed form:

$$G_1^*(z_1) = \underset{x_1}{\text{opt}}\left[F(x_1, z_0, z_1)\right] \quad (1)$$

$$G_2^*(z_2) = \underset{x_2}{\text{opt}}\left[F(x_2, z_1, z_2)\right] \quad (2)$$

....

$$G_i^*(z_i) = \underset{x_i}{\text{opt}}\left[F(x_i, z_{i-1}, z_i)\right] \quad (3)$$

....

$$G_n^*(z_n) = \underset{x_n}{\text{opt}}\left[F(x_n, z_{n-1}, z_n)\right] \quad (4)$$

Restrictions for previous relationships, in order:

$$Z_1 = g_1\,(x_1, z_0) \quad (1)'$$

$$Z_2 = g_2\,(x_2, z_1) \quad (2)'$$

....

$$Z_i = g_i\,(x_i, z_{i-1}) \quad (3)'$$

....

$$Z_n = g_n\,(x_n, z_{n-1}) \quad (4)'$$

The previous relationships with their constraints include all issues of dynamic programming with one dimension and one influence.

If cost, profit or time are neutrosophic values then the issue becomes a matter of neutrosophic dynamic programming,

If the problem of neutrosophic dynamic programming has one dimension and one influence, then we represent the neutrosophic dynamic programming method with the following relations:

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$$Z_i = NF_i(x_i) \qquad i = 1,2,\ldots,n$$

where:

Z_i is the state function that expresses the state the problem has reached at a particular partial stage.

x_i is the decision variable in stage i.

- The relationship between the state function Z_i (in a certain stage) and the state function Z_{i-1} (in the previous stage) and the decision in stage i. It takes the following form:

$$Z_i = g_i(x_i, z_{i-1})$$

The value of the influencer we are looking for is the goal function G. We calculate it from the following relationship:

$$NG_i(Z_i) = NF_i\,(x_i, z_{i-1}, z_i)$$

We note that it is calculated according to the partial functions of the goal in the previous stages.

This function should have an optimal value at each stage.

We calculate the optimal value by the relationship:

$$NG_n^*(Z_n) = \underset{x_1 \to x_n}{opt}\left[NF_1(x_1, z_0, z_1) + \ldots + NF_1(x_n, z_{n-1}, z_n)\right]$$

In addition to the general relationship of constraints:

$$Z_i = g_i\,(x_i, z_i)$$

Which can be expressed as follows:

$$NG_n^*(z_n) = \underset{n}{opt}\left[NF_n(x_n, z_n, z_{n-1}) + NG_{n-1}^*(z_{n-1})\right]$$

Where:

$NG_n^*(z_n)$: The Neutrosophic ideal goal function at stage n.

$NG_{n-1}^*(z_{n-1})$: The Neutrosophic ideal goal function at stage n-1.

The previous relations take the following detailed form:

$$\mathrm{N}\,nG_1^*(z_1) = \underset{x_1}{opt}\left[NF(x_1, z_0, z_1)\right] \tag{1}$$

$$NG_2^*(z_2) = \underset{x_2}{opt}\left[NF(x_2, z_1, z_2)\right] \tag{2}$$

....

$$NG_i^*(z_i) = \underset{x_i}{opt}\left[NF(x_i, z_{i-1}, z_i)\right] \tag{3}$$

....

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$$NG_n^*(z_n) = \underset{x_n}{opt}\left[NF(x_n, z_{n-1}, z_n)\right] \quad (4)$$

Restrictions for previous relationships, in order:

$Z_1 = g_1 (x_1, z_0)$ (1)′

$Z_2 = g_2 (x_2, z_1)$ (2)′

……………

$Z_i = g_i (x_i, z_{i-1})$ (3)′

……………

$Z_n = g_n (x_n, z_{n-1})$ (4)′

The previous relationships with their constraints include all issues of dynamic programming with one dimension and one influence.

Application example:

It requires the establishment of an expressway between cities A and F. Where this expressway must pass through cities E, D, C, B. Therefore, it will consist of five sections. For each section, the cost of the various alternatives was studied and evaluated. These alternatives are represented with their costs on the attached figure. It is required to identify the road with the lowest cost to establish this expressway.

The costs are neutrosophic values .

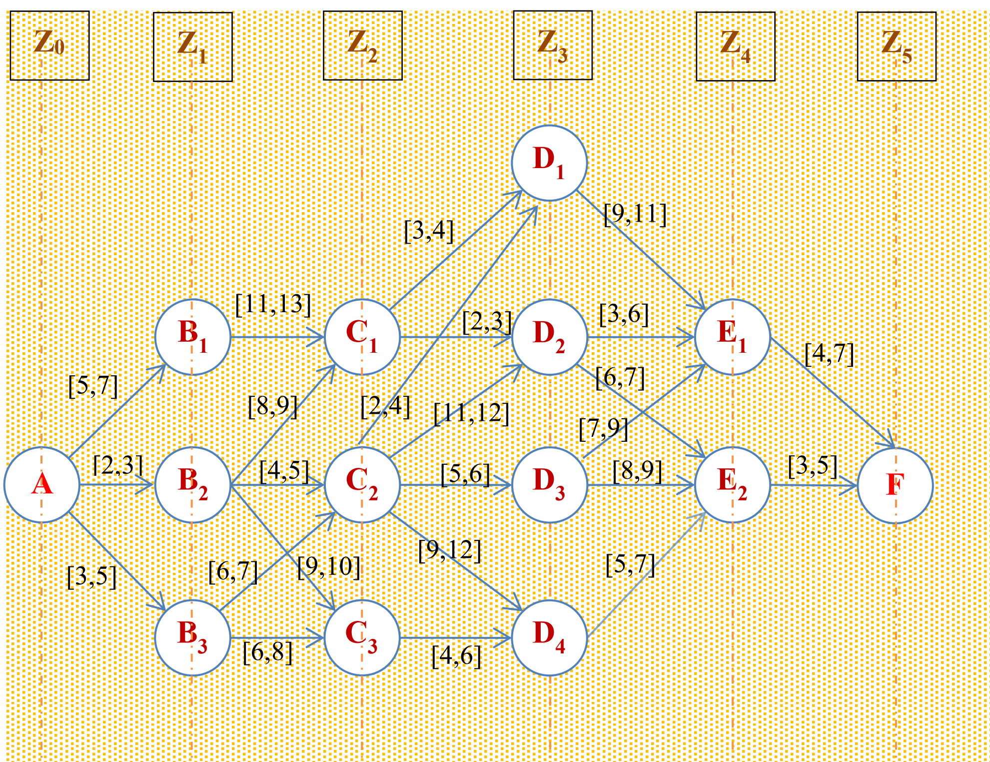

Figure 1. Application Example

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Z_i represents the state function, and it is the function that expresses the state the problem has reached at a particular partial stage, so:

Z_0 : A
Z_1: B_1 B_2 B_3
Z_2: C_1 C_2 C_3
Z_3: D_1 D_2 D_3 D_4
Z_4: E_1 E_2
Z_5: F
The relationship used in the solution:

$$NG_n(Z_n) = \text{Min}\ [NG_{n-1}(Z_{n-1}) + NF_n(Z_n, Z_{n-1})]$$

Where:

$NG_n(Z_n)$: The neutrosophic minimal cost of the fractional phase from A through Z_n.

$NG_{n-1}(Z_{n-1})$: The neutrosophic minimal cost of the fractional phase from A through Z_{n-1}.

$NF_n(Z_{n-1},Z_n)$: The neutrosophical cost of the phase or section between (Z_{n-1}, Z_n).

For n = 1. It expresses the minimum cost in the first stage:

$$NG_1(Z_1) = \text{Min}\ [NG_0(Z_0) + NF(Z_0,Z_1)]$$

For $NG_1(Z_1)$ we have three alternatives: B_1, B_2, B_3 . So we calculate the cost for each alternative:

$NG_1(B_1) = \text{Min}\ [NG_0(Z_0) + NF(A,B_1)] = Min\{0 + [5,7]\} = [5,7]$

$NG_1(B_2) = \text{Min}\ [NG_0(Z_0) + NF(A,B_2)] = Min\{0 + [2,3]\} = [2,3]$

$NG_1(B_3) = \text{Min}\ [NG_0(Z_0) + NF(A,B_3)] = Min\{0 + [3,5]\} = [3,5]$

The minimum cost in the first stage is $NG_1(B_2)$ ie the appropriate path is AB_2 at a cost $[2,3]$.

Calculating the costs for the first and second phases together:

For n = 2 the mathematical relationship is:

$$NG_2(Z_2) = \text{Min}\ [NG_1(Z_1) + NF(Z_1,Z_2)]$$

We have three alternatives for Z2 they: C1, C2, C3. We compute $G_2(Z_2)$ for each of them:

$NG_2(C_1) = \text{Min}\ [NG_1(B_1) + NF(B_1,C_1)\ , NG_1(B_2) + NF(B_2,C_1), NG_1(B_3) + NF(B_3,C_1)]$

$NG_2(C_1) = Min\{[5,7]+[11,13],[2,3]+[8,9],[3,5]+\infty\} = \{[16,20],[10,12],+\infty\} = [10,12]$

This value corresponds to $Z_1 = B_2$. The least expensive path is AB_2C_1.

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$NG_2(C_2)$ = Min [$NG_1(B_1) + NF(B_1,C_2)$, $NG_1(B_2) + NF(B_2,C_2)$, $NG_1(B_3) + NF(B_3,C_2)$]

$NG_2(C_2) = Min\{[5,7]+\infty, [2,3]+[4,5], [3,5]+[6,7]\} = \{\infty, [6,8], [9,12]\} = [6,8]$

This is for $Z_1 = B_2$. The least expensive path is AB_2C_2.

$NG_2(C_3)$ = Min [$NG_1(B_1) + NF(B_1,C_3)$, $NG_1(B_2) + NF(B_2,C_3)$, $NG_1(B_3) + NF(B_3,C_3)$]

$NG_2(C_3) = Min\{[5,7]+\infty, [2,3]+[9,10], [3,5]+[6,8]\} = \{\infty, [11,13], [9,13]\} = [9,13]$

This value corresponds to $Z_1 = B_3$. So the least expensive path is AB_3C_3.

We choose the optimal path for the first and second stages, and the least cost is: AB_2C_2 and the cost is $[6,8]$.

Calculate the costs for the first, second and third stages i.e. n = 3:

For the Z_3 we have four alternatives, we calculate the cost for each of them:

$NG_3(D_1)$ = Min [$NG_2(C_1) + NF(C_1,D_1)$, $NG_2(C_2) + NF(C_2,D_1)$, $NG_2(C_3) + NF(C_3,D_1)$]

$NG_3(D_1) = Min\{[10,12]+[3,4], [6,8]+[2,4], [9,13]+\infty\} = \{[13,16], [8,12], +\infty\} = [8,12]$

Opposite to $Z_2 = C_2$. So the least expensive path is: $AB_2C_2D_1$.

$NG_3(D_2)$ = Min [$NG_2(C_1) + NF(C_1,D_2)$, $NG_2(C_2) + NF(C_2,D_2)$, $NG_2(C_3) + NF(C_3,D_2)$]

$NG_3(D_2) = Min\{[10,12]+[2,3], [6,8]+[11,12], [11,13]+\infty\} = \{[12,15], [17,20], +\infty\} = [12,15]$

That's when $Z_2 = C_1$. So the least expensive path is $AB_2C_1D_2$.

$NG_3(D_3)$ = Min [$NG_2(C_1) + NF(C_1,D_3)$, $NG_2(C_2) + NF(C_2,D_3)$, $NG_2(C_3) + NF(C_3,D_3)$]

$NG_3(D_3) = Min\{[10,12]+\infty, [6,8]+[5,6], [11,13]+\infty\} = \{+\infty, [11,14], +\infty\} = [11,14]$

Opposite to $Z_2 = C_2$. So the least expensive path is $AB_2C_2D_3$.

$NG_3(D_4)$ = Min [$NG_2(C_1) + NF(C_1,D_4)$, $NG_2(C_2) + NF(C_2,D_4)$, $NG_2(C_3) + NF(C_3,D_4)$]

$NG_3(D_4) = Min\{[10,12]+\infty, [6,8]+[9,12], [11,13]+[4,6]\} = \{\infty, [15,20], [15,19]\} = [15,19]$

Opposite to $Z_2 = C_3$. So the least expensive path is $AB_2C_3D_4$.

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In comparison, the optimal path for the first, second and third stages is: $AB_2C_2D_1$ and cost $[8,12]$.

Calculate the costs for the first, second, third and fourth stages i.e.: n = 4:

$$NG_4(Z_4) = \text{Min}\ [NG_3(Z_3) + NF(Z_3,Z_4)]$$

For the Z_4 we have two alternatives E_1, E_2.

For E_1:

$NG_4(E_1) = \text{Min}\ [NG_3(D_1) + NF(D_1,E_1),\ NG_3(D_2) + NF(D_2,E_1),\ NG_3(D_3) + NF(D_3,E_1),\ NG_4(D_4) + NF(D_4,E_1)]$

$NG_4(E_1) = Min\{[8,12]+[9,11],[12,15]+[3,6],[11,14]+[7,9],[15,19]+\infty\} = \{[17,23],[15,21],[18,23],+\infty\} = [15,21]$

This is for $Z_3 = D_2$. So the least expensive path is $AB_2C_1D_2E_1$.

For E_2:

$NG_4(E_2) = \text{Min}\ [NG_3(D_1) + NF(D_1,E_2),\ NG_3(D_2) + NF(D_2,E_2),\ NG_3(D_3) + NF(D_3,E_2),\ NG_4(D_4) + NF(D_4,E_2)]$

$NG4(E2) = Min\{[8,12]+\infty,[12,15]+[6,7],[11,14]+[8,9],[15,19]+[5,7]\} =$

$\{+\infty,[18,22],[19,23],[20,26]\} = [18,22]$

This is for $Z_3 = D_2$. So the path has a lower cost: $AB_2C_1D_2E_2$

We proceed to the calculation of the minimum cost of the optimal total stage:

$$NG_5(Z_5) = \text{Min}\ [NG_4(Z_4) + NF(Z_4,Z_5)]$$

For the Z_5 we have one alternative which is the F.

$NG_5(F) = \text{Min}\ [NG_4(E_1) + NF(E_1,F),\ NG_4(E_2) + NF(E_2,F)]$

$NG_5(F) = Min\{[15,21]+[4,7],[18,22]+[3,5]\} = Min\{[19,28],[21,27]\} = [19,28]$

This is for $Z_4 = E_1$. So the path with the lowest cost is: $AB_2C_1D_2E_1F$ The cost is equal [19,28] .

4. Conclusion and results:

Through this study, we reached the minimum Cost for creating the expressways using the neutrosophic logic. Because of the uncertainty provided by the neutrosophic logic, we were able to find the ideal solution that achieves the lowest possible cost for creating the expressway. It takes into account all the circumstances that may encounter us in our study. This study is also used to calculate the costs of transportation and movement from one city to another city or one country to another. In the near future we are looking to use the Neutrosophical Dynamic Programming concept in other areas such as allocating a budget to projects or allocating engineers within a company's departments.

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