



Neutrosophic Two Way ANOVA

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Abstract

In this paper, well-known two way analysis of variance (Two Way ANOVA) is generalized to neutrosophic two way ANOVA dealing with neutrosophic data of the form $N=D+I$. Neutrosophic two way anova is a neutrosophic linear model test whether there is a significant difference between different treatments and whether there is a significant difference between different sectors keeping the type I error low. An algorithm is introduced to calculate fisher test statistics when we have neutrosophic data and we are testing two hypotheses, first is to test differences between treatments and second is to test differences between sectors.

Keywords: Analysis of Variance; Linear Models; Neutrosophic Statistics; Fisher Test; Type I Error

1.Introduction

Neutrosophic logic introduced by Florentin Smarandache to do not ignore indeterminacy in data in many fields of science including statistics, machine learning, artificial intelligence, operations research, queueing theory, probability theory, telecommunication systems, dynamic systems, abstract algebra, etc.[1-22]

Neutrosophic statistics has been studied well by many researchers specially Mohamad Aslam (see [16] [23] [24] [25] [26] [27] [28] [29]) where many statistical tests were generalized to neutrosophic statistical tests to take indeterminacy in hand. This extension is very useful in daily life problems because there is no certainty can be always handled. In medical field measures taken have indeterminacy, in agriculture indeterminacy rises and in all other applied sciences. F. Smarandache presented neutrosophic descriptive statistics including arithmetic mean, geometric mean, variance, standard deviation, quantiles, percentiles, etc. also, F. Smarandache presented many probability distributions with indeterminate parameters introducing what is known by neutrosophic probability distributions then used these distributions to define neutrosophic hypothesis and neutrosophic inferential statistics [7].

In [16] neutrosophic one way anova test was generalized to test whether there is significant difference between more than two groups and this generalization was applied to university students. In [21], [30] neutrosophic time series was studied and many models were presented. In [23], [25], [29], [31] many goodness of fit statistical tests were generalized to neutrosophic goodness of fit tests. In [24], [26], [27] [28] many multivariate and post hoc tests were generalized to neutrosophic statistical tests and many applications were presented. Many problems in machine learning was solved in neutrosophic environment as generalization of fuzzy machine learning algorithms like in [32] [11] [33] [34].

One of inferential statistics applications is hypothesis based on means that tests whether there is a significant difference between two samples means or more than two samples. If we have two or less samples we often use t-test, and if we have more than two samples we use ANOVA test which is one of the most important linear models. There are many ANOVA tests depending on number of factors affecting dependent variable and on number of dependent variables that are affected by studied factors. If we have one dependent variable then we are dealing with univariate ANOVA. If we have more than one dependent variable then we are dealing with multivariate analysis of variance. If we have one factor then we have a one way ANOVA, and if we have more than one factor then we are dealing with factorial ANOVA. In this paper we are going to extend a special case of ANOVA tests which is two way ANOVA where we have one dependent variable affected with two factors. This statistical test has many applications in many fields of science, e.g. in medical field, we sometimes would like to see the effect of many types of drugs on patients, and sometimes these drugs' effect differ between different sexes, e.g. if we would like to see which drugs A,B,C is better to control blood glucose, and we would like to see if these drugs effect males more than females or vice versa, this can be done using two way ANOVA. But we know that glucose itself is not certain for each person, so we will deal with neutrosophic data, i.e. glucose level will range in $[x^L, x^U]$. Before arising of neutrosophic logic this type of problems was solved by ignoring indeterminacy taking midpoint of the range $[x^L, x^U]$ and this may lead to wrong decisions. In this paper we will introduce an algorithm to generalize two way ANOVA test to take indeterminacy in hand and make reliable decisions. Also, many solved examples are introduced to show the power of the generalized test.

2. Definitions

Definition 1

Neutrosophic number or neutrosophic statistical number is defined by $N = D + I$ where D is the determinant part of the number and I is indeterminacy.

Example:

$$N = 3 + I \text{ where } I \in [0,0.1]$$

Operations on neutrosophic numbers

Let $N_1 \in [a, b], N_2 \in [d, c]$ be two neutrosophic number, then operations on neutrosophic numbers are defined as follows:

$$N_1 + N_2 = [a + d, b + c]$$

$$N_1 - N_2 = [a - c, b - d]$$

$$N_1 * N_2 = [a * c, b * d]$$

$$\frac{N_1}{N_2} = \left[\frac{a}{c}, \frac{b}{d} \right]$$

$$\sqrt{N_1} = [\sqrt{a}, \sqrt{b}] ; a \geq 0, b \geq 0$$

Some neutrosophic descriptive statistics measures:

Let $x_{1N}, x_{2N}, \dots, x_{nN}$ be neutrosophic observation then:

- Neutrosophic arithmetic mean is defined by:

$$\bar{x}_N \in \frac{\sum_{i=1}^n x_{iN}}{n}$$

Example 1

Let the observations be: $1+I, 4, 5, 6-2I$

Then:

$$\bar{x}_N \in \frac{\sum_{i=1}^n x_{iN}}{n} = \frac{1 + I + 4 + 5 + 6 - 2I}{4} = \frac{16 - I}{4} = 4 - \frac{I}{4}$$

$$\text{If } I \in [0,0.1] \text{ then } \bar{x}_N \in 4 - \frac{[0,0.1]}{4} = 4 - [0,0.025] = [3.975,4]$$

Which means that arithmetic mean will lay in the interval $[3.975,4]$

- Neutrosophic standard deviation is defined by:

$$s_N = \sqrt{\frac{\sum_{i=1}^n (x_{iN} - \bar{x}_N)^2}{n - 1}}$$

Back to example 1 we find:

$$s_N = \sqrt{\frac{\sum_{i=1}^n (x_{iN} - \bar{x}_N)^2}{n-1}} = \sqrt{\frac{\left(1 + I - 4 + \frac{I}{4}\right)^2 + \left(4 - 4 + \frac{I}{4}\right)^2 + \left(5 - 4 + \frac{I}{4}\right)^2 + \left(6 - 2I - 4 + \frac{I}{4}\right)^2}{3}}$$

$$= \sqrt{\frac{14}{3} - \frac{37}{12}I}$$

Take $I \in [0,0.1]$ then:

$$s_N = \sqrt{\frac{14}{3} - \frac{37}{12}[0,0.1]} = \left[\sqrt{\frac{523}{120}}, \sqrt{\frac{14}{3}} \right] = [2.088, 2.160]$$

3. Neutrosophic Two Way ANOVA

Suppose that we have neutrosophic observaions, (observations with indeterminacy, uncertainty or ambiguity) given in a table as below:

Table 1: Neutrosophic observaions

	Treatment 1	Treatment 2	...	Treatment t
Sector 1	x_{11N}	x_{12N}	...	x_{1m_1N}
Sector 2	x_{21N}	x_{22N}	...	x_{2m_2N}
⋮	⋮	⋮	...	⋮
Sector r	x_{r1N}	x_{r2N}	...	x_{rm_tN}

Which means that we have t treatments and r sectors affecting the data, e.g, three types of medicine with 2 concentrations, here we have 3 treatments with two sectors.

We would like to test two hypothesizes defined as follows:

H_01 : there is no significant difference between treatments.

H_02 : there is no significant difference between sectors.

Last problem is known by two way ANOVA since we have two sources of variance, treatments and sectors.

The problem arises when we have some indeterminacy with observations, and here we will present an algorithm to test whether there is significant differences in means according to different treatments and different sectors without ignoring indeterminacy:

We will consider that observations are neutrosophic observations presented by

$$x_{ij}^N \in [x_{ij}^l, x_{ij}^u]$$

The algorithm:

1. calculate general neutrosophic sum:

$$G^N \in \left[\sum_{i=1}^t \sum_{j=1}^t x_{ij}^L, \sum_{i=1}^r \sum_{j=1}^t x_{ij}^U \right] \quad (1)$$

2. calculate neutrosophic correction factor

$$CF^N = \frac{G^N}{n} \in \left[\frac{(\sum_{i=1}^r \sum_{j=1}^t x_{ij}^L)^2}{n}, \frac{(\sum_{i=1}^r \sum_{j=1}^t x_{ij}^U)^2}{n} \right] = [CF^L, CF^U] \quad (2)$$

3. calculate neutrosophic total sum of squares

$$SSo^N = \sum_{i=1}^r \sum_{j=1}^t x_{ij}^{N^2} - CF^N \in \left[\sum_{i=1}^r \sum_{j=1}^t x_{ij}^{L^2} - CF^L, \sum_{i=1}^r \sum_{j=1}^t x_{ij}^{U^2} - CF^U \right] = [SSo^L, SSo^U] \quad (3)$$

4. calculate neutrosophic sectors sums

$$R_i^N \in \left[\sum_{j=1}^t x_{ij}^L, \sum_{j=1}^t x_{ij}^U \right] = [R_i^L, R_i^U] \quad (4)$$

5. calculate neutrosophic treatments sums

$$T_j^N \in \left[\sum_{i=1}^r x_{ij}^L, \sum_{i=1}^r x_{ij}^U \right] = [T_j^L, T_j^U] \quad (5)$$

6. calculate neutrosophic sum of squares for sectors

$$SSr^N = \frac{\sum_{i=1}^r R_i^{N^2}}{t} - CF^N \in \left[\frac{\sum_{i=1}^r R_i^{L^2}}{t} - CF^L, \frac{\sum_{i=1}^r R_i^{U^2}}{t} - CF^U \right] = [SSr^L, SSr^U] \quad (6)$$

7. calculate neutrosophic sum of squares for treatments

$$SSt^N = \frac{\sum_{j=1}^t T_j^{N^2}}{r} - CF^N \in \left[\frac{\sum_{j=1}^t T_j^{L^2}}{r} - CF^L, \frac{\sum_{j=1}^t T_j^{U^2}}{r} - CF^U \right] = [SSt^L, SSt^U] \quad (7)$$

8. calculate neutrosophic within sum of squares

$$SSe^N = SSo^N - SSr^N - SSt^N \in [SSo^L - SSr^L - SSt^L, SSo^U - SSr^U - SSt^U] = [SSe^L, SSe^U] \quad (8)$$

9. calculate neutrosophic mean within sum of squares

$$mse^N = \frac{SSe^N}{(r-1)(t-1)} \in \left[\frac{SSe^L}{(r-1)(t-1)}, \frac{SSe^U}{(r-1)(t-1)} \right] = [mse^L, mse^U] \quad (9)$$

10. calculate neutrosophic mean sum of squares between treatments

$$mst^N = \frac{SSt^N}{t-1} \in \left[\frac{SSt^L}{t-1}, \frac{SSt^U}{t-1} \right] = [mst^L, mst^U] \quad (10)$$

11. calculate neutrosophic mean sum of squares between sectors

$$msr^N = \frac{SSr^N}{r-1} \in \left[\frac{SSr^L}{r-1}, \frac{SSr^U}{r-1} \right] = [msr^L, msr^U] \quad (11)$$

12. calculate neutrosophic fisher test statistic for sectors

$$F_1^N = \frac{msr^N}{mse^N} \quad (12)$$

13. calculate neutrosophic fisher test statistic for treatments

$$F_2^N = \frac{mst^N}{mse^N} \quad (13)$$

14. compare neutrosophic test statistics with critical value F^* and take decision about given hypothesizes noticing that:
 $F^N \in [F^L, F^U]$

- 15. if $F^L > F^*$ then we reject the null hypothesis.
- if $F^U < F^*$ then we accept the null hypothesis.
- if $F^L < F^*$ and $F^U > F^*$ then we have no decision (we are indeterminant).

Results can be presented in a table called neutrosophic anova table as follows:

Table 2: Neutrosophic ANOVA table

Source of Variance	df	SS	MS	F
Treatments	t-1	$[SSt^L, SSt^U]$	$[mst^L, mst^U]$	F_1^N
Sectors	r-1	$[SSr^L, SSr^U]$	$[msr^L, msr^U]$	F_2^N
Error	n-t-r+1	$[Sse^L, Sse^U]$	$[mse^L, mse^U]$	
Total	n-1			

4. Application in Medical Field

A sample of 120 diabetics was treated with 3 types of medicines, first is with treatment (C) which is mixtard insulin 70/30, second is treatment (B) which is mixtard 50/50, last is treatment (A) which is lantus + actrapid. Patients were splitted according to different treatment type and to different sexes. Foreach patient we have token a range of glucose measures which makes measuring more realiable. Data was as follows:

Table 3: Neutrosophic observaions of blood glucose

	A		B		C	
	X_{ij}^L	X_{ij}^U	X_{ij}^L	X_{ij}^U	X_{ij}^L	X_{ij}^U
Males	110	119	123	128	135	148
	129	134	128	129	149	158
	125	126	139	141	125	127
	130	144	135	138	143	153
	139	154	138	141	139	152
	133	145	137	142	133	141
	123	124	140	144	141	161
	111	128	124	126	126	136
	116	126	138	139	147	150

	136	143	121	122	139	151
Females	124	136	125	127	139	159
	135	139	134	135	123	128
	124	129	130	130	133	138
	120	137	122	123	129	145
	135	150	133	136	148	167
	112	127	133	138	126	145
	119	125	132	134	126	133
	115	132	138	139	143	156
	122	142	140	145	150	164
	136	140	130	132	137	155

We would like to test the following two hypotheses:

H_01 : there is no significant difference in glucose between different treatments.

H_02 : there is no significant difference in glucose between different sexes

Solution:

According to our proposed algorithm:

1. general neutrosophic sum using equation (1):

$$G^N \in \left[\sum_{i=1}^r \sum_{j=1}^t x_{ij}^L, \sum_{i=1}^r \sum_{j=1}^t x_{ij}^U \right] = [7865,8356]$$

2. neutrosophic correction factor using equation (2):

$$CF^N = \frac{G^{N^2}}{n} \in \left[\frac{(\sum_{i=1}^r \sum_{j=1}^t x_{ij}^L)^2}{n}, \frac{\sum_{i=1}^r \sum_{j=1}^t x_{ij}^{U^2}}{n} \right] = [1030970.417,1163712.267]$$

3. neutrosophic total sum of squares using equation (3):

$$SSo^N = \sum_{i=1}^r \sum_{j=1}^t x_{ij}^{2N} - CF^N \in \left[\sum_{i=1}^r \sum_{j=1}^t x_{ij}^{L^2} - CF^L, \sum_{i=1}^r \sum_{j=1}^t x_{ij}^{U^2} - CF^U \right] = [5214.583333,7675.733333]$$

4. neutrosophic sectors sums using equation (4):

$$R_{1.}^N \in \left[\sum_{j=1}^t x_{1j}^L, \sum_{j=1}^t x_{1j}^U \right] = [3952,4170]$$

$$R_{2.}^N \in \left[\sum_{j=1}^t x_{1j}^L, \sum_{j=1}^t x_{1j}^U \right] = [3913,4186]$$

5. neutrosophic treatments sums using equation (5):

$$T_{.1}^N \in \left[\sum_{i=1}^r x_{i1}^L, \sum_{i=1}^r x_{i1}^U \right] = [2494,2700]$$

$$T_{.2}^N \in \left[\sum_{i=1}^r x_{i2}^L, \sum_{i=1}^r x_{i2}^U \right] = [2640,2689]$$

$$T_{.3}^N \in \left[\sum_{i=1}^r x_{i3}^L, \sum_{i=1}^r x_{i3}^U \right] = [2731,2967]$$

6. neutrosophic sum of squares for sectors using equation (6):

$$SSr^N = \frac{\sum_{i=1}^r R_i^{N^2}}{t} - CF^N \in \left[\frac{\sum_{i=1}^r R_i^{L^2}}{t} - CF^L, \frac{\sum_{i=1}^r R_i^{U^2}}{t} - CF^U \right] = [4.266666667,25.35]$$

7. neutrosophic sum of squares for treatments using equation (7):

$$SSt^N = \frac{\sum_{j=1}^t T_j^{N^2}}{r} - CF^N \in \left[\frac{\sum_{j=1}^t T_j^{L^2}}{r} - CF^U, \frac{\sum_{j=1}^t T_j^{U^2}}{r} - CF^L \right] = [1429.433333,2478.233333]$$

8. neutrosophic within sum of squares using equation (8):

$$SSe^N = SS_0^N - SSr^N - SSt^N \in [SS_0^L - SSr^L - SSt^L, SS_0^U - SSr^U - SSt^U] = [3759.8,5193.233333]$$

9. neutrosophic mean within sum of squares using equation (9):

$$mse^N = \frac{SSe^N}{(r-1)(t-1)} \in \left[\frac{SSe^L}{(r-1)(t-1)}, \frac{SSe^U}{(r-1)(t-1)} \right] = [67.13928571,92.73630952]$$

10. neutrosophic mean sum of squares between treatments using equation (10):

$$mst^N = \frac{SSt^N}{t-1} \in \left[\frac{SSt^L}{t-1}, \frac{SSt^U}{t-1} \right] = [714.7166667,1239.116667]$$

11. neutrosophic mean sum of squares between sectors using equation (11):

$$msr^N = \frac{SSr^N}{r-1} \in \left[\frac{SSr^L}{r-1}, \frac{SSr^U}{r-1} \right] = [4.266666667,25.35]$$

12. neutrosophic fisher test statistic for sectors using equation (12):

$$F_1^N = \frac{msr^N}{mse^N} = [0.046008588,0.377573275]$$

13. neutrosophic fisher test statistic for treatments using equation (13):

$$F_2^N = \frac{mst^N}{mse^N} = [10.64528255, 13.3617207]$$

Table 4: Neutrosophic ANOVA table

Source of Variance	df	SS	MS	F
Treatments	2	[1429.43, 2478.23]	[714.72, 1239.12]	[10.65, 13.36]
Sectors	1	[4.27, 25.35]	[4.27, 25.35]	[0.05, 0.38]
Error	56	[3759.80, 5193.23]	[67.14, 92.74]	
Total	59			

The first critical value is $F_1^* = 4.01297$, we notice that $F_1^N = [0.046008588, 0.377573275]$, $F_1^{N^U} = 0.377573275 < 4.01297$ so we accept the null hypothesis so that there is no significant difference in glucose between different sexes.

The second critical value is $F_2^* = 3.16186$, we notice that $F_2^N = [10.64528255, 13.3617207]$, $F_2^{N^L} = 10.64528255 > 3.16186$ so we reject the null hypothesis so that there is a significant difference in glucose between different treatments.

Notice that:

$$\bar{X}_A \in [124.7, 135], \bar{X}_B \in [132, 134.45], \bar{X}_C \in [136.55, 148.35]$$

We conclude that treatment A causes mean of glucose less than treatment C since $\bar{X}_A^U < \bar{X}_C^L$, also treatment B causes mean of glucose less than treatment C for the same reason, but there is some overlap in glucose level between treatment A and treatment B.

Also, we see that $\bar{X}_{Males} \in [131.7333333, 139]$, $\bar{X}_{Females} \in [130.4333333, 139.5333333]$ which are totally overlapped and this proves the nonsignificant differences.

In crisp sets theory we cannot deal with neutrosophic data, we may take average of measures or we may take only one measure which leads to wrong decisions.

In our application, if we take average of the measures then the ANOVA table will be:

Table 5: Neutrosophic ANOVA table

Source of Variance	df	SS	MS	F
Treatments	2	1701.675	850.838	11.459
Sectors	1	2.204	2.204	0.030
Error	56	4158.033	74.251	
Total	59			

We notice from the previous table that we have same decisions in neutrosophic statistics, but comparing means of treatments gives:

$$\bar{X}_A = 129.85, \bar{X}_B = 133.225, \bar{X}_C = 142.45$$

Previous means may lead us to wrong decision where we see that the biggest mean is of treatment C then treatment B then treatment A, but using neutrosophic statistics we see that means of treatment B and C overlap so there is no significant difference between these treatments. Also from ANOVA table we see that test statistics differs. So, in some applications we maybe accept the null hypothesis but we have to reject it according to less accuracy in crisp statistics or vice versa.

5. Conclusions

In this article, we discussed and presented neutrosophic two way ANOVA test which deals with indeterminate data and show the difference between decisions made under neutrosophic environment and crisp environment. This algorithm make decisions safer and closes the lag made by ignoring indeterminacy. In future work we will study post-hoc tests which allows to check the statistical differences pairwise between samples. Also we will generalize other linear models like multivariate ANOVA, analysis of covariance and repeated measures ANOVA.

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