New Kind of Neutrosophic Crisp with Analytic Study

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Abstract: Neutrosophic is the important mathematically issue, which have a major role in applied science and pure mathematics. At the same level of importance, we have to create new kinds of Those sets, and more comprehensive than the first kind, and we named him second type of neutrosophic – Algebraic construction of this type has been done, showing it is a ring with at $Nt-$ intersection and Nt −difference operation.

Keywords: Neutrosophic triple sets (Simple $Nt-$ sets), Nt −points, Nt −function.

1. Introduction

The set theory is one of the important mathematical topics on which the rest of the branches of mathematics are based and is considered the basic foundation for it. Among this importance, scientists began to find a different formula for sets, whether on a one – dimensional, two – dimension) or three – dimensional level. Behind all this, is to finding optimal solutions to some open problems in the natural sciences as well as engineering and other sciences. There are many types of them, fuzzy sets [1], soft fuzzy sets [2], fuzzy soft sets [3], double sets [4] and neutresoploic sets [4]. If $\mathbf{X}$ is the non – empty universal set, the fuzzy set are constructed on the $\mathbf{2D}$ plane $\mathbf{X} \times [\mathbf{0}, \mathbf{1}]$, but the soft sets are constructed on $\mathbf{E} \times \mathbb{P}(\mathbf{X})$, where $\mathbf{E}$ be the set of all parameters for $\mathbf{X}$. Similarly for double sets, they are constructed in the plane $\mathbb{P}(\mathbf{X}) \times \mathbb{P}(\mathbf{X})$. But the neutrosophic crisp set are constructed on the plane space $\mathbb{P}(\mathbf{X}) \times \mathbb{P}(\mathbf{X}) \times \mathbb{P}(\mathbf{X})$. The reason for the diversity of the creation of these sets, as well as the diversity and difference of the binary operations that the identified, came as a result of the diversity and difference of problems that scientists face in the natural and engineering sciences, as well as the difference, whether in the two – dimensional or three – dimensional.

Through sharing simple and intensive study of some of the concepts presented by Salama and Florentin [5] on the topic of the Neutrosophic crisp and which are important in the process of algebraic construction of these aggregates, we have

I. Shown that De Morken's Law (proposition 1.2.2) is not entirely correct.

For any type $\mathbf{NCS}$ $\mathbf{A}$ and $\mathbf{B}$, we have the following if we take the complements $\mathbf{C_1}, \mathbf{C_2}$ and $\mathbf{C_3}$ in Definition 1.1.3,[5] $\mathbf{A}$ and $<>\mathbf{A}$ is definition 1.1.5.

1. For the complement $\mathbf{C_1}$, we have

i. $(\mathbf{A} \cap_2 \mathbf{B})^{C_1} = \mathbf{A}^{C_1} \cup_2 \mathbf{B}^{C_1}$ and $(\mathbf{A} \cup_2 \mathbf{B})^{C_1} = \mathbf{A}^{C_1} \cap_2 \mathbf{B}^{C_1}$.

ii. $(\mathbf{A} \cap_2 \mathbf{B})^{C_1} \neq \mathbf{A}^{C_1} \cup_2 \mathbf{B}^{C_1}$ and $(\mathbf{A} \cup_2 \mathbf{B})^{C_1} \neq \mathbf{A}^{C_1} \cap_2 \mathbf{B}^{C_1}$.

iii. $(\mathbf{A} \cap_1 \mathbf{B})^{C_1} \neq \mathbf{A}^{C_1} \cup_1 \mathbf{B}^{C_1}$ and $(\mathbf{A} \cup_1 \mathbf{B})^{C_1} \neq \mathbf{A}^{C_1} \cap_1 \mathbf{B}^{C_1}$.

iv. $(\mathbf{A} \cap_1 \mathbf{B})^{C_1} \neq \mathbf{A}^{C_1} \cup_2 \mathbf{B}^{C_1}$ and $(\mathbf{A} \cup_1 \mathbf{B})^{C_1} \neq \mathbf{A}^{C_1} \cup_1 \mathbf{B}^{C_1}$.

If we take $\mathbf{A} = \langle\{\mathbf{a},\mathbf{b}\},\{\mathbf{c}\},\emptyset\rangle$, $\mathbf{B} = \langle\{\mathbf{a},\mathbf{b}\},\emptyset,\{\mathbf{c}\}\rangle$, for $\mathbf{X} = \{\mathbf{a},\mathbf{b},\mathbf{c}\}$ are satisfy the condition of type $\mathbf{i-NCS}$ for all $\mathbf{i} = \mathbf{1},\mathbf{2},\mathbf{3}$, that the three cases ii, iii and iv are true.

2. For the complement $\mathbf{C_2}$, we have

i. $(\mathbf{A} \cap_1 \mathbf{B})^{C_2} = \mathbf{A}^{C_2} \cup_2 \mathbf{B}^{C_2}$ and $(\mathbf{A} \cup_2 \mathbf{B})^{C_2} = \mathbf{A}^{C_2} \cap_1 \mathbf{B}^{C_2}$.

ii. $(\mathbf{A} \cap_1 \mathbf{B})^{C_2} \neq \mathbf{A}^{C_2} \cup_1 \mathbf{B}^{C_2}$ and $(\mathbf{A} \cup_1 \mathbf{B})^{C_2} \neq \mathbf{A}^{C_2} \cap_1 \mathbf{B}^{C_2}$.

iii. $(\mathbf{A} \cap_2 \mathbf{B})^{C_2} \neq \mathbf{A}^{C_2} \cup_2 \mathbf{B}^{C_2}$ and $(\mathbf{A} \cup_1 \mathbf{B})^{C_2} \neq \mathbf{A}^{C_2} \cap_2 \mathbf{B}^{C_2}$.

iv. $(\mathbf{A} \cap_2 \mathbf{B})^{C_2} \neq \mathbf{A}^{C_2} \cup_2 \mathbf{B}^{C_2}$ and $(\mathbf{A} \cup_2 \mathbf{B})^{C_2} \neq \mathbf{A}^{C_1} \cap_2 \mathbf{B}^{C_2}$.

If we take $\mathbf{A} = \langle\{\mathbf{c}\},\emptyset,\{\mathbf{a},\mathbf{b}\}\rangle$ and $\mathbf{B} = \langle\emptyset,\{\mathbf{c}\},\{\mathbf{a},\mathbf{b}\}\rangle$, are satisfy the condition of type $\mathbf{i-NCS}$ for all $\mathbf{i} = \mathbf{1},\mathbf{2},\mathbf{3}$, that the three cases ii, iii and iv are true

i.e. $(\mathbf{A} \cap_1 \mathbf{B})^{C_2} = \langle\{\mathbf{a},\mathbf{b}\},\emptyset,\emptyset\rangle \neq \langle\{\mathbf{a},\mathbf{b}\},\emptyset,\{\mathbf{c}\}\rangle = \mathbf{A}^{C_2} \cup_1 \mathbf{B}^{C_2}$.

$(\mathbf{A} \cap_2 \mathbf{B})^{C_2} = \langle\{\mathbf{a},\mathbf{b}\},\{\mathbf{c}\},\emptyset\rangle \neq \langle\{\mathbf{a},\mathbf{b}\},\emptyset,\{\mathbf{c}\}\rangle = \mathbf{A}^{C_2} \cup_1 \mathbf{B}^{C_2}$.

3. For the complement $\mathbf{C_3}$, we have

i. $(\mathbf{A} \cap_1 \mathbf{B})^{C_3} \neq \mathbf{A}^{C_2} \cup_1 \mathbf{B}^{C_3}$ and $(\mathbf{A} \cup_1 \mathbf{B})^{C_3} \neq \mathbf{A}^{C_3} \cap_1 \mathbf{B}^{C_3}$

ii. $(\mathbf{A} \cap_1 \mathbf{B})^{C_3} \neq \mathbf{A}^{C_3} \cup_2 \mathbf{B}^{C_3}$ and $(\mathbf{A} \cup_2 \mathbf{B})^{C_3} \neq \mathbf{A}^{C_3} \cap_1 \mathbf{B}^{C_3}$

If we take $\mathbf{A} = \langle\{\mathbf{a},\mathbf{b}\},\{\mathbf{c}\},\emptyset\rangle, \mathbf{B} = \langle\{\mathbf{a},\mathbf{b}\},\emptyset,\{\mathbf{c}\}\rangle$ are satisfy the condition of type $\mathbf{i-NCS}$, $\mathbf{i} = \mathbf{1},\mathbf{2},\mathbf{3}$, that the two cases i and ii are true.

iii. $(\mathbf{A} \cap_2 \mathbf{B})^{C_3} \neq \mathbf{A}^{C_3} \cup_2 \mathbf{B}^{C_3}$ and $(\mathbf{A} \cup_2 \mathbf{B})^{C_3} \neq \mathbf{A}^{C_3} \cap_2 \mathbf{B}^{C_3}$.

iv. $(\mathbf{A} \cap_2 \mathbf{B})^{C_3} \neq \mathbf{A}^{C_3} \cup_1 \mathbf{B}^{C_3}$ and $(\mathbf{A} \cup_1 \mathbf{B})^{C_3} \neq \mathbf{A}^{C_3} \cap_2 \mathbf{B}^{C_3}$.

If we take $\mathbf{A} = \langle\emptyset,\{\mathbf{a},\mathbf{b}\},\{\mathbf{c}\}\rangle$ and $\mathbf{B} = \langle\{\mathbf{a},\mathbf{c}\},\{\mathbf{b}\},\emptyset\rangle$, are satisfy the condition of type $\mathbf{i-NCS}$ for all $\mathbf{i} = \mathbf{1},\mathbf{2},\mathbf{3}$, that the two cases iii and iv are true.

4. For the complement [**5**], the De - Morgan's Law are satisfy.

5. For the complement $<>$, we have:

a. $<>(\mathbf{A} \cap_1 \mathbf{B}) =<> \mathbf{A} \cup_2 <> \mathbf{B}$ and $<>(\mathbf{A} \cup_2 \mathbf{B}) =<> \mathbf{A} \cap_1 <> \mathbf{B}$.

b. $<>(\mathbf{A} \cap_2 \mathbf{B}) =<> \mathbf{A} \cup_2 <> \mathbf{B}$ and $<>(\mathbf{A} \cup_2 \mathbf{B}) =<> \mathbf{A} \cap_2 <> \mathbf{B}$.

c. $<>(\mathbf{A} \cap_1 \mathbf{B}) \neq<> \mathbf{A} \cup_1 <> \mathbf{B}$ and $<>(\mathbf{A} \cup_1 \mathbf{B}) \neq<> \mathbf{A} \cap_1 <> \mathbf{B}$.

d. $<>(\mathbf{A} \cap_2 \mathbf{B}) \neq<> \mathbf{A} \cup_1 <> \mathbf{B}$ and $<>(\mathbf{A} \cup_1 \mathbf{B}) \neq<> \mathbf{A} \cap_2 <> \mathbf{B}$.

If we take $\mathbf{A} = \langle\emptyset,\{\mathbf{a},\mathbf{b}\},\{\mathbf{c}\}\rangle$ and $\mathbf{B} = \langle\{\mathbf{a},\mathbf{c}\},\{\mathbf{b}\},\emptyset\rangle$ are satisfy the two cases iii and iv.

II. For any neutrosophic crisp set $\mathbf{A}$, we have:

i. For the complement $\mathbf{C_1}$, we get that:

1. $\mathbf{A} \cap_1 \mathbf{A}^{\mathbf{C}_1} = \langle \emptyset , \emptyset , \mathbf{X} \rangle = \emptyset_{\mathbf{N_1}}, \emptyset_{\mathbf{N_2}}$ and $\emptyset_{\mathbf{N_3}}$ of type 1.

2. $\mathbf{A} \cap_2 \mathbf{A}^{\mathbf{C}_2} = \langle \emptyset , \mathbf{X} , \mathbf{X} \rangle = \emptyset_{\mathbf{N_3}}$ of type 3.

ii. For the complement $\mathbf{C_2}$, we get that.

$\mathbf{A} \cap_{\mathbf{j}} \mathbf{A}^{\mathbf{C}_2} \neq \emptyset_{\mathbf{N_i}}, \forall\, \mathbf{i} = \mathbf{1}, \mathbf{2}, \mathbf{3}$ and $\forall$ type $\mathbf{i}$, $\mathbf{i} = \mathbf{1}, \mathbf{2}, \mathbf{3}$ and $\mathbf{j} = \mathbf{1}, \mathbf{2}$.

iii. For the complement $\mathbf{C_3}$, we get that:

$\mathbf{A} \cap_1 \mathbf{A}^{\mathbf{C}_3} = \emptyset_{\mathbf{N}}$ of type 3; if $\mathbf{A}$ is $\mathbf{NCS}$ $-$type1 and $\mathbf{NCS}$ $-$type2.

iv. For the complement $[\mathbf{5}\ ]$, we have:

$\mathbf{A} \cap_{\mathbf{j}} [\ ]\mathbf{A} \neq \emptyset_{\mathbf{N_i}}$, $\forall\, \mathbf{i} = \mathbf{1}, \mathbf{2}, \mathbf{3}\ \forall$ type $\mathbf{i}, \mathbf{i} = \mathbf{1}, \mathbf{2}, \mathbf{3}$ for $\mathbf{j} = \mathbf{1}, \mathbf{2}$.

v. For the complement $< >$, we have:

For $\mathbf{j} = \mathbf{1}, \mathbf{2}$, $\mathbf{A} \cap_{\mathbf{j}} < > \mathbf{A} \neq \emptyset_{\mathbf{N_i}}, \forall\, \mathbf{i} = \mathbf{1}, \mathbf{2}, \mathbf{3}, \forall$ type $\mathbf{i}$, $\mathbf{i} = \mathbf{1}, \mathbf{2}, \mathbf{3}$.

vi. For any neutrosophic crisp $\mathbf{A}$, we have:

1. For the complement $\mathbf{C_1}$

$\mathbf{A} \cup_1 \mathbf{A}^{\mathbf{C}_1} = \langle \mathbf{X} , \emptyset , \mathbf{X} \rangle = \mathbf{X}_{\mathbf{N_3}}$ type 3.

$\mathbf{A} \cup_2 \mathbf{A}^{\mathbf{C}_1} = \langle \mathbf{X} , \emptyset , \emptyset \rangle = \mathbf{X}_{\mathbf{N_i}}$ type 1 $\forall\, \mathbf{i} = \mathbf{1}, \mathbf{2}, \mathbf{3}$.

2. For the complement $\mathbf{C_2}$

$\forall\, \mathbf{j} = \mathbf{1}, \mathbf{2}$ $\mathbf{A} \cup_{\mathbf{j}} \mathbf{A}^{\mathbf{C}_2} \neq \mathbf{X}_{\mathbf{N_i}} \forall \mathbf{i} = \mathbf{1}, \mathbf{2}, \mathbf{3}$ $\forall$ type $\mathbf{k}$, $\mathbf{k} = \mathbf{1}, \mathbf{2}, \mathbf{3}$.

3. For the complement $\mathbf{C_3}$

$\forall\, \mathbf{j} = \mathbf{1}, \mathbf{2}$ $\mathbf{A} \cup_{\mathbf{j}} \mathbf{A}^{\mathbf{C}_3} \neq \mathbf{X}_{\mathbf{N_i}} \forall \mathbf{i} = \mathbf{1}, \mathbf{2}, \mathbf{3}$ $\forall$ type $\mathbf{k}$, $\mathbf{k} = \mathbf{1}, \mathbf{2}, \mathbf{3}$.

4. For the complement $[\mathbf{5}\ ]$

$\forall\, \mathbf{j} = \mathbf{1}, \mathbf{2}$ $\mathbf{A} \cup_{\mathbf{j}} [\ ]\mathbf{A} \neq \mathbf{X}_{\mathbf{N_i}} \forall \mathbf{i} = \mathbf{1}, \mathbf{2}, \mathbf{3}$ $\forall$ type $\mathbf{k}$, $\mathbf{k} = \mathbf{1}, \mathbf{2}, \mathbf{3}$.

5. For the complement $< >$

$\forall\, \mathbf{j} = \mathbf{1}, \mathbf{2}$ $\mathbf{A} \cup_{\mathbf{j}} < > \mathbf{A} \neq \mathbf{X}_{\mathbf{N_i}} \forall \mathbf{i} = \mathbf{1}, \mathbf{2}, \mathbf{3}$ $\forall$ type $\mathbf{k}$, $\mathbf{k} = \mathbf{1}, \mathbf{2}, \mathbf{3}$.

III. The theorem 1.2.2. is not entirely correct, if $\mathbf{X} = \{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$, them by Definition 1.2.1 the $\mathbf{NCP'S}$ are

$\mathbf{P_1} = \langle \{\mathbf{a}\}, \{\mathbf{b}\}, \{\mathbf{c}\} \rangle$, $\mathbf{P_2} = \langle \{\mathbf{a}\}, \{\mathbf{c}\}, \{\mathbf{b}\} \rangle$, $\mathbf{P_3} = \langle \{\mathbf{b}\}, \{\mathbf{a}\}, \{\mathbf{c}\} \rangle$,

$\mathbf{P_2} = \langle \{\mathbf{b}\}, \{\mathbf{c}\}, \{\mathbf{a}\} \rangle$, $\mathbf{P_2} = \langle \{\mathbf{c}\}, \{\mathbf{a}\}, \{\mathbf{b}\} \rangle$, $\mathbf{P_2} = \langle \{\mathbf{c}\}, \{\mathbf{b}\}, \{\mathbf{a}\} \rangle$.

For $\mathbf{A} = \langle \{\mathbf{a}\}, \emptyset , \{\mathbf{b}, \mathbf{c}\} \rangle$ is $\mathbf{NCS}$ $-$type 1,2,3 and $\mathbf{P_1}, \mathbf{P_2} \in_2$ (belong to of type 2) $\mathbf{A}$.

$\mathbf{A} \neq \mathbf{P_1} \cup_2 \mathbf{P_2}$ and $\mathbf{A} = \mathbf{P_1} \cup_1 \mathbf{P_2}$ if $\mathbf{B} = \langle \{\mathbf{a}\}, \emptyset , \{\mathbf{c}\} \rangle$, the $\mathbf{P_1} \in_2 \mathbf{B}$, but $\mathbf{P_1} \cup_1 \mathbf{P_1} = \mathbf{P} \neq \mathbf{B}$.

So for each $\mathbf{NCP}$ $\mathbf{P}$ identified by Definition 1.2.1. is $\mathbf{NCS}$ at the same time, so the only $\mathbf{NCS}$ has the $\mathbf{NCP}$ with respect to belong's type 1 but other $\mathbf{NCS}$ does not have $\mathbf{NCP}$ points with respect to belongs type 1. In other words, this tope of $\mathbf{NCP}$ points does not represent portions of $\mathbf{NCS}$ set of all types.

Also The proposition 1.2.6 is not entirely correct, if $\mathbf{X} = \{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$, then by Definition 1.2.4 and 1.2.5 the $\mathbf{VNCP}(\mathbf{P}_{\mathbf{N_N}})$ and $\mathbf{NCP}(\mathbf{P_N})$ points are:

$\{\mathbf{a}\}_{\mathbf{N_N}} = \langle \emptyset , \{\mathbf{a}\}, \{\mathbf{b}, \mathbf{c}\} \rangle$, $\{\mathbf{b}\}_{\mathbf{N_N}} = \langle \emptyset , \{\mathbf{b}\}, \{\mathbf{a}, \mathbf{c}\} \rangle$, $\{\mathbf{c}\}_{\mathbf{N_N}} = \langle \emptyset , \{\mathbf{c}\}, \{\mathbf{a}, \mathbf{b}\} \rangle$.

$\{\mathbf{a}\}_{\mathbf{N}} = \langle\{\mathbf{a}\}, \emptyset, \{\mathbf{b}, \mathbf{c}\}\rangle$, $\{\mathbf{b}\}_{\mathbf{N}} = \langle\{\mathbf{b}\}, \emptyset, \{\mathbf{a}, \mathbf{c}\}\rangle$, $\{\mathbf{c}\}_{\mathbf{N}} = \langle\{\mathbf{c}\}, \emptyset, \{\mathbf{a}, \mathbf{b}\}\rangle$.

For $\mathbf{A} = \langle\{\mathbf{a}\}, \{\mathbf{b}, \mathbf{c}\}, \emptyset\rangle$ is **NCS** −tape 1,2 and 3, also by using Definition 1.2.5 part 4 and 5 with Definition 1.1.5 part 2, we have

$\{\mathbf{a}\}_{\mathbf{N_N}}, \{\mathbf{a}\}_{\mathbf{N}}, \{\mathbf{b}\}_{\mathbf{N}}$ and $\{\mathbf{c}\}_{\mathbf{N}} \in \mathbf{A}$

but $\{\mathbf{a}\}_{\mathbf{N_N}} \cup_{\mathbf{1}} \{\mathbf{a}\}_{\mathbf{N}} \cup \{\mathbf{b}\}_{\mathbf{N}} \cup \{\mathbf{c}\}_{\mathbf{N}} = \langle\mathbf{X}, \emptyset, \mathbf{X}\rangle \neq \mathbf{A}$

and $\{\mathbf{a}\}_{\mathbf{N_N}} \cup_{\mathbf{2}} \{\mathbf{a}\}_{\mathbf{N}} \cup \{\mathbf{b}\}_{\mathbf{N}} \cup \{\mathbf{c}\}_{\mathbf{N}} = \langle\mathbf{X}, \emptyset, \emptyset\rangle \neq \mathbf{A}$

Through the above discussion, we suggest new concoptuals from the **NCP** points as follows $\mathbf{P_{N_1}} = \langle\mathbf{P_1}, \mathbf{P_2}, \mathbf{P_3}\rangle$ such that $\mathbf{P_i} \neq \emptyset$ for $\mathbf{i = 1}$ or $\mathbf{i = 2}$ or $\mathbf{i = 3}$ and the other is empty $\mathbf{P_{N_2}} = \langle\mathbf{P_1}, \mathbf{P_2}, \mathbf{P_3}\rangle$ such that $\mathbf{P_i} \neq \emptyset$ for $\mathbf{i = 1}$ or $\mathbf{i = 2}$ or $\mathbf{i = 3}$ and the other are singleton prints, and $\mathbf{P_{N_i}} \in \mathbf{A}$ iff $\mathbf{p_i} \subseteq \mathbf{A_i}, \forall\, \mathbf{i = 1, 2, 3}$.

Finally, we proposed anew kind of neutrosophic crisp sets with **NCP** point and all binary operations that qualify these sets to build a ring as well as Boolean algebra.

2. Neutrosophic Crisp Triple Sets

2.1. Definition

Let X be a nonempty set. A triple set $\ddot{A}$ is an object having the form $\ddot{A} = \langle A_1, A_2, A_3\rangle$

Where $A_1, A_2, A_3 \subseteq X$ satisfying $A_1 \subseteq A_2$ and $A_2 \cap A_3 = \emptyset$. Which we called the neutrosophic crisp triple sets (Simple NCT – sets).$T(X) = \{\ddot{A} = \langle A_1, A_2, A_3\rangle : A_1 \subseteq A_2$ and $A_2 \cap A_3 = \emptyset\}$ is the family of all $NCT -$ sets on X.

2.2. Definition

Let $\ddot{A} = \langle A_1, A_2, A_3\rangle$ and $\ddot{B} = \langle B_1, B_2, B_3\rangle$ be two $NCT -$ sets over a nonempty set X. Then

1. $\ddot{A}$ is a $NCT -$ subset of $\ddot{B}$ if $A_1 \subseteq B_1, A_2 \subseteq B_2$ and $A_3 \supseteq B_3$. We write $\ddot{A} \ddot{\subseteq} \ddot{B}$.
2. $\ddot{A} = \ddot{B}$ iff $\ddot{A} \ddot{\subseteq} \ddot{B}$ and $\ddot{B} \ddot{\subseteq} \ddot{A}$
3. The NCT −complement of $NCT -$ set $\ddot{A}$ is $C\ddot{A} = \langle A_3, A_2^c, A_1\rangle$
4. $\ddot{A} \ddot{\cup} \ddot{B} = \langle A_1 \cup B_1, A_2 \cup B_2, A_3 \cap B_3\rangle$ is the union neutrosophic crisp triple set (NCT −union set)
5. $\ddot{A} \ddot{\cap} \ddot{B} = \langle A_1 \cap B_1, A_2 \cap B_2, A_3 \cup B_3\rangle$ is the intersection neutrosophic crisp triple set (NCT −intersection sets)
6. $\ddot{A} - \ddot{B} = \ddot{A} \ddot{\cap} C\ddot{B}$.
7. $\ddot{A} \ddot{\Delta} \ddot{B} = (\ddot{A} \ddot{\cap} C\ddot{B}) \ddot{\cup} (C\ddot{A} \ddot{\cap} \ddot{B})$.

2.3. Definition

Let X be a nonempty set. Then

1. $\ddot{X} = \langle X, X, \emptyset\rangle$ is NCT −universel set
2. $\ddot{\emptyset} = \langle\emptyset, \emptyset, X\rangle$ is NCT −Null set

Clearly $C\ddot{X} = \ddot{\emptyset}$ and $C\ddot{\emptyset} = \ddot{X}$.

We list in the following properties the most important relationship the NCT −union, NCT −intersection and NCT −complement.

2.4. Proposition

Let $\ddot{A} = \langle A_1, A_2, A_3\rangle$ and $\ddot{B} = \langle B_1, B_2, B_3\rangle$ be two $Nt-$ sets over a nonempty set X. Then

1. $\ddot{A} \ddot{\cup} \ddot{A} = \ddot{A}$
2. $\ddot{A} \ddot{\cap} \ddot{A} = \ddot{A}$
3. $\ddot{A} \ddot{\cup} \ddot{\emptyset} = \ddot{A}$
4. $\ddot{A} \ddot{\cup} \ddot{X} = \ddot{X}$
5. $\ddot{A} \ddot{\cap} \ddot{\emptyset} = \ddot{\emptyset}$
6. $C(\ddot{A} \ddot{\cup} \ddot{B}) = C\ddot{A} \ddot{\cap} C\ddot{B}$
7. $C(\ddot{A} \ddot{\cap} \ddot{B}) = C\ddot{A} \ddot{\cup} C\ddot{B}$

Proof .

1. $\ddot{A} \ddot{\cup} \ddot{A} = \langle A_1 \cup A_1, A_2 \cup A_2, A_3 \cap A_3\rangle = \langle A_1, A_2, A_3\rangle = \ddot{A}$

2. $\ddot{A} \ddot{\cap} \ddot{A} = \langle A_1 \cap A_1, A_2 \cap A_2, A_3 \cup A_3\rangle = \langle A_1, A_2, A_3\rangle = \ddot{A}$

3. $\ddot{A} \ddot{\cup} \ddot{\emptyset} = \langle A_1 \cup \emptyset, A_2 \cup \emptyset, A_3 \cap X\rangle = \langle A_1, A_2, A_3\rangle = \ddot{A}$

4. $\ddot{A} \ddot{\cup} \ddot{X} = \langle A_1 \cup X, A_2 \cup X, A_3 \cap \emptyset\rangle = \langle X, X, \emptyset\rangle = \ddot{X}$

5. $\ddot{A} \ddot{\cap} \ddot{\emptyset} = \langle A_1 \cap \emptyset, A_2 \cap \emptyset, A_3 \cup X\rangle = \langle \emptyset, \emptyset, X\rangle = \ddot{A}$

6. $C(\ddot{A} \ddot{\cup} \ddot{B}) = C\langle A_1 \cup B_1, A_2 \cup B_2, A_3 \cap B_3\rangle = \langle A_3 \cap B_3, (A_2 \cup B_2)^c, A_1 \cup B_1\rangle = \langle A_3 \cap B_3, A_2^c \cap B_2^c, A_1 \cup B_1\rangle = \langle A_3, A_2^c, A_1\rangle \ddot{\cap} \langle B_3, B_2^c, B_1\rangle = C\ddot{A} \ddot{\cap} C\ddot{B}$

7. $C(\ddot{A} \ddot{\cap} \ddot{B}) = C\langle A_1 \cap B_1, A_2 \cap B_2, A_3 \cup B_3\rangle = \langle A_3 \cup B_3, (A_2 \cap B_2)^c, A_1 \cap B_1\rangle = \langle A_3 \cup B_3, A_2^c \cup B_2^c, A_1 \cap B_1\rangle = \langle A_3, A_2^c, A_1\rangle \ddot{\cup} \langle B_3, B_2^c, B_1\rangle = C\ddot{A} \ddot{\cup} C\ddot{B}.$

2.5. Proposition

Let $\ddot{A} = \langle A_1, A_2, A_3\rangle$ be $NCT-$ sets over a nonempty set X. Then:

1. $\ddot{\emptyset} \ddot{\subseteq} \ddot{A} \ddot{\cap} C\ddot{A}$.
2. $\ddot{A} \ddot{\cup} C\ddot{A} \ddot{\subseteq} \ddot{X}$.

Proof .

1. Obvious

The converse is not true in general for example, if $X = \{1, 2, 3\}$ and $\ddot{A} = \langle\{1\}, \{1,2\}, \{3\}\rangle$, then

$\ddot{A} \ddot{\cap} C\ddot{A} = \langle \emptyset, \emptyset, \{1,3\}\rangle \ddot{\nsubseteq} \ddot{\emptyset} = \langle \emptyset, \emptyset, X\rangle$.

2. Obvious

The converse is not true in general for example, if $X = \{1, 2, 3\}$ and $\ddot{A} = \langle\{1\}, \{1,2\}, \{3\}\rangle$, then

$\ddot{X} = \langle X, X, \emptyset\rangle \ddot{\nsubseteq} (\ddot{A} \ddot{\cup} C\ddot{A}) = \langle\{1,3\}, X, \emptyset\rangle$.

2.6. Proposition

Let $\ddot{A} = \langle A_1, A_2, A_3\rangle$,$\ddot{B} = \langle B_1, B_2, B_3\rangle$ and $\ddot{C} = \langle C_1, C_2, C_3\rangle$ be three NCT −sets over a nonempty set X. Then:

1. $\ddot{A} \ddot{\cup} (\ddot{B} \ddot{\cup} \ddot{C}) = (\ddot{A} \ddot{\cup} \ddot{B}) \ddot{\cup} \ddot{C}$.

2. $\ddot{A} \ddot{\cap} (\ddot{B} \ddot{\cap} \ddot{C}) = (\ddot{A} \ddot{\cap} \ddot{B}) \ddot{\cap} \ddot{C}$.

3. $\ddot{A} \ddot{\cup} (\ddot{B} \ddot{\cap} \ddot{C}) = (\ddot{A} \ddot{\cup} \ddot{B}) \ddot{\cap} (\ddot{A} \ddot{\cup} \ddot{C})$.

4. $\ddot{A} \ddot{\cap} (\ddot{B} \ddot{\cup} \ddot{C}) = (\ddot{A} \ddot{\cap} \ddot{B}) \ddot{\cup} (\ddot{A} \ddot{\cap} \ddot{C})$.

Proof .

1. $\ddot{A} \ddot{\cup} (\ddot{B} \ddot{\cup} \ddot{C}) = \langle A_1, A_2, A_3 \rangle \ddot{\cup} \langle B_1 \cup C_1, B_2 \cup C_2, B_3 \cap C_3 \rangle = \langle A_1 \cup (B_1 \cup C_1), A_2 \cup (B_2 \cup C_2), A_3 \cap (B_3 \cap C_3) \rangle = \langle (A_1 \cup B_1) \cup C_1, (A_2 \cup B_2) \cup C_2, (A_3 \cap B_3) \cap C_3 \rangle = \langle A_1 \cup B_1, A_2 \cup B_2, A_3 \cap B_3 \rangle \ddot{\cup} \langle C_1, C_2, C_3 \rangle = (\ddot{A} \ddot{\cup} \ddot{B}) \ddot{\cup} \ddot{C}$

2. $\ddot{A} \ddot{\cap} (\ddot{B} \ddot{\cap} \ddot{C}) = \langle A_1, A_2, A_3 \rangle \ddot{\cap} \langle B_1 \cap C_1, B_2 \cap C_2, B_3 \cup C_3 \rangle = \langle A_1 \cap (B_1 \cap C_1), A_2 \cap (B_2 \cap C_2), A_3 \cup (B_3 \cup C_3) \rangle = \langle (A_1 \cap B_1) \cap C_1, (A_2 \cap B_2) \cap C_2, (A_3 \cup B_3) \cup C_3 \rangle = \langle A_1 \cap B_1, A_2 \cap B_2, A_3 \cup B_3 \rangle \ddot{\cap} \langle C_1, C_2, C_3 \rangle = (\ddot{A} \ddot{\cap} \ddot{B}) \ddot{\cap} \ddot{C}$

3. $\ddot{A} \ddot{\cup} (\ddot{B} \ddot{\cap} \ddot{C}) = \langle A_1, A_2, A_3 \rangle \ddot{\cup} \langle B_1 \cap C_1, B_2 \cap C_2, B_3 \cup C_3 \rangle = \langle A_1 \cup (B_1 \cap C_1), A_2 \cup (B_2 \cap C_2), A_3 \cap (B_3 \cup C_3) \rangle = \langle (A_1 \cup B_1) \cap (A_1 \cup C_1), (A_2 \cup B_2) \cap (A_2 \cup C_2), (A_3 \cap B_3) \cup (A_3 \cap C_3) \rangle = \langle A_1 \cup B_1, A_2 \cup B_2, A_3 \cap B_3 \rangle \ddot{\cap} \langle A_1 \cup C_1, A_2 \cup C_2, A_3 \cap C_3 \rangle = (\ddot{A} \ddot{\cup} \ddot{B}) \ddot{\cap} (\ddot{A} \ddot{\cup} \ddot{C})$

4. $\ddot{A} \ddot{\cap} (\ddot{B} \ddot{\cup} \ddot{C}) = \langle A_1, A_2, A_3 \rangle \ddot{\cap} \langle B_1 \cup C_1, B_2 \cup C_2, B_3 \cap C_3 \rangle = \langle A_1 \cap (B_1 \cup C_1), A_2 \cap (B_2 \cup C_2), A_3 \cup (B_3 \cap C_3) \rangle = \langle (A_1 \cap B_1) \cup (A_1 \cap C_1), (A_2 \cap B_2) \cup (A_2 \cap C_2), (A_3 \cup B_3) \cap (A_3 \cup C_3) \rangle = \langle A_1 \cap B_1, A_2 \cap B_2, A_3 \cup B_3 \rangle \ddot{\cap} \langle A_1 \cap C_1, A_2 \cap C_2, A_3 \cup C_3 \rangle = (\ddot{A} \ddot{\cap} \ddot{B}) \ddot{\cup} (\ddot{A} \ddot{\cap} \ddot{C})$

2.7. Definition

Let $\{\ddot{A}_i : i \in I\}$ be a family of NCT −sets in X, where $\ddot{A}_i = \langle A_{i1}, A_{i2}, A_{i3} \rangle$. Then

1. $\ddot{\cup}_{i \in I} \ddot{A}_i = \langle \cup_{i \in I} A_{i1}, \cup_{i \in I} A_{i2}, \cap_{i \in I} A_{i3} \rangle$

2. $\ddot{\cap}_{i \in I} \ddot{A}_i = \langle \cap_{i \in I} A_{i1}, \cap_{i \in I} A_{i2}, \cup_{i \in I} A_{i3} \rangle$

2. 8. Proposition

1. *Let* $\ddot{A}, \ddot{B}, \ddot{C}$ *and* $\{\ddot{A}_i : i \in I\}$ *be triple sets of* X*.Then*

2. $\ddot{A} \ddot{\subseteq} \ddot{B}$ *and* $\ddot{B} \ddot{\subseteq} \ddot{C}$*, implies* $\ddot{A} \ddot{\subseteq} \ddot{C}$

3. $\ddot{A}_i \ddot{\subseteq} \ddot{B}$ *for each* $i \in I$*, then* $\ddot{\cup}_{i \in I} \ddot{A}_i \ddot{\subseteq} \ddot{B}$

4. $\ddot{B} \ddot{\subseteq} \ddot{A}_i$ *for each* $i \in I$*, then* $\ddot{B} \ddot{\subseteq} \ddot{\cap}_{i \in I} \ddot{A}_i$

5. $\ddot{A} \ddot{\subseteq} \ddot{B}$ *iff* $C\ddot{B} \ddot{\subseteq} C\ddot{A}$

Proof . Obvious

2.9. Definition

Let X be a nonempty set and $p \in X$. Then the NCT −points are the form:

1. $\ddot{p}_{\sim} = \langle \{p\}, \{p\}, \{p\}^c \rangle$
2. $\ddot{p}_{\approx} = \langle \emptyset, \{p\}, \{p\}^c \rangle$
3. $p_{\cong} = \langle \emptyset, \emptyset, \{p\}^c \rangle$

We easily note that NCT −points are NCT −sets.

2.10 . Definition

Let X be a nonempty set $p \in X$ and $\ddot{A} = \langle A_1, A_2, A_3 \rangle$. Then the NCT −belong as follows:

1. $\ddot{p}_{\sim} \ddot{\in} \ddot{A}$ iff $p \in A_1$.
2. $\ddot{p}_{\approx} \ddot{\in} \ddot{A}$ iff $p \in A_2$.
3. $p_{\cong} \ddot{\in} \ddot{A}$ iff $p \notin A_3$.

2.11. Proposition

Let $\{\ddot{A}_i : i \in I\}$ be family of NCT −set in X. Then

1. $\ddot{p}_{\sim} \ddot{\in} \ddot{\cap}_{i\in I} \ddot{A}_i$ iff $\ddot{p}_{\sim} \ddot{\in} \ddot{A}_i \, for\ each\ i \in I$
2. $\ddot{p}_{\approx} \ddot{\in} \ddot{\cap}_{i\in I} \ddot{A}_i$ iff $\ddot{p}_{\approx} \ddot{\in} \ddot{A}_i \, for\ each\ i \in I$
3. $p_{\cong} \ddot{\in} \ddot{\cap}_{i\in I} \ddot{A}_i$ iff $p_{\cong} \ddot{\in} \ddot{A}_i \, for\ each\ i \in I$
4. $\ddot{p}_{\sim} \ddot{\in} \ddot{\cup}_{i\in I} \ddot{A}_i$ iff $\exists i \in I\ such\ that\ \ddot{p}_{\sim} \ddot{\in} \ddot{A}_i$
5. $\ddot{p}_{\approx} \ddot{\in} \ddot{\cup}_{i\in I} \ddot{A}_i$ iff $\exists i \in I\ such\ that\ \ddot{p}_{\approx} \ddot{\in} \ddot{A}_i$
6. $\ddot{p}_{\cong} \ddot{\in} \ddot{\cup}_{i\in I} \ddot{A}_i$ iff $\exists i \in I\ such\ that\ p_{\cong} \ddot{\in} \ddot{A}_i$

2.12. Proposition

Let $\ddot{A}$ and $\ddot{B}$ be two NCT −set in X. Then

i. $\ddot{A} \ddot{\subseteq} \ddot{B}$ iff for each $\ddot{p}_{\sim}$ we have $\ddot{p}_{\sim} \ddot{\in} \ddot{A} \Longrightarrow \ddot{p}_{\sim} \ddot{\in} \ddot{B}$ and for each $\ddot{p}_{\approx}$ we have $\ddot{p}_{\approx} \ddot{\in} \ddot{A} \Longrightarrow \ddot{p}_{\approx} \ddot{\in} \ddot{B}$ and for each $\ddot{p}_{\cong}$ we have $\ddot{p}_{\cong} \ddot{\in} \ddot{A} \Longrightarrow \ddot{p}_{\cong} \ddot{\in} \ddot{B}$

ii. $\ddot{A} = \ddot{B}$ iff for each $\ddot{p}_{\sim}$ we have $\ddot{p}_{\sim} \ddot{\in} \ddot{A} \Leftrightarrow \ddot{p}_{\sim} \ddot{\in} \ddot{B}$ and for each $\ddot{p}_{\approx}$ we have $\ddot{p}_{\approx} \ddot{\in} \ddot{A} \Leftrightarrow \ddot{p}_{\approx} \ddot{\in} \ddot{B}$ and for each $\ddot{p}_{\cong}$ we have $\ddot{p}_{\cong} \ddot{\in} \ddot{A} \Leftrightarrow \ddot{p}_{\cong} \ddot{\in} \ddot{B}$

2. 13. proposition

Let $\ddot{A} = \langle A_1, A_2, A_3 \rangle$ be triple set in X . Then $\ddot{A} = \left(\ddot{\cup}\{\ddot{p}_{\sim} : \ddot{p}_{\sim} \ddot{\in} \ddot{A}\}\right) \ddot{\cup} \left(\ddot{\cup}\{\ddot{p}_{\approx} : \ddot{p}_{\approx} \ddot{\in} \ddot{A}\}\right) \ddot{\cup} \left(\ddot{\cup}\{\ddot{p}_{\cong} : \ddot{p}_{\cong} \ddot{\in} \ddot{A}\}\right)$

2. 14. proposition

1. Let $\ddot{A} = \langle A_1, A_2, A_3 \rangle, \ddot{B} = \langle B_1, B_2, B_3 \rangle$ and $\ddot{C} = \langle C_1, C_2, C_3 \rangle$ be NCT −sets. Then
2. $\ddot{A} - \ddot{B} \ddot{\subseteq} \ddot{A} - (\ddot{A} \ddot{\cap} \ddot{B})$ and $\ddot{A} - \ddot{B} \ddot{\subseteq} (\ddot{A} \ddot{\cup} \ddot{B}) - \ddot{B}$.
3. $(\ddot{A} \ddot{\cap} \ddot{B}) \ddot{\cup} (\ddot{B} - \ddot{A}) \ddot{\cup} (\ddot{A} - \ddot{B}) \ddot{\subseteq} \ddot{A} \ddot{\cup} \ddot{B}$.
4. $(\ddot{A} - \ddot{B}) - \ddot{C} = \ddot{A} - (\ddot{B} \ddot{\cup} \ddot{C})$.
5. $\ddot{A} - (\ddot{B} \ddot{\cap} \ddot{C}) = (\ddot{A} - \ddot{B}) \ddot{\cup} (\ddot{A} - \ddot{C})$.
6. $\ddot{A} \ddot{\cup} (\ddot{B} - \ddot{C}) = (\ddot{A} \ddot{\cup} \ddot{B}) - (\ddot{C} - \ddot{A})$.
7. $\ddot{A} \ddot{\cap} (\ddot{B} - \ddot{C}) = (\ddot{A} \ddot{\cap} \ddot{B}) - (\ddot{C}\ddot{A} \ddot{\cup} \ddot{C})$.
8. Not necessary if $\ddot{A} \ddot{\subseteq} \ddot{B}$ and $\ddot{A} \ddot{\subseteq} C\ddot{B}$, then $\ddot{A} = \ddot{\emptyset}$.
9. Not necessary if $\ddot{A} \ddot{\subseteq} \ddot{B}$ and $C\ddot{A} \ddot{\subseteq} \ddot{B}$, then $\ddot{A} = \ddot{X}$.
10. $\ddot{\cap}\{\ddot{B} \ddot{\in} T(X)\} = \ddot{\emptyset}$.
11. $\ddot{A} \ddot{\cap} C\ddot{B} \ddot{\subseteq} (\ddot{A} \ddot{\cup} \ddot{B}) \ \ddot{\cap}\ C\ddot{B}$
12. $(\ddot{A} - \ddot{B}) \ddot{\cup} (\ddot{B} - \ddot{A}) \ddot{\subseteq} (\ddot{A} \ddot{\cup} \ddot{B}) - (\ddot{B} \ddot{\cap} \ddot{A})$.

13. $\ddot{A}\,\ddot{\Delta}\,\ddot{\emptyset} = \ddot{A}$.

14. $\ddot{\emptyset} = \ddot{A}\,\ddot{\Delta}\,\ddot{A}$ if and only if $A_1 \cup A_3 = X$

15. $\ddot{A}\,\ddot{\Delta}\,\ddot{B} = \ddot{B}\,\ddot{\Delta}\,\ddot{A}$.

Proof.

16. $\ddot{A} - (\ddot{A} \ddot{\cap} \ddot{B}) = \ddot{A} \ddot{\cap} C(\ddot{A} \ddot{\cap} \ddot{B}) = \ddot{A} \ddot{\cap} (C\ddot{A} \ddot{\cup} C\ddot{B}) = (\ddot{A} \ddot{\cap} C\ddot{A}) \ddot{\cup} (\ddot{A} \ddot{\cap} C\ddot{B}) \ddot{\supseteq} \ddot{\emptyset} \ddot{\cup} (\ddot{A} \ddot{\cap} C\ddot{B}) = \ddot{A} \ddot{\cap} C\ddot{B} = \ddot{A} - \ddot{B}$.

The converse is not true in general for example, if $X = \{1, 2, 3\}$, $\ddot{A} = \langle\{1\}, \{1,2\}, \{3\}\rangle$ and $B = \langle\{2\}, \{2,3\}, \{1\}\rangle$, then:

$$\ddot{A} - (\ddot{A} \ddot{\cap} \ddot{B}) = \langle\{1\}, \{1\}, \{3\}\rangle \ddot{\nsubseteq} \ddot{A} - \ddot{B} = \langle\{1\}, \{1\}, \{2,3\}\rangle.$$

17. $(\ddot{A} \ddot{\cap} \ddot{B}) \ddot{\cup} (\ddot{A} - \ddot{B}) \ddot{\cup} (\ddot{B} - \ddot{A}) = (\ddot{A} \ddot{\cap} \ddot{B}) \ddot{\cup} (\ddot{A} \ddot{\cap} C\ddot{B}) \ddot{\cup} (\ddot{B} \ddot{\cap} \ddot{A}) \ddot{\cup} (\ddot{B} \ddot{\cap} C\ddot{A}) =$
$\left(\ddot{A} \ddot{\cap} (\ddot{B} \ddot{\cup} C\ddot{B})\right) \ddot{\cup} \left(\ddot{B} \ddot{\cap} (\ddot{A} \ddot{\cup} C\ddot{A})\right) \ddot{\subseteq} (\ddot{A} \ddot{\cap} \ddot{X}) \ddot{\cup} (\ddot{B} \ddot{\cap} \ddot{X}) = (\ddot{A} \ddot{\cup} \ddot{B}) \ddot{\cap} \ddot{X} = (\ddot{A} \ddot{\cup} \ddot{B})$.

The converse is not true in general for example, if $X = \{1, 2, 3\}$, $\ddot{A} = \langle\{1\}, \{1,2\}, \{3\}\rangle$ and $B = \langle\{2\}, \{2,3\}, \{1\}\rangle$, then:

$$\ddot{A} \ddot{\cup} \ddot{B} = \langle\{1,2\}, X, \emptyset\rangle \ddot{\nsubseteq} (\ddot{A} \ddot{\cap} \ddot{B}) \ddot{\cup} (\ddot{B} - \ddot{A}) \ddot{\cup} (\ddot{A} - \ddot{B}) = \langle\{1\}, \{1,3\}, \emptyset\rangle.$$

18. $\ddot{A} - (\ddot{B} \ddot{\cup} \ddot{C}) = \ddot{A} \ddot{\cap} C(\ddot{B} \ddot{\cap} \ddot{C}) = \ddot{A} \ddot{\cap} (C\ddot{B} \ddot{\cap} C\ddot{C}) = (\ddot{A} \ddot{\cap} C\ddot{B}) \ddot{\cap} C\ddot{C} = (\ddot{A} - \ddot{B}) \ddot{\cap} C\ddot{C} = (\ddot{A} - \ddot{B}) - \ddot{C}$.

19. $\ddot{A} - (\ddot{B} \ddot{\cap} \ddot{C}) = \ddot{A} \ddot{\cap} C(\ddot{B} \ddot{\cap} \ddot{C}) = \ddot{A} \ddot{\cap} (C\ddot{B} \ddot{\cup} C\ddot{C}) = (\ddot{A} \ddot{\cap} C\ddot{B}) \ddot{\cup} (\ddot{A} \ddot{\cap} C\ddot{C}) = (\ddot{A} - \ddot{B}) \ddot{\cup} (\ddot{A} - \ddot{C})$.

20. $(\ddot{A} \ddot{\cup} \ddot{B}) - (\ddot{C} - \ddot{A}) = (\ddot{A} \ddot{\cup} \ddot{B}) \ddot{\cap} C(\ddot{C} \ddot{\cap} C\ddot{A}) = (\ddot{A} \ddot{\cup} \ddot{B}) \ddot{\cap} (C\ddot{C} \ddot{\cup} \ddot{A}) = \ddot{A} \ddot{\cup} (\ddot{B} \ddot{\cap} C\ddot{C}) = \ddot{A} \ddot{\cup} (\ddot{B} - \ddot{C})$.

21. $\ddot{A} \ddot{\cap} (\ddot{B} - \ddot{C}) = \ddot{A} \ddot{\cap} (\ddot{B} \ddot{\cap} C\ddot{C}) = (\ddot{A} \ddot{\cap} \ddot{B}) \ddot{\cap} (\ddot{A} \ddot{\cap} C\ddot{C}) = (\ddot{A} \ddot{\cap} \ddot{B}) \ddot{\cap} C(C\ddot{A} \ddot{\cup} \ddot{C}) = (\ddot{A} \ddot{\cap} \ddot{B}) - (C\ddot{A} \ddot{\cup} \ddot{C})$.

22. Let $X = \{1,2,3\}$, $\ddot{A} = \langle\emptyset, \emptyset, \{1,2\}\rangle$ and $\ddot{B} = \langle\emptyset, \emptyset, \{1\}\rangle$, then $\ddot{A} \ddot{\subseteq} \ddot{B}$, $\ddot{A} \ddot{\subseteq} C\ddot{B}$ and $\ddot{A} \neq \ddot{\emptyset}$.

23. Let $X = \{1,2,3\}$, $\ddot{A} = \langle X, X, \{1,2\}\rangle$ and $\ddot{B} = \langle X, X, \{1\}\rangle$, then $\ddot{A} \ddot{\subseteq} \ddot{B}$, $C\ddot{A} \ddot{\subseteq} \ddot{B}$ and $\ddot{A} \neq \ddot{X}$.

24. Obvious.

25. Since $\ddot{\emptyset} \ddot{\subseteq} (\ddot{B} \ddot{\cap} C\ddot{B})$, then $\ddot{\emptyset} \ddot{\cup} (\ddot{A} \ddot{\cap} C\ddot{B}) \ddot{\subseteq} (\ddot{A} \ddot{\cap} C\ddot{B}) \ddot{\cup} (\ddot{B} \ddot{\cap} C\ddot{B}) = (\ddot{A} \ddot{\cup} \ddot{B}) \ddot{\cap} C\ddot{B}$. Hence $\ddot{A} \ddot{\cap} C\ddot{B} \ddot{\subseteq} (\ddot{A} \ddot{\cup} \ddot{B}) \ddot{\cap} C\ddot{B}$.

26. $(\ddot{A} - \ddot{B}) \ddot{\cup} (\ddot{B} - \ddot{A}) = (\ddot{A} \ddot{\cap} C\ddot{B}) \ddot{\cup} (\ddot{B} \ddot{\cap} C\ddot{A}) \ddot{\subseteq} [(\ddot{A} \ddot{\cup} \ddot{B}) \ddot{\cap} C\ddot{B}] \ddot{\cup} [(\ddot{A} \ddot{\cup} \ddot{B}) \ddot{\cap} C\ddot{A}] =$
$(\ddot{A} \ddot{\cup} \ddot{B}) \ddot{\cap} (C\ddot{B} \ddot{\cup} C\ddot{A}) = (\ddot{A} \ddot{\cup} \ddot{B}) \ddot{\cap} C(\ddot{B} \ddot{\cap} \ddot{A}) = (\ddot{A} \ddot{\cup} \ddot{B}) - (\ddot{B} \ddot{\cap} \ddot{A})$.

27. $\ddot{A}\,\ddot{\Delta}\,\ddot{\emptyset} = (\ddot{A} \ddot{\cap} C\ddot{\emptyset}) \ddot{\cup} (C\ddot{A} \ddot{\cap} \ddot{\emptyset}) = (\ddot{A} \ddot{\cap} \ddot{X}) \ddot{\cup} \ddot{A} = \ddot{A} \ddot{\cup} \ddot{A} = \ddot{A}$.

28. Suppose that $\ddot{A}\,\ddot{\Delta}\,\ddot{B} = \ddot{\emptyset}$, then

$$\ddot{A}\,\ddot{\Delta}\,\ddot{B} = (\ddot{A} \ddot{\cap} C\,\ddot{B}) \cup (C\ddot{A} \ddot{\cap} \ddot{B}) = \langle\emptyset, \emptyset, A_1 \cup A_3\rangle = \langle\emptyset, \emptyset, X\rangle$$

Hence $A_1 \cup A_3 = X$.

Conversely, suppose that $A_1 \cup A_3 = X$, then

$$\ddot{A}\,\ddot{\Delta}\,\ddot{B} = (\ddot{A} \ddot{\cap} C\,\ddot{B}) \cup (C\ddot{A} \ddot{\cap} \ddot{B}) = \langle\emptyset, \emptyset, A_1 \cup A_3\rangle = \langle\emptyset, \emptyset, X\rangle = \ddot{\emptyset}$$

29. $\ddot{A}\,\ddot{\Delta}\,\ddot{B} = (\ddot{A} \ddot{\cap} C\ddot{B}) \ddot{\cup} (C\ddot{A} \ddot{\cap} \ddot{B}) = (C\ddot{A} \ddot{\cap} \ddot{B}) \ddot{\cup} (\ddot{A} \ddot{\cap} C\ddot{B}) = (\ddot{B} \ddot{\cap} C\ddot{A}) \ddot{\cup} (C\ddot{B} \ddot{\cap} \ddot{A}) = \ddot{B}\,\ddot{\Delta}\,\ddot{A}$.

2. 15. Remark

From proposition 2.14, part 12 is, which ensures that that NCT −null set acts as the identity element for $\ddot{\Delta}$, so that by proposition 2.14, part 13 each members of $*-T(X) = \{\ddot{A} = \langle A_1, A_2, A_3\rangle : \ddot{A}$ is $Nt-$ set and $A_1 \cup A_3 = X\}$ happens to have its own inverse, finally the part 14, shows that $\ddot{\Delta}$ is commutative. All this supports contusion that $(*-T(X), \ddot{\Delta})$ constitutes a commutative group.

2. 16. Theorem

Let X be non-null set and $*-T(X) = \{\ddot{A} = \langle A_1, A_2, A_3\rangle : \ddot{A}$ is $Nt-$ set and $A_1 \cup A_3 = X\}$ on X. Then $(*-T(X), \ddot{\Delta}, \ddot{\cap})$ form a ring.

Proof.

By proposition 2.4 and 2.5, that $(*-T(X), \ddot{\cap})$ is semi group, and $(*-T(X), \ddot{\Delta})$ is commutative group. It is only necessary to check the left distribution. of the $\ddot{\cap}$ operation on $\ddot{\Delta}$.

$$
\begin{aligned}
(\ddot{A} \ddot{\cap} \ddot{B}) \ddot{\Delta} (\ddot{A} \ddot{\cap} \ddot{C}) &= \{C(\ddot{A} \ddot{\cap} \ddot{B}) \ddot{\cap} C(\ddot{A} \ddot{\cap} \ddot{C}\} \ddot{\cup} \{(\ddot{A} \ddot{\cap} \ddot{B}) \ddot{\cap} C(\ddot{A} \ddot{\cap} \ddot{C})\} \\
&= \{(\ddot{A} \ddot{\cap} \ddot{B}) \ddot{\cap} C(\ddot{A} \ddot{\cap} \ddot{C})\} \ddot{\cup} \{C(\ddot{A} \ddot{\cap} \ddot{B}) \ddot{\cap} C(\ddot{A} \ddot{\cap} \ddot{C}\} \\
&= \{(\ddot{A} \ddot{\cap} \ddot{B}) \ddot{\cap} (C\ddot{A} \ddot{\cup} C\ddot{C})\} \ddot{\cup} \{(\ddot{A} \ddot{\cap} \ddot{C}) \ddot{\cap} (C\ddot{A} \ddot{\cup} C\ddot{B})\} \\
&= \{[(\ddot{A} \ddot{\cap} \ddot{B}) \ddot{\cap} C\ddot{A}] \ddot{\cup} [(\ddot{A} \ddot{\cap} \ddot{B}) \ddot{\cap} C\ddot{C}]\} \ddot{\cup} \{[(\ddot{A} \ddot{\cap} \ddot{C}) \ddot{\cap} C\ddot{A}] \ddot{\cup} [(\ddot{A} \ddot{\cap} \ddot{C}) \ddot{\cap} C\ddot{B}]\} \\
&= \{(\ddot{A} \ddot{\cap} \ddot{B}) \ddot{\cap} C\ddot{C}\} \ddot{\cup} \{(\ddot{A} \ddot{\cap} \ddot{C}) \ddot{\cap} C\ddot{B}\} \\
&= \ddot{A} \ddot{\cap} \left((\ddot{B} \ddot{\cap} C\ddot{C}) \ddot{\cup} (\ddot{C} \ddot{\cap} C\ddot{B})\right) \\
&= \ddot{A} \ddot{\cap} (\ddot{B} \ddot{\Delta} \ddot{C})
\end{aligned}
$$

Therefore $(*-T(X), \ddot{\cap}, \ddot{\Delta})$ is a ring.

2. 17. Definition

Let $f : X \to Y$ be a function. Define the NCT −function $\mathcal{F} : T(X) \to T(Y)$ by

1. If $\ddot{A} = \langle A_1, A_2, A_3\rangle \in T(X)$, then $\mathcal{F}(\ddot{A}) = \langle f(A_1), f(A_2), f-(A_3)\rangle$ where, $f-(A_3) = Y - (f(X - A_3))$.

2 If $\ddot{B} = \langle B_1, B_2, B_3\rangle \in T(Y)$, then, $\mathcal{F}^{-1}(\ddot{B}) = \langle f^{-1}(B_1), f^{-1}(B_2), f^{-1}(B_2)\rangle$.

2. 18. Properties

Let $\mathcal{F} : T(X) \to T(Y)$ be a NCT −function and $\ddot{A}$, $\ddot{A}_i (i \in I) \in T(X), \ddot{B}, \ddot{B}_j (j \in J) \in T(Y)$, . Then

1. $\ddot{A}_1 \ddot{\subseteq} \ddot{A}_2 \Longrightarrow \mathcal{F}(\ddot{A}_1) \ddot{\subseteq} \mathcal{F}(\ddot{A}_2)$

2. $\ddot{B}_1 \ddot{\subseteq} \ddot{B}_2 \Longrightarrow \mathcal{F}^{-1}(\ddot{B}_1) \ddot{\subseteq} \mathcal{F}^{-1}(\ddot{B}_2)$

3. If $\ddot{A} \ddot{\subseteq} \mathcal{F}^{-1}\left(\mathcal{F}(\ddot{A})\right)$ and f is 1-1, then $\ddot{A} = \mathcal{F}^{-1}\left(\mathcal{F}(\ddot{A})\right)$.

4. If $\mathcal{F}\left(\mathcal{F}^{-1}(\ddot{B})\right) \ddot{\subseteq} \ddot{B}$ and f is onto, then $\mathcal{F}\left(\mathcal{F}^{-1}(\ddot{B})\right) = \ddot{B}$

5. $\mathcal{F}^{-1}(\ddot{\cup}\, \ddot{B}_j) = \ddot{\cup}\, \mathcal{F}^{-1}(\ddot{B}_j)$

6. $\mathcal{F}^{-1}(\ddot{\cap}\, \ddot{B}_j) = \ddot{\cap}\, \mathcal{F}^{-1}(\ddot{B}_j)$

7. $\mathcal{F}(\ddot{\cup}\, \ddot{A}_i) = \ddot{\cup}\, \ddot{\mathcal{F}}(A_i)$

8. $\mathcal{F}(\ddot{\cap}\, \ddot{A}_i) \ddot{\subseteq} \ddot{\cap}\, \ddot{\mathcal{F}}(A_i)$ and if f is 1-1, then $\mathcal{F}(\ddot{\cap}\, \ddot{A}_i) = \ddot{\cap}\, \ddot{\mathcal{F}}(A_i)$.

9. $\mathcal{F}^{-1}(\ddot{Y}) = \ddot{X}$

10. $\mathcal{F}^{-1}(\ddot{\emptyset}) = \ddot{\emptyset}$.

11. $\mathcal{F}(X) = Y$, if f is onto.

12. $\mathcal{F}(\emptyset) = \emptyset$

Proof.

Let $\ddot{A}_i = \langle A_{i1}, A_{i2}, A_{i3}\rangle$, $\ddot{B}_j = \langle B_{j1}, B_{j2}, B_{j3}\rangle$, $(i \in I, j \in J)$, $\ddot{A} = \langle A_1, A_2, A_3\rangle$ and $\ddot{B} = \langle B_1, B_2, B_3\rangle$.

1. Let $\ddot{A}_1 \ddot{\subseteq} \ddot{A}_2$. Since $A_{11} \subseteq A_{21}, A_{12} \subseteq A_{22}$ and $A_{23} \subseteq A_{13}$, then $f(A_{11}) \subseteq f(A_{21}), f(A_{12}) \subseteq f(A_{22})$ and

$X - A_{13} \subseteq X - A_{23} \Rightarrow f(X - A_{13}) \subseteq f(X - A_{23}) \Rightarrow$

$Y - f(X - A_{23}) \subseteq Y - f(X - A_{13}) \Rightarrow f-(A_{23}) \subseteq f-(A_{13})$. Hence $\mathcal{F}(\ddot{A}_1) \ddot{\subseteq} \mathcal{F}(\ddot{A}_2)$.

2. It is similar to (1.).

3. $\mathcal{F}^{-1}\left(\mathcal{F}(\ddot{A})\right) = \mathcal{F}^{-1}(\mathcal{F}(\langle A_1, A_2, A_3\rangle)) = \mathcal{F}^{-1}(\langle f(A_1), f(A_2), f-(A_3)\rangle) =$ $\langle f^{-1}(f(A_1)), f^{-1}(f(A_2)), f^{-1}(f-(A_3))\rangle \ddot{\supseteq} \langle A_1, A_2, A_3\rangle = \ddot{A}$.

4. It is similar to (3.).

5. $\mathcal{F}^{-1}(\ddot{\cup}\, \ddot{B}_j) = \mathcal{F}^{-1}(\langle \cup B_{j1}, \cup B_{j2}, \cap B_{j3}\rangle) = \langle f^{-1}(\cup B_{j1}), f^{-1}(\cup B_{j2}), f^{-1}(\cap B_{j3})\rangle = \langle \cup f^{-1}(B_{j1}), \cup f^{-1}(B_{j2}), \cap f^{-1}(B_{j3})\rangle = \ddot{\cup}\, \mathcal{F}^{-1}(\ddot{B}_j)$.

6. It is similar to (5.).

7. $\mathcal{F}(\ddot{\cup}\, \ddot{A}_i) = \mathcal{F}(\langle \cup A_{i1}, \cup A_{i2}, \cap A_{i3}\rangle) = \langle f(\cup A_{i1}), f(\cup A_{i2}), f-(\cap A_{i3})\rangle = \langle \cup f(A_{i1}), \cup f(A_{i2}), \cap f-(A_{i3})\rangle = \ddot{\cup}\, \ddot{\mathcal{F}}(A_i)$. Noties that $f-(\cap A_{i3}) = Y - f(X - \cap A_{i3}) = Y - f(\cup (X - A_{i3}) = Y - \cup f(X - A_{i3}) = \cap \left(Y - f(X - A_{i3})\right) = \cap f-(A_{i3})$.

8. $\mathcal{F}(\ddot{\cap}\, \ddot{A}_i) = \mathcal{F}(\langle \cap A_{i1}, \cap A_{i2}, \cup A_{i3}\rangle) = \langle f(\cap A_{i1}), f(\cap A_{i2}), f-(\cup A_{i3})\rangle \ddot{\subseteq} \langle \cap f(A_{i1}), \cap f(A_{i2}), \cup f-(A_{i3})\rangle = \ddot{\cap}\, \ddot{\mathcal{F}}(A_i)$. Noties that $f-(\cup A_{i3}) = Y - f(X - \cup A_{i3}) = Y - f(\cap (X - A_{i3}) \supseteq Y - \cap f(X - A_{i3}) = \cup \left(Y - f(X - A_{i3})\right) = \cup f-(A_{i3})$.

9. $\mathcal{F}^{-1}(\ddot{Y}) = \mathcal{F}^{-1}(\langle Y, Y, \emptyset\rangle) = \langle f^{-1}(Y), f^{-1}(Y), f^{-1}(\emptyset)\rangle = \langle X, X, \emptyset\rangle = \ddot{X}$.

(10.), (11.), (12.) are similar to (**9.**).

3. Conclusions

I. We can define topology over $T(X)$ as following:

The sub collection $Nt - T$ of $T(X)$ is called neutrosophic triple topology (simply Nt −topology), if satisfy the:

1. $\ddot{X}, \ddot{\emptyset} \in Nt - I$

2. $Nt - T$ is closed under the finite Nt −intersection.

3. $Nt - T$ is closed under the Nt −Union

In general, through these definition, and in particular we can modify the concepts and results in the papers [4,5,6,7, 8,9,10,11].

II. The sub collection $Nt - T$ of $T(X)$ is called neutrosophic triple ideal (Simply Nt −ideal) if satisfy that

1. If $\ddot{A} \in Nt - I$ and $\ddot{B} \ddot{\subseteq} \ddot{A}$, then $\ddot{B} \in Nt - I$.
2. $Nt - I$ is closed under the finite $Nt -$ Union

In general, through these definition, we can modify all the concepts, in particular we can modify the in the and results in the papers [2, 12,13,14, 15]

III. The sub collection $Nt - F$ of $T(X)$ is called neutrosophic triple filter (Nt −filter) if satisfy:

1. If $\ddot{A} \in Nt - F$ and $\ddot{A} \ddot{\subseteq} \ddot{B}$, then $\ddot{B} \in Nt - F$.
2. $Nt - F$ is closed under finite Nt −intersection, Also we can define the proximity relation on $T(X)$), and modify the concepts in the papers [3, 16,17,18, 19].

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