A new ranking function of triangular neutrosophic number and its application in integer programming

Sapan Kumar Das1* and S.A. Edalatpanah2

1Department of Mathematics, National Institute of Technology, Jamshedpur, India; cool.sapankumar@gmail.com
2Department of Applied Mathematics, Ayandegan Institute of Higher Education, Tonekabon, Iran; saedalatpanah@gmail.com
*Correspondence: cool.sapankumar@gmail.com

Abstract

Real humankind problems have different sorts of ambiguity in the creation, and amidst them, one of the significant issues in solving the integer linear programming issues. In this commitment, the conception of aggregation of ranking function has been focused on a distinct framework of reference. Here, we build up another framework for neutrosophic integer programming issues having triangular neutrosophic numbers by using the aggregate ranking function. To legitimize the proposed technique, scarcely numerical analyses are given to show the viability of the new model. At long last, conclusions are talked about.

Keywords: Neutrosophic triangular numbers, integer programming, aggregate ranking function.

1. Introduction

Professor Zadeh [1] was originally presented the idea of a fuzzy set theory (in 1965). The idea of fuzziness has a leading feature to solve efficiently in engineering and statistical problem. Applying the uncertainty theory, plentiful varieties of realistic problems can be solved, networking problems, decision-making problems, influence on social science, etc. As a result, by considering fuzzy parameters in linear programming, fuzzy linear programming is defined. Accordingly, various researchers have demonstrated their attentiveness to various sorts of fuzzy linear programming (FLP) issue and proposed a diverse system for dealing with FLP issues. If the parameters and constraints are fuzzy numbers, then it is called fully fuzzy numbers. A general class of fully FLP (FFLP) was introduced by Buckley and Feuring [46]. Many authors [2, 30-35, 37] considered issues either fuzzy linear programming implies either just the right-hand side or the constraints have been fuzzy or simply factors are fuzzy. Fuzzy IP problem is also the main part of LP problem. Allahviranloo et al. [3] offered a technique for solving IP problems. Fan et al., also [4] offered a general technique for resolving IP under fuzzy environment. Dehghan et al. [38] proposed practical methodologies to resolve a fully fuzzy linear system (FFLS) that is proportional to the remarkable systems. Lotfi et al. [39] proposed a procedure for symmetric triangular fuzzy number, gained another system for dealing with FFLP issues by changing over two relating LPs. To overcome these obstruction Kumar et al. [36] introduced another system for discovering the fuzzy ideal arrangement of FFLP issue with uniformity imperatives. After that Edalatpanah [14-15], Das [8-12], Das

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et al. [5-6] have portrayed to deal with the FFLP issue with the assistance of situating limit and lexicographic methodology. Wan and Dong [42] proposed a new approach for trapezoidal fuzzy linear programming problems thinking about the acceptance degree of fuzzy constraints violate. Pertaining the concept of Zadeh’s research paper, Atanassov [40] created phenomenally the intuitionistic fuzzy set where he meticulously elucidates the concept of membership and nonmembership function. Smarandache [16] in 1998 germinated the notion of having neutrosophic set holding three different fundamental elements (i) truth, (ii) indeterminate, and (iii) falsity. Each and every attribute of the neutrosophic sets are very relevant factors to our real-life models. Afterward, Wang et al. [44] progressed with a single typed neutrosophic set, which serves the solution to any sort of complicated problem in a very efficient way. Neutrosophic hypothesis applied in numerous fields of sciences, so as to take care of the issues identified with indeterminacy, see [20, 22, 27-28,41, 45, 48-49]. In like manner, Abdel-Baset [23] added the neutrosophic LP models the place their parameters are tended with trapezoidal neutrosophic numbers and introduced a method for getting them. Das and Jatindra [43] introduced a strategy for solving neutrosophic LP problem having triangular neutrosophic numbers by using ranking function. Edalatpanah [13, 21] presented some direct approaches of neutrosophic LP problem having the triangular fuzzy number. Again, Edalatpanah [17-20] established some aggregate ranking functions for data envelopment analysis (DEA) based on triangular neutrosophic numbers. Mohamed et al., [47] introduced another score function for neutrosophic integer programming problems having triangular neutrosophic numbers. Banerjee and Pramanik [25] added the LP problem with single objective in neutrosophic number (NN) condition with the assistance of goal programming. Likewise, Pramanik and Dey [24] detailed arrangement technique to linear bi-level programming problem in NN condition. Maiti et al. [26] introduced a strategy for multi-level-multi-objective LP problems by the assistance of goal programming. Hussian et al. [29] proposed a neutrosophic LP issue using ranking function. A IP issue under neutrosophic condition having triangular neutrosophic numbers was proposed by Nafei and Nasseri [7].

The motivation of this research paper, to develop an aggregate ranking function and usage of our function in integer programming (IP) problem. We propose IP problem based on triangular neutrosophic numbers. We likewise change the neutrosophic IP issue into a crisp IP model through the use of the aggregate ranking function. Any standard methodologies explain this crisp IP issue.

This research paper is prepared as follows: in the next segment, some fundamental concepts, mathematical operation on triangular neutrosophic numbers are introduced. In the next following segment, the proposed strategy for solving the IP problem is examined. Following this segment, the sub-section of Limitation and shortcoming of the existing method, sub-section of neutrosophic IP is discussed. In the next subsection, we discuss the proposed algorithm for solving our problem. In segment before determination, a numerical model is given to uncover the viability of the proposed model. At long last, conclusions are given in the last segment.

2. Preliminaries

Right now, some fundamental concepts and neutrosophic numbers have been examined under this segment.

**Definition 1. [16]**

Assume \( S \) be a space of objectives and \( s \in S \). A neutrosophic set \( N \) in \( S \) may be interpret via three membership functions for truth, indeterminacy along with falsity and represent by \( \rho_1(s), \beta_1(s) \) and \( \lambda_1(s) \) are real standard or
real nonstandard subsets of $[0^-,1^+]$. That is
\[
\rho_i(s) : S \rightarrow [0^-,1^+], \beta_i(s) : S \rightarrow [0^-,1^+] \quad \text{and} \quad \lambda_i(s) : S \rightarrow [0^-,1^+].
\]
There is no limitation on the sum of $\rho_i(s)$, $\beta_i(s)$, and $\lambda_i(s)$, so $0^- \leq \sup \rho_i(s) + \sup \beta_i(s) + \sup \lambda_i(s) \leq 3^+$.

**Definition 2.** [16]

A single-valued neutrosophic set (SVNS) $I$ through $S$ taking the form $I = \{s, \rho_i(s), \beta_i(s), \lambda_i(s) ; s \in S\}$, where $S$ be a space of discourse, $\rho_i(s) : S \rightarrow [0^-,1^+]$, $\beta_i(s) : S \rightarrow [0^-,1^+]$ and $\lambda_i(s) : S \rightarrow [0^-,1^+]$ with $0 < \rho_i(s) + \beta_i(s) + \lambda_i(s) < 3$ for all $s \in S$. $\rho_i(s)$, $\beta_i(s)$ and $\lambda_i(s)$ respectively represent truth membership, indeterminacy membership, and falsity membership degree of $s$ to $I$.

**Definition 3** [43]. A triangular neutrosophic number (TNNs) is signified via $I = (b^1, b^2, b^3, (\alpha, \delta, \lambda))$ is an extended version of the three membership functions for the truth, indeterminacy, and falsity of $s$ can be defined as follows:

\[
\rho_i(s) = \begin{cases} 
\frac{(s-b^1)}{(b^2-b^1)} & b^1 \leq s < b^2, \\
\alpha & s = b^1, \\
\frac{(b^3-s)}{(b^3-b^2)} & b^2 \leq s < b^3, \\
0 & \text{something else.}
\end{cases}
\]

\[
\beta_i(s) = \begin{cases} 
\frac{(b^3-s)}{(b^3-b^2)} & b^1 \leq s < b^2, \\
\delta & s = b^1, \\
\frac{(s-b^3)}{(b^3-b^2)} & b^2 \leq s < b^3, \\
1 & \text{something else.}
\end{cases}
\]

\[
\lambda_i(s) = \begin{cases} 
\frac{(b^3-s)}{(b^3-b^2)} & b^1 \leq s < b^2, \\
\lambda & s = b^1, \\
\frac{(s-b^3)}{(b^3-b^2)} & b^2 \leq s < b^3, \\
1 & \text{something else.}
\end{cases}
\]
Where, $0 \leq \rho_{1}(s) + \beta_{1}(s) + \lambda_{1}(s) \leq 3, s \in I$. Additionally, when $b^{1} \geq 0, I$ is called a nonnegative TNN. Similarly, when $b^{1} < 0, I$ becomes a negative TNN.

**Definition 5** [19]. Arithmetic Operation

Suppose $A^{1}_{1} = (b^{1}_{1}, b^{2}_{1}, b^{3}_{1}), (\alpha_{1}, \delta_{1}, \lambda_{1}) >$ and $A^{1}_{2} = (b^{1}_{2}, b^{2}_{2}, b^{3}_{2}), (\alpha_{2}, \delta_{2}, \lambda_{2}) >$ be two TNNs. Then the mathematical computation will be explained as:

(i) $A^{1}_{1} \oplus A^{1}_{2} = (b^{1}_{1} + b^{2}_{1} + b^{3}_{1}, b^{1}_{2} + b^{2}_{2} + b^{3}_{2}), (\alpha_{1}, \alpha_{2}, \delta_{1} \vee \delta_{2}, \lambda_{1} \vee \lambda_{2}) >$

(ii) $A^{1}_{1} - A^{1}_{2} = (b^{1}_{1} - b^{1}_{2}, b^{2}_{1} - b^{2}_{2}, b^{1}_{2} - b^{2}_{2}), (\alpha_{1}, \alpha_{2}, \delta_{1} \vee \delta_{2}, \lambda_{1} \vee \lambda_{2}) >$

(iii) $A^{1}_{1} \otimes A^{1}_{2} = (b^{1}_{1} b^{2}_{1} b^{2}_{1}, b^{1}_{2} b^{2}_{2} b^{2}_{2}), (\alpha_{1}, \alpha_{2}, \delta_{1} \vee \delta_{2}, \lambda_{1} \vee \lambda_{2}) >$, if $b^{1}_{1} > 0, b^{1}_{2} > 0,$

(iv) $A^{1}_{1} \cdot A^{1}_{2}^{U} = \begin{cases} (\lambda b^{1}_{1}, \lambda b^{2}_{1}, \lambda b^{3}_{1}), (\alpha_{1}, \delta_{1}, \lambda_{1}) >, & \text{if } \lambda > 0 \\ (\lambda b^{1}_{1}, \lambda b^{2}_{1}, \lambda b^{3}_{1}), (\alpha_{1}, \delta_{1}, \lambda_{1}) >, & \text{if } \lambda < 0 \end{cases}$

(v) $A^{1}_{1} / A^{1}_{2}^{U} = \begin{cases} \left( \frac{b^{1}_{1}}{b^{2}_{2}}, \frac{b^{2}_{1}}{b^{2}_{2}}, \frac{b^{3}_{1}}{b^{3}_{2}}; \alpha_{1} \land \alpha_{2}, \delta_{1} \land \delta_{2}, \lambda_{1} \land \lambda_{2} \right), (b^{1}_{1} > 0, b^{2}_{2} > 0) \\ \left( \frac{b^{1}_{1}}{b^{2}_{2}}, \frac{b^{2}_{1}}{b^{2}_{2}}, \frac{b^{3}_{1}}{b^{3}_{2}}; \alpha_{1} \land \alpha_{2}, \delta_{1} \land \delta_{2}, \lambda_{1} \land \lambda_{2} \right), (b^{1}_{1} < 0, b^{2}_{2} > 0) \\ \left( \frac{b^{1}_{1}}{b^{2}_{2}}, \frac{b^{2}_{1}}{b^{2}_{2}}, \frac{b^{3}_{1}}{b^{3}_{2}}; \alpha_{1} \land \alpha_{2}, \delta_{1} \land \delta_{2}, \lambda_{1} \land \lambda_{2} \right), (b^{1}_{1} < 0, b^{2}_{2} < 0) \end{cases}$

3. Proposed model

Before going to our main algorithm, first of all, we tend to begin a subsidiary i.e., drawback as well as restriction of the available method [7]

**3.1 Shortcoming and Limitation of the existing method**

First of all, we investigate drawback as well as restrictions of an available method [7] under exclusive ranking function.

Nafei and Nasseri [7] suggested a model for IP problems by utilizing the ranking function. However, the author uses some scientific presumption to resolve the problem that may be invalid in any case. This has been examined in Example 3.1 and Example 3.2.

**Definition 6:** One can examine any two TNNs in response to the ranking functions. Let $I^{N} = (b^{1}, b^{2}, b^{3}) ; \alpha, \delta, \lambda >$ be a triangular neutrosophic numbers (TNNs); then

$$R(I^{N}) = \frac{b^{1} + b^{3} + 2b^{2}}{4} + |\alpha - \delta - \lambda|$$
Example-3.1 Let $I^N_1 = \langle 4,8,10;0.5,0.3,0.6 \rangle$ and $I^N_2 = \langle 3,7,11;0.4,0.5,0.6 \rangle$ then $R(I^N_1) = 7.9$ and $R(I^N_2) = 7.7$

Based on definition 5:

$I^N_1 + I^N_2 = \langle 7,15,21;0.4,0.5,0.6 \rangle$ then $R(I^N_1 + I^N_2) = 15.2$

We observe that $R(I^N_1 + I^N_2) \neq R(I^N_1) + R(I^N_2)$.

Here, we observed that the author used an invalid mathematical assumption i.e. ranking function, to solve the problem. Therefore, we consider an aggregation ranking function, which was defined by Edalatpanah [19].

Definition 7. Let $I^N = \langle b^1, b^2, b^3;\alpha, \delta, \lambda \rangle$ be a triangular neutrosophic numbers (TNNs); then the aggregation ranking function is as follows:

$$R(I^N) = \frac{2 + \min \alpha - \max \delta - \max \lambda}{9} \sum (b^1 + b^2 + b^3)$$

Example-3.2 Let $I^N_1 = \langle 4,8,10;0.5,0.3,0.6 \rangle$ and $I^N_2 = \langle 3,7,11;0.4,0.5,0.6 \rangle$ then $R(I^N_1) = 3.91$ and $R(I^N_2) = 3.03$

Based on definition 5:

$I^N_1 + I^N_2 = \langle 7,15,21;0.4,0.5,0.6 \rangle$ then $R(I^N_1 + I^N_2) = \frac{(2+0.4-0.5-0.6) \times (22 + 21)}{9} = 6.94$

Hence,

$$R(I^N_1 + I^N_2) = R(I^N_1) + R(I^N_2).$$

Here, we observed from the above examples the existing method [7] uses the ranking function is invalid, and Definition-7 of ranking function is valid. Therefore, we consider the aggregation ranking function to solve integer programming.

3.2 Neutrosophic IP model

In this section, IP problem with neutrosophic elements are often described as the succeeding:

$$Max \; Z' = \sum_{j=1}^{n} \tilde{c}_j x_j$$

Subject to

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq \tilde{b}_i, \; i = 1, 2, ..., m,$$

$$x_j \geq 0, \; j = 1, 2, ..., n. \; \text{and it is an integer}$$

where $x_j$ is nonnegative neutrosophic triangular numbers and $\tilde{c}_j, \tilde{a}_{ij}, \tilde{b}_i$ represented the neutrosophic numbers.

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In our neutrosophic model we choose to maximize the degree of acceptance and limit the degree of rejection and indeterminacy of the neutrosophic objective function and constraints. Now the model are often typed as follows:

\[
\begin{align*}
\text{max } \rho(x) \\
\text{min } \beta(x) \\
\text{min } \lambda(x)
\end{align*}
\]

Subject to

\[
\begin{align*}
\rho(x) &\geq \lambda(x) \\
\rho(x) &\geq \beta(x) \\
0 &\leq \rho(x) + \beta(x) + \lambda(x) \leq 3 \\
\rho(x), \beta(x), \lambda(x) &\geq 0 \\
x &\geq 0 \text{ is integer.}
\end{align*}
\]

The problem may be typed for the equal structure as follows:

\[
\begin{align*}
\text{max } \alpha, \text{ min } \delta, \text{ min } \lambda
\end{align*}
\]

Subject to

\[
\begin{align*}
\alpha &\leq \rho(x) \\
\lambda &\leq \beta(x) \\
\delta &\leq \lambda(x) \\
\alpha &\geq \lambda \\
\alpha &\geq \delta \\
0 &\leq \alpha + \delta + \lambda \leq 3 \\
x &\geq 0.
\end{align*}
\]

the place \(\alpha\) represents the least degree of acceptance, \(\delta\) represents the largest degree of rejection and \(\lambda\) represents the largest degree of indeterminacy.

Now the model may be turned into the following model:

\[
\begin{align*}
\text{max}(\alpha - \delta - \lambda)
\end{align*}
\]

Subject to

\[
\begin{align*}
\alpha &\leq \rho(x) \\
\lambda &\leq \beta(x) \\
\delta &\leq \lambda(x) \\
\alpha &\geq \lambda \\
\alpha &\geq \delta \\
0 &\leq \alpha + \delta + \lambda \leq 3 \\
x &\geq 0, \text{ is integer.}
\end{align*}
\]

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Finally, the model may be typed as:
\[
\min(1 - \alpha) + \delta + \lambda
\]
Subject to
\[
\begin{align*}
\alpha &\leq \rho(x) \\
\lambda &\geq \beta(x) \\
\delta &\geq \lambda(x) \\
\alpha &\geq \lambda \\
\alpha &\geq \delta \\
0 &\leq \alpha + \delta + \lambda \leq 3 \\
x &\geq 0, \text{ is integer.}
\end{align*}
\]

4. **Proposed method**

Here, we propose an algorithm which is solved our problem (1) and the steps are given as:

**Step 1.** Build the problem as the model (1).

**Step 2.** Consider \( \tilde{b} = (b_i, b_m, b_r; T_a, I_a, F_c) >, \tilde{c} = (c^l, c^m, c^r; T_c, I_c, F_c) \) and using Definition 5, the LP problem (1) can be transformed into problem (6).

\[
\text{Max (or Min) } Z = \sum_{j=1}^{n} (c^l_j, c^m_j, c^r_j)x_j
\]
subject to
\[
\begin{align*}
\sum (a_{j1}, a_{j2}, a_{j3}; T_a, I_a, F_a)x_j &\leq (b^l_i, b^m_i, b^r_i; T_i, I_i, F_i), i = 1, 2, \ldots, m. \\
x_j &\geq 0, \text{ integer, } j = 1, 2, \ldots, n.
\end{align*}
\]

**Step 3.** Using arithmetic operations, defined in Section 2 and Definition 7, the problem obtained in Step-2, is converted into the following crisp IP problem.

\[
\text{Max (or Min) } Z = \max \sum_{j=1}^{n} (c^l_j, c^m_j, c^r_j)x_j
\]
subject to
\[
\begin{align*}
\max \sum (a_{j1}, a_{j2}, a_{j3}; T_a, I_a, F_a)x_j &\leq \max (b^l_i, b^m_i, b^r_i; T_i, I_i, F_i), i = 1, 2, \ldots, m. \\
x_j &\geq 0, \text{ integer, } j = 1, 2, \ldots, n.
\end{align*}
\]

**Step 4.** Find the optimal solution \( x_j \) by solving the crisp IP problem got in Step-3.

**Step 5.** Find the optimal value by placing \( x_j \) in \( \sum_{j=1}^{n} \tilde{c}_j x_j \).

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Step 6. From Step 5, solve the LP problem using the simplex method ignoring integer restrictions. If the obtained solution satisfies integer restrictions, then Stop, otherwise go to the next step by using Gomory’s cutting plane algorithm.

Step 7. Add the constraints to the given set of constraints of the problem and solve the modified problem. If its optimal solution is integral, stop, otherwise repeating the step till an optimal integer solution is obtained.

5. Numerical Example

Here, we consider a case of [7] to represent the model and to quantify the effectiveness of our proposed model, we tackle a numerical model.

Example 1

\[
\begin{align*}
\text{max} & \quad  \tilde{4} x_1 + \tilde{3} x_2 \\
\text{s.t.} & \quad \tilde{4} x_1 + \tilde{2} x_2 \leq 12 \\
& \quad \tilde{3} x_1 + \tilde{6} x_2 \leq 5 \\
& \quad x_1, x_2 \geq 0, \text{integer.}
\end{align*}
\]

where

\[
\begin{align*}
\tilde{4} &= (2, 4, 6); (0.8, 0.6, 0.4) > \\
\tilde{3} &= (1, 3, 5); (0.75, 0.5, 0.3) > \\
\tilde{4} &= (0, 4, 8); (1, 0.0, 0.5) > \\
\tilde{2} &= (1, 2, 3); (1, 0.5, 0.5) > \\
\tilde{12} &= (5, 12, 19); (1, 0.25, 0.25) > \\
\tilde{3} &= (1, 3, 5); (0.75, 0.0, 0.25) > \\
\tilde{5} &= (3, 5, 7); (0.8, 0.6, 0.4) > \\
\tilde{6} &= (1, 6, 11); (1, 0, 0) >
\end{align*}
\]

By utilizing the aggregation ranking function proposed in Definition 7 the above issue can be changed over to crisp model as follows:

\[
\begin{align*}
\text{max} & \quad 2.4 x_1 + 1.95 x_2 \\
\text{s.t.} & \quad 3.33 x_1 + 1.3 x_2 \leq 6 \\
& \quad 2.5 x_1 + 6 x_2 \leq 3 \\
& \quad x_1, x_2 \geq 0, \text{integer}
\end{align*}
\]

By following the steps introduced in the last segment, the optimal solution integer programming problem of the above problem is \( x_1 = 1, x_2 = 0 \) and the objective solution is \( Z = 2.4 \).

6. Conclusions
In this investigation, we present the neutrosophic IP and suggest a novel model to tackle it. In view of the present ranking function of triangular neutrosophic numbers are not valid, therefore a aggregation ranking function was adopted for solving the problem. A new algorithm, the use of these ranking functions, is introduced to gain the effectivity of IP problems. For calculating the integer programming, we use Gomory’s cutting plane algorithm. At long last, we utilize a numerical application to delineate the common sense and legitimacy of the proposed strategy. Also, the weaknesses of the current calculations are brought up and to show the benefits of the proposed calculations. At long last, from the acquired outcomes, it tends to be presumed that the model is proficient and advantageous.

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